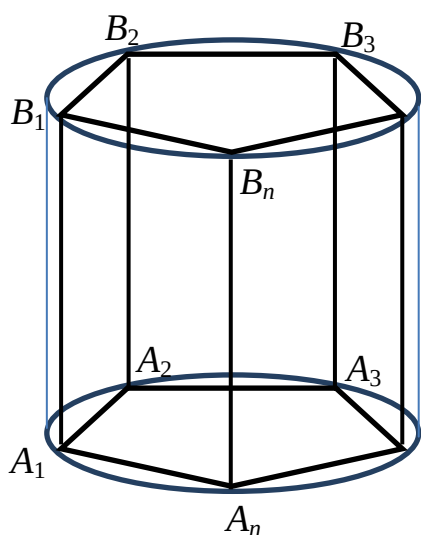


14-BOB

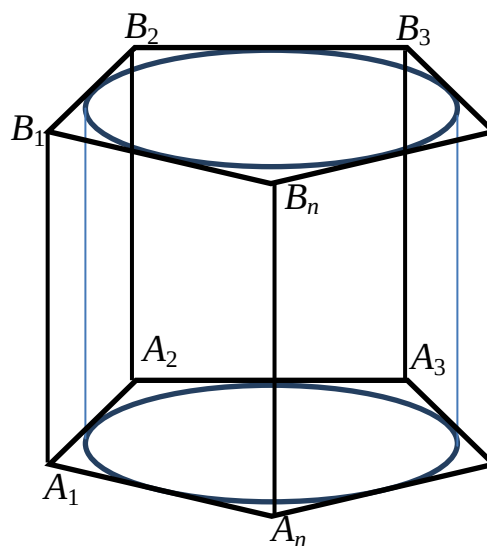
AYLANISH JISMLARI HAJMLARI

1-§. Silindrning hajmi

Ixtiyoriy silindr berilgan bo'lsin. Uning asosiga ichki $A_1A_2A_3...A_n$ ko'pburchak chizamiz (14.1-rasm). Ko'pburchakning $A_1, A_2, A_3, ..., A_n$ uchlari orqali silindrning $A_1B_1, A_2B_2, A_3B_3, ..., A_nB_n$ yasovchilarini o'tkazamiz hamda yasovchining boshqa $B_1, B_2, B_3, ..., B_n$ uchlari ketma-ket kesmalar bilan tutashtirib chiqamiz. Natijada $A_1A_2A_3...A_nB_1B_2B_3...B_n$ prizmani hosil qilamiz. Bu prizma berilgan silindrga *ichki chizilgan prizma* deyiladi. Silindr esa prizmaga tashqi chizilgan deyiladi. Agar prizma silindrga ichki chizilgan bo'lsa, unda prizmaning asosi silindr asosiga ichki chizilgan bo'ladi va prizmaning yon qirralari silindr yon sirtida yotadi.



14.1-rasm



14.2-rasm

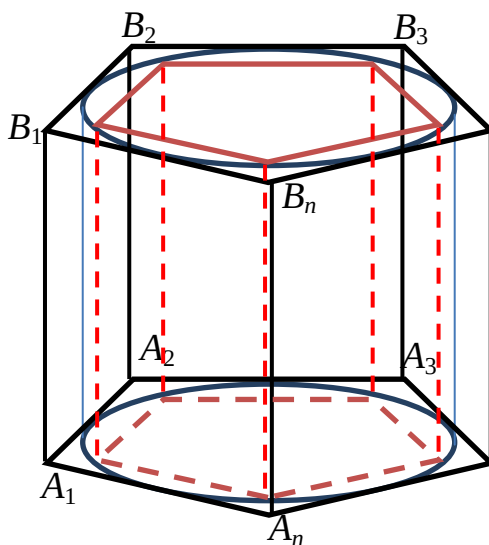
Shunga o'xshash silindrga tashqi chizilgan prizma va prizmaga ichki chizilgan silindr tushunchasini ham kiritamiz (14.2-rasm). Agar prizma silindrga

tashqi chizilgan bo'lsa, unda prizmaning asosi silindr asosiga tashqi chizilgan bo'ladi va prizmaning yon yoqlari silindr yon sirtiga urinadi.

1-teorema. Silindrning hajmi asosining yuzi bilan balandligi ko'paytmasiga teng.

$$V = S_{asos} \cdot H$$

Isbot. Asosining yuzi S , balandligi H bo'lgan silindr berilgan bo'lsin (14.3-rasm). Silindrga ichki va tashqi n burchakli muntazam prizmalar chizamiz.



14.3-rasm

Prizmalar asoslarining yuzlarini, mos ravishda, S_n va S'_n orqali belgilasak, bu prizmalarning hajmlari, $S_n \cdot H$ va $S'_n \cdot H$ ko'rinishda yoziladi. Ichki chizilgan prizma silindrning ichida, tashqi chizilgan prizma esa uning tashqarisida joylashganligidan, silindrning V hajmi uchun $S_n \cdot H < V < S'_n \cdot H$ tengsizlik o'rinli.

Agar n cheksiz orttirilsa, S_n va S'_n yuzlar silindr asosining S_{asos} yuzidan yetarlicha kichik farq qiladi. Demak, hosil qilingan qo'sh tengsizlikdagi uchta ifodaning barchasi $S_{asos} \cdot H$ dan yetarlicha kichik farq qiladi. Bu esa faqat

$$V = S_{asos} \cdot H$$

bo'lgandagina mumkin.

1-natija. Silindr asosining radiusi R bo'lsa, $S_{asos} = \pi R^2$ bo'ladi. Demak, silindrning hajmi $V = \pi R^2 H$ formula bo'yicha hisoblanadi.

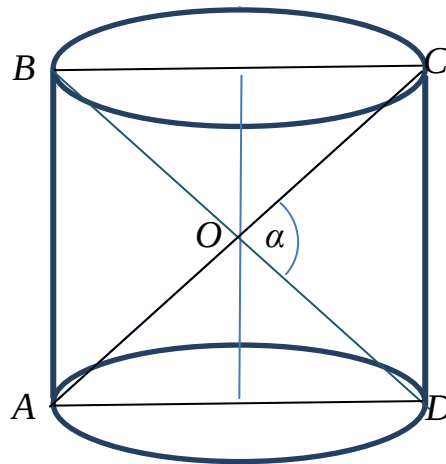
1-masala. Silindr asosining yuzi 256 ga va balandligi $\frac{9}{\sqrt{\pi}}$ ga teng bo'lsa, silindrning hajmini toping.

Yechish. Masala shartiga ko'ra $S_{asos} = \pi R^2 = 256$ ekanligidan silindr asosining radiusi $R = \frac{16}{\sqrt{\pi}}$ bo'ladi. Silindr balandligi $H = \frac{9}{\sqrt{\pi}}$. Bundan silindrning hajmi:

$$V = \pi R^2 H = \pi \cdot \frac{256}{\pi} \cdot \frac{9}{\sqrt{\pi}} = \frac{2304}{\sqrt{\pi}}.$$

2-masala. Silindrning hajmi V , uning o'q kesimi diagonallari orasidagi burchak α ga teng. Silindr asosining radiusini toping.

Yechish. Silindr balandligini H , asosining radiusini R , o'q kesim diagonalini d bilan belgilab olamiz (14.4-rasm).



14.4-rasm

Silindr hajmi formulaga ko'ra: $\pi R^2 H = V$ (1).

$ABCD$ to'rtburchakdan: $AC = \sqrt{AB^2 + AD^2} \Rightarrow d = \sqrt{H^2 + 4R^2}$ (2).

AOB uchburchakda kosinus teoremasini qo'llab, $AB^2 = 2AO^2 - 2AO^2 \cos \alpha \Rightarrow$

$$H^2 = \frac{d^2}{2}(1 - \cos \alpha) \Rightarrow d^2 = \frac{2H^2}{1 - \cos \alpha} \quad (3) \text{ ni hosil qilamiz.}$$

$$(2) \quad \text{va} \quad (3) \quad \text{dan} \quad \text{foydalanib,} \quad \frac{2H^2}{1 - \cos \alpha} = H^2 + 4R^2 \Rightarrow$$

$$(1 + \cos \alpha)H^2 = 4R^2(1 - \cos \alpha) \Rightarrow H = 2R\sqrt{\frac{(1 - \cos \alpha)}{1 + \cos \alpha}} \quad (4) \text{ ga ega bo'lamiz.}$$

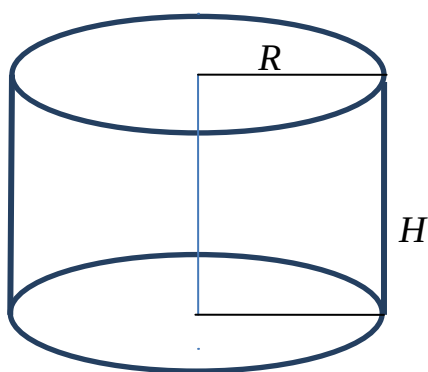
(4) ni (1) ga qo'yib, R noma'lum bo'lgan tenglamani hosil qilamiz:

$$\pi R^2 \cdot 2R \sqrt{\frac{1-\cos \alpha}{1+\cos \alpha}} = V.$$

Tenglamani yechib $R = \sqrt[3]{\frac{V}{2\pi} \sqrt{\frac{1+\cos \alpha}{1-\cos \alpha}}} = \sqrt[3]{\frac{V}{2\pi} \operatorname{ctg} \frac{\alpha}{2}}$ natijani olamiz.

3-masala. Silindrning yon sirti yuzi S ga asosining yuzi esa Q ga teng bo'lsa, uning hajmini toping.

Yechish: Berilganlardan R va H ni topib olamiz.



14.5-rasm

$$S_{\text{asos}} = \pi R^2, \quad S = \pi R^2, \quad R = \sqrt{\frac{S}{\pi}}.$$

$$S_{\text{yon}} = 2\pi RH, \quad Q = 2\pi RH, \quad H = \frac{Q}{2\pi R} = \frac{Q}{2\pi \sqrt{\frac{S}{\pi}}} = \frac{Q}{2\sqrt{\pi S}}.$$

Endi topilganlardan foydalanib, hajmini topib olish mumkin:

$$V = S_{\text{asos}} H = \pi R^2 \cdot H = S \frac{Q}{2\sqrt{\pi S}} = \frac{\sqrt{S} Q}{2\sqrt{\pi}}.$$

$$\text{Javob: } V = \frac{\sqrt{S} Q}{2\sqrt{\pi}}.$$

4-masala. Silindrning ichiga muntazam uchburchakli prizma ichki chizilgan. Prizmaning ichida esa yana silindr ichki chizilgan. Shu ikkita silindrlarning hajmlari nisbatini toping.

Yechish: Chizmadan ko'rinib turibdiki, ikkala silindrning ham va prizmaning ham balandliklari teng. Shuning uchun bu ikki silindrning hajmlari nisbati

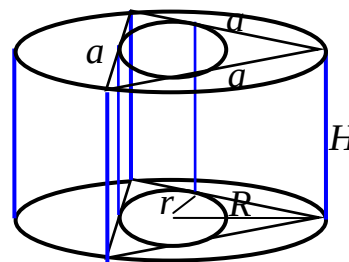
$$\frac{V_R}{V_r} = \frac{S_R H}{S_r H} = \left(\frac{R}{r}\right)^2$$

ko'rinishda bo'ladi.

Demak, muntazam uchburchakning tomonini a deb belgilab, shu tomonga bogʻliq holda radiuslarni topamiz:

$$R = \frac{a}{\sqrt{3}} = \frac{a\sqrt{3}}{3}, \quad r = \frac{a}{2\sqrt{3}} = \frac{a\sqrt{3}}{6}.$$

Endi hajmlar nisbatini topish mumkin:



14.6-rasm

$$\frac{V_R}{V_r} = \left(\frac{R}{r}\right)^2 = \left(\frac{\frac{a\sqrt{3}}{3}}{\frac{a\sqrt{3}}{6}}\right)^2 = 2^2 = 4.$$

Demak, muntazam prizmagga tashqi chizilgan silindrning hajmi unga ichki chizilgan silindr hajmidan 4 marta katta boʻlar ekan:

Javob: $\frac{V_R}{V_r} = 4.$

Mustaqil ishlash uchun masalalar

14.1. Silindr asos aylanasi uzunligi 56π ga teng va balandligi $\frac{7}{\pi}$ ga teng boʻlsa, silindr hajmini toping. javob: 5488

14.2. Silindrning diagonal kesimi diagonali uzunligi $3\sqrt{\frac{2}{\pi^2}}$ ga teng kvadratdan iborat. Silindrning hajmini toping. javob: $\frac{27\sqrt{2}}{8\pi}$

14.3. Silindrni balandligi asosi aylanasi uzunligiga teng. Agar silindrning hajmi $432\pi^2$ ga teng boʻlsa, silindrning asosi diametrik uzunligini toping. javob: 12

14.4. Silindrning toʻla sirti 156 ga teng. Boshqa bir silindrning asosi radiusi va balandligi 2 marta kichik, berilgan konusning mos oʻlchamlaridan. Ikkinchi konusning toʻla sirtini toping. javob: 39

14.5. Silindrning hajmi 16 ga teng. Bu silindrning balandligiga balandligi teng, asosi yuzi esa 4 marta katta boʻlgan boshqa bir silindrning hajmini toping. javob: 64

14.6. Silindrning hajmi 572 ga teng. Asosning diametri berilgan silindr asosining diametridan 3 marta katta, balandligi esa 3 marta kichik bo'lgan silindrning hajmini toping. javob: 1716

14.7. Silindrning balandligi uzunligi asos doirasi uzunligiga teng. Silindr yon sirtining yoyilmasi yuzi $\sqrt[3]{2304\pi^2}$ ga teng. Silindrning hajmini toping. javob: 12

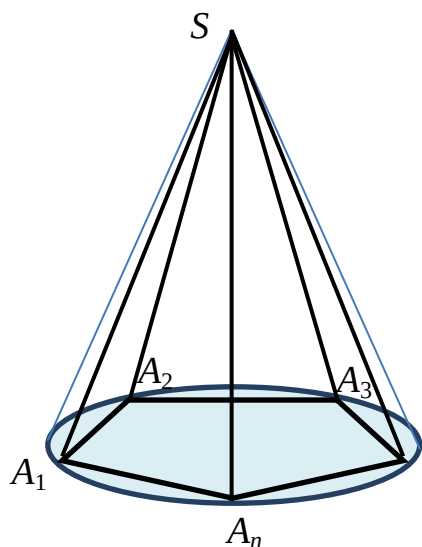
14.8. Silindrning yon sirti yoyilmasi tomoni uzunligi $\sqrt[3]{40\pi}$ ga teng bo'lgan kvadratdan iborat. Silindrning hajmini toping. javob: 10

14.9. Tomoni uzunligi $\sqrt[6]{\frac{9}{8\pi^2}}$ ga teng bo'lgan kvadrat diagonal atrofida aylantirishdan hosil bo'lgan jismning hajmini toping. javob: 0,25

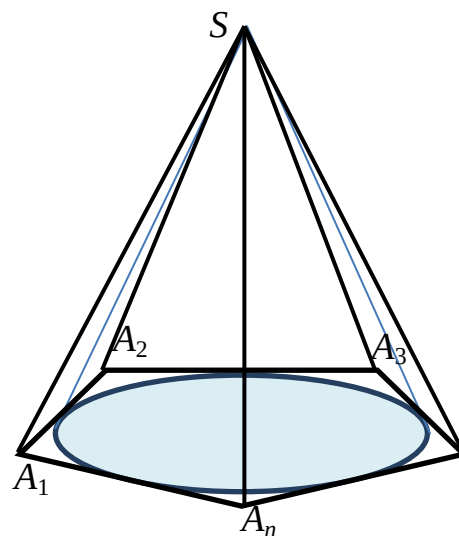
2-§. Konus va kesik konusning hajmi

Ixtiyoriy konus berilgan bo'lsin. Uning asosiga ichki $A_1A_2A_3...A_n$ ko'pburchak chizamiz. Ko'pburchakning $A_1, A_2, A_3, ..., A_n$ uchlarini konusning uchi S nuqta bilan tutashtiramiz. Natijada $SA_1A_2A_3...A_n$ piramida hosil bo'ladi. Bu piramida berilgan konusga ichki chizilgan piramida deyiladi (14.7-rasm). Konus esa piramidaga tashqi chizilgan konus deb yuritiladi. Agar piramida konusga ichki chizilgan bo'lsa, uning asosi konus asosiga ichki chizilgan bo'ladi va piramidaning yon qirralari konus yon sirtida yotadi.

Xuddi shunday konusga tashqi chizilgan piramida tushunchalari ham kiritiladi (14.8-rasm). Agar piramida konusga tashqi chizilgan bo'lsa, unda piramidaning asosi konus asosiga tashqi chizilgan bo'ladi va piramidaning yon yoqlari konus yon sirtiga urinadi. Agar piramida asosiga ichki aylana chizish mumkin bo'lsa, bu piramidaga ichki konus ham chizish mumkin.



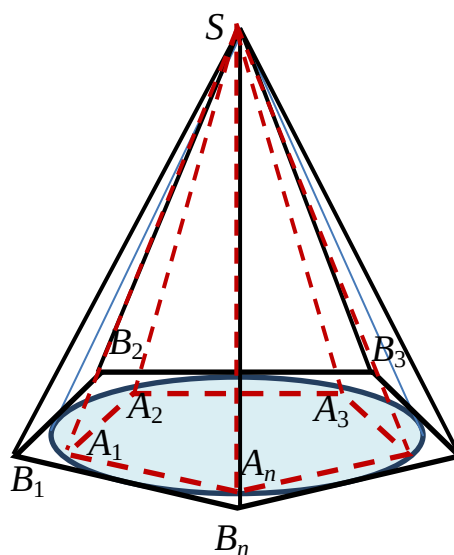
14.7-rasm



14.8-rasm

2-teorema. Konusning hajmi asosining yuzi bilan balandligi ko'paytmasining uchdan biriga teng $V = \frac{1}{3} S_{asos} \cdot H$.

Isbot. Uchi S nuqtada va balandligi H ga teng bo'lgan konus berilgan bo'lsin (14.9-rasm).



14.9-rasm

Unga ichki va tashqi $SB_1B_2B_3...B_n$ (14.9-rasm) piramidalarni chizamiz. Konus hajmini V , ichki va tashqi chizilgan piramidalar hajmini V_n va V'_n bilan belgilasak,

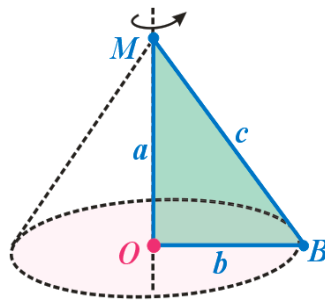
unda $V_n < V < V'_n$ qo'sh tengsizlik o'rinli bo'ladi. Piramidalar hajmi quyidagi formulalardan topiladi:

$$V_n = \frac{1}{3} S_{A_1 A_2 A_3 \dots A_n} \cdot H \text{ va } V'_n = \frac{1}{3} S_{B_1 B_2 B_3 \dots B_n} \cdot H.$$

Piramidalar asosi tomonlari sonini borgan sari oshirib boramiz. Unda ichki chizilgan piramida hajmi oshib boradi, tashqi chizilgan piramidaning hajmi esa kamayib boradi. Agar tomonlar soni n cheksiz kattalashganda, bu hajmlar orasidagi farq 0 ga intiladi. Konusga ichki va tashqi chizilgan piramidalar hajmi yaqinlashgan son berilgan konusning hajmi sifatida olinadi. Bu jarayonda ichki va tashqi chizilgan ko'pburchaklar yuzi konus asosida yotgan doira yuzi S ga yaqinlashadi. Demak, $V = \frac{1}{3} S_{asos} \cdot H$ yoki $V = \frac{1}{3} \pi R^2 H$.

To'g'ri burchakli uchburchakni a kateti atrofida aylantirishdan hosil bo'lgan konusning hajmi quyidagicha bo'ladi (14.10-rasm):

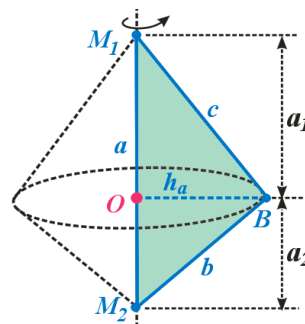
$$V = \frac{1}{3} \pi b^2 a.$$



14.10-rasm

Tomonlari ixtiyoriy a, b, c bo'lgan uchburchakni, ixtiyoriy tomoni atrofida, masalan, a tomoni atrofida aylantirishdan hosil bo'lgan aylanish jismining hajmi quyidagicha bo'ladi (14.11-rasm):

$$V = \frac{1}{3} \pi h_a^2 a.$$



14.11-rasm

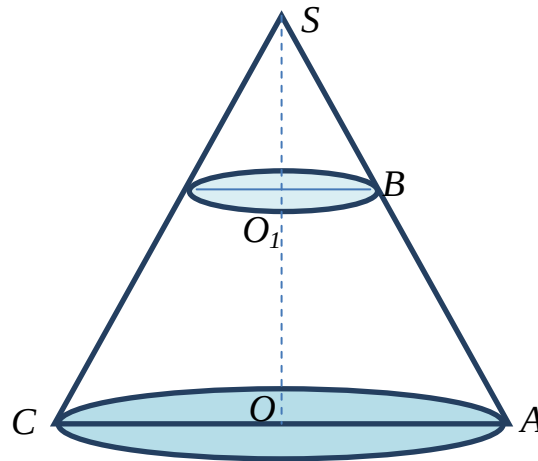
3-teorema. Balandligi H , asoslarining yuzlari S_1 va S_2 bo'lgan kesik konusning hajmi

$$V = \frac{1}{3} H (S_1 + \sqrt{S_1 S_2} + S_2)$$

formula bo'yicha hisoblanadi.

Isbot. Kesik konus ostki va ustki asoslari radiuslari, mos ravishda, R va r , uning yasovchisi $l=AB$ bo'lsin (14.12-rasm). U holda kesik konusning hajmi asosining radiusi R va r bo'lgan konuslar hajmlarining ayirmasi teng bo'ladi:

$$V_{k.k} = \frac{1}{3}\pi R^2 \cdot OS - \frac{1}{3}\pi r^2 \cdot O_1S = \frac{1}{3}\pi(R^2(OO_1 + O_1S) - r^2 O_1S). \quad (1)$$



14.12- rasm

$O_1B \parallel OA$ bo'lganligidan, ΔSO_1B va ΔSOA o'xshash. Shu sababli

$$\frac{OA}{O_1B} = \frac{OS}{O_1S} = \frac{OO_1 + O_1S}{O_1S} = \frac{R}{r}$$

va

$$OO_1 \cdot r + O_1S \cdot r = O_1S \cdot R.$$

Bundan $O_1S = \frac{OO_1 \cdot r}{R-r}$ bo'ladi. Olingan qiymatlarni (1) formulaga qo'yamiz:

$$V_{k.k} = \frac{1}{3}\pi \left(R^2 OO_1 + R^2 \frac{OO_1 \cdot r}{R-r} - r^2 \frac{OO_1 \cdot r}{R-r} \right) \Rightarrow$$

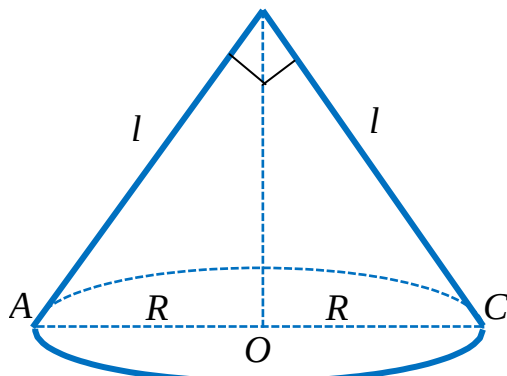
$$V_{k.k} = \frac{1}{3}\pi \cdot OO_1 \left(\frac{R^2(R-r) + R^2r - r^3}{R-r} \right) \Rightarrow V_{k.k} = \frac{1}{3}\pi \cdot OO_1 \left(\frac{R^3 - r^3}{R-r} \right) = \frac{1}{3} OO_1 (\pi R^2 + \pi Rr + \pi r^2).$$

Demak, $OO_1 = H$, $S_1 = \pi R^2$, $S_2 = \pi r^2$ ekanligini hisobga olsak:

$$V_{k.k} = \frac{1}{3}H(S_1 + \sqrt{S_1 S_2} + S_2)$$

bo'ladi.

1-masala. Konus o'q kesimi to'g'ri burchakli uchburchakdan iborat va bu uchburchakning yuzi 18 ga teng (14.13-rasm). Konusning hajmini toping.



14.13-rasm

Yechish. O'q kesimimiz teng yonli to'g'ri burchakli uchburchakdan iborat va uning yuzi

$$S_{ABC} = \frac{l^2}{2}; \quad 18 = \frac{l^2}{2}; \quad 36 = l^2 \Rightarrow l = 6$$

To'g'ri burchakli uchburchak uchun Pifagor teoremasidan foydalanib konus asosining radiusini topib olamiz:

$$AC^2 = AB^2 + BC^2; \quad 4R^2 = 72 \Rightarrow R = 3\sqrt{2}.$$

Konus hajmi $V = \frac{1}{3}\pi R^2 H$ quyidagi formula orqali topiladi. Demak, biz konus balandligini topib olishimiz kerak.

$$l^2 = H^2 + R^2; \quad H^2 = 6^2 - (3\sqrt{2})^2 \Rightarrow H = 3\sqrt{2}.$$

$$\text{Bundan } V = \frac{1}{3}\pi R^2 H = \frac{1}{3}\pi \cdot 18 \cdot 3\sqrt{2} = 18\sqrt{2}\pi.$$

$$\text{Javob: } V = 18\sqrt{2}\pi.$$

2-masala. Ushbu $y = |x+1|$, $x = -3$, $x = 0$ va $y = 0$ chiziqlar bilan chegaralangan shaklning abssissalar o'qi atrofida aylanishidan hosil bo'lgan jismning hajmini toping (14.14-rasm).

Yechish. Bu funksiya grafiklarini bitta Dekart koordinatalar sistemasida chizib olamiz va kesishish nuqtalarining koordinatalarini aniqlaymiz.

Endi bu grafiklarni abssissalar o'qi atrofida aylantirganimizda ikkita konus hosil bo'ladi.

Birinchi konus uchun $R_1 = 1$, $H_1 = 1$ va hajmi

$$V_1 = \frac{1}{3}\pi R_1^2 H_1 = \frac{1}{3}\pi.$$

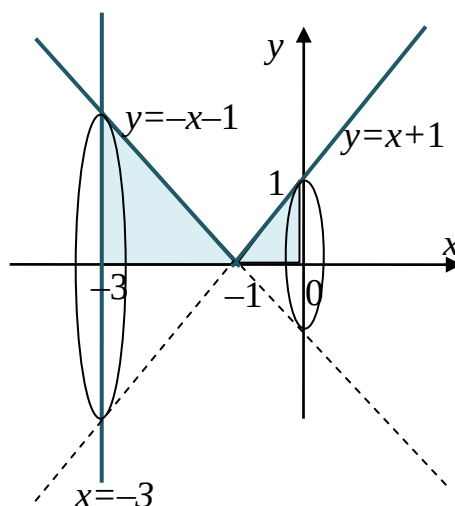
Ikkinchi konus uchun $R_2 = 2$, $H_2 = 2$ va

$$\text{hajmi } V_2 = \frac{1}{3}\pi R_2^2 H_2 = \frac{1}{3}\pi \cdot 8 = \frac{8\pi}{3}.$$

Biz izlayotgan hajm esa bu hajmlar yig'indisiga teng bo'ladi. Ya'ni

$$V = V_1 + V_2 = \frac{\pi}{3} + \frac{8\pi}{3} = 3\pi.$$

Javob. $V = 3\pi$.



14.15-rasm

Mustaqil ishlash uchun masalalar

14.10. Konusning hajmi 275π ga va asosining diametri esa $5\sqrt{2}$ ga teng bo'lsa, konusning balandligini toping. javob: 66

14.11. Konus asosi va balandligi silindr asosi va balandligiga teng. Agar silindr hajmi 447 bo'lsa, konusning hajmini toping. javob: 149

14.12. Agar konusning hajmi 48π va asosining diametri $\frac{\sqrt{3}}{2}$ ga teng. Konusning balandligini toping. javob: 768

14.13. Konusning asosi diametri va yasovchisining uzunliklari $2\sqrt[6]{\frac{3}{\pi^2}}$ ga teng bo'lsa, konusning hajmini toping. javob: 1

14.14. Konusning o'q kesimi yuzi $4\sqrt{3}$ ga teng. Agar konusning balandligi 2π ga teng bo'lsa, konusning hajmini toping. javob: 8

14.15. Konusning o'q kesimi teng tomonli, tomoni uzunligi $\sqrt[6]{\frac{3}{\pi^2}}$ ga teng uchburchakdan iborat. Konusning hajmini toping. javob: 0,125 201.

14.16. Konusning hajmi 112 ga teng. Boshqa bir konusning asosi radiusi 5 marta katta, balandligi esa 2 marta katta. Katta konusning hajmini toping. javob: 5600

14.17. Konusning hajmi 3 ga teng. Balandligi berilgan konus balandligiga teng, asosi yuzi esa 25 marta katta bo'lgan konusning hajmini toping. javob: 75

14.18. Konusning yon sirti yoyilmasi radiusi $\sqrt{\frac{990}{\pi}}$ ga teng bo'lgan uchdan bir doirani hosil qiladi. Konusning asosi yuzini toping. javob: 110

14.19. Konusning yon sirti yoyilmasi radiusi $\frac{2\sqrt{3}}{\sqrt[3]{\pi}}$ ga teng bo'lgan yarim doirani hosil qiladi. Konusning hajmini toping. javob: 3

14.20. Konusning asosi radiusi $\sqrt[6]{\frac{15}{\pi^2}}$ teng bo'lib, yon sirti yoyilmasi uchidagi burchagi esa 90° ga teng. Konusning hajmini toping. javob: 5

14.21. Konusning yon sirti yoyilmasi radiusi 60 ga teng bo'lgan doiraning chorak qismini hosil qiladi. Konusning balandligini toping. javob: $15\sqrt{15}$

14.22. Teng yonli uchburchakning asosi uzunligi $2\sqrt{\frac{5}{\pi}}$ va balandligi uzunligi esa $\frac{5}{2\sqrt{\pi}}$ ga teng. Bu uchburchak balandligi atrofida aylantirishdan hosil bo'lgan jismning to'la sirtini toping. javob: 12,5

14.23. Tomoni uzunligi $\sqrt[6]{\frac{9}{8\pi^2}}$ ga teng bo'lgan kvadratni diagonal atrofida aylantirishdan hosil bo'lgan jismning hajmini toping. javob: 0,25

14.24. Asosi tomoni uzunligi $\frac{9}{\pi}$ ga va balandligi $7\sqrt{2}$ ga teng bo'lgan teng yonli uchburchakni asosi atrofida aylantirishdan hosil bo'lgan jismning hajmini toping. javob: 294

14.25. To'g'ri burchakli uchburchakning gipotenuzasi uzunligi $\frac{2}{\sqrt[3]{\pi}}$ ga va o'tkir burchagi 30° ga teng. Bu uchburchakni gipotenuzasi atrofida aylantirishdan hosil bo'lgan jismning hajmini toping. javob: 0,5

14.26. Tomoni uzunligi $\sqrt[3]{\frac{7}{\pi}}$ ga teng bo'lgan teng tomonli uchburchakni bir tomoni atrofida aylantirishdan bo'lgan jismning hajmini toping. javob: 1,75

14.27. Diagonallari uzunligi $\sqrt{15}$ va $\frac{60}{\pi}$ ga teng bo'lgan romb katta diagonali atrofida aylantirishdan hosil bo'lgan jismning hajmini toping. javob: 75

14.28. Balandligi uzunligi $2\sqrt{\frac{19}{\pi}}$ ga teng bo'lgan teng tomonli uchburchak balandligi atrofida aylantirishdan hosil bo'lgan jismning to'la sirtini yuzini toping. javob: 76

14.29. Tomonlari uzunliklari $\sqrt{\frac{7}{\pi}}$ va $\sqrt{\frac{1}{7\pi}}$ ga teng bo'lgan to'g'ri to'rtburchakni katta tomonlari o'rtasidan o'tuvchi to'g'ri chiziq atrofida aylantirishdan hosil bo'lgan jismning to'la sirtini toping. javob: 4,5

14.30. Konusning balandligi asosining diametriga teng. Agar konusning hajmi $\frac{128\pi}{3}$ ga teng bo'lsa, konus asosining radiusini toping. javob: 4

14.31. Konusning hajmi 384 ga teng. Agar konusning asosi aylanasi uzunligi 15 ga teng bo'lsa, konusning o'q kesimi yuzini toping. javob: 153,6

14.32. Katetlari uzunliklari $\frac{6}{\sqrt{\pi}}$ va $\frac{8}{\sqrt{\pi}}$ ga teng bo'lgan to'g'ri burchakli uchburchakni kichik kateti atrofida aylantirishdan hosil bo'lgan jismning to'la sirtini toping. javob: 144

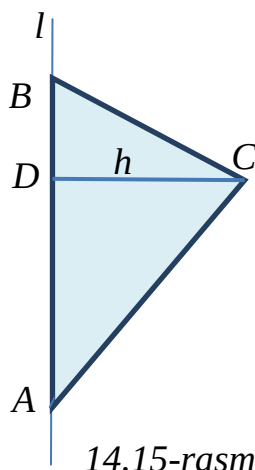
3-§. Aylanish jismlari hajmi uchun umumiy formula. Shar va uning bo'laklari hajmi

1-ta'rif. (*umumlashgan ta'rif*) Biror to'g'ri chiziqqa (aylanish o'qiga) perpendikular bo'lgan tekisliklar bilan markazi shu to'g'ri chiziqda yotgan doiralar bo'yicha kesishishidan hosil bo'lgan geometrik jismga **aylanish jismi** deyiladi.

4-teorema. Agar $\triangle ABC$ uchburchak tekisligida yotib, uning A uchi orqali o'tuvchi va BC tomonini kesib o'tmaydigan l o'q atrofida aylansa, bu aylanish natijasida hosil bo'ladigan geometrik jismning hajmi qarama-qarshi BC tomoni hosil qilgan sirt yuzining shu tomonga tushirilgan h balandlikka ko'paytmasining uchdan biriga teng.

Isbot. Teoremani isbotlashda uchta holni qaraymiz.

1-hol. l aylanish o'qi AB tomon bilan ustma-ust tushsin (14.15-rasm). $\triangle ABC$ ning AB tomon atrofida aylanishidan ikkita konus hosil qilamiz. Agar $CD \perp AB$ bo'lsa, bu konuslarning V_1 va V_2 hajmlari, mos ravishda, $V_1 = \frac{1}{3} \pi \cdot CD^2 \cdot BD$ va $V_2 = \frac{1}{3} \pi \cdot CD^2 \cdot AD$ bo'ladi.



U holda aylanish jismining hajmi uchun

$$V = V_1 + V_2 = \frac{1}{3} \pi \cdot CD^2 \cdot BD + \frac{1}{3} \pi \cdot CD^2 \cdot AD = \frac{1}{3} \pi \cdot CD^2 \cdot (BD + AD) = \frac{1}{3} \pi \cdot CD^2 \cdot AB$$

ifodani olamiz. Uchburchakning A uchidan EC tomonga uchburchakning h balandligini o'tkazamiz. U holda

$$CD \cdot AB = AB \cdot h = 2S_{\triangle ABC}$$

bo'ladi. Shunday qilib, aylanish jismining hajmi uchun hosil qilingan ifoda

$$V = \frac{1}{3} \pi \cdot CD \cdot BD \cdot h$$

ko‘rinishni oladi. Bizga ma’lumki, $\pi \cdot CD \cdot BD$ ko‘paytma BDC konus yon sirtining yuziga teng:

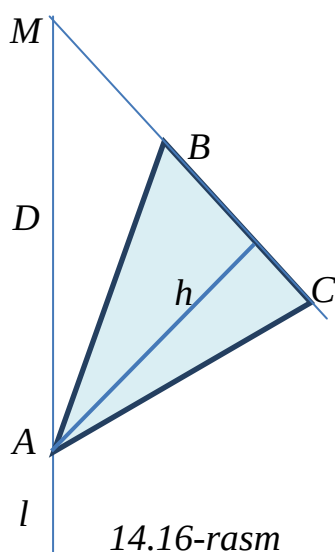
$$S_{yon} = \pi \cdot CD \cdot BD.$$

Bundan

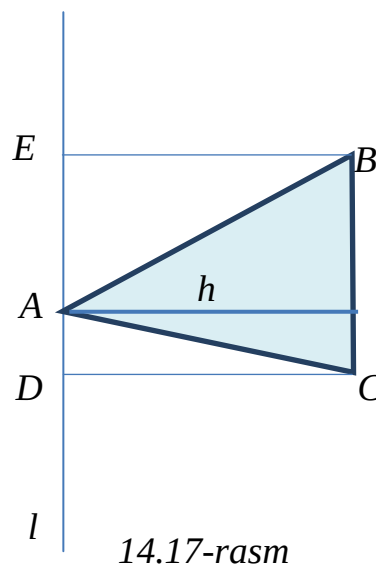
$$V = \frac{1}{3} S_{yon} \cdot h$$

ko‘rinishni oladi.

2-hol. l o‘q A nuqta orqali o‘tib, BC tomonga parallel bo‘lmasin (14.16-rasm). U holda aylanish jismining hajmi $\triangle AMC$ va $\triangle AMB$ larning aylanishidan hosil bo‘lgan geometrik jismlar hajmlarining ayirmasiga teng bo‘ladi. Birinchi holda isbotlanganiga ko‘ra, formulani $V_{AMC} = \frac{1}{3} S_{MC} \cdot h$ va $V_{AMB} = \frac{1}{3} S_{MB} \cdot h$ ko‘rinishda yozib olamiz, bunda S_{MC} va S_{MB} — mos ravishda yasovchisi MC va MB bo‘lgan konusning yon sirti.



14.16-rasm



14.17-rasm

Bundan aylanish jismining hajmi uchun

$$V_{ABC} = \frac{1}{3} h (S_{MC} - S_{MB}) = \frac{1}{3} h \cdot S_{BC}$$

ega bo‘lamiz, bunda S_{BC} — BC tomonning aylanishidan hosil bo‘lgan sirtning yuzi.

3-hol. l o‘q BC tomonga parallel bo‘lsin (14.17-rasm). U holda aylanish jismining hajmi BC tomonning aylanishidan hosil bo‘lgan silindrning hajmidan

$\triangle ABE$ va $\triangle ACD$ larning aylanishidan hosil bo'lgan ikkita konus hajmlarini ayrilganiga teng bo'ladi, ya'ni $V_{BC} = V_{BCDE} - V_{BAE} - V_{ADC}$, bunda $BE \perp ED$, $CD \perp DE$.

Bundan

$$\begin{aligned} V_{BC} &= \pi \cdot CD^2 \cdot DE - \frac{1}{3} \pi BC^2 \cdot EA - \frac{1}{3} \pi CD^2 \cdot AD = \\ &= \pi \cdot CD^2 \cdot \left(DE - \frac{1}{3} (EA + AD) \right) = \frac{2}{3} \pi CD^2 \cdot DE \end{aligned}$$

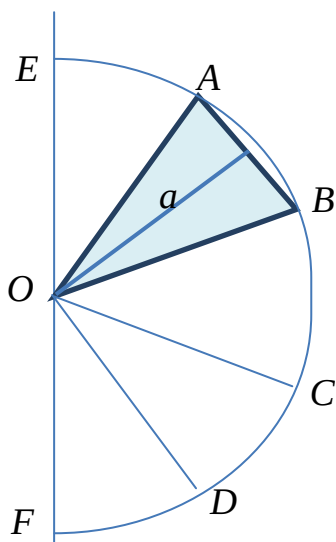
ko'rinishni oladi.

$2\pi CD \cdot DE = S_{BC}$ bo'lganligi uchun

$$V_{BC} = \frac{1}{3} S_{BC} \cdot CD = \frac{1}{3} h \cdot S_{BC}$$

ga ega bo'lamiz.

Endi shar sektorining hajmi haqida so'z yuritamiz. AOD doiraviy sektorning EF diametr atrofida aylanishidan hosil bo'lgan shar sektorining hajmi sifatida, OA va OD radiuslar va doiraviy sektorga ichki chizilgan muntazam $ABCD$ siniq chiziq bilan chegaralangan ko'pburchakli sektorning aylanishidan hosil bo'lgan jism hajmining, siniq chiziq tomonlari soni cheksiz ortgandagi limiti qabul qilinadi (14.18-rasm).



14.18-rasm

5-teorema. Shar sektorining hajmi mos shar kamari sirti (yoki mos segment sirti) yuzi bilan radius ko'paytmasining uchdan biri teng.

Isbot. Shar sektori doiraviy OAD sektorning EF diametr atrofida aylanishidan hosil bo'lsin (14.18-rasm). Uning hajmini topish uchun AD yoyga tomonlari ixtiyoriy sondagi ichki muntazam $ABCD$ sinig chiziq chizamiz. Bunda ko'pburchakli $OABCD$ sektorning aylanishi natijasida qandaydir jism hosil bo'ladi, uning hajmini V_n deb belgilaymiz. Bu jismning hajmi OAB , OBC , OCD uchburchaklarning EF o'q atrofida aylanishidan hosil bo'lgan jismlar hajmlarining yig'indisiga teng. U holda 4-teoremaga muvofiq

$$V_n = S_{AB} \frac{a}{3} + S_{BC} \frac{a}{3} + \dots = S_{ABCD} \frac{a}{3}$$

bo'ladi.

Endi sinig chiziq tomonlari sonini ikkilantiramiz. U holda $ABCD$ sirt shar kamari AD ning sirtiga, uchburchaklarning balandligi esa sharning radiusiga intiladi. Demak,

$$V = \lim V_n = S_{AD} \frac{R}{3}$$

bo'ladi.

6-teorema. Sharning hajmi uning sferasi sirti yuzi bilan radiusi ko'paytmasining uchdan biri teng.

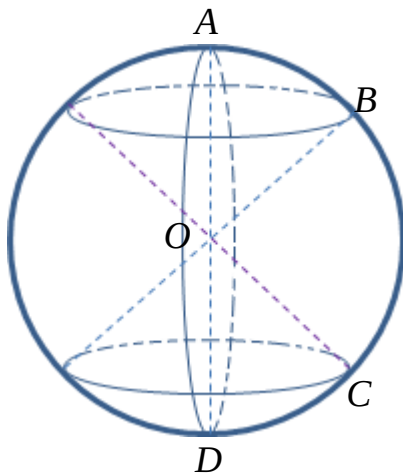
Isbot. Aylanishi natijasida shar hosil bo'ladigan $ABCD$ yarimdoirani AOB , BOC , COD doiraviy sektorlarga bo'lamiz (14.19-rasm). U holda, 5-teoremaga ko'ra,

$$V_{AOB} = \frac{1}{3} R \cdot S_{AB}, \quad V_{BOC} = \frac{1}{3} R \cdot S_{BC}, \quad V_{COD} = \frac{1}{3} R \cdot S_{CD}.$$

Olingan ifodalarni qo'shib, shar hajmi uchun

$$V = \frac{1}{3} R (S_{AB} + S_{BC} + S_{CD}) = \frac{1}{3} R \cdot S_{ABCD}$$

formulani hosil qilamiz.



14.19- rasm

1-natija. Shar segmenti yoki shar kamarining balandligi H , sharning radiusi R bo'lsin. Shar sektori va shar hajmlari uchun

$$V_{\text{shar.sek}} = \frac{2}{3}\pi R^2 H, \quad V_{\text{shar}} = \frac{4}{3}\pi R^3$$

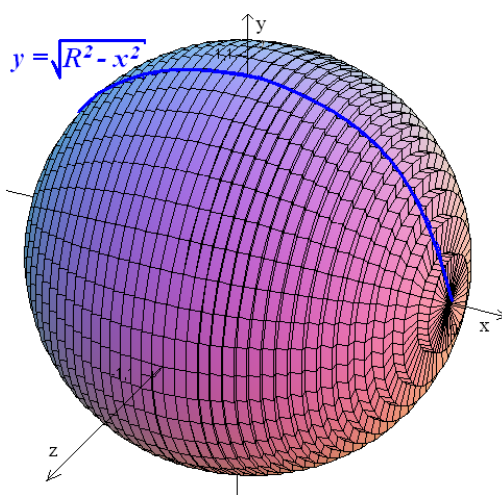
formulalar o'rinli.

Shar va shar segmenti hajmi formulalarini boshqacha usul bilan chiqaramiz.

Aylanma jismning hajmini topishning $V = \pi \int_a^b f^2(x) dx$ formulasini

isbotlaymiz.

Doira tenglamasi $x^2 + y^2 \leq R^2$ (14.20-rasm) formulasining oshkor ko'rinishini yozsak, $y_1 \leq \sqrt{R^2 - x^2}$ – yuqorigi yarimdoira hamda $y_2 \geq -\sqrt{R^2 - x^2}$ – quyi yarimdoira hosil bo'ladi. Shulardan birini, aytaylik yuqorigi yarimdoirani olib, uni Ox o'qi atrofida bir aylantirsak, shar hosil bo'ladi. Yuqori yarimdoirani $x \in [-R, R]$ kesmada integrallasak, shar hajmi kelib chiqadi, ya'ni



14.20-rasm

$$V = \pi \int_{-R}^R (R^2 - x^2) dx = \pi \left(R^2 x - \frac{x^3}{3} \right) \Big|_{-R}^R = \pi \left(R^3 - \frac{R^3}{3} \right) -$$

$$-\pi \left(R^2(-R) - \frac{(-R)^3}{3} \right) = \frac{2\pi R^3}{3} + \frac{2\pi R^3}{3} = \frac{4}{3}\pi R^3.$$

Shar segmenti hajmini aniqlash uchun shar hajmini topishdagi usuldan foydalanamiz.

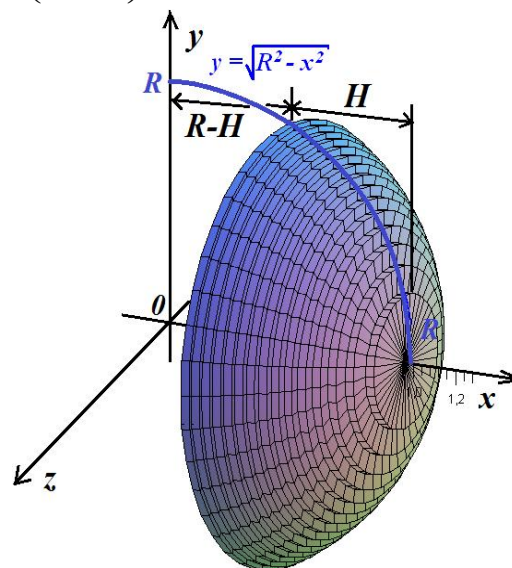
Yuqorigi $y_1 \leq \sqrt{R^2 - x^2}$ yarimdoirani $x \in [R-H, R]$ kesmada integrallasak, shar segmentining hajmi kelib chiqadi:

$$\begin{aligned} V &= \pi \int_{R-H}^R (R^2 - x^2) dx = \pi \left(R^2 x - \frac{x^3}{3} \right) \Big|_{R-H}^R = \pi \left(R^3 - \frac{R^3}{3} \right) - \\ &-\pi \left(R^2(R-H) - \frac{(R-H)^3}{3} \right) = \frac{2\pi R^3}{3} - \pi R^2 H + \pi R^2 H + \frac{\pi R^3}{3} - \\ &-\pi R^2 H + \pi R H^2 - \frac{\pi H^3}{3} = \pi H^2 \left(R - \frac{H}{3} \right). \end{aligned}$$

Shar segmenti bilan umumiy asosga ega bo'lgan va uchi shar markazida bo'lgan konusga shar **segmentiga mos konus** deyiladi.

Shar segmenti va unga mos konus asoslari umumiy doiradan iborat bo'lib, uning radiusi quyidagi formula bilan aniqlanadi:

$$r = \sqrt{2RH - H^2}.$$



14.21-rasm

Isbot. Pifagor teoremasi ko'ra

$$r^2 = R^2 - (R-H)^2 = R^2 - R^2 + 2RH - H^2, \Rightarrow r = \sqrt{2RH - H^2} \text{ bo'ladi.}$$

7-teorema. Shar segmentiga mos konusning hajmi:

$$V_{kon} = \frac{\pi}{3} H (2R^2 - 3RH + H^2)$$

ga teng.

Isbot.

$$\begin{aligned} V_{kon} &= \frac{1}{3} \pi r^2 H_{kon} = \frac{1}{3} \pi (2RH - H^2) (R-H) = \\ &= \frac{\pi}{3} (2R^2 H - 2RH^2 - RH^2 + H^3) = \frac{\pi}{3} H (2R^2 - 3RH + H^2). \end{aligned}$$

Shar segmenti va unga mos konusdan iborat aylanish jismiga **shar sektori** deyiladi.

8-teorema. Shar sektorining hajmi shar segmenti hajmi va konus hajmlarining yig'indisiga tengdir:

$$V_{sekt} = V_{segm} + V_{kon} \quad \text{yoki} \quad V_{sekt} = \frac{2}{3} \pi R^2 H.$$

Isbot. Sektor hajmi segment va konus hajmlari yig'indisiga teng ekanligidan ushbu

$$\begin{aligned} V_{sekt} &= V_{segm} + V_{kon} = \pi H^2 \left(R - \frac{H}{3} \right) + \frac{\pi}{3} H (2R^2 - 3RH + H^2) = \\ &= \frac{\pi}{3} H (3RH - H^2 + 2R^2 - 3RH + H^2) = \frac{2}{3} \pi R^2 H \end{aligned}$$

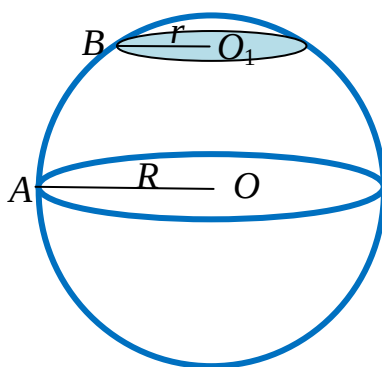
natija kelib chiqadi.

9-teorema. Shar segmentining hajmi shunday silindrning hajmiga tengki, uning asosi radiusi segmentning halandligiga, balandligi esa shar radiusining segment balandligining uchdan biriga kamaytirilganiga tengya'ni

$$V_{sigm} = \pi H^2 \left(R - \frac{H}{3} \right),$$

bunda H – segmentning balandligi, R – sharning radiusi.

1-masala. Shar kesimi radiusi shar radiusidan 3 marta kichik. Agar kesim yuzi $\frac{4}{9} \sqrt[3]{\pi}$ ga teng bo'lsa, sharning hajmini toping.



14.22-rasm

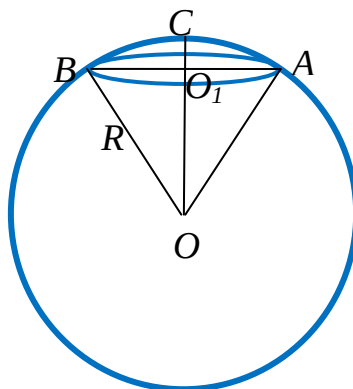
Yechish. Masala shartga ko'ra kesimning yuzi $S = \pi r^2 = \frac{4}{9} \sqrt[3]{\pi}$,

$r^2 = \frac{4}{9 \sqrt[3]{\pi^2}}, r = \frac{2}{3 \sqrt[3]{\pi}}$. Sharning radiusi kesim radiusidan 3 marta kattaligini hisbga

olsak, sharning radiusi $R = 3r = \frac{2}{\sqrt[3]{\pi}}$ bo'ladi. Bundan sharning hajmini $V_{shar} = \frac{4}{3}\pi R^3$

formula yordamida hisoblaymiz: $V_{shar} = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi \frac{8}{\pi} = \frac{32}{3}$.

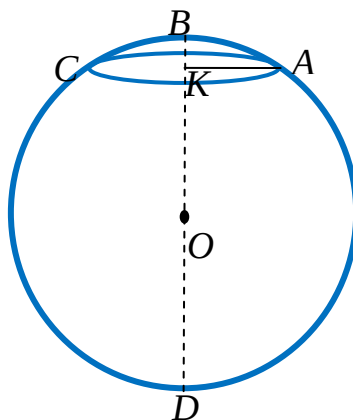
2-masala. Agar sharning radiusi 6 sm va shar sektorining balandligi 2 sm bo'lsa, shar sektorining hajmini hisoblang.



14.23-rasm

Yechish. Masala shartiga ko'ra $R = 6$, $H = 2$ ra teng. Shar sektorining hajmini $V_{sekt} = \frac{2}{3}\pi R^2 H$ formula yordamida hisoblaymiz: $V_{sekt} = \frac{2}{3}\pi R^2 H = \frac{2}{3}\pi \cdot 36 \cdot 2 = 48\pi$.

3-masala. Sharda uning diametriga perpendikular va diametrga 6 sm va 12 sm qismlarga bo'luvchi tekislik otkazilgan. Sharning hosil bo'lgan qismlari hajmlarini hisoblang.



14.24-rasm

Yechish. Shardagi AKC kesim sharni ikkita ABC va ADC shar segmentlariga bo'ladi (14.24-rasm). Bu segmentlarning balandliklari, mos ravishda, 6 sm va 12 sm. U holda sharning radiusi

$$R = \frac{BK + KD}{2} = \frac{6 + 12}{2} = 9 \text{ sm.}$$

Shar segmenti hajmi uchun yuqorida topilgan

$$V_{\text{sigm}} = \pi H^2 \left(R - \frac{H}{3} \right)$$

formuladan foydalanib, shar segmentlari hajmlarini hisoblaymiz:

$$V_{ABC} = \pi H^2 \left(R - \frac{H}{3} \right) = \pi 36 \cdot \left(9 - \frac{6}{3} \right) = 252\pi \text{ sm}^3,$$

$$V_{\text{sigm}} = \pi H^2 \left(R - \frac{H}{3} \right) = \pi 144 \cdot \left(9 - \frac{12}{3} \right) = 720\pi \text{ sm}^3.$$

4-masala. Sharni chegaralovchi sfera tenglamasi

$$x^2 + y^2 + z^2 - 6x + 4y - 6z = 3$$

ma'lum bo'lsa, shar hajmini hisoblang.

Yechish. Ma'lumki, sharning hajmi

$$V_{\text{shar}} = \frac{4}{3} \pi R^3$$

formula bo'yicha hisoblanadi. Berilgan sfera tenglamasida x, y, z lar uchun to'la kvadratlar ajratamiz:

$$(x^2 - 6x + 9) + (y^2 + 4y + 4) + (z^2 - 6z + 9) = 3 + 22$$

yoki

$$(x - 3)^2 + (y + 2)^2 + (z - 3)^2 = 25$$

Bundan sharning radiusi $R = 5$ bo'lishi kelib chiqadi. Demak, sharning hajmi

$$V = \frac{4}{3} \pi R^3 = \frac{500}{3} \pi.$$

Mustaqil ishlash uchun masalalar

14.33. Sharning hajmi $\frac{2048\pi}{3}$ ga teng bo'lsa, diametrini toping. javob: 16

14.34. Shar sirti 43 ga teng. Boshqa bir sharning hajmi bu shar hajmidan 27 marta katta bo'lsa, katta sharning sirti yuzini toping. javob: 387

14.35. Sharning hajmi 135 ga teng. Bu sharning diametridan 3 marta katta diametrli sharning hajmini toping. javob: 3645

14.36. Sharning hajmi 12 ga teng. Bu shardan sirti yuzi 9 marta katta bo'lgan sharning hajmini toping. javob: 324

14.37. Radiusi 12 ga teng shar hajmi radiusi bu shar radiusining 0,25 qismiga teng bo'lgan shar hajmidan necha marta katta? javob: 16

14.38. Sharining radiusi oxiridan radius bilan 60° burchak tashkil qiluvchi tekislik o'tkazilgan. Agar hosil bo'lgan kesim yuzi $\sqrt[3]{36\pi}$ ga teng bo'lsa, sharining hajmini toping. javob: 64

14.39. Shar kesimi radiusi shar radiusidan 2 marta kichik. Agar kesim yuzi $\frac{9}{4}\sqrt[3]{\pi}$ ga teng bo'lsa, sharining hajmini toping. javob: 36

14.40. Sharining hajmi V . Shar sektori o'q kesimining markaziy burchagi α bo'lsa, sektorning hajmi hisoblansin. javob: $\frac{V}{2\pi}$

14.41. Agar sharining radiusi 8 sm, shar sektorining o'q kesimidagi burchak 120° ga teng bo'lsa, sektorning yuzini hisoblang.

14.42. Sharining radiusi R , shar sektori o'q kesimining yoyi 120° bo'lsa, shar sektori to'la sirtning yuzini hisoblang. javob: $\frac{\pi R^2}{2}(2 + \sqrt{3})$

14.43. Agar sharining radiusi 6 sm va shar sektorining balandligi 2 sm bo'lsa, shar sektorning hajmini hisoblang.

14.44. Sharining radiusi 37 sm. Sharining markazidan 23 sm uzoqlikda kesim o'tkazilgan. Shu kesimning yuzi hisoblansin. javob: 840π

14.45. A va C nuqtalar sharining OK radiusini uchta teng qismga ajratadi. A va C nuqtalardan radiusga perpendikular bo'lgan kesimlar o'tkazilgan. Shu kesimlar yuzlarining nisbati topilsin.

14.46. M nuqtadan sharga MK urinma o'tkazilgan va $MK=12$. Agar sharining radiusi 5 bo'lsa, M nuqtadan shargacha bo'lgan masofa topilsin. javob: 13

14.47. Sharda ikkita o'zaro perpendikular va yuzlari 185 l sm^2 va 320 l sm^2 bo'lgan kesimlar o'tkazilgan. Bu kesimlar o'zaro kesishadigan vatarning uzunligi 16 sm bo'lsa, shar radiusining uzunligi topilsin.

14.48. Radiusi 18 sm ga teng bo'lgan sharga ikkita o'zaro perpendikular kesim o'tkazilgan. Agar kesimlar radiuslarining nisbati 2:3 kabi hamda kesimlar o'zaro kesishadigan vatarning uzunligi 2 sm bo'lsa, kesimlar radiuslarining uzunliklari topilsin.

14.49. Ikkita shar berilgan bo'lib, ularning radiuslari 41 sm va 5 dm, markazlari orasidagi masofa 21 sm bo'lsa, sharlar kesishish chizig'ining uzunligi topilsin.

14.50. Ikkita shar berilgan bo'lib, ularning radiuslari 25 sm va 3 dm, sharlar kesishish chizig'ining uzunligi 48π sm bo'lsa, sharlarning markazlari orasidagi masofa topilsin. javob: 25

14.51. Markazi O nuqtada bo'lgan shar a tekislikka B nuqtada urinadi va A nuqta shu tekislikda yotadi hamda $OA=26$, $AB=24$ sm. Shar sirtining yuzi hisoblansin.

14.52. Sferik kamar asoslarining radiuslari 18 va 20 m, sharning radiusi esa 24 m. Kamarning sferik qismining yuzi hisoblansin.

14.53. Radiusi 13 sm bo'lgan shar markazining har xil tomonlarida o'zaro parallel va teng qismlar o'tkazilgan. Kesimlarning har birining radiusi 5 sm bo'lsa, parallel tekisliklar orasidagi shar qismining hajmi topilsin.

14.54. Sferaning radiusi 112 sm. Sferaning A nuqtasidan urinma tekislik o'tkazilgan va shu tekislikda B nuqta olingan. Agar $AB=15$ sm bo'lsa, B nuqtadan sferagacha bo'lgan masofa topilsin.

14.55. Sharda ikkita parallel kesim o'tkazilgan. Agar kesimlarning radiuslari 9 va 12 sm, kesimlar orasidagi masofa 3 sm bo'lsa, sferaning yuzi hisoblansin.

14.56. Tenglamasi $x^2+y^2+z^2-4x+10z-35=0$ bo'lgan sferaning radiusi uzunligini aniqlang.

14.57. Agar sfera AB diametrining $A(2;-3;5)$, $B(4;1;-3)$ uchlari berilgan bo'lsa, sferaning tenglamasini toping.

14.58. Agar sferaning markazi $C(3;-2;1)$ va uning tegishli $A(2;-1;-3)$ nuqta ma'lum bo'lsa, sfera tenglamasini yozing.

14.59. Sferasining tenglamasi $x^2+y^2+z^2-4x-6y+2z+5=0$ ko'rinishida bo'lgan sharning hajmi hisoblansin.

14.60. Radiusi $R=4$ bo'lgan sharning markazi $A(2;-4;7)$ nuqtada bo'lsa, sferaning tenglamasini toping.

14.61. Markazi $A(-2;2;0)$ nuqtada, radiusi $R=2$ bo'lgan sferaning tenglamasi topilsin.

14.62. Markazi $A(-2;2;0)$ nuqtada bo'lib, $R(5;0;-1)$ nuqtadan o'tuvchi sferaning tenglamasi topilsin.

14.63. Tenglamasi $x^2+y^2+z^2+6x-2y-2z-5=0$ ko'rinishida bo'lgan sferaning markazi va radiusi topilsin.

14.64. Sferaning tenglamasi $(x+1)^2+(y-2)^2+(z-1)^2=25$ ko'rinishda berilgan bo'lsa, uning markazi va radiusini toping.

14.65. Sferaning tenglamasi $x^2+y^2+z^2-4x+6y-6z-3=0$ ko'rinishda berilgan bo'lsa, yuzini hisoblang.

14.66. Markazi $S(5;-2;7)$ nuqtada, radiusi $R=2$ bo'lgan sfera tenglamasini yozing.

14.67. Shar qatlami asoslarining radiuslari 7 sm va 20 sm, sharning radiusi esa 25 sm. Agar kesimlar shar markazidan bir tomonda o'tkazilgan bo'lsa, shar qatlami sirtning yuzini hisoblang.

14.68. Tekislik sharni hajmlari 252 l sm^3 va 720 l sm^3 bo'lgan qismlarga ajratadi. Shu qismlarning sferik sirtlarini hisoblang.

14.69. K nuqtadan sferagacha bo'lgan eng qisqa masofa 6 sm, eng uzoq masofa esa 16 sm. Berilgan sfera bilan chegaralangan shar katta doirasining yuzini hisoblang.

14.70. R nuqtadan shargacha bo'lgan eng qisqa masofa 12 sm, eng uzoq masofa esa 30 sm. Shar sirtining yuzini hisoblang. javob: 324π

14.71. Sharni ikkita parallel tekislik bilan kesganda hosil bo'lgan kesimlarning yuzlari 81π va 36π . Tekisliklar shar markazidan bir tomonda yotadi va ular orasidagi masofa 5 sm ga teng. Shar sirtining yuzini va tekisliklar orasidagi shar kamarining yuzini hisoblang.

14.72. Sharlarning radiuslari 17 sm va 10 sm bo'lib, ular uzunligi 16π sm bo'lgan chiziq bo'ylab kesishadi. Sharlarning markazlari orasidagi masofani toping.

14.73. Silindr o'q kesimining diagonalini 12sm va asos tekisligi bilan 60° li burchak tashkil qiladi. Silindr yon sirtining yuzini va hajmini hisoblang. javob: $36\sqrt{3}\pi\text{ sm}^2$, $54\sqrt{3}\pi\text{ sm}^3$.

14.74. Silindr asos aylanasi uzunligi l , balandligi asosining radiusiga teng. Silindrning hajmini hisoblang. javob: $\frac{l^3}{8\pi^2}$.

14.75. Silindr asos doirasining yuzi $49\pi\text{ sm}^2$, balandligi asosining diametriga teng. Silindrning hajmini hisoblang. javob: $686\pi\text{ sm}^3$.

14.76. Bir to'p shag'al konus shakilda bo'lib uning asosining radiusi 2 m, yasovchisi esa 2,5 m. Bu to'p shag'alning hajmini toping.

14.77. Konus yasovchisining uzunligi L , asos aylanasi uzunligi C . Konusning hajmini toping.

14.78. Konus yon sirtining yuzi 32π ga teng, uning yasovchisi asos tekisligiga 60° li burchak ostida og'gan. Konusning hajmini hisoblang. Javob: $\frac{128}{3}\pi$.

14.79. Kesik konus yuqori va pastki asoslarining radiuslari 18sm va 10sm, o'q kesimining diagonalari o'zaro perpendikular. Kesik konusning hajmini hisoblang. javob: $\frac{2114}{3}\pi\text{ sm}^2$.

14.80. Kesik konusning yasovchisi 10sm va asos tekisligiga 60° li burchak ostida og'gan, o'q kesimining diagonalini bu burchakni teng ikkiga bo'ladi. Kesik konusning hajmini hisoblang. javob: $\frac{875\sqrt{3}}{3}\pi\text{ sm}^2$.

14.81. Sharning markazidan 6sm masofada joylashgan kesimning yuz $64\pi\text{ sm}^2$ bo'lsa, sharning hajmini toping. javob: $\frac{4000\pi}{3}\text{ sm}^3 = \frac{4}{3}\pi\text{ dm}^3$.

14.82. Agar sharning radiusi 6sm va shar sektorining balandligi 2sm bo'lsa, shar sektorining hajmini hisoblang. javob : $48\pi\text{ sm}^3$.

14.83. Tekislik sharni hajmlari $252\pi\text{ sm}^3$ va $720\pi\text{ sm}^3$ bo'lgan qismlarga ajratadi. Shu qismlarning sferik sirtlarini hisoblang. javob: $1810\pi\text{ sm}^2$ va $216\pi\text{ sm}^2$.

