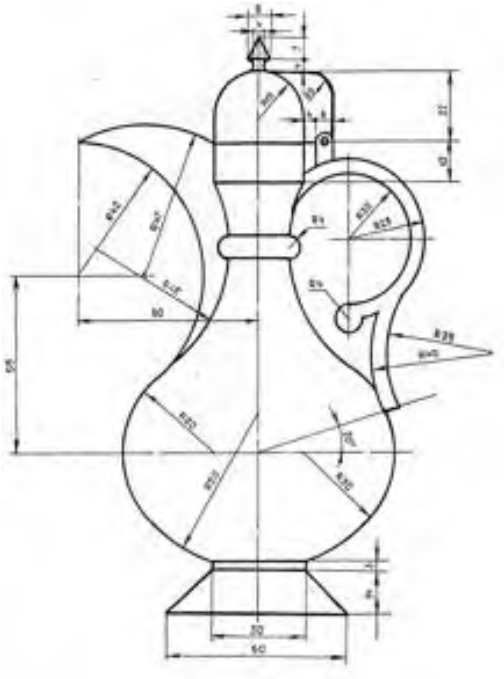
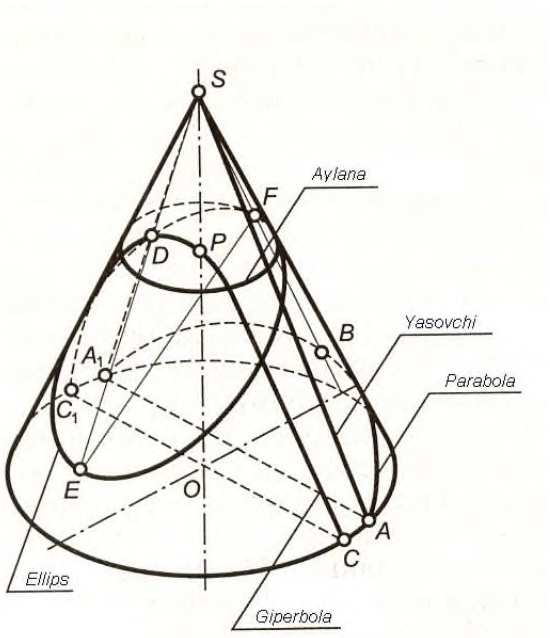
MINISTRY OF EDUCATION AND SCIENCE OF THE REPUBLIC OF UZBEKISTAN
GULISTON STATE UNIVERSITY
DEPARTMENT OF APPLIED ARTS AND DESIGN

CONNECTIONS AND CURVED CONTOURS

TUTASHMALAR VA LEKALO EGRI CHIZIQLAR





Gulistan-2026

D.N.Bo‘riboyeva Connections and curved contours (Tutashmalar va lekalo egri chiziqlar) – Gulistan, 2026.- 35. p

Ushbu ma‘ruza matni Muhandislik kompyuter grafikasi fanidan o‘tiladigan “Tutashmalar va lekalo egri chiziqlar” mavzusiga bag‘ishlangan bo‘lib, unda tutashma tushunchasi, ikki to‘g‘ri chiziqni radiusi berilgan yoy bilan tutashtirish, parallel to‘g‘ri chiziqlarning tutashmalari, ichki urinma, tashqi urunma va lekalo egri chiziqlarning qayerda uchrashi, episikloida, giposikloida, evol’venta, arximed spirallari haqidagi ma‘umotlar ingliz va o‘zbek tillarida bayon etilgan. Amaliy grafik uchun namuna keltirilgan va talabalar bajarishi uchun topshiriqlar berilgan hamda asosiy atamalar izohli-glossariy keltrilgan. Ma’ruza 2 - bosqich, 60210400 - Dizayn: interyer dizayni ta‘lim yo‘nalishi talabalariga mo‘ljallangan.

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Ma’ruza matni San’atshunoslik fakulteti Ilmiy Kengashi qaroriga asosan 21.09. 2026 yil 2 -bayonnoma) nashrga tavsiya qilindi.

CONNECTIONS AND CURVED CONTOURS

TUTASHMALAR VA LEKALO EGRI CHIZIQLAR

Objective: To provide students with theoretical and practical knowledge regarding geometric transitions and complex curves to teach methods for achieving smooth and precise connections between geometric elements; and to develop skills in constructing such curves, identifying their properties, and accurately representing them in technical drawings.

Plan:

1. Connections.
2. Curved contours.

Maqsad: Talabalarga tutashmalar va lekalo egri chiziqlar haqida nazariy va amaliy bilim berish, geometrik elementlarni o‘zaro silliq va aniq tutashtirish usullarini o‘rgatish hamda lekalo egri chiziqlarni qurish, ularning xususiyatlarini aniqlash va chizmalarda to‘g‘ri tasvirlash ko‘nikmalarini shakllantirish.

Reja:

1. Tutashmalar.
2. Lekalo egri chiziqlar.

1. CONNECTIONS

In technical drawings, the surfaces of many objects are represented by lines that transition smoothly from one to another. Such smooth transitions are determined by the object’s design characteristics, manufacturing technology, its function within a machine assembly, aesthetic requirements, and other factors. In a drawing, the smooth transition from one line to another is called a “junction” (or “blend”). The following types of junctions exist:

1. A junction between two circular arcs via a straight line.
2. A junction between two straight lines via a circular arc.
3. A junction between two circular arcs via an intermediate straight line.
4. A junction between two circular arcs via a third circular arc.

To create a junction, the following elements are required: the point of tangency (junction point), the center of the junction arc, and its radius. The construction of such junctions is based on three fundamental theoretical principles:

1. A straight line is tangent to a circle if it is perpendicular to the radius drawn to the point of tangency (Figure 1, a). To construct a line tangent to a circle from a given point A, a right angle OAK is drawn (Figure 1, b). This is defined as an inscribed angle within an auxiliary circle, subtended by the diameter OA (Figure 1, b).

2. Two circles can be tangent to each other. If the point of tangency K lies on the straight line connecting their centers, O_1 and O_2 (Figure 1, v). Tangency between circles can be external (Figure 1, v) or internal (Figure 1, g).

3. The center O of an arc tangent to two circles of equal radius lies on a line that is perpendicular to the straight line connecting the centers of the circles, O_1 and O_2, and passes through its midpoint. An arc serving as a connection between two circles can be external (Figure 1, d) or internal (Figure 1, e).

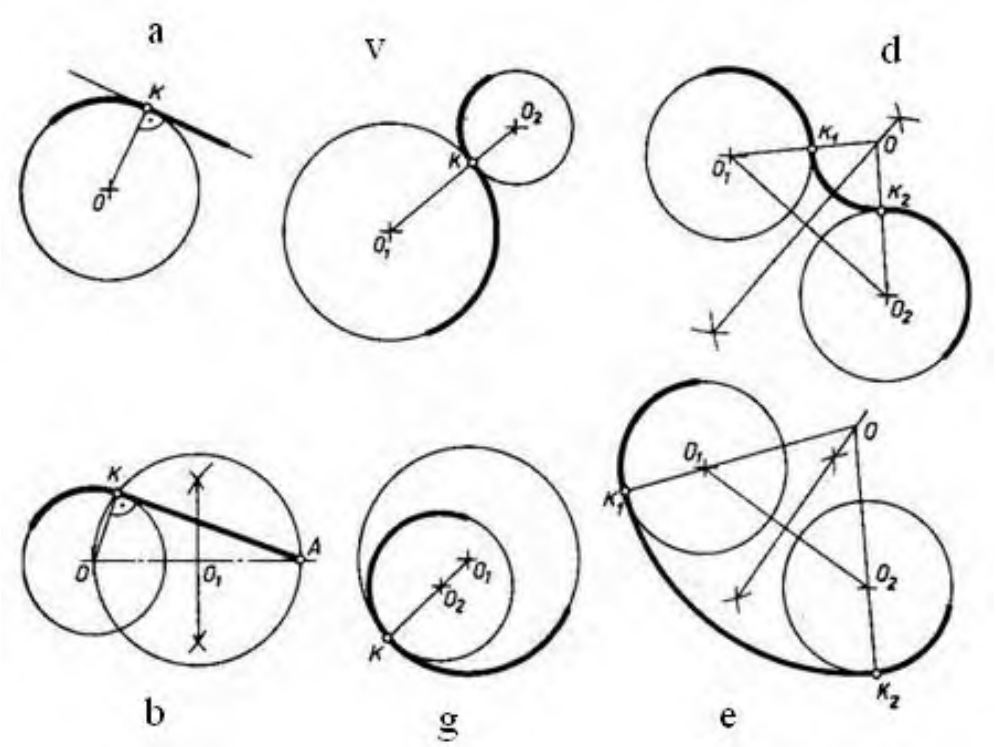


Figure 1

Connecting two straight lines with an arc of a given radius

Connecting two straight lines that form an acute or obtuse angle is performed as follows. First, we determine the center of the connection, O. To do this, we draw auxiliary lines parallel to the sides of the angle at a distance equal to the given radius, R.

Next, by drawing perpendiculars from the center O to the sides of the angle, we identify points K and K_1; these points are then connected to the center O using the radius R.

(Figure 2, a, b) When determining the center of an arc that connects lines forming a right angle, it is quite convenient to use a compass. (Figure 2, c, d) If one of the points of tangency, K, lies on one of the given straight lines, then the center of the connection O-as well as the second point of tangency, K_1-must be determined on the second straight line. For this, we apply the first geometric principle: a perpendicular is drawn from point K, and the bisector of the angle is determined.

The point where the perpendicular and the bisector intersect is the center of the connection, O. A perpendicular drawn from point O to the second side of the angle identifies point K_1. Points K and K_1 are then connected using the radius OK (Figure 2, g).

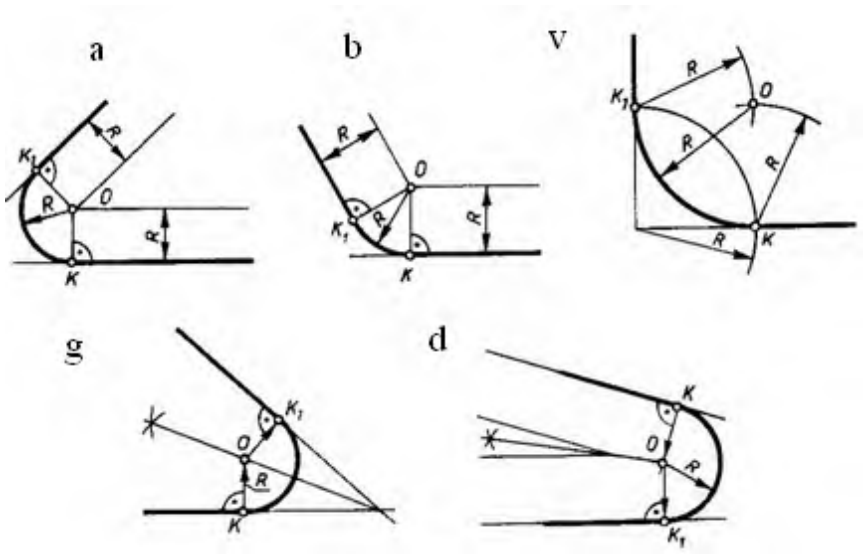


Figure 2

Connections between parallel straight lines

First, let us consider the case where a point of tangency, K, is given on one of two parallel straight lines (Figure 3, a). To determine the corresponding point of tangency, K_1, on the second line, we draw a line through point K that is perpendicular to the given parallel lines. The center and radius of the connecting arc are determined by bisecting the line segment KK_1. Using O as the center, we draw an arc that connects points K and K_1.

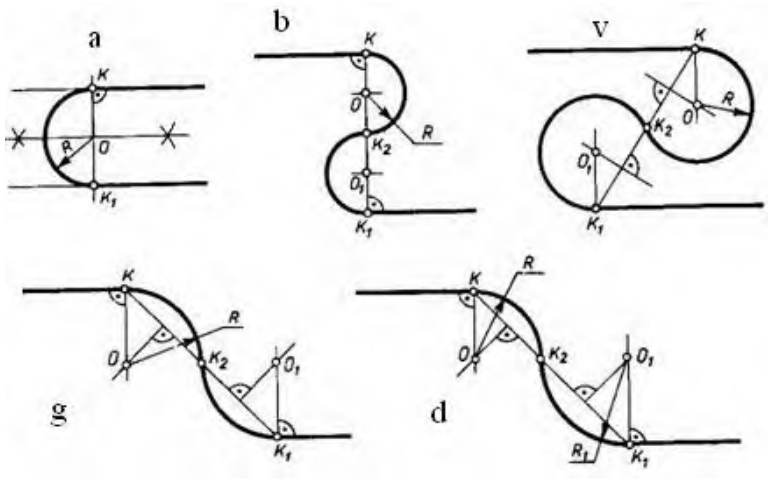


Figure 3

Connecting two parallel straight lines via a point of tangency located on a single perpendicular

The connections are formed using arcs of the same radius (Figure 4, b). The distance between the parallel straight lines must be equal $\frac{1}{4}$. The point of tangency, K_2, is determined by bisecting the segment K K_1. The centers of the connecting arcs, O and O_1, are located at the midpoints of the segments K K_2 and K_1 K_2, respectively. Using centers O and O_1, draw the connecting arcs with radius R.

Connecting mutually parallel straight lines using arcs

If the connecting arcs have the same radius, the point of tangency K_2 is determined by bisecting the line segment K K_1. To find the radius of the connecting arc and its center, perpendiculars are drawn from points K and K_1 to the given lines. A

perpendicular drawn from the center of a circular arc to its chord passes through the midpoint of that chord. Therefore, we draw perpendiculars from the midpoints of the segments (chords) K K_2 and K_1 K_2; the connecting arcs, centered at O and O_1, are then drawn through points K, K_2, and K_1

Connecting a straight line to a circular arc using an arc of a given radius

There are two types of connections between a straight line and a circular arc: internal and external.

External tangency (Figure 4, a). The center of the connection, O, is determined. We draw a line parallel to the given straight line at a distance equal to the radius R. From the center O_1 of the given circle, an auxiliary arc with a radius of R + R_1 is drawn. Using the first theorem, we determine the point of tangency, K. We find the second point of tangency, K_1, by applying the second theorem. We connect points K and K_1, with the center O_1 and radius R.

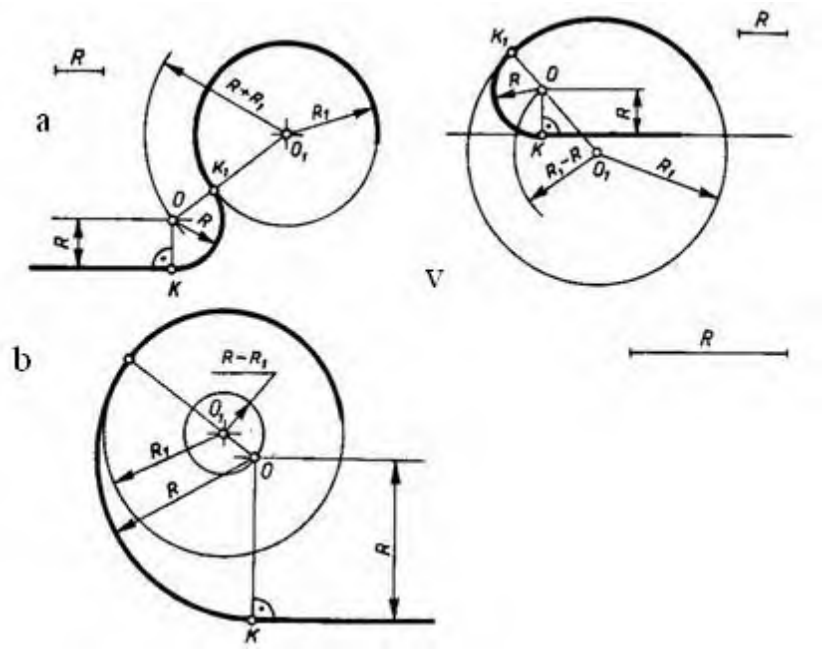


Figure 4

Internal tangency. A straight line may intersect a circle or pass outside it. Figure 4 b illustrates the connection of a circle (with radius R_1) to an intersecting straight line using an arc of radius R. To achieve this, we draw an auxiliary straight line parallel to

the given line at a distance equal to the radius R of the connecting arc. From the center O_1, we draw an auxiliary arc with a radius of R_1-R. The intersection of this auxiliary arc and the auxiliary straight line defines point O. The first point of tangency, K, is determined according to the first principle, and the second point of tangency, K, is determined according to the second principle. Figure 4v shows the connection of a circle to a straight line that does not intersect it. Constructing this connection is similar to the previous case; however, the radius of the auxiliary arc is equal to the difference between the radius of the connecting arc and the radius of the given circle—that is, R-R_1.

Connecting two circular arcs with a straight line

This process involves drawing internal and external tangents to given circular arcs (Figure 5, a, b). To draw an external tangent line to circular arcs, we apply the first theoretical principle. We begin by drawing an auxiliary circle centered at O, with a radius equal to the difference between the radii of the given circles (R_1-R). Next, we draw a tangent line from the center O_1 to this auxiliary circle. We then draw a straight line passing through points O and K_0 (the point of tangency on the auxiliary circle) that intersects the circle of radius R at point K; point K serves as the point of tangency. To locate the corresponding point of tangency K_1 on the circle with radius R1, we draw a line $O_1 K_1$ that is parallel to the line OK. By connecting points K and K_1 with a straight line segment, we obtain the tangent line. Constructing an internal tangent line to the circles follows a similar procedure, with the exception that the radius of the auxiliary circle is equal to the sum of the radii of the given circles (R+R_1; Figure 5, b).

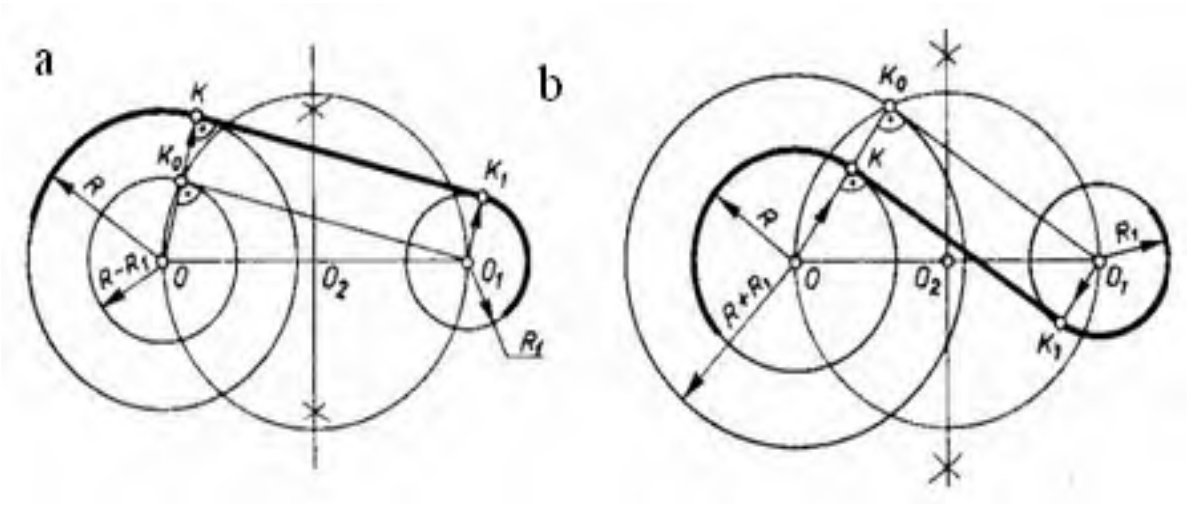


Figure 5

Connecting two circular arcs with a third arc

In this case, both internal and external connections are possible. Given the radius R of the connecting arc and the radii R_1 and R_2 of the arcs being connected, the center of the connecting arc and the points of tangency are determined.

External tangency. (Figure 6, a) First, the center of the connecting arc is determined. To do this, we draw auxiliary arcs from centers O_1 and O_1 with radii of R-R_1 and R+R_2, respectively. Using the central lines O O_1 and O O_2, we identify the points of tangency, K and K_1, by applying the second theoretical principle. We then draw the connecting arc with radius R from center O.

Internal tangency. (Figure 6, b) The center O of the connecting arc is the intersection point of arcs drawn from centers O_1 and O_2 with radii of R-R_1 and R- R_2. By applying the second theoretical principle and extending the central lines, we determine the points of tangency, K and K_1. To calculate the sum or difference of the radii, we use an auxiliary straight line, upon which we mark off the respective radii using a compass.

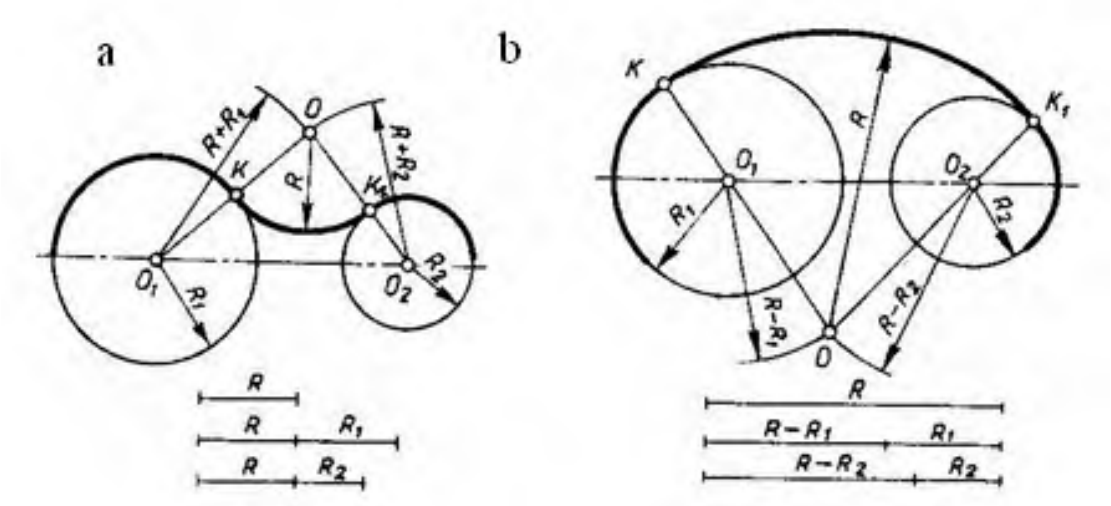


Figure 6

Drawing internal and external connections using an arc of a given radius R

(Figure 7, a, b): In this case, the distance from the center of the connecting arc to the center of one circle is equal to the sum of the radii, $R+R_2$ (external connection), while the distance to the center of the other circle is equal to the difference between the connecting radius and the radius of that circle (internal connection). These points are determined through the drawing process.

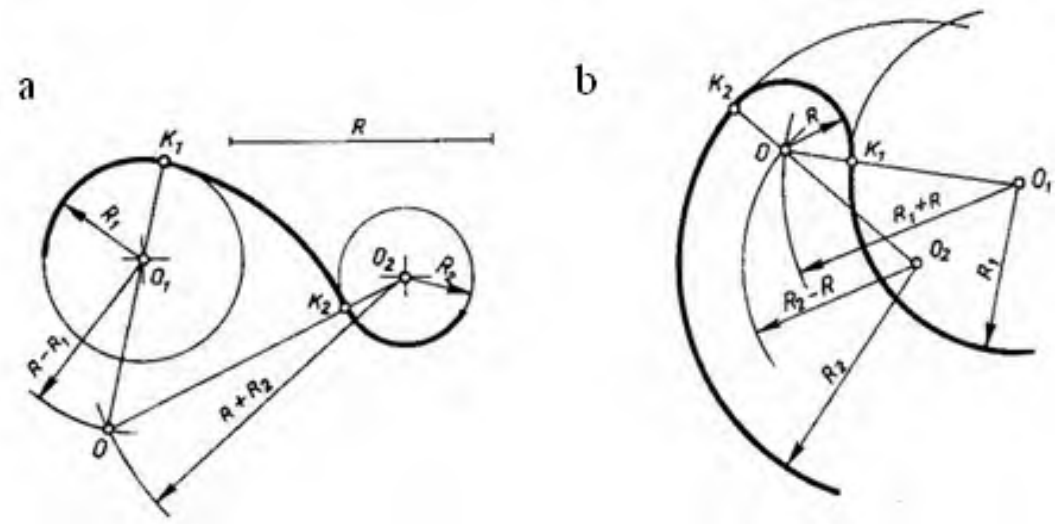


Figure 7

1. TUTASHMALAR

Chizmada ko‘p buyumlarning sirti biridan ikkinchisiga silliq tutashgan chiziqlar bilan tasvirlanadi. Silliq o‘tish buyumning konstruksiyasini hislatlari, uni tayyorlash texnologiyasi, mashina tarkibida, qanday vazifa bajarishini estetik talablar va boshqalar bilan aniqlanadi. Chizmada bir chiziqni ikkinchi chiziqqa silliq o‘tishi tutashma deyiladi. Tutashmalarni quyidagi ko‘rinishlari mavjud:

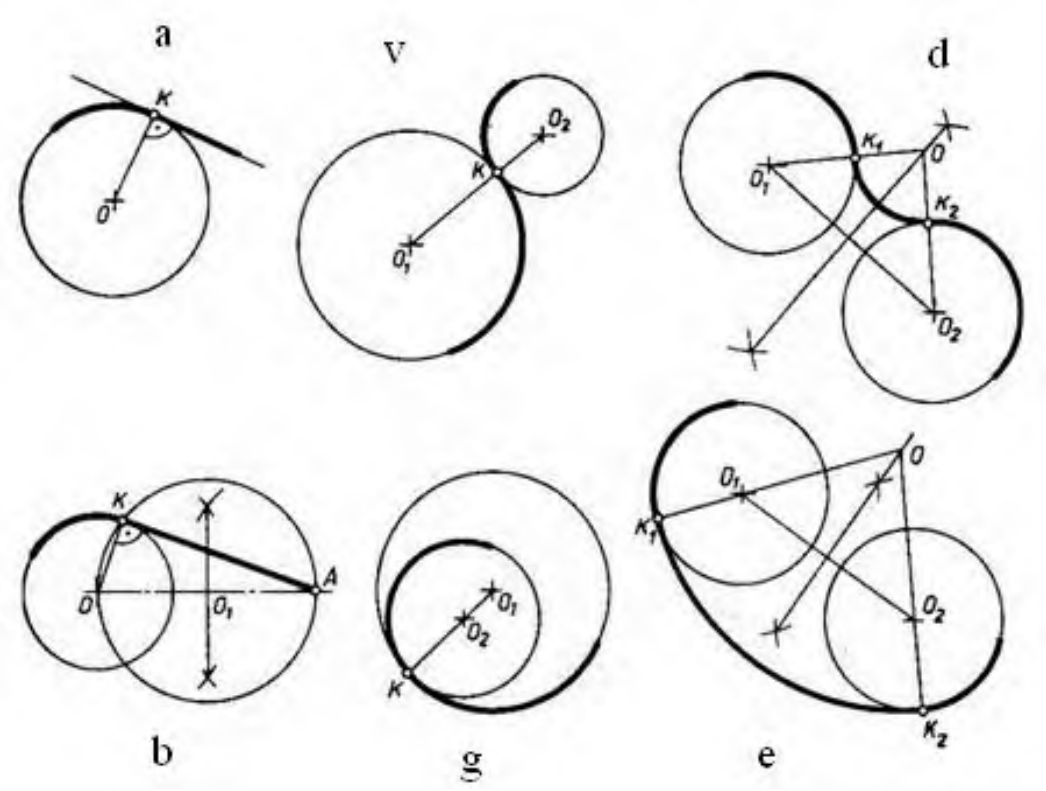
1. Ikki aylana yoyining to‘g‘ri chiziq bilan.
2. Ikki to‘g‘ri chiziqni aylana yoyi bilan.
3. Aylana yoyini to‘g‘ri chiziq orqali ikkinchi aylana yoyi bilan.
4. Aylananing ikki yoyini uchinchi yoy bilan.

Tutashma hosil qilish uchun quyidagi elementlar kerak: tutash nuqtasi, tutashma yoyining markazi va uning radiusi. Tutashmalarni chizish uchta asosiy nazariyaga asoslangan.

1. **To‘g‘ri chiziq aylanaga urinma** bo‘ladi, agar to‘g‘ri chiziq urunish nuqtasidan o‘tkazilgan radiusga perpendikulyar bo‘lsa (1-shakl, a). Berilgan A nuqtadan aylanaga urinma to‘g‘ri chiziq o‘tkazish uchun to‘g‘ri burchak OAK chiziladi (1-shakl, b). Uni O_1 dan o‘tkazilgan OA diametriga (1-shakl, b) suyangan qo‘shimcha aylananing ichki burchagi sifatida aniqlanadi.

2. **Ikki aylana urinma bo‘lishi mumkin.** Agar urinish nuqtasi K ularning markazi O_1 va O_2 ni birlashtiruvchi to‘g‘ri chiziqda joylashgan bo‘lsa. (1-shakl, v). Aylanalarning urinma bo‘lishi tashqi (1-shakl, v) va ichki (1-shakl, g) bo‘lishi mumkin.

3. **Ikki bir xil radiusdagi aylanaga urinma bo‘lgan** yoyning markazi O, aylanalarnig markazi O_1 va O_2 birlashtiruvchi to‘g‘ri chiziqning o‘rtasiga perpendikulyar bo‘lgan to‘g‘ri chiziqda bo‘ladi. Ikki aylanaga tutashma bo‘lgan yoy tashqi (1-shakl, d) va ichki bo‘lishi mumkin (1-shakl, e).



1-shakl

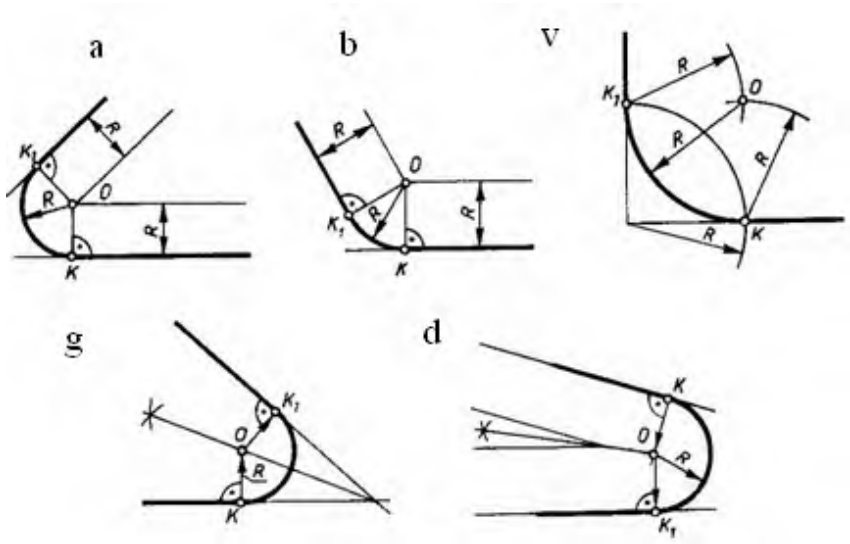
Ikki to‘g‘ri chiziqni radiusi berilgan yoy bilan tutashtirish

O‘tkir va o‘tmas burchak bilan o‘zaro joylashgan ikki to‘g‘ri chiziqni tutashtirish quyidagi tartibda bajariladi. Tutashmalar markazi O ni aniqlaymiz. Buning uchun berilgan R teng masofada burchakni tomonlariga parallel yordamchi to‘g‘ri chiziq o‘tkazamiz.

Buning uchun O markazidan burchak tomonlariga perpendikulyar o‘tkazib K va K_1 nuqtalarini aniqlaymiz va aniqlangan nuqtalarni R radius bilan O markazidan tutashtiramiz.

(2-shakl, a, b) to‘g‘ri burchakni tutashtiruvchi yoyning markazini aniqlashda sirkuldan foydalanish ancha qulay. (2-shakl, v, d) tutashmaning bitta tutashish nuqtasi K berilgan to‘g‘ri chiziqni bittasida yotsa u holda, tutashish markazi O tutashish markazi R va ikkinchi tutashish nuqtasi K_1 ni ikkinchi to‘g‘ri chiziqda aniqlanadi. Buning uchun birinchi nazariya holatidan foydalanamiz. Bunda K nuqtadan perpendikulyar o‘tkaziladi va burchak bissektrisasi aniqlanadi.

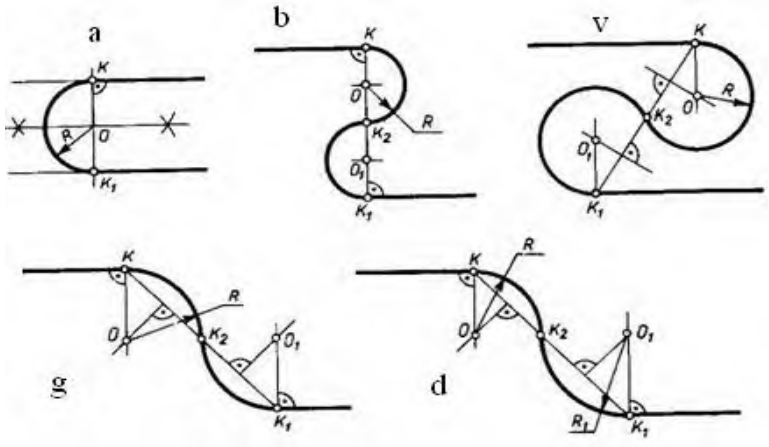
Perpendikulyar bilan bissektrisa kesishgan nuqta O tutashma markazi bo‘ladi. O nuqtadan burchakning ikkinchi tomoniga o‘tkazilgan perpendikulyar tomoniga K_1 nuqta aniqlanadi. K va K_1 nuqtalarini OK radius bilan tutashtiriladi. (2-shakl, g).



2-shakl

Parallel to‘g‘ri chiziqlarning tutashmalari

Avval ikkita parallel to‘g‘ri chiziqlardan birida tutashish nuqtasi K ni berilgan holini ko‘ramiz (3-shakl, a). Ikkinchi chiziqdan tutashish nuqtasi K_1 aniqlash uchun K nuqtadan berilgan parallel to‘g‘ri chiziqlarga perpendikulyar chiziq o‘tkazamiz. Tutashayotgan yoyning markazi va radiusi K K_1 kesmani ikkiga bo‘lib aniqlangan. O markazidan K va K_1 nuqtalarini birlashtiruvchi yoy o‘tkazamiz.



3-shakl

Tutashish nuqtasi bitta perpendikulyarda joylashgan o‘zaro parallel ikkita to‘g‘ri chiziqlarni tutashtirish

Tutashmalar bir xil radiusdagi yoylar bilan bajariladi (4-shakl, b). Parallel to‘g‘ri chiziqlar orasidagi masofa $\frac{1}{4}$ teng bo‘lishi shart. K K_1 kesmani teng ikkiga bo‘lib tutashayotgan yoylarni tutashish nuqtasi K_2 aniqlanadi. Tutashayotgan yoylarni O va O_1 markazi K K_2 va K_1 K_2 kesmalarini har birini o‘rtasidan bo‘ladi. O va O_1 markazdan tutashish yoylarini R radiusda o‘tkazamiz.

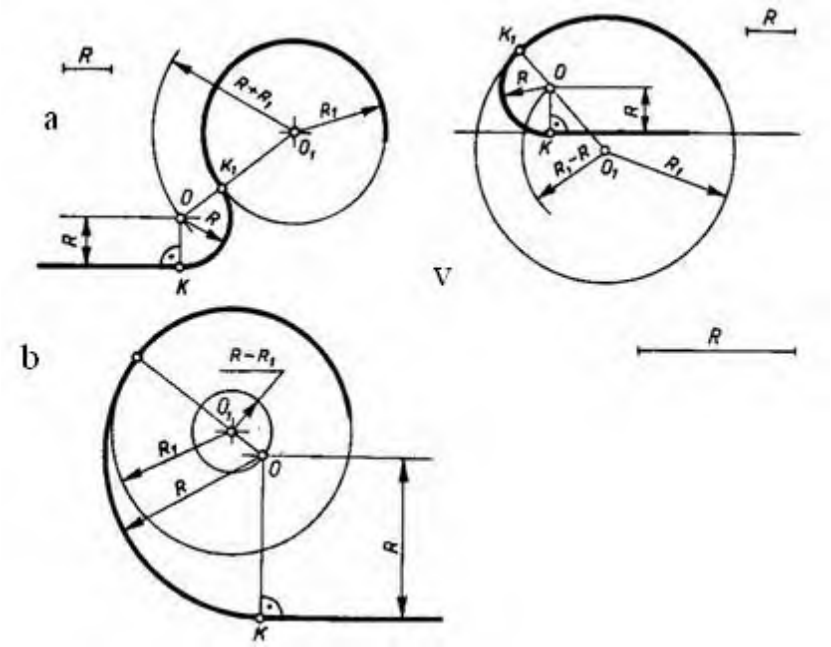
Tutashish nuqtasi har xil perpendikulyarda joylashgan o‘zaro parallel to‘g‘ri chiziqlarni tutashtirish

Agar tutashuvchi yoylar bir xil radiusda bo‘lsa u holda urunish nuqtasi K_2 ni K K_1 ni to‘g‘ri chiziq kesmasini ikkiga bo‘lib aniqlanadi. Tutashuvchi yoyning radiusi va ularning markazini aniqlash uchun K va K_1 nuqtadan berilgan chiziqlarga perpendikulyar o‘tkaziladi. Aylana yoyining markazidan uning vatariga o‘tkazilgan perpendikulyarlarning o‘rtasidan bo‘ladi. Shuning uchun K K_2 va K_1 K_2 (vatarlarni) kesmalarning o‘rtasidan ularga perpendikulyar o‘tkazamiz, O va O_1 markazidan tutashuvchi yoylarni K K_2 va K_1 nuqtalar orqali o‘tkaziladi

Aylana yoyini to‘g‘ri chiziq bilan radiusi berilgan yoy yordamida tutashtirish

To‘g‘ri chiziqni aylana yoyi bilan tutashtirishda ichki va tashqi hollari mavjud.

Tashqi urinma (4-shakl, a). Tutashmalar markazi O aniqlanadi. Tutashmani R radiusga teng masofada berilgan to‘g‘ri chiziqqa parallel to‘g‘ri chiziq o‘tkazamiz. Berilgan aylananing markazi O_1 dan $R+R_1$ yordamchi yoy chiziladi. Birinchi nazariyadan foydalanib tutashish nuqtasi K ni aniqlaymiz. K_1 tutashish nuqtasini ikkinchi nazariyani qo‘llab topamiz. K va K_1 nuqtalarni O_1 markazdan R radius bilan tutashtiramiz.



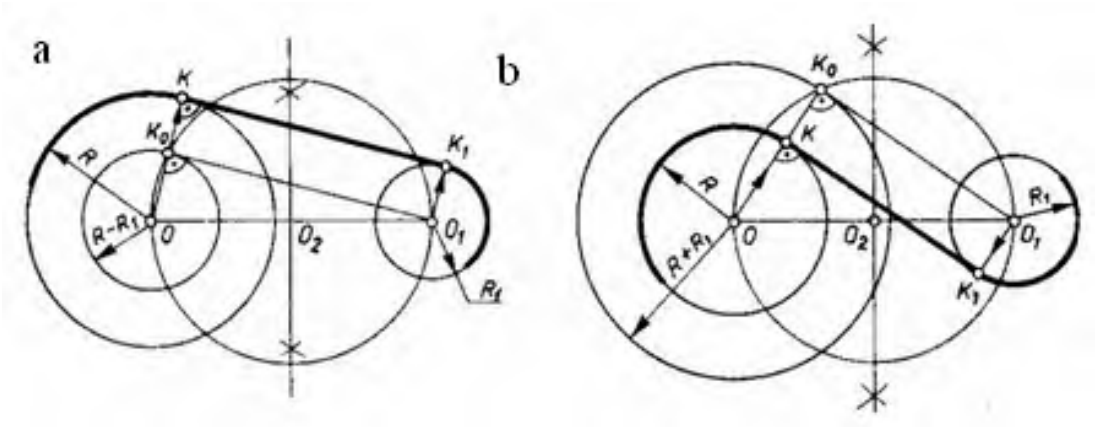
4-shakl

Ichki urunma. To‘g‘ri chiziq aylanani kesib o‘tishi yoki uni tashqaridan o‘tishi mumkin. 4-shakl, b, da R_1 radiusli aylanani uning kesib o‘tuvchi to‘g‘ri chiziq bilan R radiusli yoy bilan tutashtirish ko‘rsatilgan. Buning uchun tutashma yoyining radiusi R teng masofada yordamchi to‘g‘ri chiziqni berilgan parallel vaziyatda o‘tkazamiz. O_1 markazdan R_1-R yordamchi aylana yoyini chizamiz. O‘tkazilgan aylana yoyi o‘tkazilgan yordamchi to‘g‘ri chiziq bilan kesishib O nuqtani hosil qiladi. Tutashish nuqtasi K ni birinchi nazariya, ikkinchi tutashish nuqtasi K ni ikkinchi nazariya orqali aniqlaymiz. 4-shakl, v, da aylana yoyini kesmaydigan to‘g‘ri chiziq bilan aylanani tutashishi ko‘rsatilgan. Bu tutashmani chizish oldingi tutashmaga o‘xshash bo‘lib yordamchi aylananing radiusi tutashma radiusdan berilgan aylananing radiusining ayirmasiga teng bo‘ladi ya’ni $R-R_1$.

Ikkita aylana yoyini to‘g‘ri chiziq bilan tutashtirish

Bu ichki va tashqi urinmalarni berilgan aylana yoylari chizishdan iboratdir (5-shakl, a,b). Aylana yoylariga tashqi tutash chizig‘ini chizish uchun birinchi nazariya holatidan foydalanamiz. Buning uchun O markazdan yordamchi aylanani berilgan aylanalarning radiuslar ayirmasiga (R_1-R) teng radiusda chizamiz. O_1 markazdan

yordamchi aylanaga urunma chiziq o‘tkazamiz. O va K_0 nuqtalar orqali R radiusli aylanani K nuqtadan kesib o‘tuvchi to‘g‘ri chiziq o‘tkazamiz. K nuqta tutashish nuqtasi bo‘ladi. R_1 radiusli aylanadagi K_1 tutashish nuqtasini O_1 K_1 chizini OK chiziqa parallel chizib o‘tkazamiz. K va K_1 nuqtalarini to‘g‘ri chiziq kesamasi bilan tutashtirib urinma chiziqni hosil qilamiz. Aylanalarga ichki urinadigan to‘g‘ri chizqni o‘tkazish yuqorida keltirilgan misolga o‘xshash bo‘ladi, lekin bunda yordamchi aylananing radiusi berilgan aylananlarning radiusni yig‘indisiga teng bo‘ladi (R+ R_1, 5-shakl,b).



5-shakl

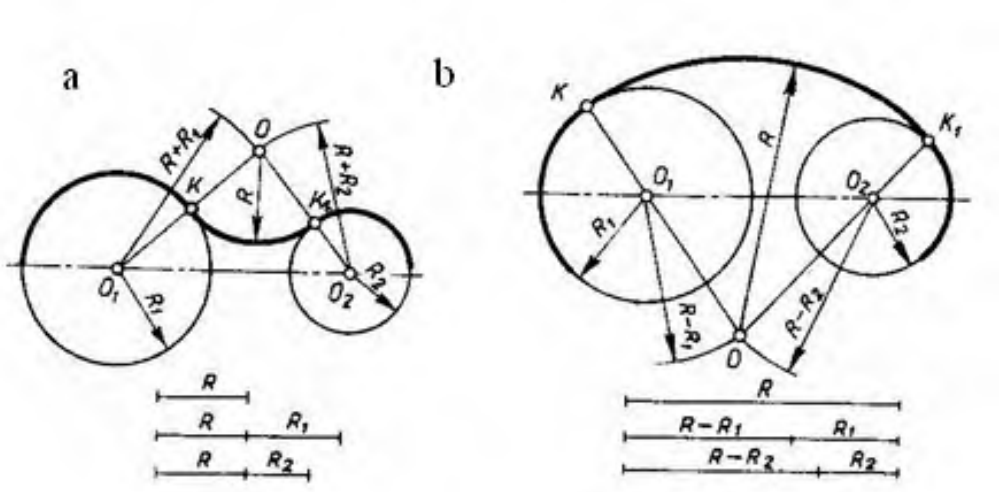
Ikkita aylana yoyini uchinchi yoy bilan tutashtirish

Bu holda ichki va tashqi tutashmalar bo‘lishi mumkin. Tutashtirilayotgan yoyni radiusi R va tutashayotgan yoylarning radiusi R_1 va R_2 lar berilganda tutashmaning markazi va tutashish markazi aniqlanadi.

Tashqi urinma. (6-shakl, a) avval tutashma yoyining markazi aniqlanadi. Buning uchun O_1 va O_2 markazdan R-R_1 va R+R_2 radiusda tegishlicha yordamchi yoylar o‘tkazamiz. O O_1 va O O_2 markaziy chiziqlardan foydalanib K va K_1 tutashish nuqtalarini ikkinchi nazariya holatidan foydalanib aniqlaymiz. Tutashtiriluvchi yoyni R radius bilan O markazdan o‘tkazamiz.

Ichki tutashma. (6-shakl, b) tutashtiriluvchi yoyning markazi O_1 ni R- R_1 va R-R_2 radiuslar bilan O_1 va O_2 markazlardan o‘tkazilgan yoylarning kesishish nuqtasi bo‘ladi. Ikkinchi nazariya holatini qo‘llab markaziy chiziqlarni davom ettirib tutashish

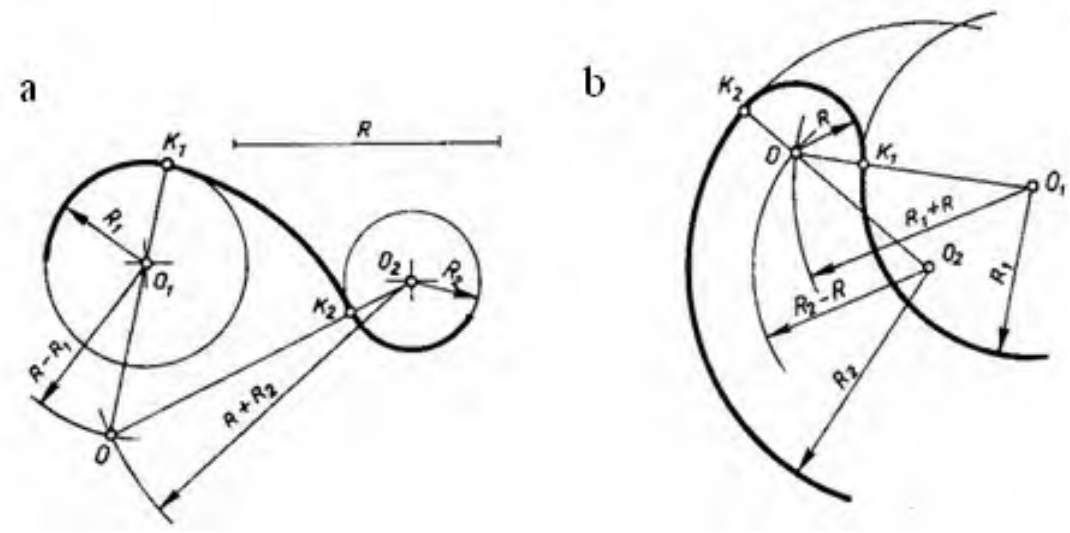
nuqtasini K va K_1 aniqlaymiz. Radiuslarni yig‘indisini yoki ayirmasini aniqlash uchun yordamchi to‘g‘ri chiziqdan foydalanamiz. Bu to‘g‘ri chiziqqa sirkul yordamida tegishli radiuslarni o‘lchab qo‘yamiz.



6-shakl

Berilgan R radiusli yoy bilan ichki va tashqi tutashmalarni birga chizish

(7-shakl, a,b) bu holda tutashma markazidan bitta aylananing markazigacha bo‘lgan masofa radiuslar yig‘indisi $R+R_2$ ga teng (tashqi tutashma), boshqa aylana markazigacha bo‘lgan masofa tutashma radiusi bilan shu aylana radiusini ayirmasiga teng (ichki tutashma). Bularni chizish bilan aniqlanadi.



7-shakl

2. FRENCH CURVES.

When it is not possible to connect the points of a curve using a compass, French curves (templates) are used to join them smoothly. For this reason, such curves are called “template curves.” These types of curves are widely used in engineering-in various machines and apparatuses, such as locators, searchlights, antennas, and the tooth profiles of gears.

We begin by studying how to draw these curves-specifically, the conic sections (associated with Hypatia).

Hypatia (c. 370–415) was a prominent scholar-a mathematician, astronomer, and mechanician-who lived in Alexandria. Like her father, she dedicated her life to science. She was among the first to study conic sections. A crater located in the “Sea of Tranquility” on the visible side of the Moon is named “Hypatia.” She made numerous scientific discoveries. In 415, a group of Christian zealots, upon discovering that she was a woman, burned her to death.

1. Conic section curves. If all the generators (slant lines) of a circular cone are intersected by a plane (P_v) inclined to its axis (Figure 8, a), an ellipse is formed. If the cone is intersected by a plane (T_v) parallel to one of its generators (1 S) (Figure 8, a), a parabola is formed. If it is intersected by a plane (Q_w) parallel to the axis (OS) (Figure 8, a), a hyperbola is formed.

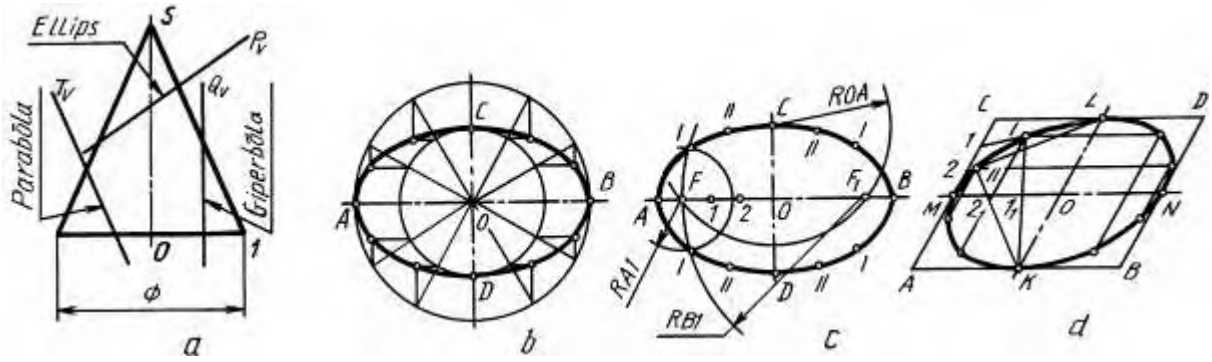


Figure 8

Key properties of conic section curves: For an ellipse, the sum of the distances from any point on the curve to two fixed points-denoted as F and F1 and located on the

major axisis a constant value equal to the length of the ellipse's major axis (Figure 8, c). For a parabola, the distance from any point on the curve to its focus F is equal to the distance from that same point to its directrix (Figure 8, a).

Cyclic curves. The term “cycle”-derived from the Greek “kyklos” (circle)-refers to a set of processes within a system of periodically recurring events, wherein an object, having undergone changes in a specific sequence, returns to its initial state.

If a circle rolls along a fixed straight line without slipping, a specific point on that circle-for instance, the point of contact with the straight line-traces a smooth, open, continuous curve known as a cycloid. In this context, the straight line is called the directrix (or guiding line), and the circle is called the generatrix (or generating circle). To construct it, a circle with diameter d is drawn and divided into 12 equal parts. A horizontal line tangent to the circle is drawn through point A, and the circumference of the circle-calculated as $\pi \cdot d$-is measured out along this line, starting from point A. The segment AA_{12} is then divided into 12 equal parts, and a vertical line is drawn through each of these division points. Points O_1, O_2, O_3, ..., O_{12} are identified along a dashed-dotted line passing through the center O. Using these points (O, O_1, O_2, ..., O_{12})-which serve as the centers of the circles-auxiliary circles, each with diameter d, are drawn. As the points on the circle move, their paths intersect with the arcs of these auxiliary circles; these points of intersection are labeled I, II, ..., XII. The points of the cycloid are smoothly connected using French curves (Figure 9).

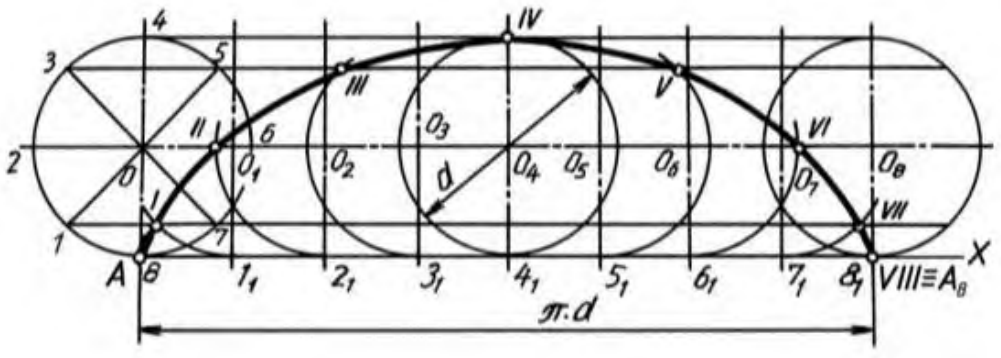


Figure 9

Epicycloid. If a circle rolls without slipping along the outside of a second, stationary circle, a specific point on the moving circle traces an open, smooth curve known as an epicycloid. In this context, the moving circle is called the generating circle, while the stationary circle is referred to as the directing circle (or base circle).

To construct an epicycloid, draw an arc of a circle with radius R, determining its central angle alpha α=d (2R-360°) (Figure 10). A circle with diameter d is drawn such that it is tangent to the arc at point A; the segment AA_{12}along the arc of radius R is then divided into 12 equal parts. A line representing the path of the centers of the circle with diameter d is drawn through the center O, and points O1,…O12 are marked along it. From each of these points (O_1,O_2,O_3…O_{12}), an auxiliary circular arc-with a radius equal to that of the circle with diameter d-is drawn. The points of intersection between these arcs and the path of the centers are labeled I, II, ..., XII.

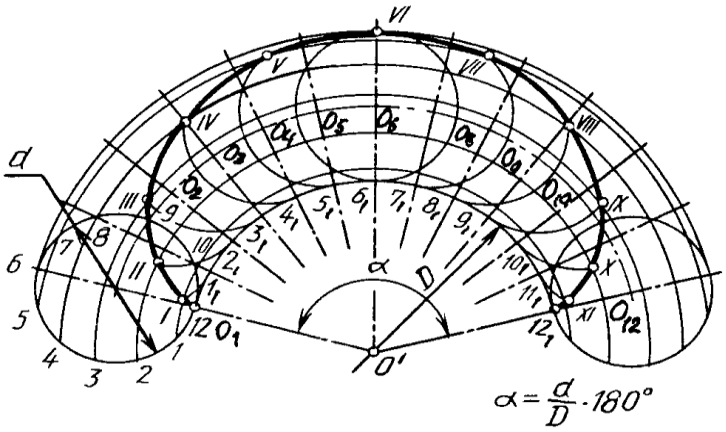


Figure 10

Hypocycloid. If a generating circle rolls without slipping along the inner side of a directing (fixed) circle, any point on the generating circle-for instance, the point of contact with the directing circle-traces a smooth, open, continuous curve known as a hypocycloid.

The method used to construct an epicycloid is also applied to construct a hypocycloid. To do this, an arc of a circle with radius R and its corresponding central angle (α=d (2Rx360°) are determined. A circle with diameter d is drawn such that it is

tangent to the inner side of the arc of radius R at point A; both this circle and the segment A A12 of the larger circle are then each divided into 12 equal parts (Figure 11). The remaining steps of the construction are clearly illustrated in the drawing.

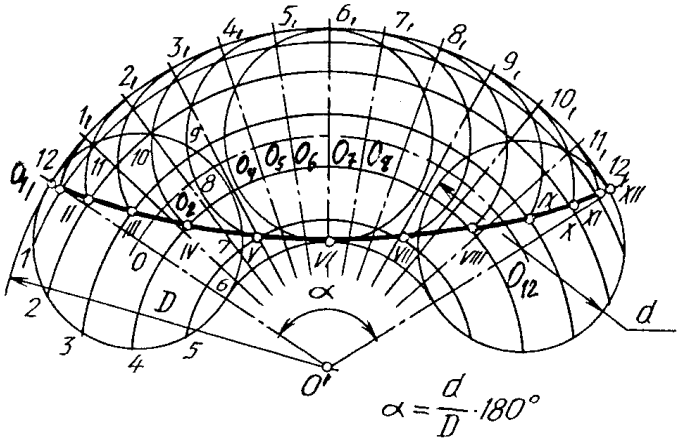


Figure 11

Involute. If a straight line rolls along a fixed circle without slipping, then each point on that straight line traces a curve-an open, smooth, coil-like path-known as the involute of the circle (Figure 12).

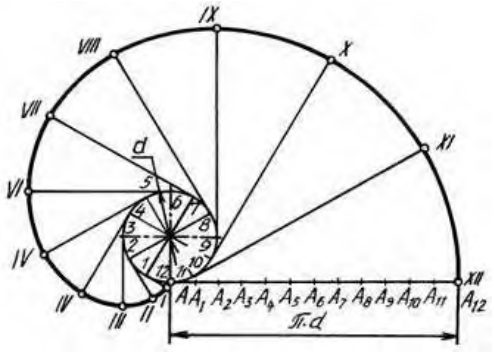


Figure 12

To draw an involute, a circle with diameter d is constructed, divided into 12 equal parts, and each division point is connected to the center O. This results in six diameters. From both ends of each diameter, auxiliary lines are drawn perpendicular to the diameter. Starting from point A on the circle, a line segment equal to the circumference ($\pi \cdot d$) is laid out and divided into 12 equal parts A A_{12}. By marking off one segment A A_1 on the auxiliary line corresponding to point 1, two segments on the line for point 2,

and so on, the points of the involute (I, II, ..., XII) are determined; these points are then smoothly connected using a French curve.

Archimedes' spiral. If a point moves with uniform linear motion along a straight line that is simultaneously rotating uniformly around a fixed center, the point traces a smooth, open, continuous curve. This curve is called an Archimedes' spiral. The distance covered by the point along the straight line during one full rotation of that line is known as the pitch of the Archimedes' spiral (Figure 13).

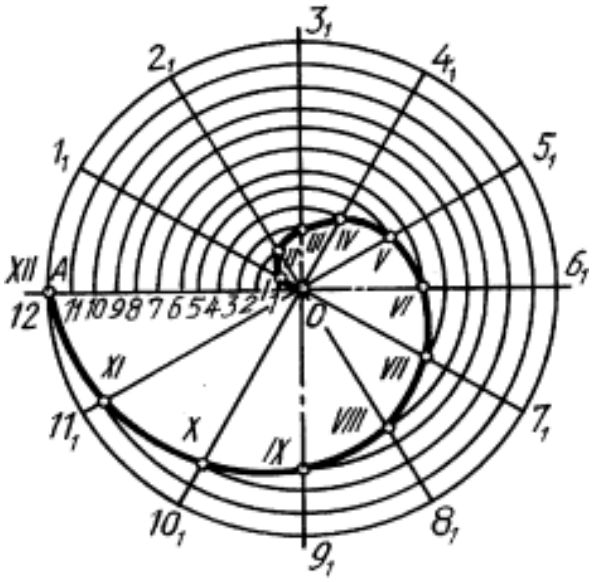


Figure 13

To draw it, a circle is constructed using the given segment OA as the radius; both the segment OA and the circle are then divided into 12 equal parts. The points on the circle-labeled 1, 2, ..., 12-are connected to the center O. Using a compass, the points on OA (1_1,2_1, ...) are transferred to the corresponding radial lines (O1, O2, ...). This results in a series of points (I, II, ..., XII), which are then joined together with a smooth curve using a French curve (template).

2. LEKALO EGRI CHIZIQLARI

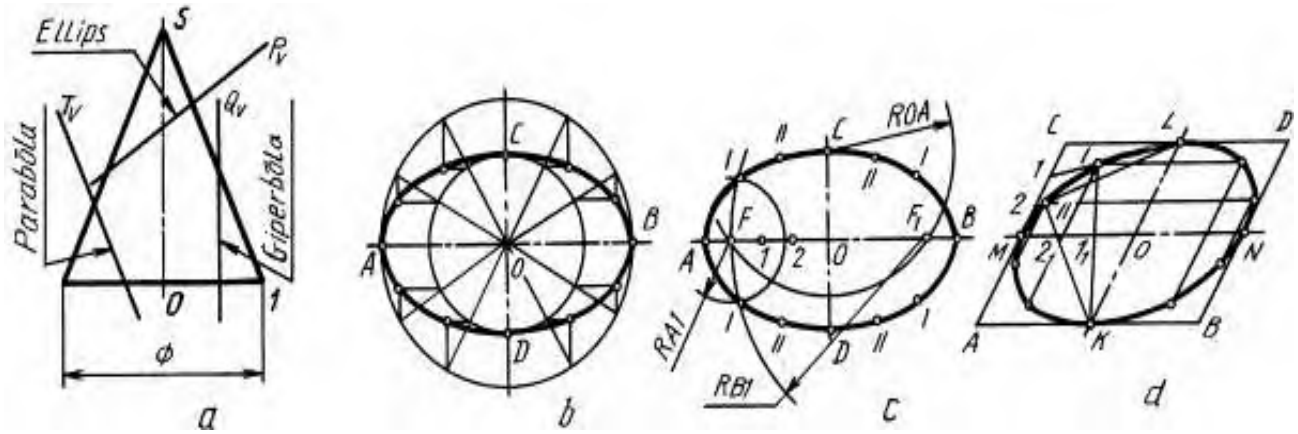
Egri chiziq nuqtalarini sirkul yordamida tutashtirishning imkoni bo‘lmasa, ularni ravon tutashtirish uchun lekalolardan foydalaniladi. Shuning uchun bunday egri

chiziqlar lekalo egri chiziqlari deyiladi. Bunday egri chiziqlar texnikada turli mashina va aparatlarda, lokatorlar, projektorlar, antennalar, tishli g‘irdiraklarning tish profillarida keng qo‘llaniladi.

Quyida shunday egri chiziqlar, ya’ni Ipatiya konus kesimlarini chizishni o‘rganishdan boshlanadi.

Ipaitya-novcha degani (370-415) ko‘zga ko‘ringan yirik olim Aleksandriyada yashagan matematik, astronom, mexanik Teon qizi. Otasi kabi ilm bilan shug‘ullangan. U birinchi bo‘lib konus kesimlari bilan shug‘ullangan. Oyning ko‘rinadigan tomonidagi “Xotirjamlik” dengizi krateri Ipatiya deb nomlangan. Juda ko‘p ilmiy kashfiyotlar ochgan. 415-yili xristian dindorlari uning ayol ekanligini bilib qolishadi va olovda yoqib yuborishadi.

1. Konus kesimi chiziqlari. Doiraviy konusning barcha yasovchilari o‘qiga qiya tekislik P_v bilan kesilsa (8-shakl, a) ellips, bitta 1 S yasovchisi parallel tekislik T_v bilan kesilsa (8-shakl, a) parabola, o‘qi O S ga parallel tekislik bilan (Q_w) kesilsa (8-shakl, a) giperbola hosil bo‘ladi.

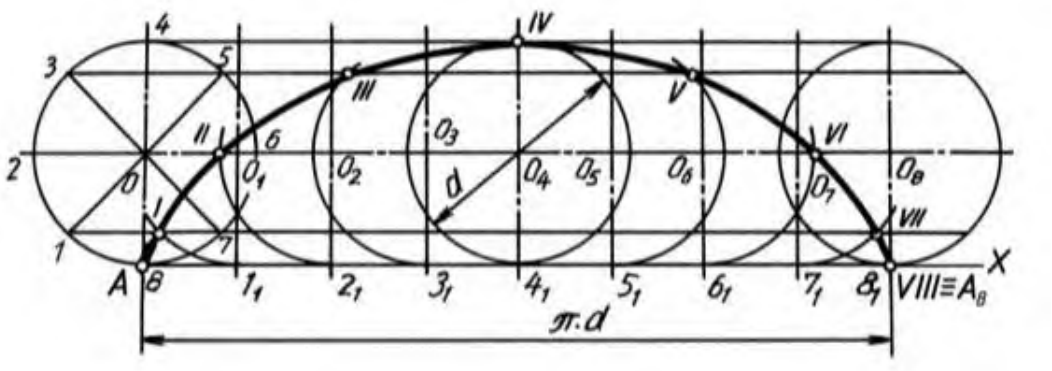


8-shakl

Konus kesimi chiziqlarining asosiy hususiyatlari: ellipsning har bir nuqtasidan katta o‘qida joylashgan vaq F, F1 fokuslar deb ataladigan 2 doimiy nuqtasiga qadar bo‘lgan masofalarning yig‘indisi o‘zgarmas kattalik bo‘lib, u ellips katta o‘qiga teng (8-shakl, c). Parabolaning qaysi bir nuqtasi olinmasin, bu nuqtadan parabola fokusi F ga derektrissasigacha bo‘lgan masofalar o‘zaro teng bo‘ladi (8-shakl, a).

Siklik egri chiziqlar. Sikl-yunoncha seklos-doira-davriy qaytariladigan hodisalar tizimidagi jarayonlar majmui; bunda ma’lum ketma-ketlikda o‘zgaradigan ob`yekt yana dastlabki holatiga qaytadi.

Aylana qo‘zg‘almas to‘g‘ri chiziq bo‘yicha surilmasadan yumalasa, u holda bu aylananing biror, masalan, to‘g‘ri chiziqdagi urinish nuqtasi tekis, ochiq va ravon egri chiziq-sikloida hosil qiladi. Bu yerda to‘g‘ri chiziqni yo‘naltiruvchi, aylanani esa yasovchi deyiladi. Uni chizish uchun d diametrli aylana chizib olinadi va uni o‘zaro teng 12 qismga ajratiladi. A nuqtasidan aylanaga urinma gorizontal chiziq chiziladi va unga aylana uzunligi $\pi \cdot d$ aniqlanib, A nuqtadan boshlabo‘lchab qo‘yiladi. AA_{12} oraliq teng 12 qismga bo‘linib, har bir nuqtasidan vertical chiziqlar chiziladi. O dan o‘tkazilgan shtrix punktir chiziqda $O_1,O_2,O_3…O_{12}$ nuqtalar aniqlanadi. Ushbu O, O_1, O_2,…nuqtalardan (aylanalar markazlaridan) d diametrli aylanaga teng yordamchi aylanalar chiziladi. Aylana nuqtalarining harakat yo‘nalishlari hosil bo‘lib, ular yordamchi aylana yoylari bilan mos ravishda kesishgan nuqtalari I,II,… XII deb belgilanadi. Sikloida nuqtalari lekalolar yordamida ravon tutashtiriladi (9-shakl).

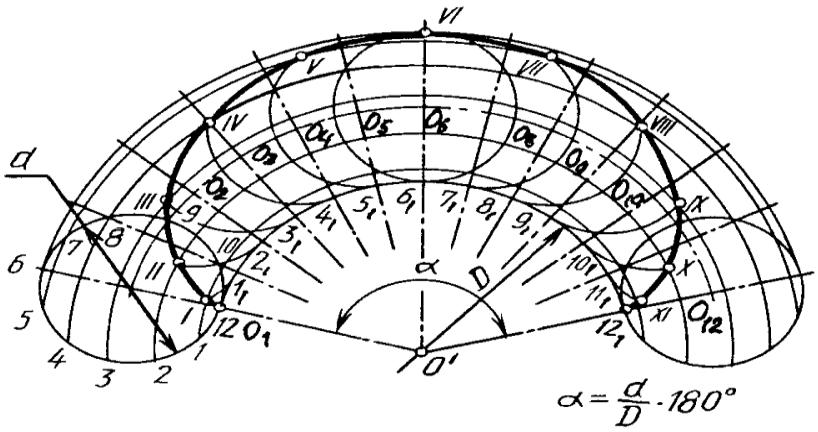


9-shakl

Episikloida. Agar biror aylana qo‘zg‘almas ikkinchi aylananing ustida surilmasdan yumalab siljisa, u holda bu harakatlanuvchi aylananing biror, ochiq va ravon egri chiziq-episikloida hosil qiladi. Bu yerda harakatlanuvchi aylanani yasovchi, qo‘zg‘almas aylanani esa yo‘naltiruvchi deyiladi.

Episikloidani chizish uchun R radiusli aylana yoyini chizib, uning markaziy

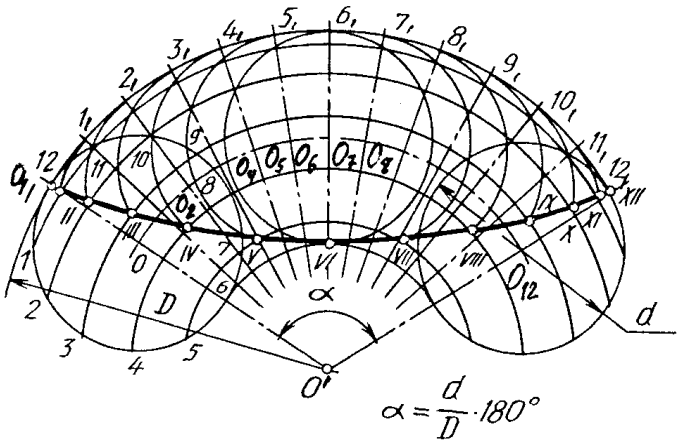
burchagi (α=d (2R-360°) aniqlanadi (10-shakl). d diametrli aylana A nuqtadan urinadigan qilib chiziladi va u bilan R radiusli aylana yoyining AA12 oralig‘i teng 12 qismga bo‘linib chiqiladi. D diametrli aylana nuqtalarining harakat yo‘nalishlari markaziy chizig‘ini O markaz orqali chizib olinadi va unda O1,…O12 markazlar belgilanadi. O1, O2,… O12 nuqtalardan d diametrli aylanaga teng yordamchi aylana yoylari chizilib, O‘ markazdan chizilgan aylana nuqtalarining harakat yo‘nalishlari bilan mos ravishda kesishib hosil bo‘lgan nuqtalar I, II,…XII deb belgilanadi.



10-shakl

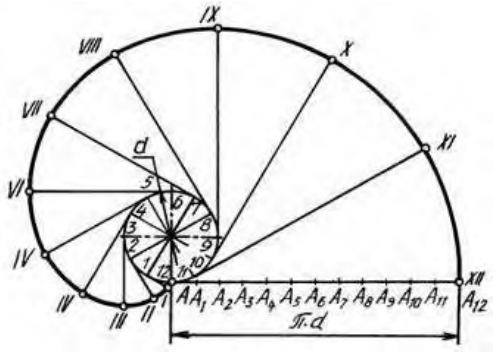
Giposikloida. Agar yasovchi aylana yo‘naltiruvchi aylananing ichki tomoni bo‘yicha srpanmasdan yumalab siljisa, bu holda yasovchi aylananing biror, masalan, yo‘naltiruvchi aylanadagi urinish nuqtasi tekis, ochiq va ravon egri chiziq- giposikloida chizadi.

Giposikloida yasash uchun episikloidani chizishda tatbiq qilingan usuldan foydalaniladi. Buning uchun R radiusli aylana yoyi va uning markaziy burchagi (α=d (2Rx360°) aniqlanadi. D diametrli aylana A nuqtadan R radiusli aylana yoyiga uning ichki tomoniga urinadigan qilib chiziladi va u hamda katta aylananing AA12 bo‘lagi o‘zaro teng 12 qismga bo‘lib olinadi (11-shakl). Qolgan yasashlar chizmadan yaqqol ko‘rinib turibdi.



11-shakl

Evol’venta. Agar to‘g‘ri chiziq qo‘zg‘almas aylana bo‘yicha surilmasdan yumalasa, u holda bu to‘g‘ri chiziqning har bir nuqtasi shaklan o‘ramaga o‘xshash ochiq va ravon aylana evol’ventasi (yoyilmasi) deb ataluvchi egri chiziq chiziladi (12-shakl).

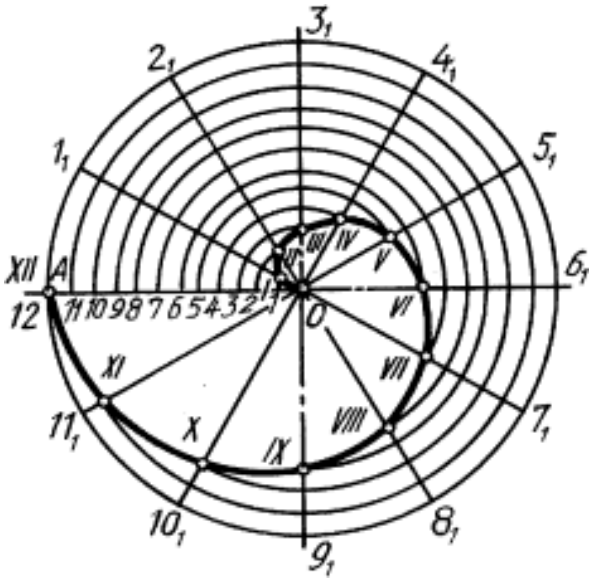


12-shakl

Evol’ventani chizish uchun d diametrli aylana chizilib, u teng 12 qismga bo‘lib oilinadi va markazi O bilan tutashtirilib chiqiladi. Shunda oltita aylana diametri hosil bo‘ladi. Aylana diametrlarining har ikkala uchlaridan diametrlariga perpendikulyar yordamchi chiziqlar chiziladi. Aylananing A nuqtasidan uning uzunligi ($\pi \cdot d$) aniqlanib chiziladi va u AA_{12} teng 12 qismga bo‘lib olinadi. Aylana diametrlariga chizilgan yordamchi chiziqlarga 1 nuqtadan bir AA_1 bo‘lak, 2 nuqtadan ikki AA_1 bo‘lak,... o‘lchab qo‘yish orqali evolventa nuqtalari I,II,...XII lar aniqlanadi va ular lekalolar yordamida ravon tutashtiriladi

Arximed spirali. Nuqta biror markaz atrofida teks aylanma harakat qiluvchi

to‘g‘ri chiziq bo‘yicha bir vaqtda teks ilgarilama harakat qilsa, bu nuqta tekis, ochiq ravon egri chiziq chizadi. Bu egri chiziq Arximed spirali deb ataladi. Agar to‘g‘ri chiziqning bir mata aylanishi vaqtida nuqtaning to‘g‘ri chiziq bo‘yicha bosgan yo‘lini Arximed spiralning qadami deyiladi (13-shakl).



13-shakl

Uni chizish uchun uning berilgan qadami OA ni radius qilib aylana chiziladi va qadami OA hamda aylana bir xil teng 12 qismga bo‘lib chiziladi. Aylanadagi 1_1,2_1,... nuqtalar markazi O bilan tutashtiriladi. OA dagi 1 nuqta sirkul yordamida $O1_1$ga,2 nuqta $O2_1$ ga,... kabi olib o‘tiladi. Shunda I,II,...XII nuqtalar hosil bo‘ladi va ular lekalolar yordamida ravon tutashtiriladi.

Takrorlash uchun savollar

1. Tutashma deb nimaga aytiladi?

2. Tutashma turlarini sanab bering?

3. Lekalo egri chiziqlari deganda nimani tushunasiz?

4. Ellipsning qanday turlari mavjud?

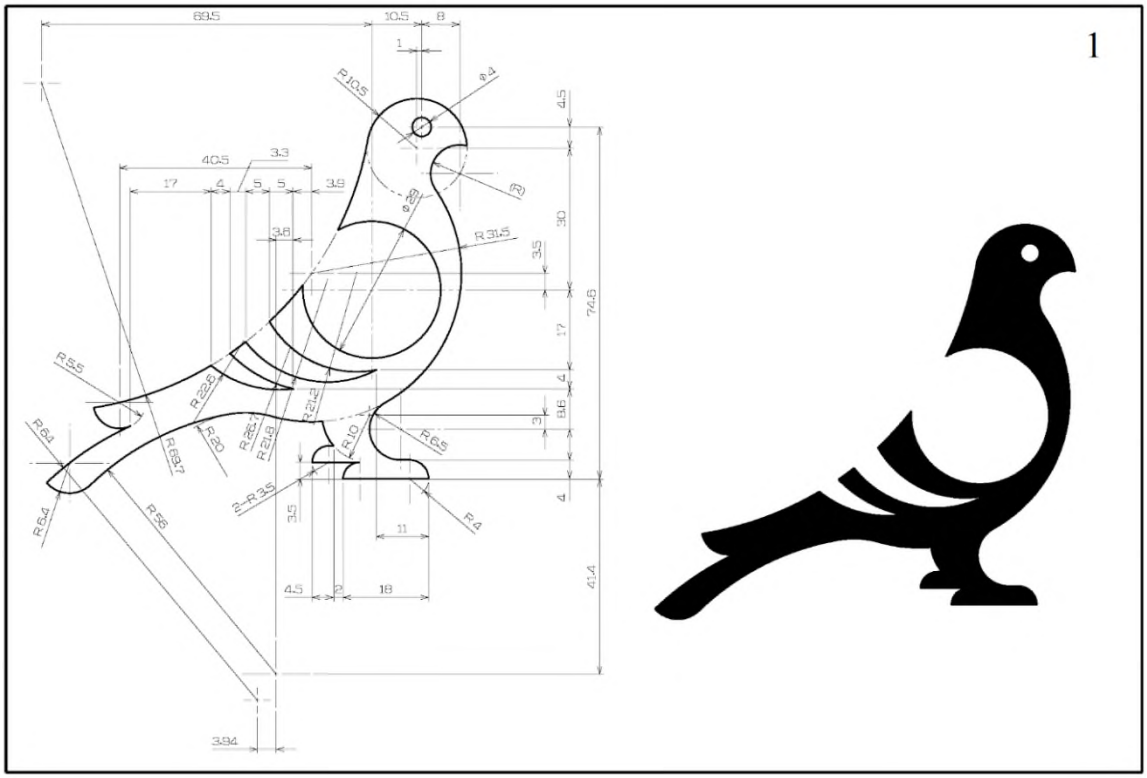
5. Parabola qanday turlari mavjud?

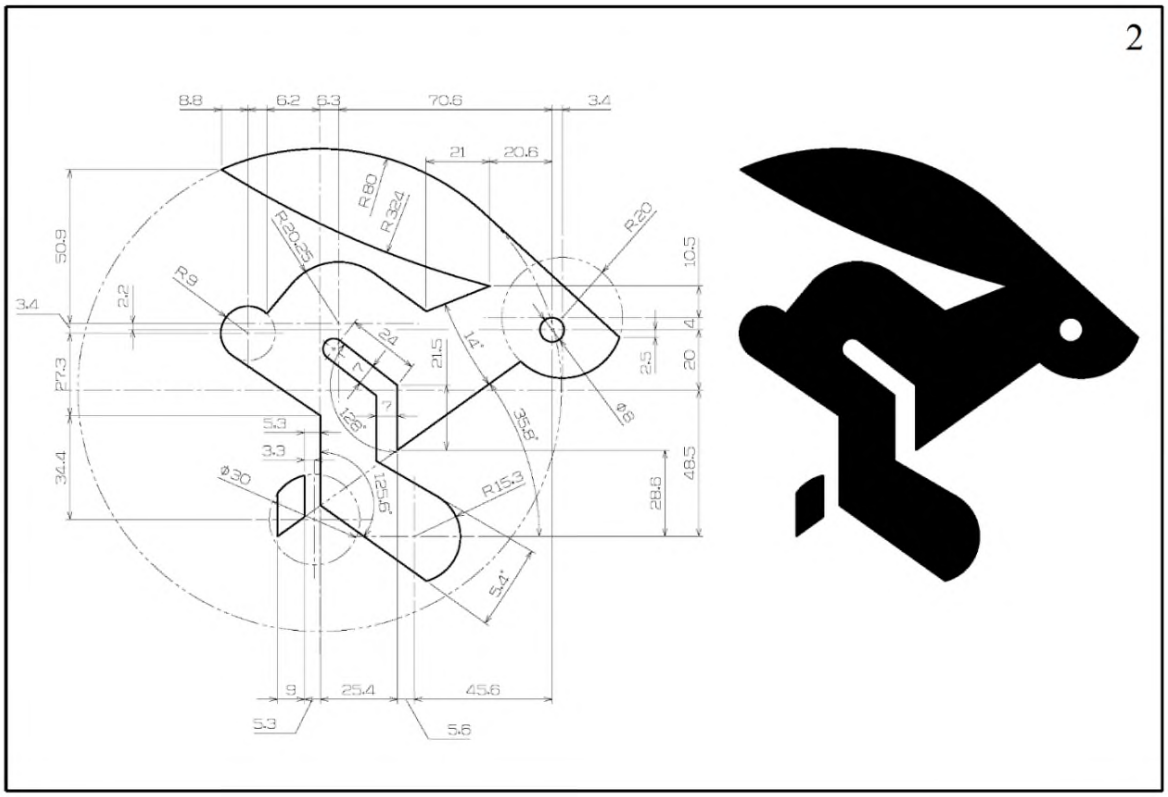
6. Giperbola qanday hosil bo‘ladi?

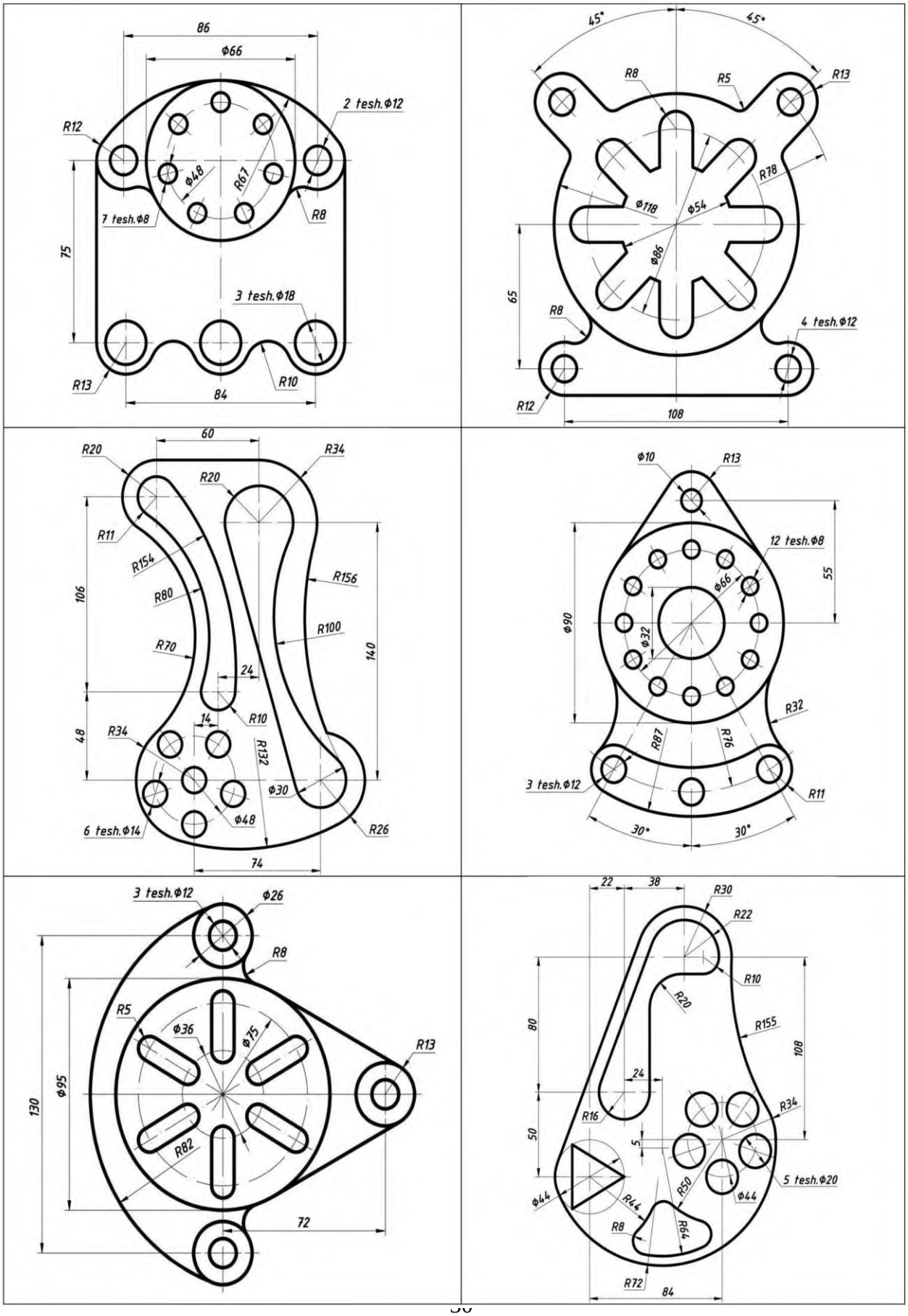
7. Siklik egri chiziqlarni ta’riflang.

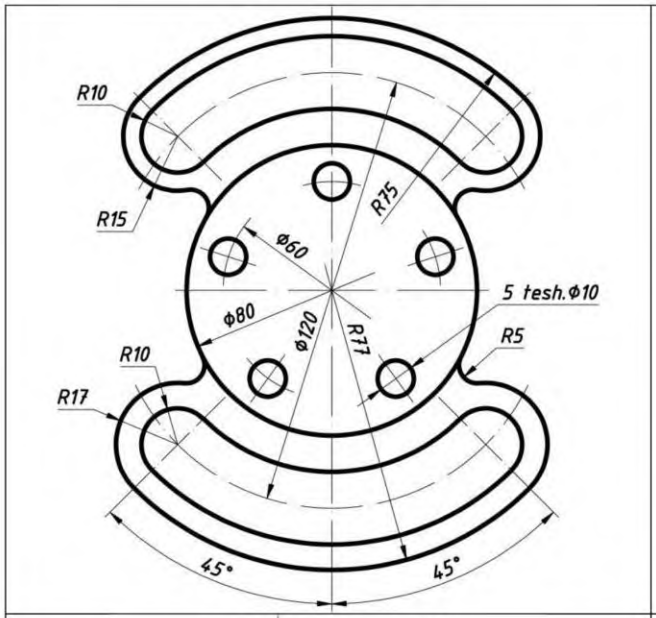
8. Evol’ventani hosil bo‘lishi.

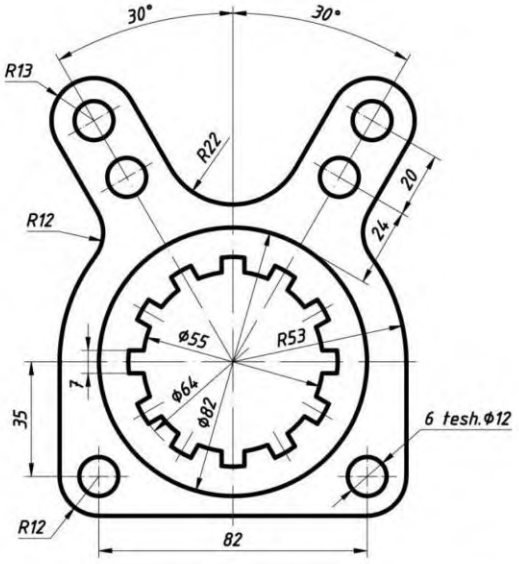
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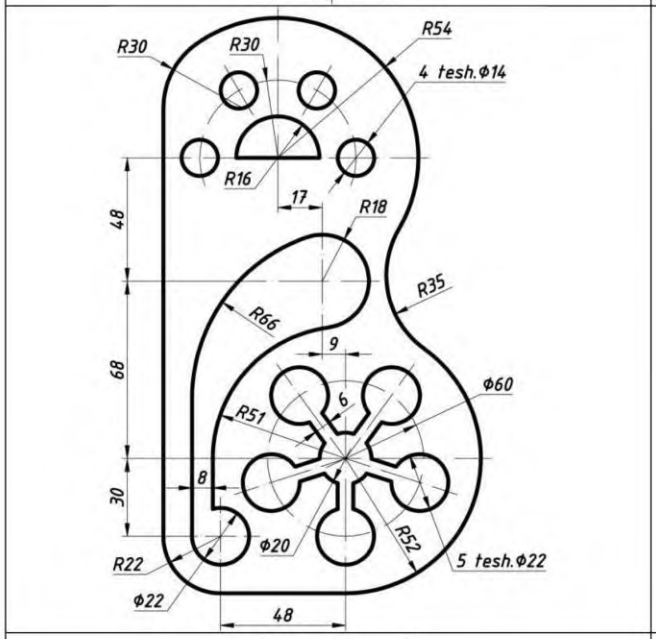

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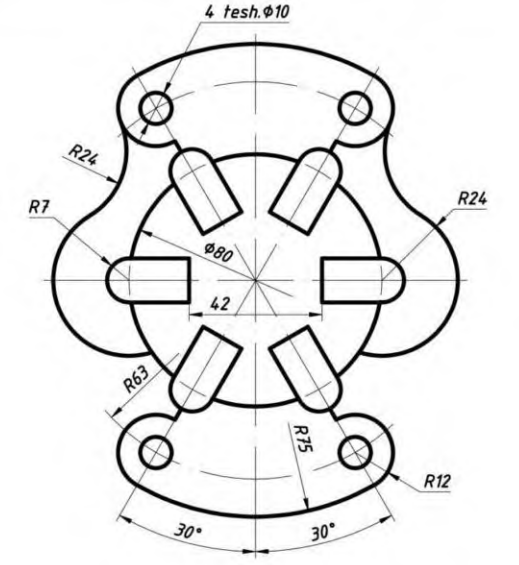

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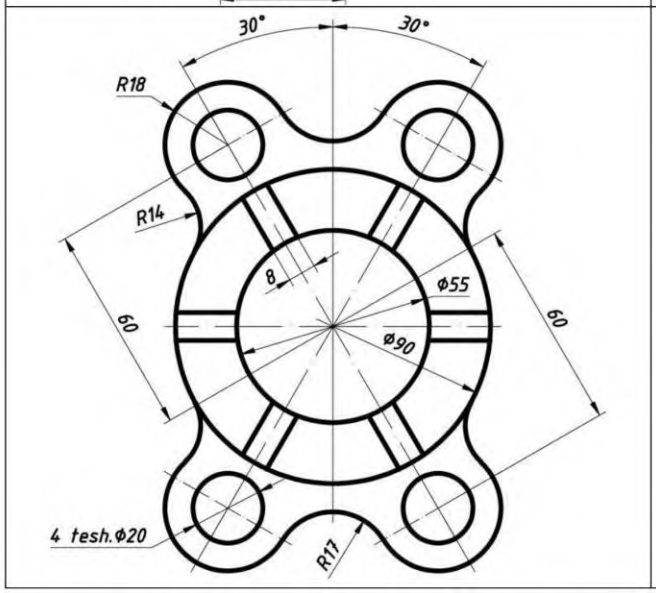

86
ϕ66
2 tesh.ϕ12
R12
ϕ48
R67
7 tesh.ϕ8
R8
75
3 tesh.ϕ18
R13
R10
84
45°
45°
R8
R5
R13
R78
ϕ118
ϕ54
ϕ86
65
R8
4 tesh.ϕ12
R12
108
60
R20
R34
R20
R11
R154
R156
106
R80
R100
R70
140
24
R10
14
48
R34
R132
ϕ30
ϕ48
R26
6 tesh.ϕ14
74
ϕ10
R13
12 tesh.ϕ8
55
ϕ66
ϕ90
ϕ32
R32
R87
R76
3 tesh.ϕ12
R11
30°
30°
3 tesh.ϕ12
ϕ26
R8
R5
ϕ36
ϕ75
R13
ϕ95
130
R82
72
22
38
R30
R22
R10
R20
80
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R50
5 tesh.ϕ20
ϕ44
ϕ44
R44
R8
R64
R72
84

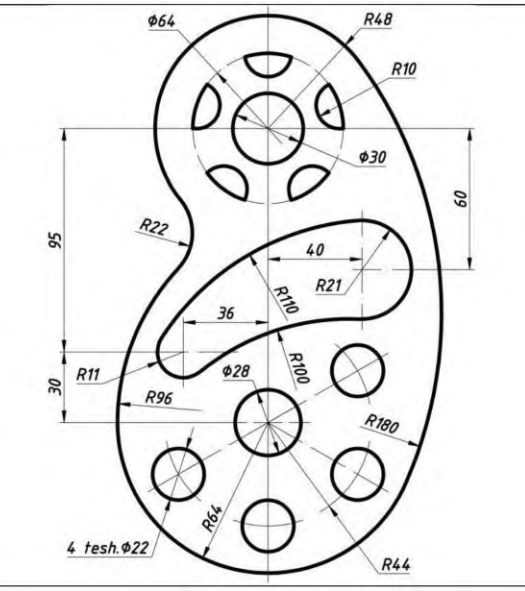

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R15
R75
ϕ60
ϕ80
ϕ120
R77
5 tesh.ϕ10
R5
R10
R17
45°
45°

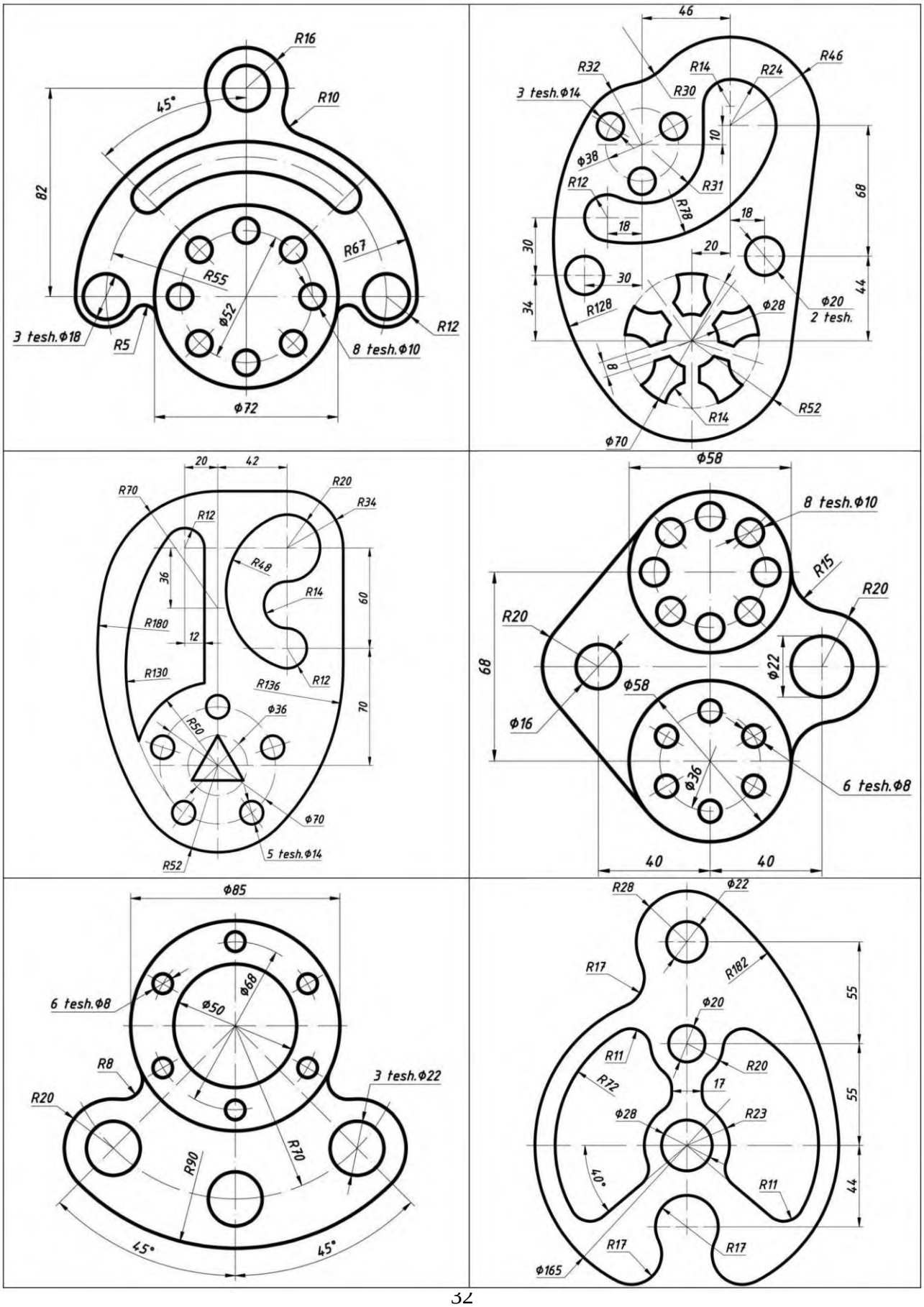

30°
30°
R13
R22
20
24
R12
ϕ55
R53
7
ϕ64
ϕ82
35
6 tesh.ϕ12
R12
82

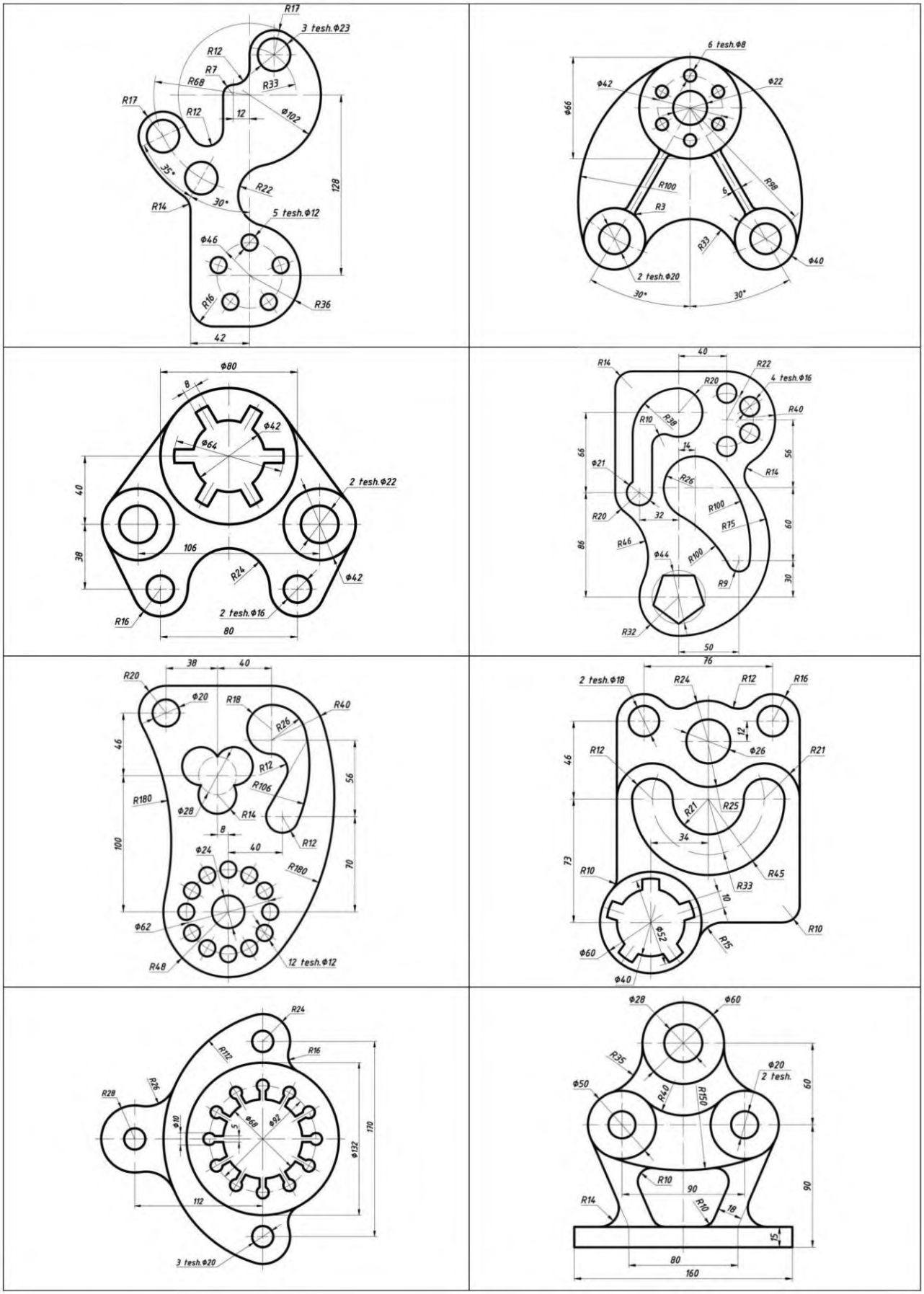

R30
R30
R54
4 tesh.ϕ14
R16
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R18
48
R35
R66
9
68
ϕ60
R51
6
8
30
R22
ϕ20
R52
5 tesh.ϕ22
ϕ22
48


4 tesh.ϕ10
R24
R24
R7
ϕ80
42
R63
R75
R12
30°
30°


30°
30°
R18
R14
8
ϕ55
60
ϕ90
60
4 tesh.ϕ20
R17


ϕ64
R48
R10
ϕ30
60
95
R22
40
R21
R110
36
30
R11
R100
ϕ28
R96
R180
R64
4 tesh.ϕ22
R44


R16
R10
45°
82
R67
R55
ϕ52
3 tesh.ϕ18
R5
8 tesh.ϕ10
R12
ϕ72
46
R32
R14
R24
R46
3 tesh.ϕ14
R30
10
ϕ38
R31
R12
R78
68
18
18
30
20
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34
R128
ϕ28
ϕ20
2 tesh.
8
R14
R52
ϕ70
20
42
R70
R20
R34
R12
R48
36
R14
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R180
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R130
R136
R12
70
ϕ36
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ϕ70
5 tesh.ϕ14
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ϕ58
8 tesh.ϕ10
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ϕ85
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ϕ68
ϕ50
R8
3 tesh.ϕ22
R20
R70
R90
45°
45°
R28
ϕ22
R17
R182
55
ϕ20
R11
R20
R72
17
55
ϕ28
R23
40°
R11
44
R17
R17
ϕ165


R17
3 tesh.Φ23
R12
R7
R68
R33
R17
R12
12
Φ102
35°
R22
128
R14
30°
5 tesh.Φ12
Φ46
R16
R36
42
6 tesh.Φ8
Φ42
Φ22
Φ66
R100
R28
6
R3
R33
Φ40
2 tesh.Φ20
30°
30°
Φ80
8
Φ42
Φ64
2 tesh.Φ22
40
38
106
R24
Φ42
R16
2 tesh.Φ16
80
40
R14
R22
R20
4 tesh.Φ16
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R38
R40
14
66
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Φ21
R26
R14
R20
32
R100
R75
60
86
R46
Φ44
R100
R9
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R32
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76
R20
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Φ20
R18
R40
R26
46
R12
56
R106
R180
Φ28
R14
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R12
100
40
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Φ24
R180
Φ62
R48
12 tesh.Φ12
2 tesh.Φ18
R24
R12
R16
12
Φ26
R12
R21
46
R21
R25
34
73
R10
R45
R33
10
R10
Φ52
R15
Φ60
Φ40
R24
R112
R16
R28
R26
Φ10
Φ68
Φ92
170
Φ132
112
3 tesh.Φ20
Φ28
Φ60
R35
Φ20
2 tesh.
Φ50
R40
R150
60
R10
90
90
R14
R10
18
15
80
160

GLOSSARY (ГЛОССАРИЙ)

Atamaning nomlanishi			Atamaning ma’nosi
O‘zbek tilida	Ingliz tilida	Rus tilida	
Aylanish o‘qi	Axis of rotation	Ось вращения	Fazodagi shaklni biror proyeksiyalar tekisligiga qulay holda keltirishda uni aylantirish uchun tanlangan to‘g‘ri chiziq.
Aylanish radiusi	Radius of gyration	Радиус поворота	Aylanish markazidan harakatlanuvchi nuqtagacha bo‘lgan masofa.
Aylanma yoki aylanish sirti	Surface of revolution	Поверхность вращения или обращения	Biror to‘g‘ri chiziqni, tekis yoki fazoviy egri chiziqni qo‘zg‘almas o‘q atrofida aylanishidan hosil bo‘lgan sirt
Aylantirish markazi	Center of rotation	Центр вращения	Aylanish o‘qi bilan aylantirish tekisligining kesishuv nuqtasi.
Ikkinchi tartibli aylanish sirti	Surface of revolution of the second order	Поверхность вращения второго порядка	Ikkinchi tartibli egri chiziqlarning o‘z o‘qlaridan biri atrofida aylanishidan hosil bo‘lgan sirtlar.
Ikkinchi tartibli sirtlar	Second-order surfaces	Поверхности второго порядка	Biror to‘g‘ri chiziq bilan maksimum ikki nuqtada kesishgan sirtlar yoki tenglamasining darajasi ikkiga teng sirtlar