2-BOB
UCHBURCHAKLAR GEOMETRIYASI

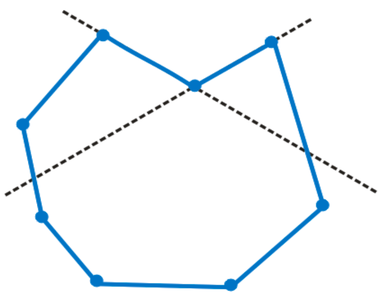
1-§. Uchburchak mavjudligi shartlari. Uchburchak turlari va elementlari

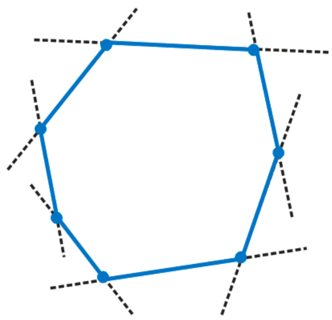
Tekislikdagi ko‘pburchaklarni o‘rganish bilan bog‘liq masalalar uchburchaklarning xossalarini o‘rganish bilan bog‘liq. Uchburchaklarning xossalarini o‘rganishdan oldin turli xildagi ko‘pburchaklarning shakllari bilan tanishamiz.

1-ta’rif. Tekislikda yotgan har qanday shaklga **yassi yoki tekis shakl** deyiladi.

2-ta’rif. Uchtasi bir to‘g‘ri chiziqda yotmagan tekislikdagi nuqtalarni ketma-ket tutashtirishdan hosil bo‘lgan yassi shaklga ***siniq chiziq*** deyiladi. Agar siniq chiziqning boshi va ohiri tutashtirilgan bo’lsa ***yopiq siniq chiziq*** yoki ***ko‘pburchak*** deyiladi.

3-ta’rif. Ko‘pburchakning tomoni orqali o‘tuvchi to‘g‘ri chiziq ko‘pburchakning biror tomonini kesib o‘tsa, u holda bunday ko‘pburchakga ***botiq ko‘pburchak*** (2.1-*a* rasm) va aksincha bo‘lsa, ***qavariq ko‘pburchak*** deyiladi (2.1-*b* rasm).


a)


b)

2.1-rasm

4-ta’rif. Ko‘pburchakning bir tomonda yotmagan ikki uchini tutashtiruvchi kesma ko‘pburchakning ***diagonali*** deyiladi.

1-teorema. Qavariq ko‘pburchakning ichki burchaklari yig‘indisi quyidagi formula orqali aniqlanadi (2.2- rasm):

$$\alpha+\beta+\gamma+\delta+\varphi+\omega+\rho+...=(n-2)\pi.$$

a) *b)*

2.2-rasm

Isbot. Ko‘pburchakning ixtiyoriy bitta uchidan boshqa uchlariga diagonallar o‘tkazib tutashtiramiz. Agar ko‘pburchak tomonlari soni n ga teng bo‘lsa, u holda bitta uchdan o‘tkazilgan diagonallar soni $n-3$ ta bo‘ladi. Bu diagonallar esa berilgan n burchakli qavariq ko‘pburchakni $n-2$ ta uchburchaklarga ajratib tashlaydi. Ko‘pburchakning ichki burchaklari yig‘indisi $n-2$ ta uchburchakning ichki burchaklar yig‘indisiga teng, ya’ni $\sum\alpha_i=(n-2)\pi$ bo‘ladi.

Qavariq ko‘pburchakning ichki burchagiga qo‘shni bo‘lgan burchakka shu uchdagi tashqi burchak deyiladi. Ko‘pburchakning har bir uchida ikkitadan tashqi burchak bo‘ladi.

2-teorema. Qavariq ko‘pburchakning har bir uchidan bittadan olingan tashqi burchaklari yig‘indisi har doim 2π ga teng bo‘ladi (2.2-*a* rasm):

$$\alpha'+\beta'+\gamma'+\delta'+\varphi'+\omega'+\rho'+...=2\pi.$$

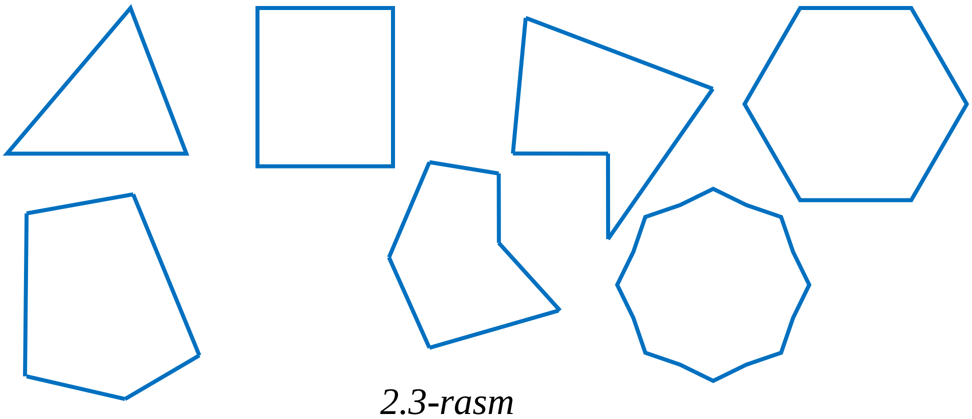
Isbot. Ko‘pburchakning tomonlari bo‘ylab aylanib chiqishda har bir uchida tashqi burchakka teng burchakka buriladi. Qavariq ko‘pburchak necha burchakli bo‘lmasin barcha uchidagi burilish burchaklari qo‘shilib bir aylanani beradi. Bir aylana esa 360^0 yoki 2π burchak deganidir.

n burchakli qavariq ko‘pburchakning diagonallar soni k quyidagicha bo‘ladi:

$$k=\frac{n(n-3)}{2}.$$

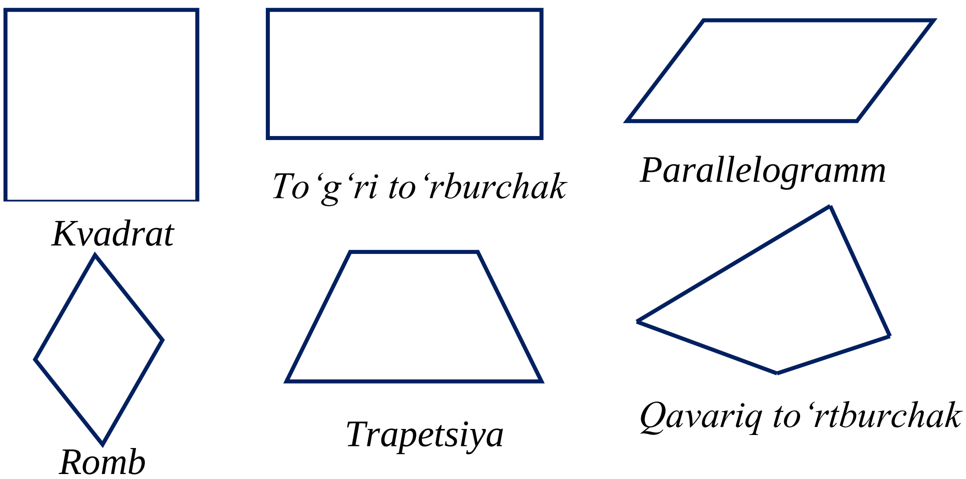
Isbot. Bunda n ta nuqtadan ixtiyoriy bittasini olsak, ta’rifga ko‘ra tanlangan nuqtaning o‘zi, shu nuqta bilan bir tomonda yotgan ikki qo‘shnisiga diagonal o‘tkazib bo‘lmaydi. Demak, har bir nuqtadan $n-3$ ta diagonal o‘tkazish mumkin. Ko‘pburchakning n ta uchidan $n-3$ tadan diagonal o‘kazilsa, jami $n(n-3)$ ta bo‘ladi. Lekin ko‘pburchakning A va C uchlarini tutashtiruvchi diagonal ham A uchidan, ham C uchindan chiquvchi diagonallar sonida hisoblangani uchun ikkiga bo‘lib qo‘yamiz $\frac{(n-3)n}{2}$.

Ko‘pburchaklarga quyidagilar misol bo‘ladi:



2.3-rasm

To‘rtburchaklar ham ko‘pburchakka misol bo‘ladi:

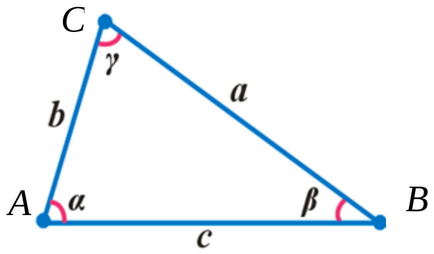


2.4-rasm

5-ta'rif. Bir to'g'ri chiziqda yotmaydigan uchta nuqtani tutashtiruvchi kesmalardan iborat shaklga ***uchburchak*** deyiladi. Uchta nuqtani ***uchburchak uchlari***, bu nuqtalarni tutashtiruvchi kesmalarni esa ***uchburchak tomonlari*** deyiladi.

Uchburchak uchlarini lotin alifbosining katta A, B, C harflar bilan, bu uchlardagi burchaklar (ichki)ni mos ravishda grek alifbosidagi α, β, γ harflar bilan, bu burchaklar qarshisidagi tomonlar esa lotin alifbosining kichik a, b, c harflar bilan belgilanadi (2.5-rasm). Tomonlar va burchaklar quyidagi ko'rinishda bo'ladi:

$$\boxed{\begin{cases}\angle A = \alpha \\ \angle B = \beta, \\ \angle C = \gamma\end{cases} \quad \begin{cases} BC = a \\ AC = b. \\ AB = c\end{cases}}$$



2.5-rasm

Uchburchakning katta burchagi qarshisida katta tomon yotadi, kichik burchagi qarshisida kichik burchak yotadi, teng burchaklari qarshisida esa teng tomonlar yotadi.

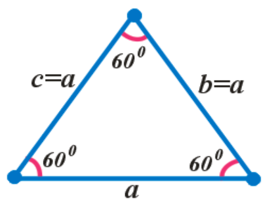
6-ta'rif. Uchburchakning barcha tomonlar uzunliklari yig'indisi uning ***perimetri*** deyiladi va p harfi bilan belgilanadi:

$$\boxed{p = a + b + c.}$$

Uchburchakning ixtiyoriy ikki tomoni yig'indisi har doim uchinchi tomondan katta bo'ladi:

$$\boxed{\begin{matrix} a + b > c, \\ b + c > a, \\ a + c > b. \end{matrix}}$$

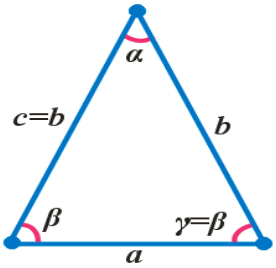
7-ta'rif. Barcha tomonlari teng bo'lgan uchburchakka ***teng tomonli (muntazam) uchburchak*** deyiladi. Teng tomonli uchburchakning uchala burchaklari o'zaro teng bo'lib, ular 60^0 ni tashkil etadi (2.6-rasm).



2.6-rasm

$$\boxed{\begin{aligned} &a = b = c, \\ &\alpha = \beta = \gamma = 60^0. \end{aligned}}$$

8-ta'rif. Ikkita tomoni o'zaro teng bo'lgan uchburchakka ***teng yonli uchburchak*** deyiladi. O'zaro teng tomonlarni yon tomonlar, uchinchi tomonni esa uchburchak asosi deyiladi (2.7-rasm).



2.7-rasm

$$\boxed{\begin{aligned} &b = c, \\ &\beta = \gamma,\ \ \alpha = 180^0 - 2\beta, \\ &p = a + 2b. \end{aligned}}$$

Barcha ichki burchaklari o'tkir bo'lgan uchburchakni ***o'tkir burchakli*** uchburchak deyiladi (2.8-*a* rasm).

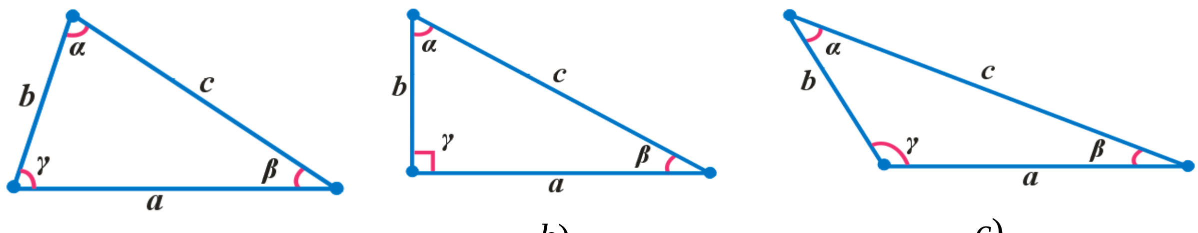
Agar uchburchakning birorta burchagi 90^0 ga teng bo'lsa, bunday uchburchakka ***to'g'ri burchakli*** uchburchak deyiladi. Bunda to'g'ri burchak qarshisidagi tomonni gipotenuza, qolgan tomonlarni esa katetlar deyiladi (2.8-*b* rasm).

Agar uchburchakning birorta burchagi 90^0 dan katta bo'lsa, bunday uchburchakni ***o'tmas burchakli*** uchburchak deyiladi (2.8-*c* rasm).

$$\boxed{\alpha, \beta, \gamma < 90^0.}$$

$$\boxed{\begin{cases} \gamma = 90^0, \\ \alpha + \beta = 90^0. \end{cases}}$$

$$\boxed{\begin{cases} \gamma > 90^0, \\ \alpha + \beta < 90^0. \end{cases}}$$



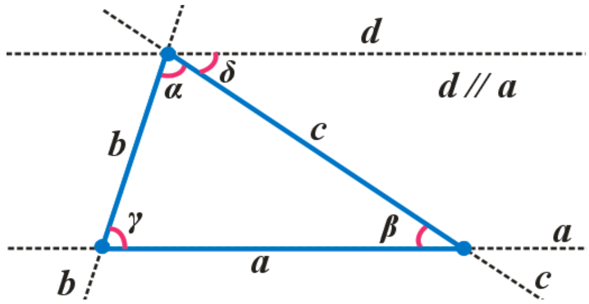
a) *b*) *c*)

2.8-rasm

3-teorema. Uchburchakning ichki burchaklari yig'indisi har doim 180^0ga teng bo'ladi:

$$\boxed{\alpha + \beta + \gamma = 180^0.}$$

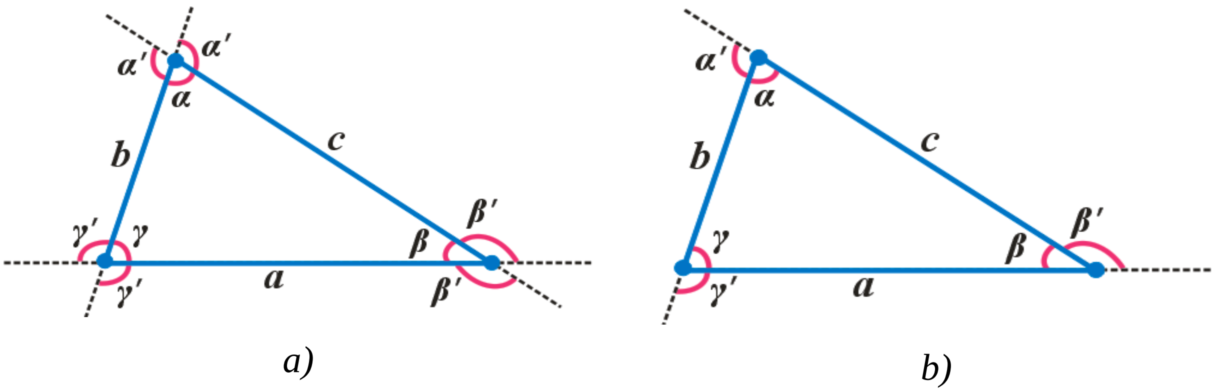
Isbot. Tomonlar orqali a, b, c to‘g‘ri chiziqlar va A uch orqali $d \| a$ to‘g‘ri chiziq o‘tkazamiz. δ va β burchaklar o‘zaro ichki almashinuvchi burchaklar bo‘lgani uchun ular o‘zaro teng, ya’ni $\delta = \beta$ (2.9-rasm). $\alpha + \delta$ va γ burchaklar ichki bir yoqli burchaklar bo‘lgani uchun ularning yig‘indisi 180^0 ga teng. Bundan



2.9-rasm

$$\alpha + \beta + \gamma = 180^0 .$$

Uchburchakning ixtiyoriy uchidagi ichki burchagiga qo‘shni bo‘lgan burchakka shu uchidagi ***tashqi burchak*** deyiladi. Uchburchakning har bir uchida ikkitadan tashqi burchak mavjud (2.10-*a* rasm).



2.10-rasm

Uchburchakning ixtiyoriy uchidagi tashqi burchagi o‘ziga qo‘shni bo‘lmagan ikkita ichki burchagi yig‘indisiga teng bo‘ladi (2.10-*b* rasm).

$$\boxed{\alpha' = \beta + \gamma, \quad \beta' = \alpha + \gamma, \quad \gamma' = \alpha + \beta.}$$

4-teorema. Uchburchakning har bir uchidan bittadan olingan tashqi burchaklari yig‘indisi 360^0ga tengdir.

$$\boxed{\alpha' + \beta' + \gamma' = 360^0.}$$

Isbot. Uchburchakning har bir uchidagi ichki va tashqi burchagi o‘zaro qo‘shni burchaklar bo‘lgani uchun ular yig‘indisi 180^0ga teng. Shuning uchun

$$\begin{cases} \alpha+\alpha'=180^0 \\ \beta+\beta'=180^0, \\ \gamma+\gamma'=180^0 \end{cases} \Rightarrow (\alpha+\beta+\gamma)+(\alpha'+\beta'+\gamma')=3\cdot180^0.$$

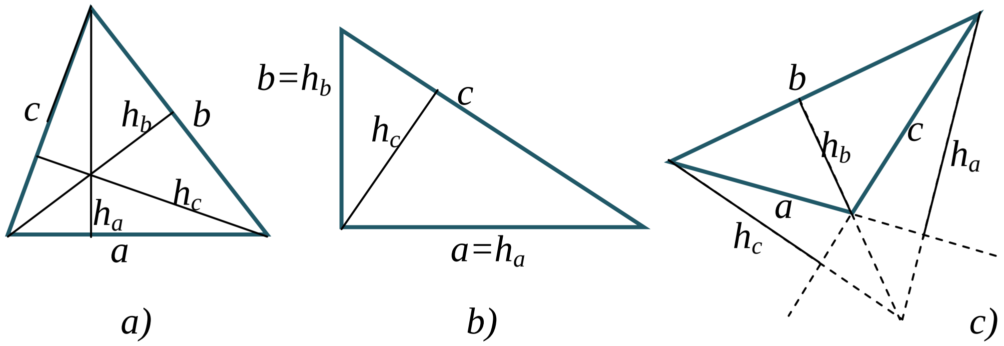
Bundan $\alpha'+\beta'+\gamma'=3\cdot180^0-(\alpha+\beta+\gamma)=3\cdot180^0-180^0=2\cdot180^0=360^0$ kelib chiqadi.

Uchburchak uchi bilan shu uch qarshisidagi tomonning ixtiyoriy nuqtasini tutashtiruvchi kesma uchburchak ***chevianasi*** deyiladi. Uchburchak balandligi, medianasi va bissektrisasi uchburchakning chevianasi bo‘ladi.

9-ta’rif. Uchburchakning ixtiyoriy uchidan qarshisidagi tomon yotgan to‘g‘ri chiziqqa tushirilgan perpendikular kesma shu tomonga tushirilgan ***balandlik*** deyiladi.

Tomonlari a,b,c bo‘lgan uchburchak tomonlariga tushirilgan balandliklar mos ravishda h_a,h_b,h_c lar bilan belgilanadi.

Uchburchak qanday bo‘lmasin, uchta tomonga tushirilgan balandliklar har doim bitta nuqtada kesishadi. Bu balandliklar kesishish nuqtasi o‘tkir burchakli uchburchakda uchburchak ichida, to‘g‘ri burchakli uchburchakda to‘g‘ri burchakning o‘zida va o‘tmas burchakli uchburchakda esa uchburchak tashqarisida bo‘ladi (2.11-rasm).



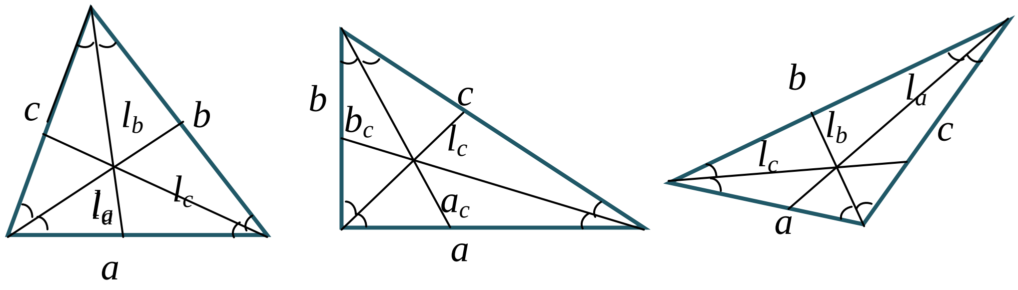
2.11-rasm

Umumiy holda uchta tomonga tushirilgan balandliklar uzunliklari uch xil bo‘ladi. Eng katta tomonga tushirilgan balandlik eng qisqa va eng qisqa tomonga tushirilgan balandlik eng uzun bo‘ladi. Agar tomonlar uzunliklari $a<b<c$ bo‘lsa, u holda shu tomonga tushirilgan balandliklar $h_a>h_b>h_c$ bo‘ladi.

10-ta’rif. Uchburchakning ixtiyoriy uchidan chiqib burchakni teng ikkiga bo‘luvchi va qarshisidagi tomon bilan tutashuvchi kesmaga shu tomonga tushirilgan ***bissektrisa*** deyiladi.

Tomonlari a,b,c bo‘lgan uchburchak tomonlariga tushirilgan bissektrisalar mos ravishda ℓ_a, ℓ_b, ℓ_c lar bilan belgilanadi.

Uchburchak qanday bo‘lmasin, uchta tomonga tushirilgan bissektrisalar har doim bitta nuqtada kesishadi va bu kesishish nuqtasi har doim uchburchak ichida yotadi (2.12-rasm).



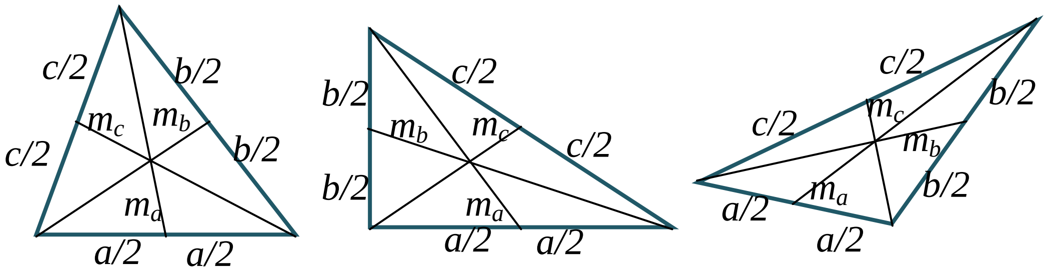
a) *b)* *c)*

2.12-rasm

11-ta’rif. Uchburchakning ixtiyoriy uchi bilan qarshisidagi tomonning o‘rtasiga tutashuvchi kesmaga shu tomonga tushirilgan ***mediana*** deyiladi.

Tomonlari a,b,c bo‘lgan uchburchak tomonlariga tushirilgan medianalar mos ravishda m_a, m_b, m_c lar bilan belgilanadi.

Uchburchak qanday bo‘lmasin, uchta tomonga tushirilgan medianalar har doim bitta nuqtada kesishadi va bu kesishish nuqtasi har doim uchburchak ichida yotadi (2.13-rasm).

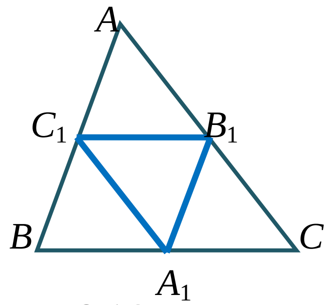


a) *b)* *c)*

2.13-rasm

12-ta'rif. Uchburchakning ixtiyoriy ikki tomonining o'rtalarini tutashtiruvchi kesma uchburchakning ***o'rta chizig'i*** deyiladi (2.14-rasm). Uchburchaknin o'rta chizig'i o'ziga parallel bo'lgan tomonning yarmiga teng bo'ladi.

$$AB = 2A_1B_1,\ AC = 2A_1C_1,\ \ BC = 2B_1C_1.$$



2.14-rasm

Mustaqil ishlash uchun masalalar

2.1. Uchburchakning burchaklari berilgan: 1) 65° va 35°; 2) 75° va 82°. Noma'lum burchakni toping. javob: 80º, 23º

2.2. Uchburchakning burchaklari berilgan: 1) 70° va 20°; 2) 45° va 60°. Noma'lum burchakni toping. javob: 90º, 75º

2.3. Teng yonli *ABC* uchburchakning *AC* asosidagi tashqi burchaklari bissektrisalari va uchburchak asosi bilan 126° hosil qilsa, *ABC* burchakni qiymatini toping. javob: 36°

2.4. Uchburchakning bir burchagi 30° ga teng va ikkinchi burchagi uchinchi burchagidan 2 marta katta bo'lsa, uchburchakning eng kichik burchagini toping. javob: 30°

2.5. Uchburchakning ikkita teng ichki burchaklari yig'indisi uchinchi burchagidan 10° ga katta bo'lsa, uchburchakning katta burchagini toping. javob: 85°

2.6. Uchburchakning ikkita teng ichki burchaklari yig'indisi uchinchi burchagidan 1,5 marta katta bo'lsa, uchburchakning katta burchagini toping. javob: 72°

2.7. To‘g‘ri burchakli uchburchakda bitta o‘tkir burchagi ikkinchi bir o‘tkir burchagidan 2 marta katta bo‘lsa, katta o‘tkir burchakni toping. javob: 60°

2.8. Teng yonli uchburchakda ikkita teng bo‘lmagan burchaklar farqi 90° ga teng bo‘lsa, katta burchakni toping. javob:120°

2.9. Uchburchakning ichki burchaklari 1:2:3 nisbatda bo‘lsa, kichik burchagini toping. javob: 30°

2.10. Uchburchakning bitta burchagi 60° ga teng va qolgan ikkitasi esa 2: 3 nisbatda bo‘lsa, uchburchakning katta burchagini toping. javob: 72°

2.11. Uchburchakning bitta tashqi burchagi 150° ga teng va qolgan ikkita ichki burchaklari o‘zaro teng bo‘lsa, uchburchakning kichik burchagini toping. javob: 30°

2.12. Uchburchakning ichki burchaklari 2:3:5 nisbatda bo‘lsa, kichik burchagiga qo‘shni tashqi burchagini toping. javob: 144°

2.13. Uchburchakning bitta ichki burchagi 50° ga teng va qolgan ikkitasining farqi 10° ga teng bo‘lsa, uchburchakning katta burchagiga qo‘shni tashqi burchagini toping. javob: 110°

2.14. Uchburchakning ikkita teng ichki burchaklari yig‘indisi uchinchi burchagidan 20° ga katta bo‘lsa, uchburchakning katta burchagini toping. javob: 80°

2.15. Teng yonli uchburchakning uchidagi burchagi 96° ga teng. Asosidagi burchaklarning bissektrisalari kesishishidan hosil bo‘lgan o‘tkir burchakni toping. javob: 42º

2.16. Uchburchakning burchaklari 1) 3, 4, 8; 2) 4, 5, 11 sonlariga proporsional. Uchburchakning burchaklarini toping. javob: 1) 36º,48º,96º; 2) 36º, 45º, 99º

2.17. Teng yonli uchburchakning asosidagi burchagi berilgan:1) 52°; 2) 18°. Teng yonli uchburchakning yon tomonlari orasidagi burchakni toping. javob: 76º, 144º

2.18. Teng yonli uchburchakning yon tomonlari orasidagi burchak berilgan: 1) 34°; 2) 72°. Teng yonli uchburchakning asosidagi burchakni toping. javob: 73º, 54º

2.19. Uchburchakning ikkita tashqi burchaklari 96° va 145°. Uchinchi tashqi burchakni toping. javob: 119º

2.20. Uchburchak ichki burchaklaridan biri 40° ga , tashqi burchaklaridan biri esa 60° ga teng. Uchburchakning qolgan burchaklarini toping. javob: 20º, 40º, 120º

2.21. Teng yonli uchburchakning yon tomoni 23 ga teng. Agar bu uchburchakning perimetri 71 boʻlsa, asosi uzunligini toping. javob: 25

2.22. Uchburchakning perimetri 156 boʻlsa, uchlari bu uchburchakning tomonlari oʻrtasida boʻlgan uchburchakning perimetrini toping. javob: 78

2.23. Toʻgʻri burchakli uchburchakning gipotenuzasi oʻrtasidan katetlarigacha boʻlgan masofalar 26 va 33 boʻlsa, bu uchburchakning katetlari uzunligining yigʻindisini toping. javob: 118

2.24. Teng yonli uchburchakning asosi uzunligi 17 ga va perimetri 93 ga teng boʻlsa, bu uchburchakning yon tomoni uzunligini toping. javob: 38

2.25. Teng yonli uchburchakning yon tomoni uzunligi 16 ga va perimetri 57 boʻlsa, asosiga parallel oʻrta chizigʻini toping. javob: 12,5

2.26. Teng tomonli uchburchakning tomoni uzunligi 10 ga teng. Bu uchburchakning balandliklari asoslarini tutashtirishdan hosil boʻlgan uchburchakning perimetrini toping. javob: 15

2.27. Teng yonli ABC ($AB=BC$) uchburchakning perimetri 70 ga teng, BM balandlik. Agar AMB uchburchakning perimetri 50 ga teng boʻlsa, BM balandlik uzunligini toping. javob: 15

2.28. Teng yonli uchburchakning perimetri 7 ga teng va yon tomonlari uzunliklari yigʻindisi asosi uzunligidan 2,5 marta katta boʻlsa, yon tomoni uzunligini toping. javob: 2,5

2.29. Teng yonli uchburchakda yon tomoni uzunligining asosi uzunligiga nisbati 3:4 va perimetri 20 ga teng bo‘lsa, bu uchburchakning asosi uzunligini toping. javob:8

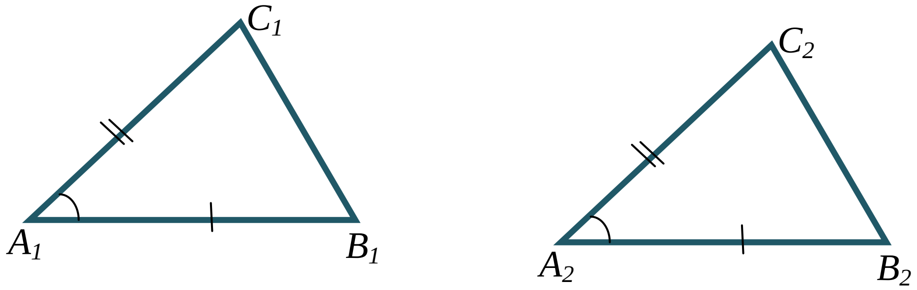
2-§. Uchburchakning tengligi va uning alomatlari. To‘g‘ri burchakli uchburchakda burchak sinusi, kosinusi, tangensi va kotangensi. Pifagor teoremasi

Agar tekislikda ikki kesma bir xil uzunlikka ega bo’lsa, ular teng kesmalar deyiladi. Agar ikki burchakning burchak kattaliklari teng bo’lsa, bu burchaklar teng deyiladi.

1-ra’rif. Agar uchburchaklarning mos burchaklari va mos tomonlari teng bo’lsa, bunday ***uchburchaklar teng*** deyiladi, ya’ni ABC va $A_1B_1C_1$ uchburchaklarda $\angle A = \angle A_1,\ \angle B = \angle B_1,\ \angle C = \angle C_1,$ $AB = A_1B_1,\ AC = A_1C_1,\ BC = B_1C_1$ bo’lsa, bu uchburchaklar teng deyiladi.

Uchburchaklar tengligining uchta alomati bor: 1) ikki tomon va ular orasidagi burchakka ko‘ra, 2) bir tomon va unga yopishgan ikkita burchakka ko‘ra, 3) uchta tomonga ko‘ra. Bularning har biriga alohida-alohida to‘xtalib o‘tamiz.

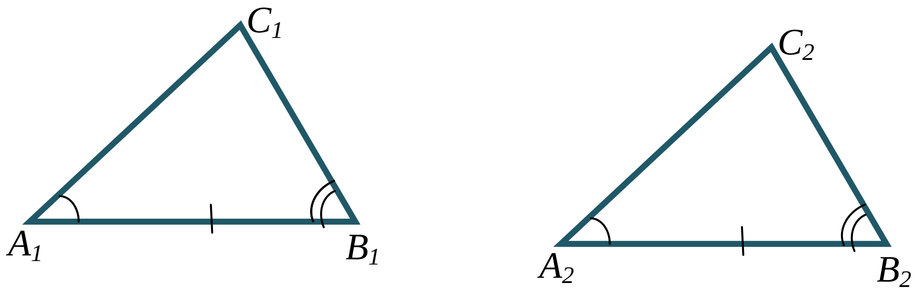
1-alomat. Agar bir uchburchakning ikki tomoni va ular orasidagi burchagi mos holda ikkinchi bir uchburchakning ikki tomoni va ular orasidagi burchagiga teng bo‘lsa, u holda bu uchburchaklar tengdir (2.14-rasm).



2.14-rasm

Agar $\begin{cases} A_1B_1 = A_2B_2 \\ A_1C_1 = A_2C_2 \\ \angle A_1 = \angle A_2 \end{cases}$ bo‘lsa, u holda $\Delta A_1B_1C_1 = \Delta A_2B_2C_2$ bo‘ladi.

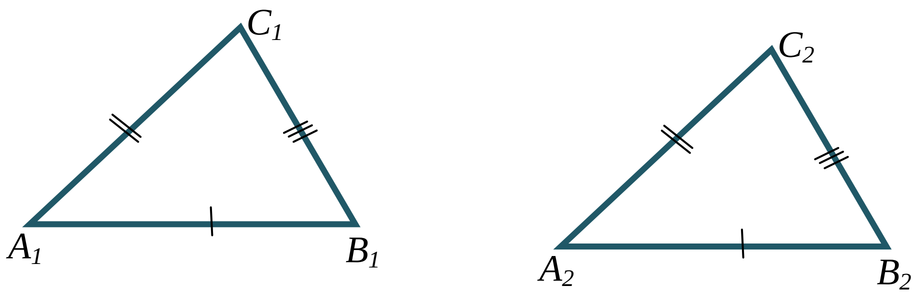
2-alomat. Agar bir uchburchakning bir tomoni va unga yopishgan burchaklari boshqa bir uchburchakning mos tomoni va unga yopishgan burchaklariga teng bo‘lsa, u holda bu uchburchaklar tengdir (2.15-rasm).



2.15-rasm

Agar $\begin{cases} A_1B_1 = A_2B_2 \\ \angle A_1 = \angle A_2 \\ \angle B_1 = \angle B_2 \end{cases}$ bo‘lsa, u holda $\Delta A_1B_1C_1 = \Delta A_2B_2C_2$ bo‘ladi.

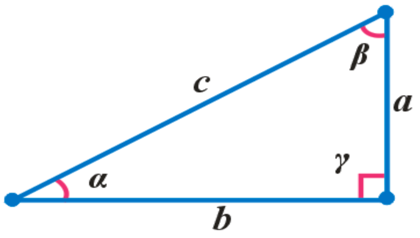
3-alomat. Agar bir uchburchakning uchta tomoni mos holda ikkinchi bir uchburchakning uchta tomoniga teng bo‘lsa, u holda bu uchburchaklar tengdir (2.16-rasm).



2.16-rasm

Agar $\begin{cases} A_1B_1 = A_2B_2 \\ A_1C_1 = A_2C_2 \\ B_1C_1 = B_2C_2 \end{cases}$ bo‘lsa, u holda $\Delta A_1B_1C_1 = \Delta A_2B_2C_2$ bo‘ladi.

Katetlari a va b, gipotenuzasi c bo‘lgan to‘g‘ri burchakli uchburchak berilgan bo‘lsin. Bu to‘g‘ri burchakli uchburchakning o‘tkir burchaklirini α va β deb belgilaymiz (2.18-rasm).



2.18-rasm

2-ta’rif. α burchak qarshisidagi katetning gipotenuzaga nisbatiga ***α burchakning sinusi,*** α burchakka yopishgan katetning gipotenuzaga nisbatiga ***α burchakning kosinusi,*** α burchak qarshisidagi katetning yopishgan katetga nisbatiga ***α burchakning tangensi,*** α burchakka yopishgan katetning qarshisidagi katetga nisbatiga ***α burchakning kotangensi*** deyiladi va quyidagicha belgilanadi:

$$\boxed{\sin\alpha=\frac{a}{c};\cos\alpha=\frac{b}{c};\mathrm{tg}\alpha=\frac{a}{b}} \text{ va } \boxed{\mathrm{ctg}\alpha=\frac{b}{a}.}$$

Burchak sinusi, cosinusi, tangensi va kotangensi o‘lchamsiz kattalik bo‘lib, burchakka yopishgan katet burchak qarshisidagi katetning qanday qismini tashkil etishini bildiradi.

Quyidagi munosabatlarni keltirib chiqaramiz:

$$\boxed{1.\begin{cases}\sin(90°-\alpha)=\cos\alpha,\\ \cos(90°-\alpha)=\sin\alpha,\\ \mathrm{tg}(90°-\alpha)=\mathrm{ctg}\alpha,\\ \mathrm{ctg}(90°-\alpha)=\mathrm{tg}\alpha,\end{cases}} \text{ va } \boxed{2.\begin{cases}\mathrm{tg}\alpha=\dfrac{\sin\alpha}{\cos\alpha},\\ \mathrm{ctg}\alpha=\dfrac{\cos\alpha}{\sin\alpha},\\ \mathrm{tg}\alpha\,\mathrm{ctg}\alpha=1.\end{cases}}$$

Isbot. 1) To‘g‘ri burchakli uchburchakda $\alpha+\beta=90° \Rightarrow \beta=90°-\alpha$.

Bundan

$$\begin{cases}\sin\alpha=\dfrac{a}{c}\\ \cos\beta=\dfrac{a}{c}\end{cases}, \Rightarrow \sin\alpha=\cos\beta \Rightarrow \sin\alpha=\cos(90°-\alpha).$$

$$\begin{cases}\mathrm{tg}\alpha=\dfrac{a}{b}\\ \mathrm{ctg}\beta=\dfrac{a}{b}\end{cases}, \Rightarrow \mathrm{tg}\alpha=\mathrm{ctg}\beta \Rightarrow \mathrm{tg}\alpha=\mathrm{ctg}(90°-\alpha).$$

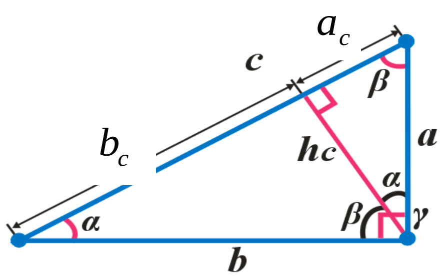
2) Ta'rifga ko'ra quyidagi kelib chiqadi:

$$\frac{\sin\alpha}{\cos\alpha}=\frac{a/c}{b/c}=\frac{a}{b}=\mathrm{tg}\alpha\ ;$$

$$\frac{\cos\alpha}{\sin\alpha}=\frac{b/c}{a/c}=\frac{b}{a}=\mathrm{ctg}\alpha\,;$$

$$\mathrm{tg}\alpha\cdot\mathrm{ctg}\alpha=\frac{a}{b}\cdot\frac{b}{a}=1\,.$$

Gipotenuzaga tushirilgan balandlikni h_c bilan, bu balandlikning gipotenuzadan ajratgan kesmalarini esa a_c va b_c deb belgilaylik (2.19-rasm). Bu yerda a_c ni a katetning gipotenuzadagi proyeksiyasi deb, b_c ni esa b katetning gipotenuzadagi proyeksiyasi deb ham atashimiz mumkin.



2.19-rasm

h_c balandlikning gipotenuzadan ajratgan kesmalari a_c va b_c quyidagicha topiladi:

$$\boxed{a_c=\frac{a^2}{c},\qquad b_c=\frac{b^2}{c}.}$$

Isbot. Uchburchakning α burchak sinusidan foydalanib a_c ni topamiz.

$$\begin{cases}\sin\alpha=\dfrac{a}{c}\\[2mm] \sin\alpha=\dfrac{a_c}{a}\end{cases},\ \Rightarrow\ \frac{a_c}{a}=\frac{a}{c},\ \Rightarrow\ a_c=\frac{a^2}{c}.$$

Uchburchakning α burchak kosinusidan foydalanib b_c ni topamiz.

$$\begin{cases}\cos\alpha=\dfrac{b}{c}\\[2mm] \cos\alpha=\dfrac{b_c}{b}\end{cases},\ \Rightarrow\ \frac{b_c}{b}=\frac{b}{c},\ \Rightarrow\ b_c=\frac{b^2}{c}.$$

6-teorema. Katetlarning gipotenuzadagi proyeksiyalari nisbati katetlar nisbatining kvadratiga teng, ya'ni burchak tangensi kvadratiga teng bo'ladi.

$$\boxed{\frac{a_c}{b_c}=\left(\frac{a}{b}\right)^2=\mathrm{tg}^2\alpha.}$$

Isbot. Bundan oldingi topilgan formulalar nisbatidan foydalanamiz. Unga ko‘ra $\frac{a_c}{b_c}=\frac{a^2/c}{b^2/c}=\frac{a^2}{b^2}=\left(\frac{a}{b}\right)^2=\mathrm{tg}^2\alpha$

bo‘ladi.

Yuqoridagi formuladan shunday xulosa qilish mumkinki, agar katetlar nisbati 2 ga teng bo‘lsa, katetlarning gipotenuzadagi proyeksiyalari nisbati esa $2^2=4$ ga teng bo‘ladi.

Gipotenuzaga tushirilgan balandlik h_c ushbu formulalardan aniqlanadi:

$$\boxed{h_c=\frac{a\,b}{c}\quad \text{yoki}\quad h_c=\sqrt{a_c b_c}\,.}$$

Isbot. 2.19-rasmdan ko‘rinib turibdiki, gipotenuzaga tushirilgan balandlikni aniqlash uchun $h_c^2=a^2-a_c{}^2$ yoki $h_c^2=b^2-b_c{}^2$ dan foydalanamiz. Natijada, $h_c=\sqrt{a^2-a_c^2}=\sqrt{a^2-\left(\frac{a^2}{c}\right)^2}=\sqrt{\frac{a^2c^2-a^4}{c^2}}=\frac{a}{c}\sqrt{c^2-a^2}=\frac{a\,b}{c}$ ekanligi kelib chiqadi. Agar $h_c=\frac{a\,b}{c}$ formulada $a=\sqrt{a_c\,c}$ va $b=\sqrt{b_c c}$ ekanini e’tiborga olsak, u holda biz $h_c=\frac{a\,b}{c}=\frac{\sqrt{a_c\,c}\cdot\sqrt{b_c\,c}}{c}=\sqrt{a_c b_c}$ formulaga ega bo‘lamiz.

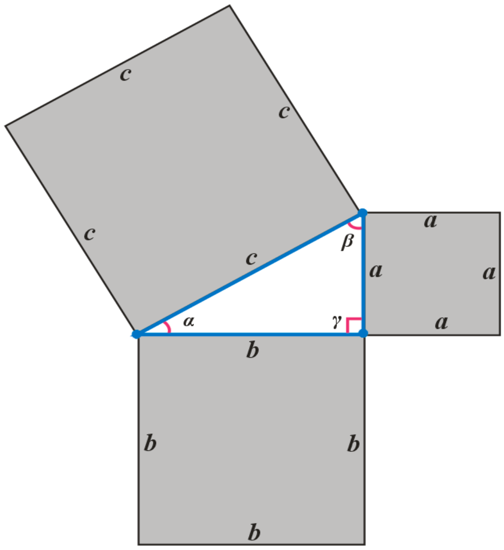
To‘g‘ri burchakli uchburchak gipotenuzasi va katetlari orasidagi bog‘lanish qadimgi yunon faylasufi Pifagor tomonidan aniqlangan bo‘lib, bu bog‘lanishni hozirgi kungacha uning sharafiga *Pifagor teoremasi* deb ataladi (2.20-rasm).

7-teorema. (***Pifagor teoremasi***) Katetlar kvadratlarining yig‘indisi gipotenuzaning kvadratiga tengdir.

$$\boxed{c^2=a^2+b^2.}$$

Bu teoremadan katetlarga qurilgan kvadratlar yuzalarining yig‘indisi gipotenuzaga qurilgan kvadrat yuzasiga tengligi kelib chiqadi (2.20-rasm).

Isbot. Katetlari *a*, *b* va gipotenuzasi *c* ga teng bo‘lgan to‘g‘ri burchakli uchburchak berilgan. Bu uchburchak uchun Pifagor teoremasi o‘rinli ekanini isbot qilamiz, ya’ni $a^2+b^2=c^2$ ekanini ko‘rsatamiz.



2.20-rasm

Buning uchun *a* va *b* katetlarning proyeksiyasi $a_c=\frac{a^2}{c}$, $b_c=\frac{b^2}{c}$ formulasidan foydalanamiz. a_c va b_c ni qo‘shamiz va $a_c+b_c=\frac{a^2}{c}+\frac{b^2}{c}=\frac{a^2+b^2}{c}$.

$a_c+b_c=c$ ekanligini inobatga olsak $c=\frac{a^2+b^2}{c}\Rightarrow c^2=a^2+b^2$.

Pifagor teoremasidan foydalanib, noma’lum katetni ham aniqlash mumkin.

$$\boxed{a=\sqrt{c^2-b^2}, \qquad b=\sqrt{c^2-a^2}.}$$

Pifagor teoremasidan foydalanib, quyidagi munosabatlarni isbotlaymiz:

1. $\sin^2\alpha+\cos^2\alpha=1;$
2. $1+\mathrm{tg}^2\alpha=\frac{1}{\cos^2\alpha};$
3. $1+\mathrm{ctg}^2\alpha=\frac{1}{\sin^2\alpha}.$

Isbot. Ayniyatlarni Pifagor teoremasidan foydalanib isbotlaymiz.

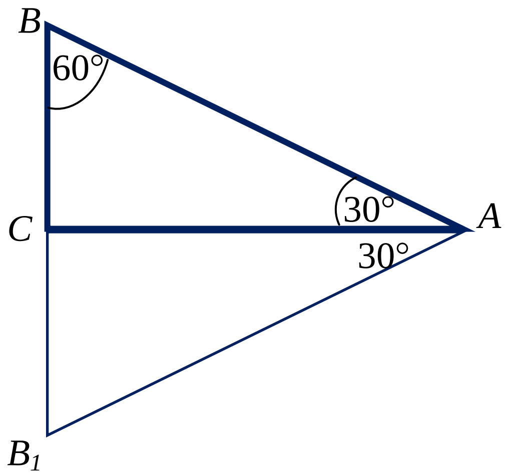
1. $\sin^2\alpha+\cos^2\alpha=\left(\frac{a}{c}\right)^2+\left(\frac{b}{c}\right)^2=\frac{a^2+b^2}{c^2}=\frac{c^2}{c^2}=1;$

2. $1+\mathrm{tg}^2\alpha=1+\frac{\sin^2\alpha}{\cos^2\alpha}=\frac{\cos^2\alpha+\sin^2\alpha}{\cos^2\alpha}=\frac{1}{\cos^2\alpha};$

3. $1+\mathrm{ctg}^2\alpha = 1+\dfrac{\cos^2\alpha}{\sin^2\alpha} = \dfrac{\sin^2\alpha+\cos^2\alpha}{\sin^2\alpha} = \dfrac{1}{\sin^2\alpha}$.

8-teorema. To‘g‘ri burchakli uchburchakda 30 gradus qarshisidagi katet gipotenuzaning yarmiga teng.

Isbot. C burchagi to‘g‘ri bo‘lgan ABC to‘g‘ri burchakli uchburchak berilgan bo‘lsin (2.17-rasm). BC tomonni davom ettirib $BC=CB_1$ kesmani yasaymiz. B_1 nuqtani A nuqta bilan tutashtiramiz va ABB_1 teng tomonli uchburchakni hosil qilamiz. Bundan $BB_1=AB=AB_1$ ekanligi kelib chiqadi.



2.17-rasm

$$AB=BB_1=BC+CB_1=2BC \Rightarrow BC=\frac{AB}{2}.$$

9-teorema. To‘g‘ri burchakli uchburchakda o‘tkir burchak kattaligi quyidagilarga teng bo‘ladi:

1. $\sin 30° = \cos 60° = \dfrac{1}{2}$;
2. $\sin 45° = \cos 45° = \dfrac{1}{\sqrt{2}}$;
3. $\cos 30° = \sin 60° = \dfrac{1}{2}$;
4. $\mathrm{tg}30° = \mathrm{ctg}60° = \dfrac{1}{\sqrt{3}}$;
5. $\mathrm{tg}45° = \mathrm{ctg}45° = 1$;
6. $\mathrm{tg}60° = \mathrm{ctg}30° = \sqrt{3}$.

Isbot. α o‘tkir burchak 30° ga teng bo‘lsa, bu burchak qarshisidagi katet gipotenuzaning yarmiga teng bo‘lgani uchun $c=2a$. Bundan $\sin\alpha = \dfrac{a}{c} = \dfrac{a}{2a} = \dfrac{1}{2}$ kelib chiqadi. $\cos(90°-\alpha) = \sin\alpha \Rightarrow \cos 60° = \sin 30° = \dfrac{1}{2}$.

α o‘tkir burchak 45° ga teng bo‘lsa, uchburchak teng yonli bo‘lib $a=b$ va Pifagor teoremasiga ko‘ra $c=\sqrt{2}a$ ga teng. Bundan $\sin\alpha=\frac{a}{c}=\frac{a}{\sqrt{2}a}=\frac{1}{\sqrt{2}}$ kelib chiqadi. $\cos(90°-\alpha)=\sin\alpha \Rightarrow \cos 45°=\sin 45°=\frac{1}{\sqrt{2}}$.

α o‘tkir burchak 60° ga teng bo‘lsa, β=30° bo‘ladi va β burchak qarshisidagi katet gipotenuzaning yarmiga teng bo‘lgani uchun $c=2b$. Bundan $\sin 30=\frac{b}{c}=\frac{b}{2b}=\frac{1}{2}$ kelib chiqadi. $\cos(90°-\beta)=\sin\beta \Rightarrow \cos 60°=\sin 30°=\frac{1}{2}$.

$$\sin 60°=\sqrt{1-\cos^2 60°}=\sqrt{1-\frac{1}{4}}=\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}\Rightarrow \cos 30°=\sin 60°=\frac{\sqrt{3}}{2}.$$

Mustaqil yechish uchun masalalar

2.30. Teng yonli uchburchakning uchidagi burchagi 120° va bu uchidan tushirilgan balandligi uzunligi 19,5 bo‘lsa, yon tomonlari uzunliklari yig‘indisini toping. javob: 78

2.31. To‘g‘ri burchakli uchburchakning perimetri 40 ga teng, hamda bir kateti uzunligi 8 ga teng bo‘lsa, bu uchburchakning gipotenuzasi uzunligini toping. javob: 17

2.32. To‘g‘ri burchakli uchburchakning bir kateti uzunligi gipotenuzadan 8 ga kam, ikkinchi katetning uzunligi 20 ga teng. Bu uchburchakning perimetrini toping. javob: 70

2.33. To‘g‘ri burchakli uchburchakning bir kateti va gipotenuza uzunliklari yig‘indisi 9 ga teng, ularning farqi esa 4 ga teng. Ikkinchi katetini toping. javob:6

2.34. Teng yonli to‘g‘ri burchakli uchburchakning perimetri $3(\sqrt{2}+1)$ ga teng bo‘lsa, bu uchburchakning gipotenuzasi uzunligini toping. javob: 3

2.35. To‘g‘ri burchakli uchburchakning gipotenuzasi kichik kateti uzunligidan 3 marta katta, hamda katta kateti uzunligi $4\sqrt{2}$ ga teng bo‘lsa, bu uchburchakning gipotenuzasiga o‘tkazilgan medianasi uzunligini toping. javob: 3

Quyidagi masalalarda to‘g‘ri burchakli uchburchakning ikkita tomoni uzunligi ma’lum bo‘lsa, uchburchakning uchinchi tomoni uzunligini toping.

2.36. 5 va 4 javob: 3; $\sqrt{41}$

2.37. 12 va 13 javob: 5; $\sqrt{313}$

2.38. $4\sqrt{2}$ va 7 javob: 9; $\sqrt{17}$

2.39. 8 va 6 javob: 10; $2\sqrt{7}$

2.40. $6\sqrt{3}$ va 12 javob: 6; $2\sqrt{63}$

2.41. 3 va 6 javob: $3\sqrt{3}$; $3\sqrt{5}$

2.42. $\sqrt{13}$ va 7 javob: 6; $\sqrt{62}$

2.43. 5 va 12 javob: 13; $\sqrt{119}$

2.44. 5 va 6 javob: $\sqrt{11}$; $\sqrt{61}$

2.45. To‘g‘ri burchakli uchburchakning bir kateti uzunligi 4 ga teng, hamda gipotenuzasiga o‘tkazilgan medianasi uzunligi 2,5 ga teng bo‘lsa, bu uchburchakning perimetrini toping. javob: 12

2.46. To‘g‘ri burchakli uchburchakning katetlari uzunligi 30 va 40 ga teng bo‘lsa, gipotenuzasiga o‘tkazilgan medianasi uzunligini toping. javob: 25

2.47. To‘g‘ri burchakli uchburchakning perimetri 17,5 ga teng. Agar bu uchburchakning bir kateti uzunligi 5 ga teng bo‘lsa, gipotenuzasiga o‘tkazilgan medianasi uzunligini toping. javob: 3,625

2.48. To‘g‘ri burchakli uchburchakning katetlari yig‘indisi 10,25 ga teng, hamda gipotenuzasiga o‘tkazilgan medianasi uzunligi esa 3,625 ga teng bo‘lsa, katta kateti uzunligini toping. javob: 5,25

2.49. To‘g‘ri burchakli uchburchakda gipotenuza uzunligi bir katetdan 2 marta katta. Agar ikkinchi katet uzunligi 4 ga teng bo‘lsa, gipotenuzasiga o‘tkazilgan medianasi uzunligini toping. javob: $\frac{4\sqrt{3}}{3}$

2.50. Gipotenuzasiga o‘tkazilgan medianasi uzunligi 25 ga teng, hamda katetlari ayirmasi 10 ga teng. Katta katetning uzunligini toping. javob: 40

2.51. To‘g‘ri burchakli uchburchakning bir kateti uzunligi 3 ga, gipotenuzasiga o‘tkazilgan medianasi uzunligi esa 2,5 ga teng. Gipotenuza va ikkinchi katetning uzunligini toping. javob: 4; 5

2.52. To‘g‘ri burchakli uchburchakning gipotenuzasiga o‘tkazilgan medianasi uzunligi 6 ga teng. Agar katetlari uzunliklari nisbati $\frac{3}{4}$ bo‘lsa, bu uchburchakning perimetrini toping. javob: 28,8

2.53. To‘g‘ri burchakli uchburchakning gipotenuzasiga o‘tkazilgan medianasi uzunligi 6,5 ga teng. Agar kichik katet va gipotenuza uzunliklari yig‘indisi 18 bo‘lsa, bu uchburchakning katetlarini toping. javob: 5; 12

2.54. To‘g‘ri burchakli uchburchakning to‘g‘ri burchagi uchidan chiqqan balandligi uzunligi $2\sqrt{5}$ ga teng. Uchburchakning bir kateti uzunligi 6 ga teng bo‘lsa, gipotenuzasi uzunligini toping. javob:9

2.55. To‘g‘ri burchakli uchburchakning katetlari $2\sqrt{21}$ va $4\sqrt{7}$ ga teng. To‘g‘ri burchagidan gipotenuzasiga tushirilgan balandlik uzunligini toping. javob: $4\sqrt{3}$

2.56. To‘g‘ri burchakli uchburchakning bir kateti 15 ga teng, ikkinchi katetining gipotenuzadagi proyeksiyasi 16 ga teng. Bu uchburchakning perimetrini toping. javob: 60

2.57. To‘g‘ri burchakli uchburchakda katetlariga tushirilgan medianalari uzunligi $\sqrt{52}$ va $\sqrt{73}$ ga teng. Gipotenuzaning uzunligini toping. javob: 10

2.58. To‘g‘ri burchakli uchburchak *ABC* da katetlari *AC*=15 va *BC*=20. Gipotenuzaga tushirilgan balandlik asosi *H* bo‘lsa, *H* nuqtadan *AC* katetgacha bo‘lgan masofani toping. javob: 7,2

2.59. To‘g‘ri burchakli uchburchak *ABC* ning to‘g‘ri burchagi uchi *B* dan tushirilgan balandlik BP. Agar *AB*=*PC* bo‘lsa, *A* burchakning kosinus qiymatini toping. javob: $\frac{\sqrt{5}-1}{2}$

2.60. To‘g‘ri burchakli uchburchakning bir kateti 18 ga teng. Bu katetga tegishli bo‘lgan nuqta ikkinchi katet va gipotenuzadan 5 masofada joylashgan. Bu uchburchakning perimetrini toping. javob: 45

3-§. Uchburchakning balandligi, bissektrisasi va medianasining xossalari

10-teorema. Uchburchakning uchta tomoni uzunliklari ham ma’lum bo‘lganda uchta tomonga tushirilgan balandlikni topish formulalari quyidagicha bo‘ladi:

$$\boxed{\begin{aligned} h_a &= \frac{1}{2a}\sqrt{(2ab)^2-\left(a^2+b^2-c^2\right)^2};\\ h_b &= \frac{1}{2b}\sqrt{(2ab)^2-\left(a^2+b^2-c^2\right)^2};\\ h_c &= \frac{1}{2c}\sqrt{(2ab)^2-\left(a^2+b^2-c^2\right)^2}.\end{aligned}}$$

Isbot. Bitta tomonga tushirilgan balandlik formulasini keltirib chiqarishimiz yetarlidir. Pifagor teoremasiga ko‘ra (2.21-rasm):

$$\begin{matrix}\Delta CAD \Rightarrow \\ \Delta ADB \Rightarrow\end{matrix}\begin{cases} h_a^2 = b^2 - x^2, \\ h_a^2 = c^2 - (a-x)^2 \end{cases} \Rightarrow b^2 - x^2 = c^2 - (a-x)^2 \Rightarrow b^2 - x^2 = c^2 - a^2 + 2ax - x^2 \Rightarrow$$

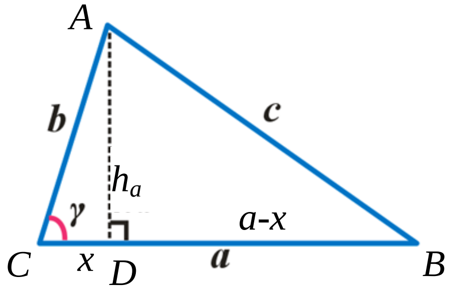
$$2ax = b^2 + a^2 - c^2 \Rightarrow x = \frac{b^2 + a^2 - c^2}{2a}.$$

a tomonga tushirilgan balandlik

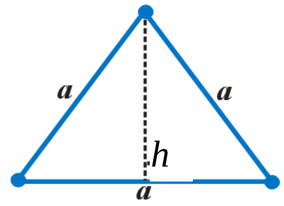
$$h_a=\sqrt{b^2-x^2}=\sqrt{b^2-\left(\frac{b^2+a^2-c^2}{2a}\right)^2}=b\sqrt{1-\left(\frac{a^2+b^2-c^2}{2ab}\right)^2}=$$

$$=b\sqrt{\frac{(2ab)^2-\left(a^2+b^2-c^2\right)^2}{(2ab)^2}}=\frac{1}{2a}\sqrt{(2ab)^2-\left(a^2+b^2-c^2\right)^2}$$

bo‘ladi. Qolgan balandliklar ham xuddi shu tartibda topiladi.



2.21-rasm



2.22-rasm

Yuqorida topilgan formulalarni muntazam, teng yonli va to‘g‘ri burchakli uchburchak kabi xususiy hollarga tatbiq etishimiz mumkin.

Tomoni a ga teng bo‘lgan muntazam uchburchakning uchta balandligi ham o‘zaro teng va quyidagicha bo‘ladi:

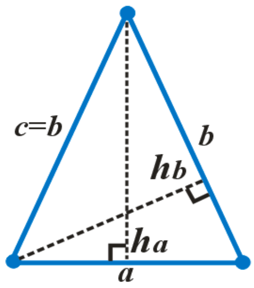
$$\boxed{h=\frac{\sqrt{3}}{2}a.}$$

Isbot. Muntazam uchburchakda (*2.22-rasm*) $a=b=c$ hamda $h_a=h_b=h_c=h$ bo‘ladi. Balandlik esa

$$h=\frac{1}{2a}\sqrt{(2aa)^2-\left(a^2+a^2-a^2\right)^2}=\frac{1}{2a}\sqrt{4a^4-a^4}=\frac{\sqrt{3}a^2}{2a}=\frac{\sqrt{3}a}{2}$$ bo‘ladi.

Asosi a ga, yon tomonlari b ga teng bo‘lgan teng yonli uchburchakning asosiga va yon tomoniga tushirilgan balandliklari quyidagicha bo‘ladi:

$$\boxed{h_a=\frac{1}{2}\sqrt{4b^2-a^2},\qquad h_b=\frac{a}{2b}\sqrt{4b^2-a^2}.}$$



2.23-rasm

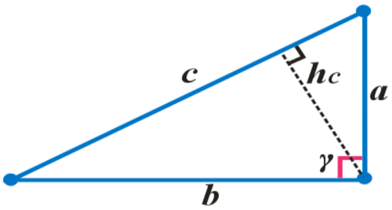
Isbot. Teng yonli uchburchakda (*2.23-rasm*) $b=c$ hamda $h_b=h_c$ bo‘ladi. a tomonga tushirilgan balandlik

$$h_a=\frac{1}{2a}\sqrt{(2ab)^2-\left(a^2+b^2-b^2\right)^2}=\frac{1}{2a}\sqrt{4a^2b^2-a^4}=\frac{a}{2a}\sqrt{4b^2-a^2}=\frac{1}{2}\sqrt{4b^2-a^2}$$

bo‘ladi. b tomonga tushirilgan balandlik

$$h_b=\frac{1}{2b}\sqrt{(2ab)^2-\left(a^2+b^2-b^2\right)^2}=\frac{1}{2b}\sqrt{4a^2b^2-a^4}=\frac{a}{2b}\sqrt{4b^2-a^2}$$ bo‘ladi.

Katetlari a va b ga hamda gipotenuzasi c ga teng bo‘lgan to‘g‘ri burchakli uchburchakning (2.24-rasm) katetlariga va gipotenuzasiga tushirilgan balandliklari quyidagicha bo‘ladi:



2.24-rasm

$$\boxed{h_a=b,\quad h_b=a,\quad h_c=\frac{ab}{c}.}$$

Isbot. Bunda Pifagor teoremasidan foydalanib, ildiz ostidagi ifoda $\sqrt{(2ab)^2-\left(a^2+b^2-c^2\right)^2}=2ab$ ekanligini bilgan holda so‘ralgan kattaliklarni aniqlaymiz. Shunga ko‘ra $h_a=\frac{1}{2a}2ab=b$, $h_b=\frac{1}{2b}2ab=a$, $h_c=\frac{1}{2c}2ab=\frac{ab}{c}$ formulalarni hosil qilamiz.

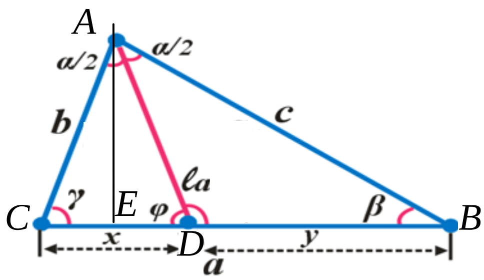
a tomonga tushirilgan bissektrisa l_a shu tomonni x va y kesmalarga ajratsa, bu kesmalarning uzunliklari quyidagi formula orqali topiladi (2.25-rasm):

$$\boxed{x=\frac{ab}{b+c},\qquad y=\frac{ac}{b+c}.}$$

Isbot. Bissektrisa ajratgan kesmalar nisbatini qolgan ikki tomonlar nisbatiga teng va bu kesmalar uzunliklari yig‘indisi esa a tomon uzunligini beradi. Shu kattaliklarni sistema qilib ishlasak, so‘ralgan kattaliklar

$$\begin{cases}\dfrac{x}{y}=\dfrac{b}{c}\\ x+y=a\end{cases};\ \Rightarrow\ \begin{cases}y=\dfrac{c}{b}x\\ x+\dfrac{c}{b}x=a\end{cases};\ \Rightarrow\ \begin{cases}y=\dfrac{c}{b}x\\ \dfrac{b+c}{b}x=a\end{cases};\ \Rightarrow\ \begin{cases}y=\dfrac{c}{b}x=\dfrac{ac}{b+c}\\ x=\dfrac{ab}{b+c}\end{cases}$$

ekanligi kelib chiqadi.



2.25-rasm

11-teorema. Uchburchakning uchta tomoni uzunliklari ham ma’lum bo‘lganda uchta tomonga tushirilgan bissektrisani topish formulalari quyidagicha bo‘ladi:

$$\boxed{\begin{aligned} \ell_a &= \frac{1}{b+c}\sqrt{bc\left((b+c)^2-a^2\right)}; \\ \ell_b &= \frac{1}{a+c}\sqrt{ac\left((a+c)^2-b^2\right)}; \\ \ell_c &= \frac{1}{a+b}\sqrt{ab\left((a+b)^2-c^2\right)}. \end{aligned}}$$

Isbot. 2.25-rasmda ΔACE va ΔADE da Pifagor teoremasidan foydalanamiz:

$$\Delta ADE \Rightarrow$$
$$\Delta CEA \Rightarrow$$
$$\begin{cases} \ell_a^2 = AE^2 + DE^2 \\ b^2 = AE^2 + CE^2 \end{cases} \Rightarrow \ell_a^2 = b^2 + DE^2 - CE^2 \Rightarrow \ell_a^2 = b^2 + (x-CE)^2 - CE^2 \Rightarrow$$
$$\ell_a^2 = b^2 + x^2 - 2xCE.$$

2.25-rasmda ΔACE va ΔAEB da Pifagor teoremasidan foydalanib, CE ni topamiz:

$$\begin{matrix} \Delta AEB \Rightarrow \\ \Delta CEA \Rightarrow \end{matrix} \begin{cases} c^2 = AE^2 + (a-CE)^2 \\ b^2 = AE^2 + CE^2 \end{cases} \Rightarrow c^2 - b^2 = (a-CE)^2 - CE^2 \Rightarrow CE = \frac{b^2+a^2-c^2}{2a}.$$

Bunda

$$\ell_a^2 = b^2 + x^2 - 2\,xCE = b^2 + \left(\frac{ab}{b+c}\right)^2 - 2\cdot\frac{ab}{b+c}\cdot\frac{a^2+b^2-c^2}{2a} =$$

$$= \frac{1}{(b+c)^2}\left[b^2(b+c)^2 + a^2b^2 - b(b+c)\left(a^2+b^2-c^2\right)\right] =$$

$$= \frac{1}{(b+c)^2}\left[b^4 + 2b^3c + b^2c^2 + a^2b^2 - b\left(a^2b + b^3 - bc^2 + a^2c + b^2c - c^3\right)\right] =$$

$$= \frac{1}{(b+c)^2}\cdot\cdot\left[b^4 + 2b^3c + b^2c^2 + a^2b^2 - a^2b^2 - b^4 + b^2c^2 - a^2bc - b^3c + bc^3\right] =$$

$$= \frac{1}{(b+c)^2}\left[2b^2c^2 + b^3c + bc^3 - a^2bc\right] = \frac{1}{(b+c)^2}\left[bc\left((b+c)^2 - a^2\right)\right]$$

kelib chiqadi. Bundan ildiz olsak,

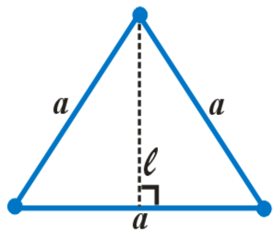
$$\ell_a = \frac{1}{b+c}\sqrt{bc\left((b+c)^2 - a^2\right)}$$

hosil bo‘ladi. Qolgan bissektrisalar ham xuddi shu yo‘sinda hisoblab topiladi.

Yuqorida topilgan formulalarni muntazam, teng yonli va to‘g‘ri burchakli uchburchak kabi xususiy hollarga tatbiq etishimiz mumkin.

Tomoni *a* ga teng bo‘lgan muntazam uchburchakning uchta bissektrisasi ham o‘zaro teng va quyidagicha bo‘ladi (2.26-rasm):

$$\boxed{\ell = \frac{\sqrt{3}}{2}a = h.}$$



2.26-rasm

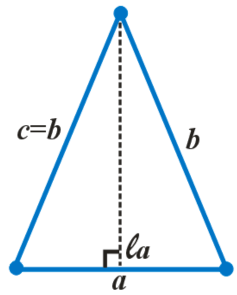
Isbot. Muntazam uchburchakda $a = b = c$ hamda $\ell_a = \ell_b = \ell_c = \ell$ bo‘ladi. Bissektrisa esa formuladan

$$\ell = \ell_a = \frac{1}{b+c}\sqrt{bc\left((b+c)^2 - a^2\right)} =$$

$$= \frac{1}{a+a}\sqrt{aa\left((a+a)^2 - a^2\right)} = \frac{1}{2a}\sqrt{a^2\left(4a^2 - a^2\right)} = \frac{1}{2a}\sqrt{3a^4} = \frac{\sqrt{3}}{2}a = h$$

ga tengligi kelib chiqadi.

Asosi a ga, yon tomonlari b ga teng bo‘lgan teng yonli uchburchakning asosiga tushirilgan bissektrisalari quyidagicha bo‘ladi:



2.27-rasm

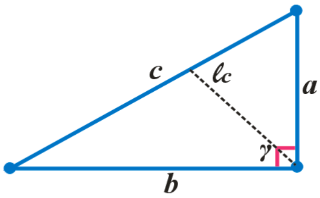
$$\boxed{\ell_a = \frac{1}{2}\sqrt{4b^2 - a^2} = h_a.}$$

Isbot. Teng yonli uchburchakda $b = c$ hamda $\ell_b = \ell_c$ bo‘ladi. a tomonga tushirilgan bissektrisa esa formulaga asosan

$$\ell_a = \frac{1}{b+c}\sqrt{bc\left((b+c)^2 - a^2\right)} = \frac{1}{b+b}\sqrt{bb\left((b+b)^2 - a^2\right)} = \frac{1}{2}\sqrt{4b^2 - a^2} = h_a$$

ga teng ekanligi kelib chiqadi (2.27-rasm).

Katetlari a va b ga hamda gipotenuzasi c ga teng bo‘lgan to‘g‘ri burchakli uchburchakning gipotenuzasiga tushirilgan bissektrisa quyidagicha bo‘ladi (2.28-rasm):



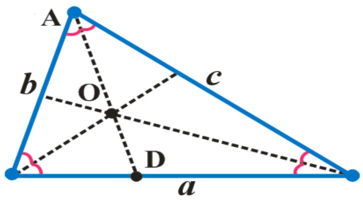
2.28-rasm

$$\boxed{\ell_c = \frac{\sqrt{2}\,ab}{a+b}.}$$

Isbot. Bunda Pifagor teoremasidan foydalansak,

$$\ell_c = \frac{1}{a+b}\sqrt{ab\left((a+b)^2 - c^2\right)} = \frac{1}{a+b}\sqrt{ab\left(a^2 + b^2 + 2ab - c^2\right)} = \frac{\sqrt{2}\,ab}{a+b}$$ ga teng bo‘ladi (2.28-rasm).

Bissektrisalar kesishish nuqtasi burchak uchidan boshlab hisoblaganda quyidagi nisbatda bo‘linadi (2.29-rasm):

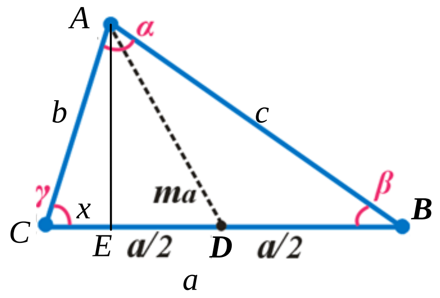


2.29-rasm

$$\boxed{\frac{AO}{OD} = \frac{b+c}{a}.}$$

12-teorema. Uchburchakning uchta tomoni uzunliklari ham ma’lum bo‘lganda uchta tomonga tushirilgan medianalarni topish formulalari quyidagicha bo‘ladi:

$$\boxed{\begin{aligned} m_a &= \frac{1}{2}\sqrt{2\left(b^2+c^2\right)-a^2};\\ m_b &= \frac{1}{2}\sqrt{2\left(a^2+c^2\right)-b^2};\\ m_c &= \frac{1}{2}\sqrt{2\left(a^2+b^2\right)-c^2}. \end{aligned}}$$



2.30-rasm

Isbot. 2.30-rasmda ΔACD dan Pifagor teoremasidan foydalanamiz:

$$\begin{matrix}\Delta ADE \Rightarrow\\ \Delta ACE \Rightarrow\end{matrix}\begin{cases} m_a^2 = AE^2 + \left(\dfrac{a}{2} - x\right)^2\\ b^2 = AE^2 + x^2\end{cases} \Rightarrow m_a^2 = b^2 + \frac{a^2}{4} - ax.$$

$$\Delta ACB \text{ dan } \begin{cases} c^2 = AE^2 + (a-x)^2\\ b^2 = AE^2 + x^2\end{cases} \Rightarrow x = \frac{a^2+b^2-c^2}{2a}.$$

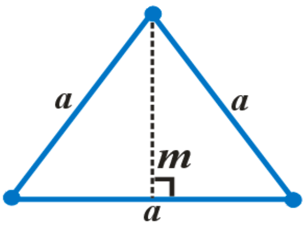
Bunda

$$m_a^2 = b^2 + \left(\frac{a}{2}\right)^2 - ax = b^2 + \frac{a^2}{4} - a\cdot\frac{a^2+b^2-c^2}{2a} =$$

$$= \frac{1}{4}\left[4b^2 + a^2 - 2a^2 - 2b^2 + 2c^2\right] = \frac{1}{4}\left[2b^2 + 2c^2 - a^2\right].$$

Tenglikning har ikki tomonini ildizga olsak, $m_a = \frac{1}{2}\sqrt{2\left(b^2+c^2\right)-a^2}$ kelib chiqadi. Qolgan medianalar ham xuddi shu yo‘sinda hisoblab topiladi.

Yuqorida topilgan formulalarni muntazam, teng yonli va to‘g‘ri burchakli uchburchak kabi xususiy hollarga tatbiq etishimiz mumkin.



2.31-rasm

Tomoni a ga teng bo‘lgan muntazam uchburchakning uchta medianasi ham o‘zaro teng va quyidagicha bo‘ladi (2.31-rasm):

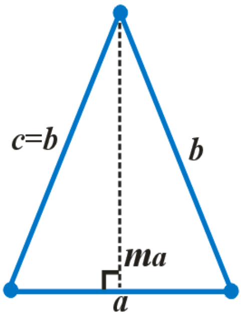
$$\boxed{m = \frac{\sqrt{3}}{2}a = \ell = h.}$$

Isbot. Muntazam uchburchakda $a = b = c$ hamda $m_a = m_b = m_c = m$ bo‘ladi. Mediana esa formuladan

$$\ell = \ell_a = m_a = \frac{1}{2}\sqrt{2\left(b^2+c^2\right)-a^2} = \frac{1}{2}\sqrt{2(a^2+a^2)-a^2} = \frac{1}{2}\sqrt{4a^2-a^2} = \frac{\sqrt{3}}{2}a = \ell = h.$$

ga tengligi kelib chiqadi. Demak, muntazam uchburchakning asosga tushirilgan balandligi, bissektrisasi va medianasi o‘zaro teng bo‘lar ekan.

Asosi a ga, yon tomonlari b ga teng bo‘lgan teng yonli uchburchakning asosiga hamda yon tomoniga tushirilgan medianalari quyidagicha bo‘ladi (2.32-rasm):



2.32-rasm

$$\boxed{m_a = \frac{1}{2}\sqrt{4b^2-a^2} = \ell_a = h_a, \qquad m_b = \frac{1}{2}\sqrt{2a^2+b^2}.}$$

Isbot. Teng yonli uchburchakda $b=c$ hamda $m_b = m_c$ bo‘ladi. a tomonga tushirilgan mediana esa formulaga asosan

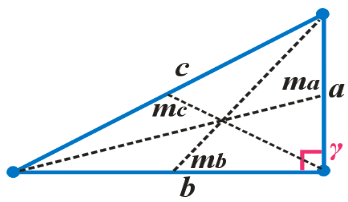
$$m_a = \frac{1}{2}\sqrt{2\left(b^2+c^2\right)-a^2} = \frac{1}{2}\sqrt{4b^2-a^2} = \ell_a = h_a$$

ga teng ekanligi kelib chiqadi. Demak, teng yonli uchburchakning asosga tushirilgan balandligi, bissektrisasi va medianasi o‘zaro teng bo‘lar ekan. Yon tomonga tushirilgan mediana esa

$$m_b = \frac{1}{2}\sqrt{2\left(a^2+c^2\right)-b^2} = \frac{1}{2}\sqrt{2\left(a^2+b^2\right)-b^2} = \frac{1}{2}\sqrt{2a^2+b^2}$$

ga teng ekanligi kelib chiqadi.

Katetlari a va b ga hamda gipotenuzasi c ga teng bo‘lgan to‘g‘ri burchakli uchburchakning gipotenuzasi va katetlariga tushirilgan medianalar quyidagicha bo‘ladi (2.33-rasm):



2.33-rasm

$$\boxed{m_c = \frac{c}{2}, \qquad m_a = \frac{1}{2}\sqrt{4b^2+a^2}, \qquad m_b = \frac{1}{2}\sqrt{4a^2+b^2}.}$$

Isbot. Bunda Pifagor teoremasidan foydalansak,

$m_c = \frac{1}{2}\sqrt{2\left(a^2+b^2\right)-c^2} = \frac{1}{2}\sqrt{2c^2-c^2} = \frac{c}{2}$ ga teng bo‘ladi. a katetga tushirilgan mediana $m_a = \frac{1}{2}\sqrt{2\left(b^2+c^2\right)-a^2} = \frac{1}{2}\sqrt{2\left(b^2+a^2+b^2\right)-a^2} = \frac{1}{2}\sqrt{4b^2+a^2}$ va b katetga tushirilgan mediana $m_b = \frac{1}{2}\sqrt{2\left(a^2+c^2\right)-b^2} = \frac{1}{2}\sqrt{2\left(a^2+a^2+b^2\right)-a^2} = \frac{1}{2}\sqrt{4a^2+b^2}$ ga teng bo‘ladi.

To‘g‘ri burchakli uchburchakning katetlariga tushirilgan medianalari m_a va m_b ma’lum bo‘lsa, gipotenuzaga tushirilgan mediana m_c quyidagicha bo‘ladi:

$$\boxed{m_c = \sqrt{\frac{m_a^2+m_b^2}{5}}.}$$

Isbot. Bunda Pifagor teoremasidan foydalansak,

$$\begin{cases} m_a^2 = \left(\frac{a}{2}\right)^2 + b^2 = \frac{a^2}{4} + b^2 \\ m_b^2 = a^2 + \left(\frac{b}{2}\right)^2 = a^2 + \frac{b^2}{4} \end{cases}$$

bo‘ladi. Bu ikkala tenglamani bir-biriga qo‘shsak,

$$m_a^2 + m_b^2 = \frac{5}{4}a^2 + \frac{5}{4}b^2 = \frac{5}{4}\left(a^2+b^2\right) = \frac{5}{4}c^2$$

kelib chiqadi. Bundan gipotenuza uzunligi $c = 2\sqrt{\frac{m_a^2+m_b^2}{5}}$ hosil bo‘ladi. Gipotenuzaga tushirilgan mediana esa gipotenuza uzunligining yarmiga teng, ya’ni $m_c = \frac{c}{2} = \sqrt{\frac{m_a^2+m_b^2}{5}}$ bo‘ladi.

Medianalar va tomonlar orasidagi bog‘lanish ushbu ko‘rinishda bo‘ladi:

$$\boxed{m_a^2 + m_b^2 + m_c^2 = \frac{3}{4}\left(a^2+b^2+c^2\right).}$$

Isbot. Yuqorida topilgan mediana formulalarini kvadratga oshirib qo‘shib chiqamiz. Natijada

$$m_a^2 + m_b^2 + m_c^2 = \frac{1}{4}\left(2b^2+2c^2-a^2\right) + \frac{1}{4}\left(2a^2+2c^2-b^2\right) + \frac{1}{4}\left(2a^2+2b^2-c^2\right) =$$

$$= \frac{1}{4}\cdot(2b^2+2c^2-a^2+2a^2+2c^2-b^2+2a^2+2c^2-b^2) = \frac{3}{4}\left(a^2+b^2+c^2\right)$$

ekanligi kelib chiqadi.

13-teorema. Agar uchburchakning uchta medianasi ma'lum bo'lsa, u holda uchta tomonini aniqlash mumkin. Tomonlarning medianalar orqali berilishi quyidagicha bo'ladi:

$$a=\frac{2}{3}\sqrt{2\left(m_b^2+m_c^2\right)-m_a^2};$$
$$b=\frac{2}{3}\sqrt{2\left(m_a^2+m_c^2\right)-m_b^2};$$
$$c=\frac{2}{3}\sqrt{2\left(m_a^2+m_b^2\right)-m_c^2}.$$

Isbot. Medianani topish formulalarini kvadratga oshirib, so'ngra uch noma'lumli tenglamalar sistemasini ishlaymiz:

$$\begin{cases} m_a^2=\frac{1}{4}\left(2b^2+2c^2-a^2\right)=\frac{1}{2}b^2+\frac{1}{2}c^2-\frac{1}{4}a^2 \\ m_b^2=\frac{1}{4}\left(2a^2+2c^2-b^2\right)=\frac{1}{2}a^2+\frac{1}{2}c^2-\frac{1}{4}b^2 \\ m_c^2=\frac{1}{4}\left(2a^2+2b^2-c^2\right)=\frac{1}{2}a^2+\frac{1}{2}b^2-\frac{1}{4}c^2 \end{cases};\Rightarrow$$

$$\begin{cases} c^2=2m_a^2+\frac{1}{2}a^2-b^2 \quad (*) \\ m_b^2=\frac{1}{2}a^2+m_a^2+\frac{1}{4}a^2-\frac{1}{2}b^2-\frac{1}{4}b^2=m_a^2+\frac{3}{4}a^2-\frac{3}{4}b^2 \\ m_c^2=\frac{1}{2}a^2+\frac{1}{2}b^2-\frac{1}{2}m_a^2-\frac{1}{8}a^2+\frac{1}{4}b^2=-\frac{1}{2}m_a^2+\frac{3}{4}b^2+\frac{3}{8}a^2 \end{cases};\Rightarrow$$

$$\begin{cases} m_b^2-m_a^2=\frac{3}{4}a^2-\frac{3}{4}b^2 \quad (**) \\ m_c^2+\frac{1}{2}m_a^2=\frac{3}{4}b^2+\frac{3}{8}a^2 \quad (***) \end{cases};\Rightarrow$$

$$(**)+(***),\ \rightarrow\ m_b^2-m_a^2+m_c^2+\frac{1}{2}m_a^2=\frac{9}{8}a^2,\ \rightarrow$$

$$\frac{1}{2}\left(2m_b^2+2m_c^2-m_a^2\right)=\frac{9}{8}a^2,\ \rightarrow a^2=\frac{4}{9}\left(2m_b^2+2m_c^2-m_a^2\right)$$

hosil bo'ladi. Bu ifodani (**)ga qo'ysak,

$$m_b^2-m_a^2=\frac{3}{4}\cdot\frac{4}{9}\left(2m_b^2+2m_c^2-m_a^2\right)-\frac{3}{4}b^2\Rightarrow m_b^2-m_a^2=\frac{2}{3}m_b^2+\frac{2}{3}m_c^2-\frac{1}{3}m_a^2-\frac{3}{4}b^2\Rightarrow$$

$$\frac{1}{3}m_b^2-\frac{2}{3}m_c^2-\frac{2}{3}m_a^2=-\frac{3}{4}b^2\Rightarrow \quad b^2=\frac{4}{9}\left(2m_a^2+2m_c^2-m_b^2\right)$$

ifodaga ega bo'lamiz. Bu ifodani (*)ga qo'yganda esa

$$c^2=2m_a^2+\frac{1}{2}a^2-b^2=2m_a^2+\frac{2}{9}\left(2m_b^2+2m_c^2-m_a^2\right)-\frac{4}{9}\left(2m_a^2+2m_c^2-m_b^2\right)=$$

$$=\frac{4}{9}\left(\frac{9}{2}m_a^2+m_b^2+m_c^2-\frac{1}{2}m_a^2-2m_a^2-2m_c^2+m_b^2\right)=\frac{4}{9}\left(2m_a^2+2m_b^2-m_c^2\right)$$

hosil bo‘ladi. Bulardan kvadrat ildiz chiqarilsa, izlanayotgan

$$a=\frac{2}{3}\sqrt{2\left(m_b^2+m_c^2\right)-m_a^2},\ b=\frac{2}{3}\sqrt{2\left(m_a^2+m_c^2\right)-m_b^2},\ c=\frac{2}{3}\sqrt{2\left(m_a^2+m_b^2\right)-m_c^2}$$

formulalar hosil bo‘ladi.

1-misol. Tomonlari 7 sm, 9 sm, 14 sm bo‘lgan uchburchakning balandligi, bissektrisasi va medianalarini toping.

$a=7, b=9$ va $c=14$ bo‘lsa, balandliklarni hisoblaymiz:

$$h_a=\frac{1}{2a}\sqrt{(2ab)^2-\left(a^2+b^2-c^2\right)^2}=\frac{1}{2\cdot 7}\sqrt{(2\cdot 7\cdot 9)^2-\left(7^2+9^2-14^2\right)^2}=$$

$$=\frac{1}{14}\sqrt{126^2-66^2}=\frac{1}{14}\cdot 48\sqrt{5}=\frac{24\sqrt{5}}{7};$$

$$h_b=\frac{1}{2b}\sqrt{(2ab)^2-\left(a^2+b^2-c^2\right)^2}=\frac{1}{2\cdot 9}\sqrt{(2\cdot 7\cdot 9)^2-\left(7^2+9^2-14^2\right)^2}=$$

$$=\frac{1}{18}\sqrt{126^2-66^2}=\frac{1}{18}\cdot 48\sqrt{5}=\frac{8\sqrt{5}}{3};$$

$$h_c=\frac{1}{2c}\sqrt{(2ab)^2-\left(a^2+b^2-c^2\right)^2}=\frac{1}{2\cdot 14}\sqrt{(2\cdot 7\cdot 9)^2-\left(7^2+9^2-14^2\right)^2}=$$

$$=\frac{1}{28}\sqrt{126^2-66^2}=\frac{1}{28}\cdot 48\sqrt{5}=\frac{12\sqrt{5}}{7}.$$

$a=7, b=9$ va $c=14$ bo‘lsa, bissektrisalarni hisoblaymiz:

$$\ell_a=\frac{1}{c+b}\sqrt{bc\left((b+c)^2-a^2\right)}=\frac{1}{14+9}\sqrt{9\cdot 14\left((9+14)^2-7^2\right)}=\frac{1}{23}\sqrt{9\cdot 14\left(23^2-7^2\right)}=$$

$$=\frac{1}{23}\sqrt{9\cdot 14\cdot 30\cdot 16}=\frac{4\cdot 3\cdot 2}{23}\sqrt{105}=\frac{24\sqrt{105}}{23};$$

$$\ell_b=\frac{1}{a+c}\sqrt{ac\left((a+c)^2-b^2\right)}=\frac{1}{7+14}\sqrt{7\cdot 14\left((7+14)^2-9^2\right)}=\frac{1}{21}\sqrt{7\cdot 14\cdot 30\cdot 12}=$$

$$=\frac{7\cdot 3\cdot 4\sqrt{5}}{21}=4\sqrt{5};$$

$$\ell_c=\frac{1}{a+b}\sqrt{ab\left((a+b)^2-c^2\right)}=\frac{1}{7+9}\sqrt{7\cdot 9\left((7+9)^2-14^2\right)}=\frac{1}{16}\sqrt{7\cdot 9\cdot 30\cdot 2}$$

$$==\frac{3\cdot 2\sqrt{105}}{16}=\frac{3\sqrt{105}}{8}.$$

$a=7, b=9$ va $c=14$ bo‘lsa, medianalarni hisoblaymiz:

$$m_a=\frac{1}{2}\sqrt{2\left(b^2+c^2\right)-a^2}=\frac{1}{2}\sqrt{2\left(9^2+14^2\right)-7^2}=\frac{1}{2}\sqrt{2(81+196)-49}=\frac{1}{2}\sqrt{505};$$

$$m_b=\frac{1}{2}\sqrt{2\left(a^2+c^2\right)-b^2}=\frac{1}{2}\sqrt{2\left(7^2+14^2\right)-9^2}=\frac{1}{2}\sqrt{2(49+196)-81}=\frac{1}{2}\sqrt{409};$$

$$m_c=\frac{1}{2}\sqrt{2\left(b^2+a^2\right)-c^2}=\frac{1}{2}\sqrt{2\left(9^2+7^2\right)-14^2}=\frac{1}{2}\sqrt{2(81+49)-196}=\frac{1}{2}\sqrt{64}=\frac{8}{2}=4.$$

Mustaqil ishlash uchun masalalar

2.61. Teng yonli uchburchakning asosi $4\sqrt{2}$ sm ga, yon tomonining medianasi esa 5 sm ga teng. Yon tomonlarning uzunligini toping. javob: 6

2.62. Teng yonli uchburchakning uchidagi burchagi 120^0 ga, shu uchidan tushirilgan balandligi esa 3 sm ga teng. Yon tomoni va asosining o‘rtasini tutashtiruvchi kesmaning uzunligini toping. javob: 6, $6\sqrt{3}$

2.63. Teng yonli uchburchakda asosga tushirilgan balandlik 15, yon tomoniga tushirilgan balandlik esa 20 ga teng. Uchburchak asosini toping. javob: $12\sqrt{5}$

2.64. Teng yonli uchburchak asosi 15 sm, yon tomonga tushirilgan balandligi 12 sm bo‘lsa, uchburchak yon tomonini toping. javob: 12,5

2.65. Teng yonli uchburchakning uchidagi burchagi 120° ga teng. Yon tomoni 4 ga teng. Uning yon tomoniga tushirilgan mediana uzunligining kvadratini toping. javob: $2\sqrt{7}$

2.66. Uchburchakning ikkita tomoni 20 va 15, shu tomonlarga mos medianalar to‘g‘ri burchak ostida kesishadi. Uchinchi tomonining uzunligini toping. javob: $5\sqrt{5}$

2.67. Teng yonli uchburchakda yon tomoni 4 sm, yon tomonga tushirilgan mediana 3 sm. Uchburchak asosini toping. javob: $\sqrt{10}$

2.68. Teng yonli uchburchakning yon tomonlari 20, asosidagi burchak kosinusi 0,8. Uchburchak perimetrini toping. javob: 72

2.69. Teng yonli ΔABC ning BC asosiga AD mediana o‘tkazilgan. Agar ΔABC ning perimetri 64 sm, ΔABD ning perimetri 52 sm bo‘lsa, AD mediana uzunligi topilsin. javob: 20

2.70. Teng yonli uchburchakning perimetri 26 sm, asosi yon tomonidan 6 marta kichik. Uchburchakning tomonlari topilsin. javob: 12

2.71. Teng yonli uchburchakning asosidagi burchagi, uchidagi burchagining 75% ini tashkil etadi. Uchidagi burchagining kattaligini toping. javob: 72º

2.72. Uchburchakning ichki burchaklari 1:1:2 nisbatda, katta tomoni 13 ga teng. Katta tomoniga o‘tkazilgan balandligini toping. javob: 6,5

2.73. Uchburchakning ichki burchaklari 3:1:2 nisbatda, kichik tomoni 5 ga teng. Katta tomonini toping. javob: 10

2.74. ABC uchburchakda BC=8 sm, AC va BC tomonlarga tushirilgan balandliklar mos ravishda 6,4 sm va 4 sm. AC va AB tomonlarni toping. javob: AC=5, $AB=\sqrt{41}$

2.75. Katetlari 24 va 18 sm bo‘lgan to‘g‘ri burchakli uchburchak o‘tkir burchaklarining bissektrisalarini toping. javob: $8\sqrt{10}, 9\sqrt{5}$

2.76. Teng yonli uchburchakning asosi 4 sm ga teng. Asosidagi burchagi 45^0 ga teng bo‘lsa, yon tomonini toping. javob: $2\sqrt{2}$

2.77. Agar teng yonli uchburchakning asosiga tushirilgan balandligi 35 sm ga teng bo‘lib, asosi bilan yon tomoni 48:25 nisbatda bo‘lsa, uchburchakning tomonlarini toping. javob: 125;125;240

2.78. Teng yonli uchburchakning asosi 8 sm ga teng. Asosidagi burchagi 60^0 ga teng bo‘lsa, yon tomonini toping. javob:8

2.79. Teng yonli uchburchakning yon tomoni va asosi mos ravishda 17 sm va 16 sm ga teng. Uchburchakning balandligini toping. javob: 15

2.80. Agar teng yonli uchburchakning balandligi 8 sm ga teng bo‘lib, asosi bilan yon tomoni 6:5 nisbatda bo‘lsa, uchburchakning tomonlarini toping. javob: 10;10;12

2.81. Teng yonli uchburchakning yon tomoni va asosi mos ravishda 5 sm va 6 sm ga teng. Uchburchakning balandligini toping. javob: 4

2.82. Teng yonli uchburchakning yon tomoni va asosi mos ravishda 55 sm va 66 sm ga teng. Uchburchakning balandligini toping. javob: 44

2.83. Agar teng yonli uchburchakning asosiga tushirilgan balandligi $2\sqrt{14}$ sm ga teng bo‘lib, asosi bilan yon tomoni $1:\sqrt{2}$ nisbatda bo‘lsa, uchburchakning tomonlarini toping. javob: 8;8; $4\sqrt{2}$

2.84. Teng yonli uchburchakning asosi 12 sm ga teng. Asosidagi burchagi 30^0 ga teng bo‘lsa, yon tomonini toping. javob: $4\sqrt{3}$

2.85. Teng yonli uchburchakning yon tomoni va asosi mos ravishda 26 sm va 48 sm ga teng. Uchburchakning balandligini toping. javob: 10

2.86. Tomonlari 7 sm, 11 sm, 12 sm bo‘lgan uchburchakning katta tomoniga tushirilgan medianani toping. javob: 7

2.87. Tomonlari 20, 20, 32 bo‘lgan teng yonli uchburchakning yon tomoniga tushirilgan balandlikni toping. javob: 19,2

2.88. Teng yonli uchburchak asosi 30 sm, balandligi 20 sm. Yon tomoniga tushirilgan balandlik uzunligini toping. javob: 24

2.89. *ABC* teng yonli uchburchak (*AB*=*BC*), yon tomoniga tushirilgan *AD* mediana 4 sm, asosidagi burchagi 30^0. Uchburchak yon tomonini toping. javob: $\frac{8\sqrt{7}}{7}$

2.90. Teng yonli uchburchak yon tomoni a=b=2 sm, uchidagi burchak 60^0. Yon tomoniga tushirilgan balandlik uzunligini toping. javob: $\sqrt{3}$

2.91. Uchburchak tomoni uzunligi 14 sm, unga tushirilgan balandlik 12 sm, boshqa ikkita tomoni yig‘indisi 28 sm. Shu tomonlar uzunligini toping. javob: 13;15

2.92. ABC uchburchakda AB=12 sm, BC=16 sm. Uchburchak medianalari AA_1 va CC_1 kesmalar 90° li burchak ostida kesishadi. AC tomon uzunligini toping. javob: $4\sqrt{5}$

2.93. Teng yonli uchburchakning asosiga parallel o‘rta chizig‘i uzunligi d sm ga, perimetri esa r sm ga teng. Shu uchburchakning tomonlari uzunliklarini toping. javob: $2d;\frac{r-2d}{2}$

2.94. Uchburchakning AB va AC tomonlari o‘rtalarini birlashtiruvchi kesma BC tomondan p sm, AC tomondan q sm, AB tomondan esa d sm qisqa. ΔABC ning perimetrini toping. javob: $4p+q+d$

2.95. Uchburchakning perimetri p sm ga teng, tomonlarining uzunliklari $k{:}m{:}n$ kabi nisbatda. Bu uchburchak tomonlarining o‘rtalari tutashtirib yangi uchburchak hosil qilindi. Hosil bo‘lgan uchburchakning tomonlari uzunliklari va perimetrini toping. javob: $\frac{kp}{2(k+m+n)};\frac{mp}{2(k+m+n)};\frac{np}{2(k+m+n)};\frac{p}{2}.$

2.96. Uchburchakning perimetri p ga teng. Uning o‘rta chiziqlari hosil qilgan uchburchaklarning tomonlari $k{:}m{:}n$ kabi nisbatda. Berilgan uchburchakning tomonlarini toping. javob: $\frac{2kp}{(k+m+n)};\frac{2mp}{(k+m+n)};\frac{2np}{(k+m+n)}.$

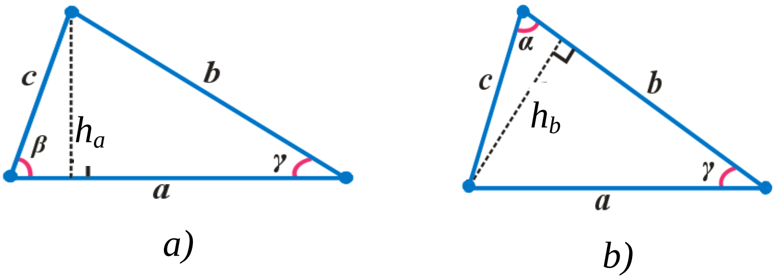
2.97. Uchburchaklarning tomonlari $k{:}m{:}n$ kabi nisbatda. Uchburchakning o‘rta chiziqlari hosil qilgan uchburchak perimetri esa q ga teng. Berilgan uchburchak tomonlarini toping. javob: $\frac{2kq}{(k+m+n)};\frac{2mq}{(k+m+n)};\frac{2nq}{(k+m+n)}.$

4-§. Uchburchak elementlari orasidagi burchaklar. Kosinuslar, sinuslar va tangenslar teoremasi

Ixtiyoriy berilgan uchburchakning noma'lum tomoni yoki noma'lum burchagini aniqlash masalasi asosan ikkita teoremadan – sinuslar va kosinuslar teoremasidan hal qilinadi.

14-teorema. (***Sinuslar teoremasi***) Uchburchak ixtiyoriy tomonining shu tomon qarshisidagi burchak sinusiga nisbati berilgan uchburchak uchun o'zgarmas kattalikdir.

$$\boxed{\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}.}$$



2.34-rasm

Isbot. a tomonga tushirilgan balandlik h_a ni burchak sinuslari orqali ifodalaymiz va ushbu $\begin{cases} h_a = c\cdot\sin\beta \\ h_a = b\cdot\sin\gamma \end{cases}$, $\Rightarrow$ $c\cdot\sin\beta = b\cdot\sin\gamma \Rightarrow$ $\frac{c}{\sin\gamma}=\frac{b}{\sin\beta}$ ga ega bo'lamiz (2.34-a rasm). b tomonga tushirilgan balandlik h_b ni burchak sinuslari orqali ifodalaymiz va ushbu $\begin{cases} h_b = c\cdot\sin\alpha \\ h_b = a\cdot\sin\gamma \end{cases}$, $\Rightarrow$ $c\cdot\sin\alpha = a\cdot\sin\gamma$ $\Rightarrow$ $\frac{c}{\sin\gamma}=\frac{a}{\sin\alpha}$ ga ega bo'lamiz (2.34-b rasm). Demak h_a va h_b lardan foydalanib $\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}$ ekanligini aniqladik.

15-teorema. (**Kosinuslar teoremasi**) Uchburchak ixtiyoriy tomonining kvadrati qolgan ikkita tomonlari kvadratlarining yig'indisidan shu tomonlar va ular orasidagi burchak kosinusi ko'paytmasining ikkilanganligini ayirish natijasiga tengdir:

$$\boxed{\begin{aligned} c^2 &= a^2+b^2-2ab\cos\gamma;\\ b^2 &= a^2+c^2-2ac\cos\beta;\\ a^2 &= b^2+c^2-2bc\cos\alpha.\end{aligned}}$$

Isbot. Tomonlari *a*, *b* va *c* ga teng bo‘lgan uchburchak berilgan bo‘lsin. Uchburchakning AD balandlik tushirib ΔADC va ΔABD to‘g‘ri burchakli uchburchaklarga ega bo‘lamiz (2.35-rasm).

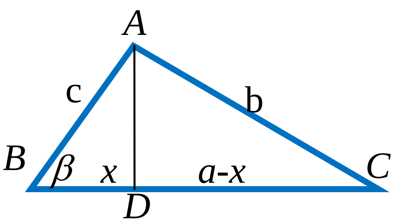
Pifagor teoremasidan foydalanib ushbu formulani keltirib chiqaramiz:

ΔABD dan $AD^2 = c^2 - x^2$ va $\cos\beta = \frac{x}{c}$, $x = c\cos\beta$.

ΔADC dan $AD^2 = b^2-(a-x)^2 \Rightarrow$

$c^2 - x^2 = b^2 - a^2 + 2ax - x^2 \Rightarrow$

$b^2 = c^2 + a^2 - 2ax \Rightarrow b^2 = c^2 + a^2 - 2ac\cos\beta$.



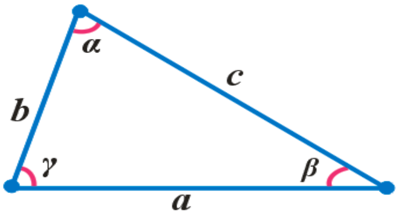
2.35-rasm

Qolgan tomonlarning formulasi ham shu usulda keltirib chiqariladi.

Kosinuslar teoremasini uchburchakning ikki tomoni va ular orasidagi burchak ma’lum bo‘lganda uchinchi tomonni aniqlash formulasidir deb ham aytish mumkin.

Kosinuslar teoremasidan foydalanib uchburchakning uchta tomoni berilganda uchta burchakni ham aniqlash mumkin bo‘ladi (2.36-rasm).

$$\boxed{\begin{aligned} \cos\alpha &= \frac{b^2+c^2-a^2}{2bc};\\ \cos\beta &= \frac{a^2+c^2-b^2}{2ac};\\ \cos\gamma &= \frac{a^2+b^2-c^2}{2ab}.\end{aligned}}$$



2.36-rasm

Kosinuslar teoremasidan foydalanib uchburchakning qachon o‘tkir burchakli, qachon to‘g‘ri burchakli va qachon o‘tmas burchakli bo‘lishi haqida aytish mumkin. Agar γ burchak uchburchakning eng katta burchagi deb hisoblasak, u holda uchburchakning o‘tkir burchakli, to‘g‘ri burchakli va o‘tmas burchakli bo‘lish shartlari quyidagicha bo‘ladi:

$$\begin{cases} c^2 < a^2 + b^2 \text{ da o'tkir burchakli;} \\ c^2 = a^2 + b^2 \text{ da to'g'ri burchakli;} \\ c^2 > a^2 + b^2 \text{ da o'tmas burchakli.} \end{cases}$$

Sinuslar va kosinuslar teoremasidan foydalanib ixtiyoriy uchburchakning noma’lum burchagi va noma’lum tomonini aniqlash mumkin.

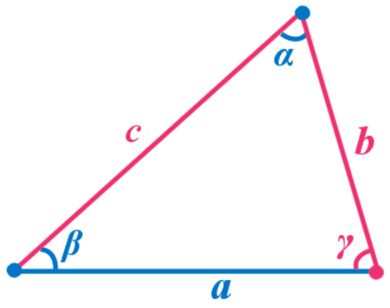
Uchburchakning tomonlar va burchaklariga oid masalalarni yechishda quyidagi holatlar bo‘lishi mumkin:

1) uchburchakning bir tomoni va ikkita burchagi berilgan;
2) uchburchakning ikki tomoni va ular orasidagi burchagi berilgan;
3) uchburchakning ikki tomoni va bu tomonlardan birining qarshisidagi burchak berilgan;
4) uchburchakning uchta tomoni berilgan.

Yuqorida keltirilgan 4 ta holning har biriga misollar yechish orqali to‘xtalib o‘tamiz.

1-masala. Uchburchakning bitta tomoni $a = 32$ va ikkita burchagi $\alpha = 64^0, \beta = 42^0$ ga teng. Bu uchburchakning qolgan ikkita b va c tomonlari hamda uchinchi burchagi γ ni aniqlang.

Yechish. Uchburchak ichki burchaklari yig‘indisi 180^0 ga tengligidan γ burchakni topamiz. Unga ko‘ra $\gamma = \pi - (\alpha + \beta) = 180^0 - (64^0 + 42^0) = 74^0$ bo‘ladi. Sinuslar teoremasi $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$ dan foydalanib, b va c tomonlarini aniqlaymiz. Unga ko‘ra noma’lum tomonlar:



2.37-rasm

$$b = \frac{\sin\beta}{\sin\alpha}a = \frac{\sin 42^0}{\sin 64^0}a = \frac{0{,}66913}{0{,}89879}\cdot 32 = 23{,}823,\ c = \frac{\sin\gamma}{\sin\alpha}a = \frac{\sin 74^0}{\sin 64^0}a = \frac{0{,}96126}{0{,}89879}\cdot 32 = 34{,}224$$

bo‘ladi.

2-masala. Uchburchakning ikkita tomoni $a=24, c=18$ va bu tomonlar orasidagi burchagi $\beta=25^0$ ga teng. Bu uchburchakning uchinchi b tomonini hamda qolgan ikkita α va γ burchaklarini aniqlang.

Yechish. Kosinuslar teoremasidan foydalanib, b tomonini aniqlaymiz. Unga ko‘ra

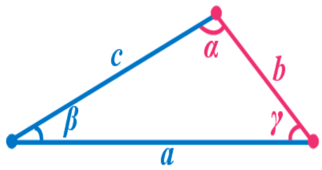
$$b^2=a^2+c^2-2ac\cos\beta=24^2+18^2-2\cdot24\cdot18\cdot\cos25^0=576+324-783=117\Rightarrow$$

$$b=\sqrt{117}=10{,}814$$

bo‘ladi. Sinuslar teoremasi $\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}$ dan foydalanib α va γ burchaklarni aniqlaymiz.

Unga ko‘ra noma’lum burchaklar

$$\sin\alpha=\frac{a}{b}\sin\beta=\frac{24}{10{,}814}\sin25^0=0{,}93794\Rightarrow\ \alpha=110{,}295^0$$



2.38-rasm

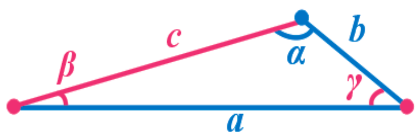
va

$$\sin\gamma=\frac{c}{b}\sin\beta=\frac{18}{10{,}814}\sin25^0=0{,}70345\Rightarrow\ \gamma=44{,}705^0$$

bo‘ladi. Topilgan burchaklardan ichki burchaklar yig‘indisi $\alpha+\beta+\gamma=110{,}295^0+25^0+44{,}705^0=180^0$ ekaniga ishonch hosil qilish mumkin.

3-masala. Uchburchakning ikkita tomoni $a=27, b=9$ va bu tomonlardan birining qarshisidagi burchak $\alpha=128^0$ ga teng. Bu uchburchakning qolgan ikkita β va γ burchaklari hamda uchinchi tomoning qiymatini toping.

Yechish. Sinuslar teoremasi $\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}$ dan foydalanib, β burchakni aniqlaymiz. Unga ko‘ra

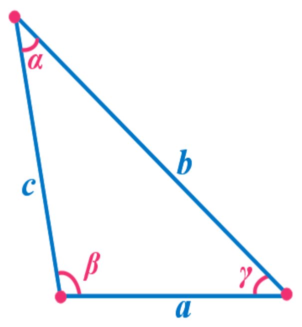


2.39-rasm

$\sin\beta=\frac{b}{a}\sin\alpha=\frac{9}{27}\sin128^0=0{,}26267,\ \rightarrow\beta=15{,}2286^0$ bo‘ladi. Uchburchak ichki burchaklari yig‘indisi 180^0 ga tengligidan γ burchakni topamiz. Unga ko‘ra $\gamma=\pi-(\alpha+\beta)=180^0-(128^0+15{,}2286^0)=36{,}7714^0$ bo‘ladi. Uchinchi c tomonni kosinuslar teoremasidan topamiz. Unga ko‘ra $c^2 = a^2 + b^2 - 2ab\cos\gamma = 27^2 + 9^2 - 2\cdot 27\cdot 9\cdot \cos 36,7714^0 = 729 + 81 - 389,3 = 420,7 \Rightarrow$

$c = \sqrt{420,7} = 20,51$ bo‘ladi.

4-masala. Uchburchakning uchta tomoni $a = 16, b = 26, c = 18$ ekani ma’lum bo‘lsa, qolgan uchta burchaklarini aniqlang.



2.40-rasm

Yechish. Kosinuslar teoremasidan foydalanib, uchta burchakni ham aniqlaymiz. Unga ko‘ra

$$\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc} = \frac{26^2 + 18^2 - 16^2}{2\cdot 26\cdot 18} = \frac{676 + 324 - 256}{2\cdot 26\cdot 18} = \frac{744}{2\cdot 26\cdot 18} = 0,79487,$$

$$\cos\beta = \frac{a^2 + c^2 - b^2}{2ac} = \frac{16^2 + 18^2 - 26^2}{2\cdot 16\cdot 18} = \frac{256 + 324 - 676}{2\cdot 16\cdot 18} = \frac{-96}{2\cdot 16\cdot 18} = -0,16667,$$

$$\cos\gamma = \frac{a^2 + b^2 - c^2}{2ab} = \frac{16^2 + 26^2 - 18^2}{2\cdot 16\cdot 26} = \frac{256 + 676 - 324}{2\cdot 16\cdot 26} = \frac{608}{2\cdot 16\cdot 26} = 0,73077.$$

Bulardan burchaklar $\alpha = \arccos 0,79487 = 37,357^0$, $\beta = \arccos(-0,16667) = 99,594^0$, $\gamma = \arccos 0,73077 = 43,049^0$ bo‘ladi. Topilgan burchaklardan ichki burchaklar yig‘indisi $\alpha + \beta + \gamma = 37,357^0 + 99,594^0 + 43,049^0 = 180^0$ ekaniga ishonch hosil qilish mumkin.

Mustaqil ishlash uchun masalalar

2.98. Uchburchakning tomonlari 4, 5 va 6 ga teng. 5 ga teng bo‘lgan tomon qarshisidagi burchakning kosinusini toping. javob: 9/16

2.99. Uchburchakning tomonlari 8 sm, 15 sm, 17 sm ga teng. Katta tomon qarshisidagi burchak topilsin. javob: 90º

2.100. Uchburchakning tomonlari 3; 5 va 6 ga teng. 5 ga teng bo‘lgan tomon qarshisidagi burchakning kosinusini toping. javob: 5/9

2.101. *ABC* uchburchakda *AB*=3, *BC*=4 va cos*B*=2/3, bo‘lsa, *AC* ning qiymatini toping. javob: 3

2.102. *ABC* uchburchakda *AB*=3, *CB*=4 va cos*B*=-11/24 bo‘lsa, *AC* ning qiymatini toping. javob: 6

2.103. Uchburchakning tomonlari *a*, *b* va *c* ga teng. Bu uchburchakning tomonlari orasida $a^2 = b^2 + c^2 + bc$ munosabat o‘rinli bo‘lsa, uzunligi *a* ga teng tomon qarshisidagi burchakni toping. javob: 120º

2.104. Uchburchakning tomonlarining uzunliklari *m*, *p* va *k* ga teng bo‘lib, $m^2 = n^2 + k^2 + \sqrt{2}nk$ tenglikni qanoatlantiradi. Uzunligi *m* ga teng tomon qarshisidagi burchakni toping. javob: 135º

2.105.Uchburchakning *a*, *b* va *c* ga teng tomonlari $a^2 = b^2 + c^2 + \sqrt{3}bc$ tenglikni qanoatlantiradi. Uzunligi *a* ga teng tomon qarshisidagi burchakni toping. javob: 150º

2.106.Uchburchakning *a*, *b* va *c* tomonlari orasida $a^2 = b^2 + c^2 - \sqrt{3}bc$ bog‘lanish mavjud. Uzunligi *a* ga teng bo‘lgan tomon qarshisidagi burchakni toping. javob: 30º

2.107. Uchburchakning *b* va *c* teng tomonlari orasidagi burchagi 30° ga teng. Uchburchakning uchinchi tomoni 12 ga teng bo‘lsa hamda uning tomonlari $c^2 = b^2 + 12b + 144$ shartni qanoatlantirsa, c ning qiymatini toping. javob: $12\sqrt{3}$

2.108.Uchburchakning *b* va *c* ga teng tomonlari orasidagi burchagi 30° ga teng. Uchburchakning uchinchi tomoni 16 ga teng bo‘lsa, hamda uning tomonlari $c^2 = b^2 + 16b + 256$ shartni qanoatlantirsa, *c* ning qiymati qanchaga teng bo‘ladi? javob: $16\sqrt{3}$

2.109. Uchburchakning ikki burchagi 30° va 45°. Agar 30° li burchak qarshisidagi tomoni $\sqrt{2}$ ga teng bo‘lsa, 45° li burchak qarshisidagi tomonni toping. javob: 2

2.110. ABC uchburchakda $\angle BAC$=45° ga, $\angle ACB$=30° ga va $CB = 14\sqrt{2}$ ga teng. AB tomonning uzunligini toping. javob: 14

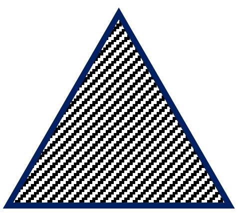
2.111.ABC uchburchakning AK medianasi AC tomon bilan 30° burchak tashkil qiladi. Agar $AK = \frac{13\sqrt{2}}{4}$ va $\angle BCA$ = 45° bo‘lsa, BC tomonning uzunligini toping. javob: 6,5

2.112. Uchburchakning tomonlari 35, 60 va 61 ga teng. Katta tomonga tushirilgan mediana nechaga teng? javob: 38,5

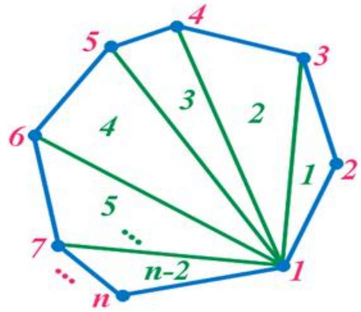
5-§. Yuza tushunchasi. Uchburchakning yuzi

Tekislikning chegaralangan qismining kattaligini o‘lchash uchun yuza tushunchasini kiritamiz. Masalani osonlashtirish uchun oldin sodda shakllarni qaraymiz.

Agar shaklni chekli sondagi uchburchaklarga bo‘lish mumkin bo‘lsa, bunday shaklni **sodda shakl** deyiladi (2.41-rasm). Tekislikning uchburchak bilan chegaralangan chekli qismi uchburchak yuzi deyiladi. Qavariq yassi ko‘pburchak sodda shaklga misol bo‘ladi. Bu ko‘pburchak bir uchidan o‘tkazilgan diagonallari bilan uchburchaklarga bo‘linadi. Sodda shakllar yuzining ta’rifini beramiz.



a)



b)

2.41-rasm

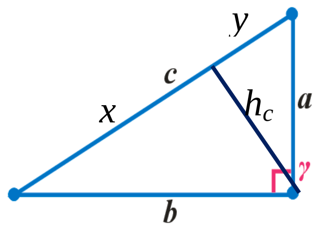
Sodda shakllar uchun yuza – bu musbat miqdor (kattalik) bo‘lib, uning son qiymati quyidagi xossalarga ega:

1) teng shakllarning yuzalari teng;
2) agar shakllar sodda shakllardan iborat qismlarga bo‘linsa, u holda bu shaklning yuzi qismlari yuzlari yig‘indisiga teng;
3) tomoni o‘lchov birligiga teng bo‘lgan kvadratning yuzi birga teng;
4) shakl yuzi kattaligi unga joylashtirilgan birlik kvadratlan soniga teng.

Agar ta’rifda so‘z borayotgan kvadratning tomoni 1 m ga teng bo‘lsa, u holda yuz kvadrat metrlarda (m^2) ifodalanadi. Agar kvadratning tomoni 100 m bo‘lsa, u holda yuz gektarlarda ifodalanadi. Agar kvadratning tomoni 1km bo‘lsa, yuza kvadrat kilometrlarda ifodalanadi.

Uchburchak yuzasini aniqlay olish turmushda juda ham zarurdir. Uchburchaklar umumiy holda yoki xususiy holda berilishi mumkin. Endi uchburchak yuzasini aniqlash uchun bir necha formulalar keltirib chiqaramiz.

Katetlari a va b ga hamda gipotenuzasi c ga teng bo‘lgan to‘g‘ri burchakli uchburchakning yuzasini aniqlash formulalari quyidagicha bo‘ladi (2.42-rasm):



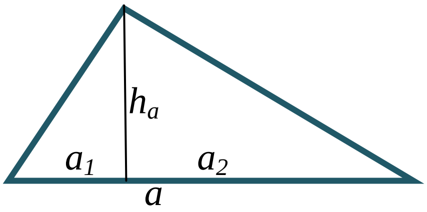
2.42-rasm

$$\boxed{S=\frac{1}{2}ab=\frac{1}{2}a\sqrt{c^2-a^2}=\frac{1}{2}b\sqrt{c^2-b^2},\quad S=\frac{1}{2}ch_c=\frac{1}{2}c\sqrt{xy}.}$$

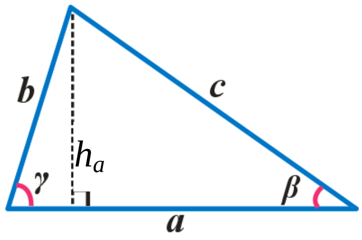
16-teorema. Uchburchak yuzasini tomon va tomonga tushirilgan balandlik orqali ifodalash formulalari quyidagicha bo‘ladi (2.43-*a* rasm):

$$\boxed{S=\frac{1}{2}ah_a,\quad S=\frac{1}{2}bh_b,\quad S=\frac{1}{2}ch_c.}$$

Isbot. Aytaylik balandlik uchburchakning ixtiyoriy a tomoniga tushirilgan bo‘lsin. Tushirilgan h_a balandlik a asosni ikkita a_1 va a_2 kesmalarga, uchburchakni esa ikkita to‘g‘ri burchakli uchburchakka ajratadi. Har bir to‘g‘ri burchakli uchburchak yuzasi bu to‘g‘ri burchakli uchburchakni qamragan to‘g‘ri to‘rtburchak yuzasining yarmiga teng, ya’ni $S_\Delta = S_1 + S_2 = \frac{1}{2}a_1 h_a + \frac{1}{2}a_2 h_a = \frac{1}{2}(a_1 + a_2)h_a = \frac{1}{2}ah_a$ bo‘ladi.



a)



b)

2.43-rasm

Uchburchakning yuzasini qolgan *b* va *c* tomonlariga tushirilgan balandliklari h_b va h_c lar orqali topish ham xuddi yuqoridagi kabi aniqlanadi.

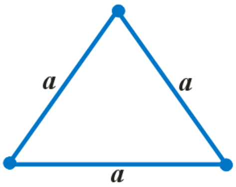
17-teorema. Uchburchak yuzasini ikki tomon va ular orasidagi burchak orqali aniqlash formulalari quyidagicha bo‘ladi (2.43-*b* rasm):

$$\boxed{S = \frac{1}{2}ab\sin\gamma, \quad S = \frac{1}{2}ac\sin\beta, \quad S = \frac{1}{2}bc\sin\alpha.}$$

Isbot. *a* tomoniga tushirilgan h_a balandlikni $h_a = b\sin\gamma$ yoki $h_a = c\sin\beta$ deb ifodalaymiz. Bundan oldingi chiqarilgan formuladan foydalansak, $S = \frac{1}{2}ah_a = \frac{1}{2}ab\sin\gamma$ yoki $S = \frac{1}{2}ah_a = \frac{1}{2}ac\sin\beta$ formulalar hosil bo‘ladi. Shuningdek, *b* tomoniga tushirilgan h_b balandlikni $h_b = a\sin\gamma$ yoki $h_a = c\sin\alpha$ deb ifodalash orqali $S = \frac{1}{2}bc\sin\alpha$ yuzani chiqarish mumkin.

Tomoni *a* bo‘lgan muntazam uchburchakning yuzasini topish formulasi quyidagicha bo‘ladi:

$$\boxed{m = \frac{\sqrt{3}}{4}a^2.}$$



2.44-rasm

Isbot. Muntazam uchburchakda $a = b = c$ hamda $\alpha = \beta = \gamma = 60^0$ bo‘ladi. Ikki tomon va ular orasidagi burchakka ko‘ra uchburchak yuzasi

$S=\frac{1}{2}ab\sin\gamma=\frac{1}{2}aa\sin 60^0=\frac{\sqrt{3}}{4}a^2$ bo‘ladi (2.44-rasm).

Uchburchak yuzasini tomonlariga ko‘ra aniqlash formulasi quyidagicha bo‘ladi:

$$\boxed{S=\frac{1}{4}\sqrt{(2ab)^2-\left(a^2+b^2-c^2\right)^2}.}$$

Isbot. Bu masalani kosinuslar teoremasi hamda ikki tomon va ular orasidagi burchakka ko‘ra uchburchak yuzasini topish formulalaridan foydalanib hal qilamiz. Unga ko‘ra yuza uchun

$$S=\frac{1}{2}ab\sin\gamma=\frac{1}{2}ab\sqrt{1-\cos^2\gamma}=\frac{1}{2}ab\sqrt{1-\left(\frac{a^2+b^2-c^2}{2ab}\right)^2}=$$

$$=\frac{1}{2}ab\cdot\frac{1}{2ab}\sqrt{(2ab)^2-\left(a^2+b^2-c^2\right)^2}=\frac{1}{4}\sqrt{(2ab)^2-\left(a^2+b^2-c^2\right)^2}$$

formula hosil bo‘ladi.

18-teorema. (***Geron formulasi***) Uchburchak uchta tomoni berilgan bo‘lsa, uning yuzi

$$\boxed{S=\sqrt{p(p-a)(p-b)(p-c)}.}$$

ga teng. Bu yerda: $p=\frac{a+b+c}{2}$ – yarimperimetr.

Isbot. Bu masalani kosinuslar teoremasi hamda ikki tomon va ular orasidagi burchakka ko‘ra uchburchak yuzasini topish formulalaridan foydalanib hal qilamiz. Unga ko‘ra yuza uchun

$$S=\frac{1}{2}ab\sin\gamma=\frac{1}{2}ab\sqrt{1-\cos^2\gamma}=\frac{1}{2}ab\sqrt{1-\left(\frac{a^2+b^2-c^2}{2ab}\right)^2}=$$

$$=\frac{1}{2}ab\sqrt{\left(1-\frac{a^2+b^2-c^2}{2ab}\right)\cdot\left(1+\frac{a^2+b^2-c^2}{2ab}\right)}=\frac{1}{2}ab\cdot\frac{1}{2ab}\cdot$$

$$\cdot\sqrt{\left(2ab-a^2-b^2+c^2\right)\cdot\left(2ab+a^2+b^2-c^2\right)}=\frac{1}{4}\sqrt{\left(c^2-(a-b)^2\right)\cdot\left((a+b)^2-c^2\right)}=\frac{1}{4}\cdot$$

$$\cdot\sqrt{(c-a+b)\cdot(c+a-b)\cdot(a+b-c)\cdot(a+b+c)}=\frac{1}{4}\sqrt{2(p-a)\cdot 2(p-b)\cdot 2(p-c)\cdot 2p}=$$

$$\sqrt{p(p-a)(p-b)(p-c)}$$

formula hosil bo‘ladi.

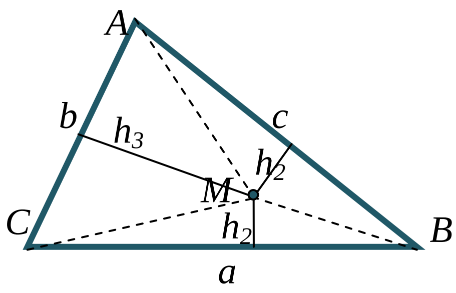
Masalalar ishlaganda tomonlar uzunliklari ancha katta sonlar bo‘lganda Geron formulasidan foydalanish, tomonlar uzunliklarida irratsional sonlar qatnashganda esa yuqoridagi formuladan foydalanish ancha qulaydir.

Agar uchburchak ichida olingan ixtiyoriy nuqtadan tomonlargacha bo‘lgan masofalar h_1, h_2, h_3 hamda uchburchak tomonlari ma’lum bo‘lsa, uchburchak yuzasini topish formulasi quyidagi ko‘rinishda bo‘ladi (2.45-rasm):

$$\boxed{S = \frac{1}{2}\left(ah_1 + bh_2 + ch_3\right).}$$

Isbot. Uchburchak ichida olingan M nuqtani uchburchak uchlari bilan tutashtirish natijasida uchta $\Delta MAB, \Delta MBC, \Delta MCA$ uchburchak hosil bo‘ladi. Bu uchburchak yuzalari mos holda

$$S_{MAB} = \frac{1}{2}ah_1, S_{MBC} = \frac{1}{2}bh_2, S_{MCA} = \frac{1}{2}ch_3$$



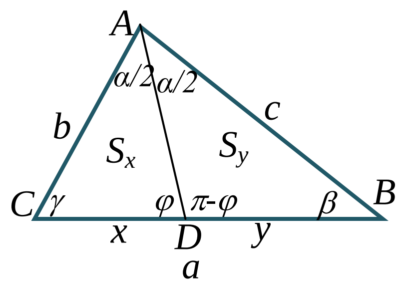
2.45-rasm

ga teng bo‘ladi. Bu uchburchaklar yuzalarining yig‘indisi esa berilgan ABC uchburchak yuzasiga teng bo‘ladi, ya’ni

$$S = S_{MAB} + S_{MBC} + S_{MCA} = \frac{1}{2}ah_1 + \frac{1}{2}bh_2 + \frac{1}{2}ch_3 = \frac{1}{2}\left(ah_1 + bh_2 + ch_3\right)$$ bo‘ladi.

19-teorema. *a* tomonga tushirilgan bissektrisa ℓ_a shu tomonni x va y kesmalarga, uchburchakning to‘la yuzasini esa S_x va S_y yuzalarga ajratadi (2.46-rasm). Bunda bissektrisa ajratgan kesmalar nisbati mos yon tomonlar nisbatiga yoki mos ajratgan yuzalar nisbatiga teng bo‘ladi:

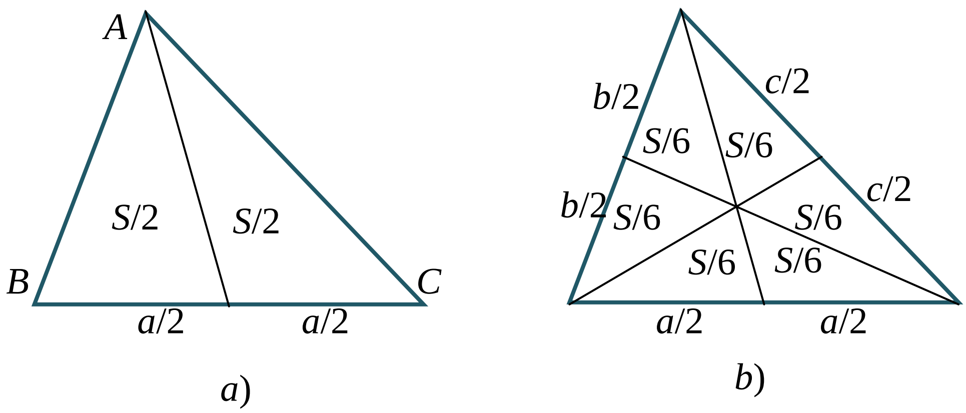
$$\boxed{\frac{x}{y} = \frac{b}{c} = \frac{S_x}{S_y}.}$$



2.46-rasm

Isbot. ΔACD dan $\frac{x}{\sin\alpha/2}=\frac{b}{\sin\varphi}$ va ΔABD dan $\frac{y}{\sin\alpha/2}=\frac{c}{\sin(\pi-\varphi)}=\frac{c}{\sin\varphi}$ bo‘ladi. Ularning nisbati $\frac{x}{y}=\frac{b}{c}$ kelib chiqadi. Bissektrisa ajratgan yuzalar $S_x=\frac{1}{2}b\,\ell_a\sin\alpha/2$ va $S_y=\frac{1}{2}c\,\ell_a\sin\alpha/2$ bo‘ladi.

Ularning nisbati $\frac{S_x}{S_y}=\frac{b}{c}$ hosil bo‘ladi. Shunday qilib, $\frac{x}{y}=\frac{b}{c}=\frac{S_x}{S_y}$ bo‘lar ekan.

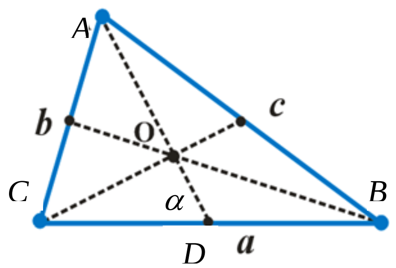


2.47-rasm

Uchburchak tomoniga tushirilgan mediana uchburchakni ikkita teng yuzali uchburchaklarga ajratadi (2.47-*a* rasm). Uchburchakning uchta tomoniga tushirilgan medianalar uchburchakni oltita teng yuzali uchburchaklarga ajratadi (2.47-*b* rasm). Shuning uchun ham uchburchak medianalar kesishgan nuqtasi uning og‘irlik markazi hisoblanadi.

Boshqacha aytganda, uchburchakni medianalar kesishgan nuqtasidan osib qo‘yilganda biror tomonga og‘masdan gorizontal holda turadi.

Medianalar kesishish nuqtasi burchak uchidan boshlab hisoblaganda $2:1$ nisbatda bo‘linadi (2.48-rasm):



$$\boxed{\frac{AO}{OD}=2.}$$

2.48-rasm

Isbot. $S_{ADC}=\frac{1}{2}CD\cdot DA\cdot\sin\alpha=\frac{S_{ABC}}{2}$ va $S_{ODC}=\frac{1}{2}CD\cdot DO\cdot\sin\alpha=\frac{S_{ABC}}{6}$

bo'lgani uchun $\frac{S_{ADC}}{S_{ODC}}=\frac{\frac{1}{2}CD\cdot DA\cdot\sin\alpha}{\frac{1}{2}CD\cdot DO\cdot\sin\alpha}=\frac{DA}{DO}=\frac{\frac{S_{ABC}}{2}}{\frac{S_{ABC}}{6}}=3.$

$AD=AO+OD$ va $AD=3OD$ ekanligidan $3DO=AO+OD\Rightarrow 2DO=AO\Rightarrow\frac{AO}{OD}=2.$

Mustaqil ishlash uchun masalalar

2.113. To‘g‘ri burchakli uchburchakning to‘g‘ri burchagi bissektrisasi gipotenuzani 3 sm va 4 sm li kesmalarga bo‘ladi. Uchburchakning yuzini toping. javob: $11\frac{19}{25}$

2.114. Uchburchakning ikkita tomoni mos ravishda 1 sm va $\sqrt{15}$ sm ga, uchinchi tomoniga tushirilgan mediana 2 sm ga teng bo‘lsa, uchburchakning yuzini toping. javob: $\frac{\sqrt{15}}{2}$

2.115. *ABC* to‘g‘ri burchakli uchburchakda $\angle C=30°$, $\angle B$ to‘g‘ri burchak uchidan *BK* mediana o‘tkazilgan. Agar *AB* katet 4 sm ga teng bo‘lsa, uchburchak *BCK* ning yuzini toping. javob: $4\sqrt{3}$

2.116. Yuzi 12 sm^2 bo‘lgan uchburchakda tomonlarining o‘rtalari tutashtirildi, yangi hosil bo‘lgan uchburchakda xuddi shunday yo‘l bilan yana yangi uchburchak hosil qilindi va hokazo. Hosil bo‘lgan barcha uchburchaklar yuzalari yig‘indisini toping. javob:16

2.117. To‘g‘ri burchakli uchburchak gipotenuzasi 15 sm, perimetri 36 sm. Katetlarini toping. javob: 9;12

2.118. Teng yonli uchburchak asosiga tushirilgan balandlik 10 sm, yon tomoniga tushirilgan balandlik 12 sm. Uchburchak yuzini toping. javob:75

2.119. To‘g‘ri burchakli uchburchak perimetri 60 sm. Gipotenuzaga tushirilgan balandlik 12 sm. Uchburchak yuzini toping. javob:150

2.120. Teng tomonli uchburchakning *a* tomoni o‘rtasidan yon tomonlariga perpendikularlar o‘tkazilgan. Perpendikularlar asosi tutashtirilgan. Hosil bo‘lgan uchburchak yuzini toping. javob: $\frac{3\sqrt{3}}{64}a^2$

2.121. To‘g‘ri burchak ichida tomonlaridan 4 va 8 sm masofada joylashgan nuqta berilgan. Shu nuqtadan o‘tuvchi to‘g‘ri chiziq, to‘g‘ri burchakdan yuzi 100 sm^2 bo‘lgan uchburchak kesadi. Uchburchak katetlarini toping. javob:(5;40), (10;20)

2.122. To‘g‘ri burchakli uchburchak katetlaridan biri 24 sm, uchburchak yuzi 216 sm^2. Gipotenuzani toping. javob:30

2.123. Agar *ABC* uchburchakda *AC*=11 sm, *AD* mediana 10 sm, uchburchak yuzi *S*=66 sm^2 bo‘lsa, *BC* tomonni toping. javob: $6\sqrt{5}$

2.124. *ABC* uchburchakda *BC*=2 sm, yuzi 8 sm^2, sin*B*=0,8. *AC* ga o‘tkazilgan balandlikni toping. javob: $\frac{2\sqrt{5}}{5}$

2.125. To‘g‘ri burchakli uchburchak gipotenuzasi 4 sm. Gipotenuzaga tushirilgan balandlik to‘g‘ri burchakni 1:2 nisbatda bo‘ladi. Uchburchak yuzini toping. javob: $\frac{8\sqrt{2}}{3}$

2.126. To‘g‘ri burchakli uchburchak gipotenuzasi 74 sm, perimetri 168 sm. Uchburchak yuzini toping. javob: 840

2.127. O‘tkir burchakli uchburchakning ikkita tomoni mos ravishda $2\sqrt{2}$ va 3 ga teng. Uchburchakning yuzi 3 ga teng bo‘lsa, uning uchinchi tomonini toping. javob: $\sqrt{3}; \sqrt{23}$

2.128. *CD* to‘g‘ri burchak bissektrisasi gipotenuzani 30 va 40 sm li kesmalarga bo‘ladi. Uchburchak yuzini toping. javob:1176

2.129. Teng yonli uchburchakda yon tomonlari 34, asosidagi burchak kotangensi $\frac{15}{8}$. Uchburchak yuzini toping. javob:480

2.130. Tomoni $2\sqrt[4]{3}$ bo‘lgan teng tomonli uchburchak yuzini toping. javob:3

2.131. *ABC* uchburchakda $AB = 5$ sm, $AC = 10$ sm va $\angle A = 45°$ ga teng. Shu uchburchakning yuzini toping. javob: $\frac{25\sqrt{2}}{2}$

2.132. *ABC* uchburchakda $AB = 4$ sm, $AC = 5$ sm va $\angle A = 45°$. Shu uchburchakning yuzini toping. javob: $5\sqrt{2}$

2.133. *ABC* uchburchakda $AB = 3$ sm, $AC = 6$ sm va A burchak 30° ga geng. Shu uchburchakning yuzini toping. javob:4,5

2.134. Uchburchakning yuzi 6 ga teng. Shu uchburchakning 3 va 8 ga teng tomonlari orasidagi burchakni toping. javob:30º

2.135. Muntazam uchburchakning yuzi $25\sqrt{3}$ ga teng. Uning tomonini toping. javob: 10

2.136. Muntazam uchburchakning yuzi 64 ga teng. Uning perimetrini toping. javob: $16\sqrt[4]{27}$

2.137. Tomonning uzunligi $4\sqrt[4]{3}$ bo‘lgan muntazam uchburchakning yuzini toping. javob:12

6-§. Burchakka bog‘liq proporsional kesmalar. Uchburchakning o‘xshashligi alomatlari. Fales teoremasi

Agar berilgan shaklning har bir nuqtasi biroz siljitilsa, yangi shakl hosil qilinadi. Bu shakl berilgan shakldan almashtirish natijasida hosil qilindi deyiladi.

1-ta'rif. Agar shaklni siljitish natijasida ikki shakldan biri ikkinchisiga o'tsa, ular ***teng shakllar*** deyiladi. Agar ikki shaklnins yuzalari teng bo'lsa ***dengdosh shakllar*** deyiladi.

2-ta'rif. Agar bir shaklni ikkinchi shaklga almashtirishda nuqtalar orasidagi masofalar bir xil nisbatda o'zgarsa, bunday almashtirish ***o'xshash almashtirish*** deyiladi.

3-ta'rif. Agar ikki shakl o'xshashlik almashtirishida bir-biriga o'tsa, ular ***o'xshash shakllar*** deyiladi. Shakllarning o'xshashligini belgilash uchun maxsus ~ belgidan foydalaniladi.

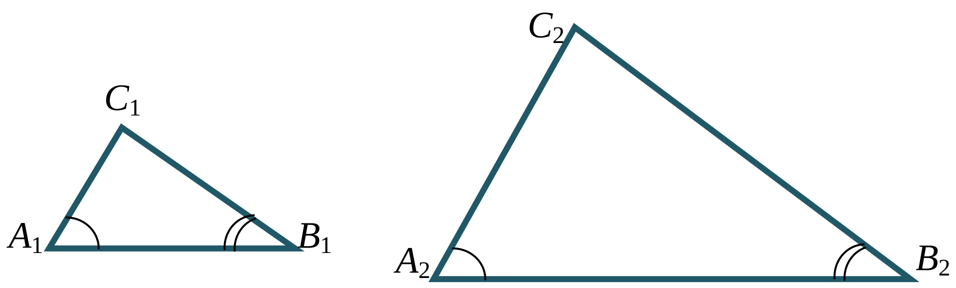
O'xshashlik almashtirishning xossalaridan ushbu xulosa chiqadi: o'xshash shakllarning mos burchaklari teng, mos kesmalari proporsional. Masalan: $\Delta ABC \sim \Delta A_1B_1C_1$ o'xshash uchburchaklardan:

$$\angle A=\angle A_1,\ \angle B=\angle B_1,\ \angle C=\angle C_1;$$

$$\frac{AB}{A_1B_1}=\frac{BC}{B_1C_1}=\frac{AC}{A_1C_1}.$$

Uchburchaklar o'xshashligining uchta alomati bor: 1) ikkita burchakka ko'ra; 2) ikki tomon va ular orasidagi burchakka ko'ra; 3) uchta tomonga ko'ra. Bularning har biriga alohida-alohida to'xtalib o'tamiz.

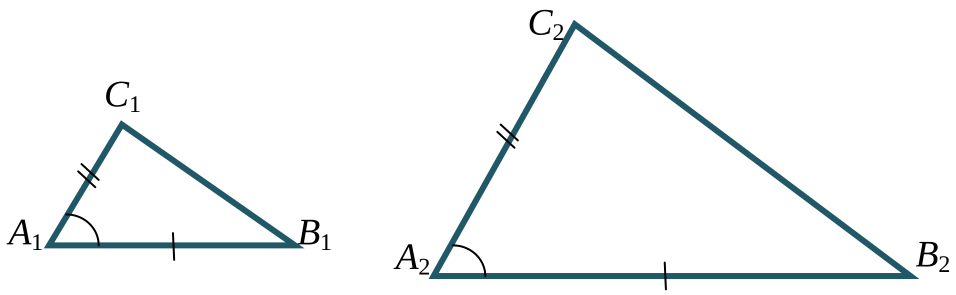
1-alomat. Agar bir uchburchakning ikkita burchagi mos holda ikkinchi bir uchburchakning ikkita burchagiga teng bo'lsa, u holda bu uchburchaklar o'xshashdir (2.49-rasm).



2.49-rasm

$$\text{Agar } \begin{cases} \angle A_1 = \angle A_2 \\ \angle B_1 = \angle B_2 \end{cases} \text{ bo‘lsa, u holda } \Delta A_1B_1C_1 \sim \Delta A_2B_2C_2 \text{ bo‘ladi.}$$

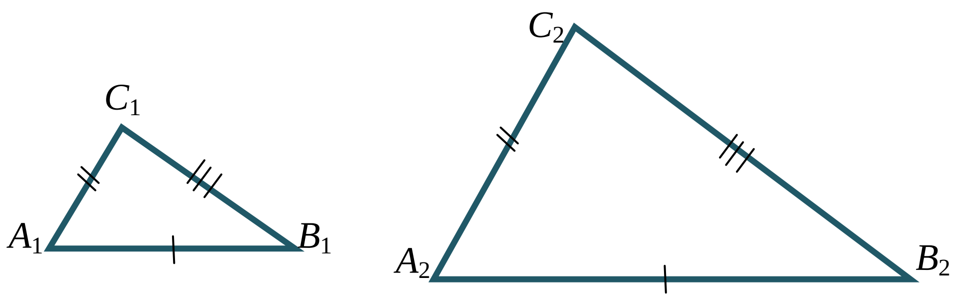
2-alomat. Agar bir uchburchakning ikkita tomoni mos holda ikkinchi uchburchakning ikkita tomoniga proporsional bo‘lib, ular orasidagi burchaklar o‘zaro teng bo‘lsa, u holda bu uchburchaklar o‘xshashdir (2.50-rasm).



2.50-rasm

$$\text{Agar } \begin{cases} A_2B_2 = k \cdot A_1B_1 \\ A_2C_2 = k \cdot A_1C_1 \\ \angle A_2 = \angle A_1 \end{cases} \text{ bo‘lsa, u holda } \Delta A_1B_1C_1 \sim \Delta A_2B_2C_2 \text{ bo‘ladi.}$$

3-alomat. Agar bir uchburchakning uchta tomoni mos holda ikkinchi bir uchburchakning uchta tomoniga proporsional bo‘lsa, u holda bu uchburchaklar o‘xshashdir (2.51-rasm).



2.51-rasm

$$\text{Agar } \begin{cases} A_2B_2 = k \cdot A_1B_1 \\ A_2C_2 = k \cdot A_1C_1 \\ B_2C_2 = k \cdot B_1C_1 \end{cases} \text{ bo‘lsa, u holda } \Delta A_1B_1C_1 \sim \Delta A_2B_2C_2 \text{ bo‘ladi.}$$

Uchinchi alomatdan quyidagi natija kelib chiqadi:

Agar ikkita uchburchakning tomonlari, perimetrlari va yuzalari ma’lum bo‘lsa, u holda quyidagi munosabatlar o‘rinlidir.

$$\boxed{\frac{a_2}{a_1}=\frac{b_2}{b_1}=\frac{c_2}{c_1}=\frac{p_2}{p_1}=\sqrt{\frac{S_2}{S_1}}=k \quad \text{yoki} \quad \frac{S_2}{S_1}=\left(\frac{a_2}{a_1}\right)^2=\left(\frac{b_2}{b_1}\right)^2=\left(\frac{c_2}{c_1}\right)^2=\left(\frac{p_2}{p_1}\right)^2=k^2.}$$

Bu yerda: $k-$proporsionallik koeffitsiyenti bo‘lib, uni o‘xshashlik koeffitsiyenti deb yuritiladi. O‘xshashlik koeffitsiyenti bir uchburchakning ixtiyoriy chiziqli o‘lchami (tomon, ichki chizilgan radius, tashqi chizilgan radius, balandlik, bissektrisa, mediana va h.k.) boshqa uchburchakning shunga mos chiziqli o‘lchamidan necha marta farq qilishini bildiradi.

Xususiy hollarni qarab chiqamiz:

1. Bir muntazam uchburchak ikkinchi muntazam uchburchakka o‘xshashdir. Boshqacha aytganda, muntazam uchburchaklar o‘xshash bo‘lishi uchun shart qo‘yilmaydi.

2. Teng yonli uchburchaklar o‘xshash bo‘lishi uchun ularning uchidagi burchaklari teng bo‘lishi kifoyadir.

3. To‘g‘ri burchakli uchburchaklar o‘xshash bo‘lishi uchun bu uchburchaklarning ixtiyoriy bitta o‘tkir burchaklari teng bo‘lishi kifoyadir.

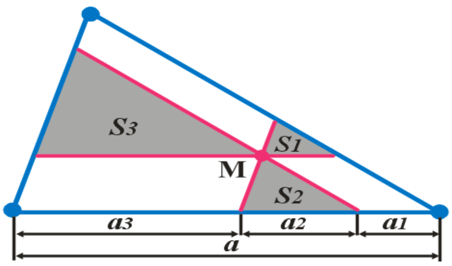
20-teorema. Agar ixtiyoriy uchburchakning ichidan olingan M nuqtadan uchburchak tomonlariga o‘tkazilgan parallel to‘g‘ri chiziqlar yuzalari S_1, S_2, S_3 bo‘lgan uchburchaklar ajratsa, berilgan uchburchak yuzi

$$\boxed{S=\left(\sqrt{S_1}+\sqrt{S_2}+\sqrt{S_3}\right)^2.}$$

ga teng.

Isbot. Hosil bo‘lgan uchta uchburchak berilgan katta uchburchakka o‘xshash uchburchaklardir (2.52-rasm).

Berilgan uchburchakning asosi a ga, hosil bo‘lgan uchburchaklarning asoslari esa mos holda a_1, a_2, a_3 ga teng. Hosil bo‘lgan uchburchaklarning berilgan uchburchaklar bilan o‘xshashlik koeffitsiyentlari mos holda



2.52-rasm

$k_1 = \sqrt{\frac{S_1}{S}} = \frac{a_1}{a},\ k_2 = \sqrt{\frac{S_2}{S}} = \frac{a_2}{a},\ k_3 = \sqrt{\frac{S_3}{S}} = \frac{a_3}{a}$ ga teng. O‘xshash uchburchaklar asoslari $a_1 = \sqrt{\frac{S_1}{S}}\,a,\ a_2 = \sqrt{\frac{S_2}{S}}\,a,\ a_3 = \sqrt{\frac{S_3}{S}}\,a$ bo‘ladi. Berilgan uchburchak asosi hosil bo‘lgan uchta o‘xshash uchburchaklar asoslarining yig‘indisiga teng, ya’ni $a = a_1 + a_2 + a_3$ bo‘ladi.

Bundan esa

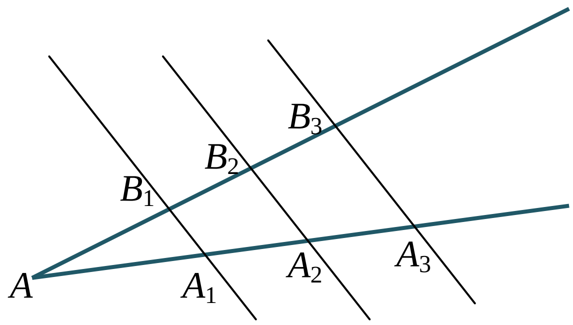
$$a = \sqrt{\frac{S_1}{S}}\,a + \sqrt{\frac{S_1}{S}}\,a + \sqrt{\frac{S_1}{S}}\,a = \left(\sqrt{S_1} + \sqrt{S_2} + \sqrt{S_3}\right)\cdot \frac{a}{\sqrt{S}},\ \Rightarrow$$

$$\sqrt{S} = \sqrt{S_1} + \sqrt{S_2} + \sqrt{S_3},\ \Rightarrow S = \left(\sqrt{S_1} + \sqrt{S_2} + \sqrt{S_3}\right)^2$$

formula kelib chiqadi.

21-teorema (*Fales teoremasi*). Agar burchak tomonini kesuvchi parallel to‘g‘ri chiziqlar burchakning tomonidan teng kesmalar ajratsa, ikkinchi tomondan ham teng kesmalar ajratadi.

Isbot. Shartga ko‘ra $A_1B_1 \| A_2B_2 \| A_3B_3 \|$ … parallel to‘g‘ri chiziqlar burchakning bir tomonidan $A_1A_2 = A_2A_3 = \ldots$ teng kesmalar ajratsa, ikkinchi tomondan ajratgan $B_1B_2 = B_2B_3 = \ldots$ kesmalar ham teng ekanligini isbotlash kerak (2.53-rasm).

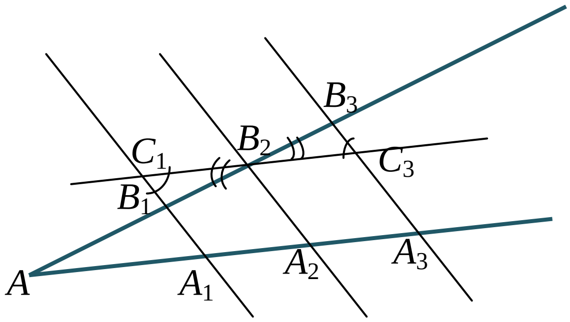


2.53-rasm

Isbot qilish uchun B_2 nuqtadan AA_1 tomonga parallel to‘g‘ri chiziq o‘tkazamiz. To‘g‘ri chiziqlarning parallellik alomatiga ko‘ra, A_1B_1, A_2B_2 va A_3B_3 parallel to‘g‘ri chiziqlar orasidagi parallel kesmalar teng (2.54-rasm).

$$A_1A_2=B_2C_1 \text{ va } A_2A_3=C_3B_2.$$

$\angle B_1B_2C_1=\angle B_3B_2C_3$ – vertikal burchaklar, A_1C_1 va A_3C_3 to‘g‘ri chiziqlarni C_1C_3 to‘g‘ri chiziq kesib o‘tganda $\angle B_1C_1B_2=\angle B_2C_3B_3$ ichki almashinuvchi burchaklar teng.



2.54-rasm

Uchburchaklar tengligining 1-alomatiga ko‘ra: $\Delta B_1C_1B_2=\Delta B_2C_3B_3$. Bundan mos tomonlari tengligi kelib chiqadi: $B_1B_2=B_2B_3$.

Teorema isbotlandi.

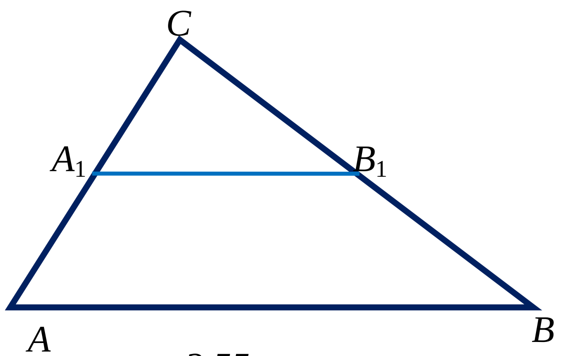
22-teorema. (*Umumlashgan Fales teoremasi*). Berilgan burchak tomonlarini kesuvchi parallel to‘g‘ri chiziqlar burchak tomonlaridan proporsional kesmalar ajratadi.

23-teorema. (*Falesga teskari teorema*). Agar burchak tomonlarini kesuvchi to‘g‘ri chiziqlar burchak tomonlaridan proporsional kesmalar ajratsa, bu to‘g‘ri chiziqlar parallel bo‘ladi.

24-teorema. Uchburchak o‘rta chizig‘i asosiga parallel va uning yarmiga teng.

Isbot. ΔABC uchburchak berilgan bo‘lsin. A_1 va B_1 mos ravishda AC va AB tomonlarning o‘rtalari bo‘lsin (2.55-rasm). Ta’rifga ko‘ra $CA_1=A_1A$ va $CB_1=B_1B$. Bundan $\frac{CA_1}{A_1A}=\frac{CB_1}{B_1B}$ bo‘lgani uchun Fales teoremasiga teskari teoremaga ko‘ra A_1B_1 va AB to‘g‘ri chiziqlar o‘zaro parallel bo‘ladi.

ΔABC va ΔA_1B_1C uchun $\angle C$ umumiy, to‘g‘ri chiziqlarning parallellik alomatiga ko‘ra $\angle A=\angle A_1$ va $\angle B=\angle B_1$ burchaklar teng. Uchburchak o‘xshashligining ikkinchi alomatiga ko‘ra ΔABC va ΔA_1B_1C uchburchaklar o‘xshash (2.56-rasm).

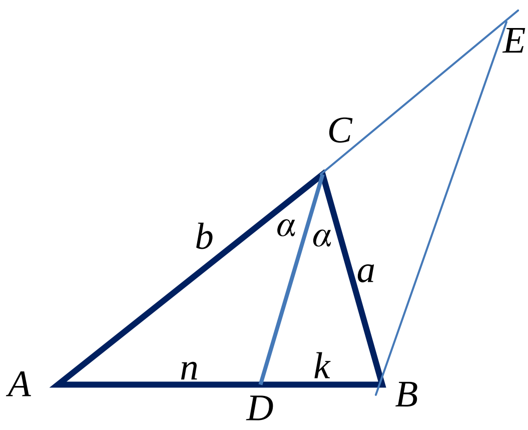


2.55-rasm

Bundan $CA_1=A_1A \Rightarrow CA=2CA_1$ va o‘xshashlik $\frac{CA_1}{CA}=\frac{A_1B_1}{AB}=\frac{CB_1}{CB}$ shartiga ko‘ra $\frac{A_1B_1}{AB}=\frac{CA_1}{2CA_1}=\frac{1}{2}$.

Uchburchakning ichki bissektirsasi, shu burchak qarshisidagi tomonni yon tomonlariga proporsional kesmalarga ajratadi. Fales teoremasidan foydalanib quyidagilarni isbotlaymiz.

1-masala. Agar uchburchak yon tomonlari a va b bo’lsa va shu tomonlar tutashgan uchdagi bissektrisasi qarshisidagi tomonni n va k uzunlikdagi kesmalarga ajratsa, u holda $\frac{n}{b}=\frac{k}{a}$ munosabat o’rinli bo’lishini isbotlang.



Isbot. Isbotlash uchun ABC uchburchakning B uchidan CD bissektrisaga parallel qilib BE chiziq o’tkazamiz. $CD \parallel BE$ bo’lgani uchun $\angle ACD=\angle CEB$ va ichki almashinuvchi burchaklar $\angle BCD=\angle CBE$. Demak, ΔBCE teng yonli va $BC=CE=a$.

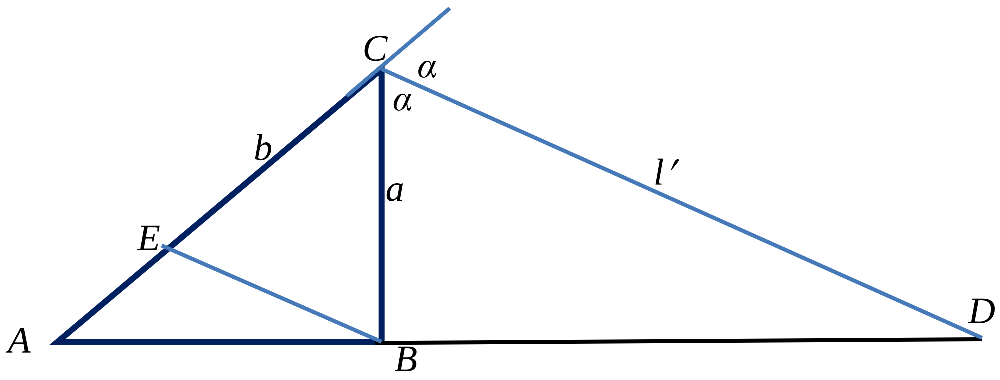
Fales teoremasiga ko’ra A burchak tomonlaridan parallel to’g’ri chiziqlar proporsional kesmalar ajratadi.

$$\frac{AC}{CE}=\frac{AD}{DB} \Rightarrow \frac{b}{a}=\frac{n}{k} \Rightarrow \frac{b}{a}=\frac{n}{k}.$$

Bizga ma’lumki, ikki qo’shni burchak bissektrisalari o’zaro perpendikular bo’ladi.

Ta'rif. Uchburchakning biror burchagi bissektrisasiga perpendikular bo'lgan va shu uchidan chiquvchi to'g'ri chiziq uchburchakning tashqi bissektrisasi deyiladi.

2-masala. Agar uchburchak yon tomonlari *a* va *b* bo'lsin. Shu tomonlar tutashgan uchdagi tashqi bissektrisasi qarshisidagi tomonni davomini D nuqtada kesib o'tsa, u holda $\frac{a}{b}=\frac{BD}{AD}$ munosabat o'rinli bo'lishini isbotlang.

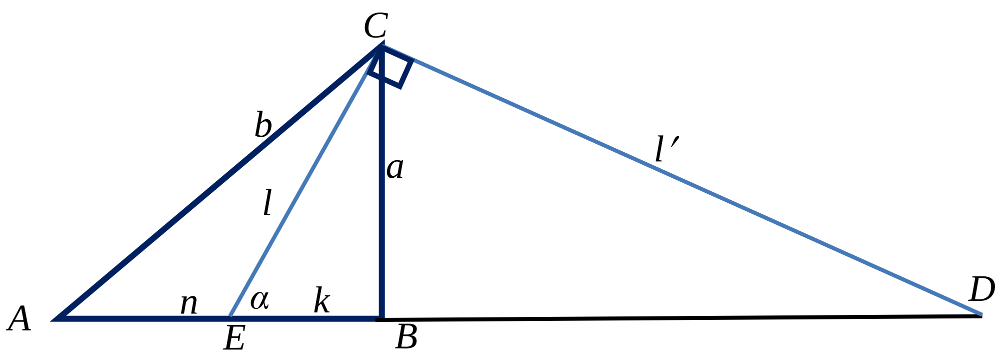


Isbot. Isbotlash uchun *ABC* uchburchakning *B* uchidan *CD* bissektrisaga parallel qilib *BE* chiziq o'tkazamiz. $CD \parallel BE$ bo'lgani uchun $\angle AEB=\angle ACD$ va ichki almashinuvchi burchaklar $\angle BCD=\angle CBE$. Demak, ΔBCE teng yonli va $BC=CE=a$.

Fales teoremasiga ko'ra *A* burchak tomonlaridan parallel to'g'ri chiziqlar proporsional kesmalar ajratadi.

$$\frac{AE}{EC}=\frac{AB}{BD}\Rightarrow\frac{b-a}{a}=\frac{AB}{BD}\Rightarrow\frac{b-a}{a}+1=\frac{AB}{BD}+1\Rightarrow\frac{b}{a}=\frac{AB+BD}{BD}=\frac{AD}{BD}\Rightarrow\frac{a}{b}=\frac{AB+BD}{BD}=\frac{BD}{AD}.$$

3-masala. Uchburchak *ABC* ning tomonlari *AC*=*a*, *BC*=*b*, *AB*=*c* bo'lsin. Aytaylik, *C* burchak bissektrisasi *l* ga perpendikular bo'lgan va *C* uchidan chiquvchi l' to'g'ri chiziq (tashqi bissektrisa) *AB* tomon davomini *D* nuqtada kesib o'tsin. *BD* kesma uzunligini toping.



Yechish. Shakldagidek belgilashlar kiritamiz. U holda

$$n=\frac{bc}{a+b};\ k=\frac{ac}{a+b};\ l=\sqrt{ab-nk}=\sqrt{ab-\frac{abc^2}{(a+b)^2}}=\frac{\sqrt{ab}}{a+b}\sqrt{(a+b)^2-c^2}$$

bo'ladi. Kosinuslar teoremasiga ko'ra:

$$a^2=l^2+k^2-2kl\cos\alpha;$$

$$a^2=\frac{ab}{(a+b)^2}\left((a+b)^2-c^2\right)+\frac{a^2c^2}{(a+b)^2}-2\frac{\sqrt{ab}}{(a+b)}\sqrt{(a+b)^2-c^2}\,\frac{ac}{(a+b)}\cos\alpha\Rightarrow$$

$$a^2(a+b)^2=ab\left((a+b)^2-c^2\right)+a^2c^2-2ac\sqrt{ab}\sqrt{(a+b)^2-c^2}\cos\alpha\Rightarrow$$

$$2ac\sqrt{ab}\sqrt{(a+b)^2-c^2}\cos\alpha=(a+b)^2\left(ab-a^2\right)+c^2\left(a^2-ab\right)\Rightarrow$$

$$2ac\sqrt{ab}\sqrt{(a+b)^2-c^2}\cos\alpha=\left(ab-a^2\right)\left((a+b)^2-c^2\right)\Rightarrow$$

$$\cos\alpha=\frac{a(b-a)\left((a+b)^2-c^2\right)}{2ac\sqrt{ab}\sqrt{(a+b)^2-c^2}}=\frac{(b-a)\sqrt{(a+b)^2-c^2}}{2c\sqrt{ab}}.$$

Bundan tashqari $\cos\alpha=\frac{l}{k+x}$ bo'lgani uchun

$$\frac{(b-a)\sqrt{(a+b)^2-c^2}}{2c\sqrt{ab}}=\frac{1}{k+x}=\frac{\frac{\sqrt{ab}}{a+b}\sqrt{(a+b)^2-c^2}}{\frac{ac}{a+b}+x}\Rightarrow$$

$$\frac{(b-a)\sqrt{(a+b)^2-c^2}}{2c\sqrt{ab}}=\frac{\sqrt{ab}\sqrt{(a+b)^2-c^2}}{ac+x(a+b)}\Rightarrow\frac{(b-a)}{2c}=\frac{ab}{ac+x(a+b)}\Rightarrow$$

$$(b-a)\left(ac+x(a+b)\right)=2abc\Rightarrow\left(b^2-a^2\right)x=ac(a+b)\Rightarrow x=\frac{ac}{b-a}.$$

Shu o'rinda, topilgan bog'lanishlar asosida quyidagilarni keltiramiz:

$$AD=AB+BD=c+\frac{ac}{b-a}=\frac{bc}{b-a}.$$

1) $\frac{AD}{BD}=\frac{bc}{b-a}:\frac{ac}{b-a}=\frac{b}{a}$, bu bizga ichki bissektrisa xossasini beradi.

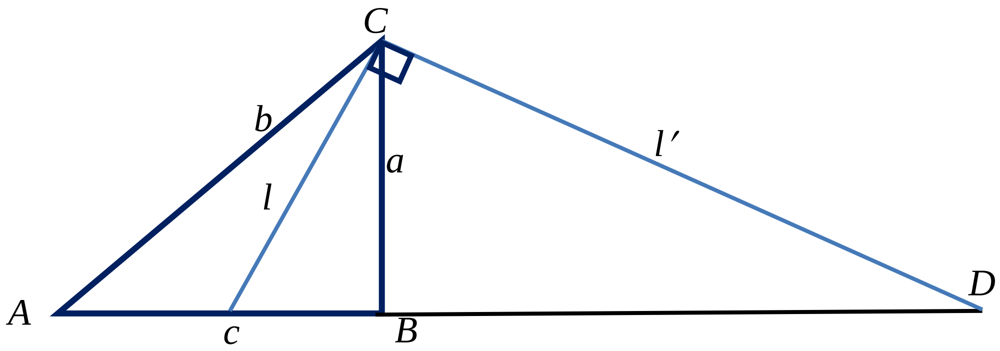
2) $\frac{AB}{BD}=c:\frac{ac}{b-a}=\frac{b-a}{a}=\frac{b}{a}-1$, bu ham ichki bissektrisa xossasiga bog'liq.

3) $\frac{AB}{AD}=c:\frac{bc}{b-a}=\frac{b-a}{b}=1-\frac{a}{b}$, yana yuqoridagi kabi o'xshashlik.

ABC uchburchakning *AC*=*a*, *BC*=*b*, *AB*=*c* bo'lsin.

Demak, quyidagi munosabatlar o'rinli ekan:

$$BD=\frac{ac}{b=a};\ \frac{AD}{BD}=\frac{b}{a};\ \frac{AB}{BD}=\frac{b}{a}-1;\ \frac{AB}{AD}=1-\frac{a}{b}.$$



4-masala. Uchburchak tomonlari *AB*=6, *BC*=5, *AC*=7 bo'lib, *B* burchak tashqi bissektrisasi *AC* tomon davomini *D* nuqtada kesib o'tadi. *CD* kesma uzunligini toping.

Yechish. $BD=\frac{ac}{b-a}=\frac{5\cdot 7}{6-5}=35$.

5-masala. Uchburchak tomonlari *BC*=2, *AC*=4 bo'lib, *C* burchak tashqi bissektrisasi *AB* tomon davomini *D* nuqtada kesib o'tadi. Agar *CD*-*BD*=1 ekanligi ma'lum bo'lsa, *CD* kesma uzunligini toping.

Yechish. $\frac{AB}{BD}=\frac{AC}{BC}-1=\frac{4}{2}-1=2-1=1$. Demak, *AB*=*BD*. Uni *x* orqali belgilaymiz. U holda $EB=\frac{2}{2+4}x=\frac{x}{3}$, $AE=\frac{2x}{3}$, $l=\sqrt{4\cdot 2-\frac{x}{3}\cdot\frac{2x}{3}}=\sqrt{\frac{72-2x^2}{9}}$, $ED=\frac{x}{3}+x=EC=\frac{4x}{3}$ bo'ladi. Uchburchak *ECD* to'g'ri burchakli ekanligidan:

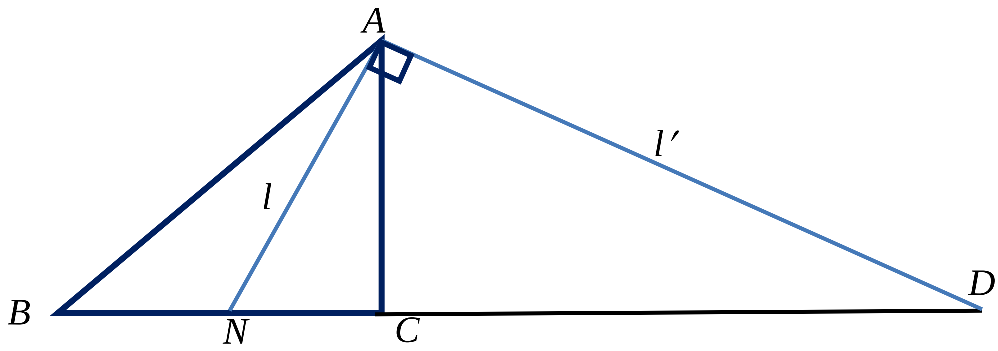
$$(x+1)^2+\frac{72-2x^2}{9}=\frac{16x^2}{9}.$$

Bu tenglamani yechib $x=\sqrt{10}+1$ bo'lishini aniqlaymiz. Demak, $CD=\sqrt{10}+1$.

6-masala. *ABC* uchburchakning *A* uchidan tushirilgan ichki bissektrisasi *BC* tomonni *N* nuqtada, tashqi bissektrisasi esa *BC* tomon davomini *D* nuqtada kesib o'tadi. Agar *BN*=6, *NC* kesma *CD* kesmadan 1 ga qisqa bo'lsa, *NC* kesma uzunligini toping.

Yechish. Topilishi kerak bo'lgan *NC* kesma uzunligini *x* orqali belgilaymiz. Yuqoridagi xossalarga ko'ra $\frac{BD}{CD}=\frac{BN}{CN}$ ekanligidan foydalanamiz.

Demak, $\frac{2x+7}{x+1}=\frac{6}{x}$ budan esa $2x^2+7x=6x+6$ tenglamani hosil qilamiz. Bu tenglamani yechib $x=\frac{3}{2}$ natijani olamiz.



Mustaqil ishlash uchun masalalar

2.138. Ikkita o‘xshash uchburchakning yuzlari 6 va 24, ulardan birining perimetri ikkinchisinikidan 6 ga ortiq. Katta uchburchakning perimetrini toping. javob: 12

2.139. Uchburchak *ABC* ning tomonlari *MN*||*AC* to‘g‘ri chiziq bilan kesildi. *ABC* va *MBN* uchburchakning perimetrlari 3:1 kabi nisbatda. *ABC* uchburchakning yuzi 144 ga teng. *MNB* uchburchakning yuzini toping. javob:16

2.140. *ABC* muntazam uchburchakning perimetri 3 ga teng. *AB* va *AC* tomonlarining davomida $AB_1=2AB$ va $AC_1=2AC$ shartlarni qanoatlantiradigan B_1 va C_1 nuqtalar olingan AB_1C_1 uchburchakning perimetrini toping. javob:6

2.141. *ABC* uchburchakda *AB* va *BC* yon tomonlarida *AC*||*DE* shartni qanoatlantiruvchi *D* va *E* nuqtalar olingan. *AB*=8, *AD*=2 bo‘lsa, *ABC* va *DBE* uchburchaklar yuzlarining nisbatini toping. javob:16/9

2.142. Ikkita o‘xshash uchburchakning perimetrlari 18 va 36 ga, yuzlarining yig‘indisi 30 ga teng. Katta uchburchakning yuzini toping. javob: 24

2.143. Yuzlari 8 va 32 bo‘lgan ikkita o‘xshash uchburchak perimetrlarining yig‘indisi 48 ga teng. Kichik uchburchakning perimetrini toping. javob:16

2.144. Perimetri 84 sm bo‘lgan uchburchakning asosiga parallel qilib o‘tkazilgan to‘g‘ri chiziq undan perimetri 42 sm ga, yuzi 27 sm^2 ga teng uchburchak ajratadi. Berilgan uchburchakning yuzini toping. javob: 108

2.145. Uchburchakning perimetri unga o‘xshash uchburchak perimetrining qismini 11/13 tashkil etadi. Agar katta uchburchakning bir tomoni va kichik uchburchakning unga mos tomoni ayirmasi 1 ga teng bo‘lsa, katta uchburchakning shu tomonini toping. javob:6,5

2.146. ΔABC ning tomonlari $MN \| AC$ to‘g‘ri chiziq bilan kesiladi. *ABC* va *MBN* uchburchakning perimetrlari 3:1 kabi nisbatda. *ABC* uchburchakning yuzi 504 ga teng. *MBN* uchburchakning yuzini toping. javob:56

2.147. *ABC* uchburchakning tomonlari $A_1B_1C_1$ uchburchakning mos tomonlaridan $2\sqrt{3}$ marta katta. *ABC* uchburchakning yuzi $A_1B_1C_1$ uchburchakning yuzidan necha marta katta? javob:12

2.148. *ABC* uchburchakning *AB* tomoni $MN \| AC$ to‘g‘ri chiziq yordamida BM=2 va MA=4 bo‘lgan kesmalarga ajratildi. Agar *MBN* uchburchakning yuzi 16 ga teng bo‘lsa, ABC uchburchakning yuzi qanchaga teng bo‘ladi? javob:144

2.149. Uchburchakning asosiga parallel to‘g‘ri chiziq uning yuzini teng ikkiga bo‘lsa, asosidan boshlab hisoblaganda, uning yon tomonlari qanday nisbatda bo‘linadi? javob: $\sqrt{2}-1$

2.150. Uchburchakning yon tomoni uchidan boshlab hisoblaganda 2:3:4 kabi nisbatda bo‘lindi va bo‘linish nuqtalari orqali asosiga parallel to‘g‘ri chiziqlar o‘tkazildi. Hosil bo‘lgan shakllar yuzlarining nisbatini toping. javob:4:21:56

2.151. Uchburchakning asosiga parallel to‘g‘ri chiziq uning yon tomoni uchidan boshlab hisoblaganda 8:3 kabi nisbatda, yuzani esa yuzlarining ayirmasi 72 ga teng bo‘lgan ikki qismga ajratadi. Berilgan uchburchakning yuzini toping. javob: $1244\frac{4}{7}$

2.152. ΔABC ning AB tomoni $MN \| AC$ to‘g‘ri chiziq yordamida BM=2 va AM=4 bo‘lgan kesmalarga ajratildi. Agar ΔMBN ning yuzi 18 ga teng bo‘lsa, ΔABC ning yuzi qanchaga teng bo‘ladi? javob:162

2.153. ABC uchburchak tomonlari 12, 8 va 6. ABC uchburchakka o‘xshash uchburchakning kichik tomoni 9. DKE uchburchak perimetrini toping. javob:39

2.154. ABC uchburchak yuzini, uning o‘rta chizig‘i ajratgan uchburchak yuziga nisbatini toping. javob:4

2.155. ABC uchburchakning AD balandligi uning EF o‘rta chizig‘ini K nuqtada kesib o‘tadi. Agar FK:KE=5:3 va BD kesma CD kesmaga qaraganda 14 sm uzun bo‘lsa, BC tomon uzunligini toping. javob: 62;31,6

2.156. ABC uchburchakning AD balandligi uning EF o‘rta chizig‘ini K nuqtada kesib o‘tadi. Agar FK:KE=k:p (k>p) va BD kesma CD kesmaga qaraganda d sm uzun bo‘lsa, BC tomon uzunligini toping. javob: $\dfrac{2dk + pd}{p}$

2.157. Burchakning bir tomonida uzunliklari a sm va b sm bo‘lgan (a<b) ikkita kesma olingan. Bu kesmalarning uchlari orqali parallel to‘g‘ri chiziqlar o‘tkazilgan. Ular burchakning ikkinchi tomonidan ikkita kesma ajratadi. Hosil bo‘lgan kesmalardan biri ikkinchisiga qaraganda c sm ortiq bo‘lsa, ularning uzunliklarini toping. javob: $\dfrac{ac}{b-a};\dfrac{bc}{b-a}$

2.158. A va B nuqtalar l to‘g‘ri chiziqdan bir tomonda yotadi. $AC \perp l, BD \perp l$ o‘tkazilgan va AC=a, BD=b ekani ma’lum. AB kesma o‘rtasidan l to‘g‘ri chiziqqacha bo‘lgan masofani toping. javob: $\dfrac{a+b}{2}$

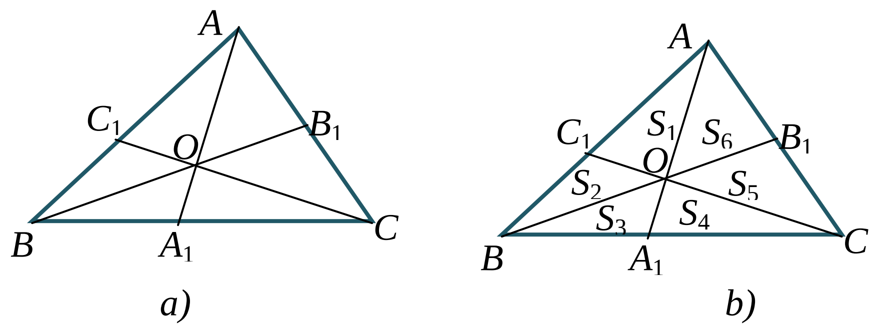
2.159. A va B nuqtalar l to‘g‘ri chiziqdan turli tomonda yotadi. $AC \perp l, BD \perp l$ o‘tkazilgan va AC=a, BD=b ekani ma’lum. AB kesma o‘rtasidan l gacha bo‘lgan x masofani toping. javob: $\dfrac{|b-a|}{2}$

2.160. Uchburchakning uchala medianasi bir nuqtada kesishishini va kesishish nuqtasida uchburchakning uchidan hisoblaganda 2:1 nisbatga bo‘lishini isbotlang.

7-§. Cheva teoremasi. Menelay teoremasi. Styuart teoremasi. Karno teoremasi. Uchburchakka bog‘liq tengsizliklar

25-teorema. (*Cheva teoremasi*) Agar O nuqta ABC uchburchakning ichida yotuvchi ixtiyoriy nuqta bo‘lsa, AO, BO, CO to‘g‘ri chiziqlar BC, CA, AB tomonlarini mos ravishda A_1, B_1, C_1 nuqtalarda kesib o‘tsa, ushbu munosabat o‘rinlidir (2.56-*a* rasm):

$$\frac{BA_1}{A_1C}\cdot\frac{CB_1}{B_1A}\cdot\frac{AC_1}{C_1B}=1.$$



2.56-rasm

AA_1, BB_1, CC_1 to‘g‘ri chiziqlar uchburchakni umumiy O uchli 6 ta uchburchakka ajratadi. Ularning yuzlarini S_1, S_2, S_3, S_4, S_5, S_6 lar bilan belgilaylik (2.56-*b* rasm).

$\triangle AA_1B$ va $\triangle AA_1C$ uchburchaklar teng balandlikka ega bo‘lgani uchun ularning yuzalar nisbati asoslar nisbatiga tengdir.

$$\frac{BA_1}{A_1C}=\frac{S_{\triangle AA_1B}}{S_{\triangle AA_1C}}=\frac{S_1+S_2+S_3}{S_4+S_5+S_6}. \qquad (1)$$

Shunga o‘xshash $\triangle OA_1B$ va $\triangle OA_1C$ uchburchaklar yuzalar nisbati asoslar nisbatiga tengdir

$$\frac{BA_1}{A_1C}=\frac{S_{\triangle OA_1B}}{S_{\triangle OA_1C}}=\frac{S_3}{S_4}. \qquad (2)$$

Bu so‘nggi ikkita proporsiyadan quyidagi hosilaviy proporsiya kelib chiqadi.

(1) va (2) tengliklar teng nisbatlar bo‘lgani uchun ularni tenglashtiramiz va hosil bo‘lgan proporsiyani bajaramiz.

$$\frac{S_1+S_2+S_3}{S_4+S_5+S_6}=\frac{S_3}{S_4} \;=>\; (S_1+S_2)\cdot S_4+\cancel{S_4\cdot S_3}=\cancel{S_4\cdot S_3}+(S_5+S_6)\cdot S_3$$

$$\frac{BA_1}{A_1C}=\frac{S_{\triangle AA_1B}}{S_{\triangle AA_1C}}=\frac{S_1+S_2+S_3}{S_4+S_5+S_6}=\frac{S_1+S_2+S_3-S_3}{S_4+S_5+S_6-S_4} \text{ yoki } \frac{BA_1}{A_1C}=\frac{S_1+S_2}{S_5+S_6}. \;(3)$$

Shunga o‘xshash $\frac{CB_1}{B_1A}=\frac{S_3+S_4}{S_1+S_2}$ (4) va $\frac{AC_1}{C_1B}=\frac{S_5+S_6}{S_3+S_4}$ (5) tengliklarni ko‘paytirsak

$$\frac{BA_1}{A_1C}\cdot\frac{CB_1}{B_1A}\cdot\frac{AC_1}{C_1B}=\frac{S_1+S_2}{S_5+S_6}\cdot\frac{S_3+S_4}{S_1+S_2}\cdot\frac{S_5+S_6}{S_3+S_4}=1$$

$$\frac{BA_1}{A_1C}\cdot\frac{CB_1}{B_1A}\cdot\frac{AC_1}{C_1B}=1.$$

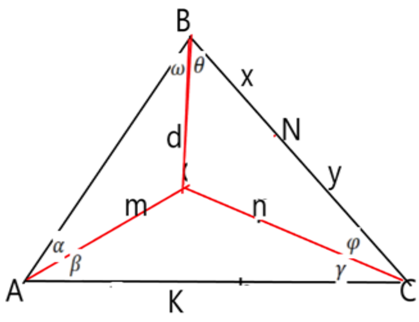
Teorema isbotlandi.

Cheva teoremasi uch to‘g‘ri chiziqning bir nuqtada kesishishini isbot etishda yoki uch chiziqdan biri uchburchakning biror uchidan chiqib, uchburchak tekisligida yotgan umumiy bir nuqtadan o‘tib, uchburchak tomonlarini kesganda hosil bo‘lgan kesmalar orasidagi nisbatlarni aniqlashda qo‘llaniladi.

Chevaning uchburchaklar burchaklari uchun sinuslar teoremasini qo‘llaymiz:

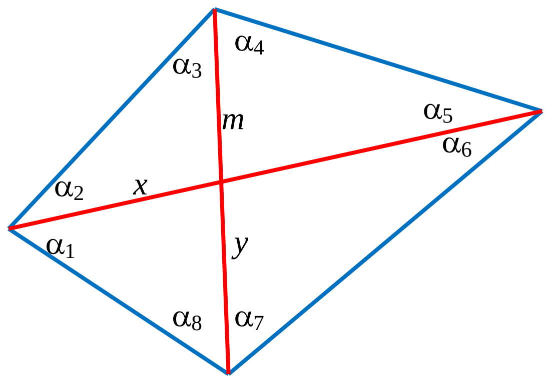
$$\begin{cases} \dfrac{n}{\sin\beta} = \dfrac{m}{\sin\gamma} \\ \dfrac{n}{\sin\theta} = \dfrac{d}{\sin\varphi} \\ \dfrac{d}{\sin\alpha} = \dfrac{m}{\sin\omega} \end{cases} \Rightarrow \begin{cases} \dfrac{m}{n} = \dfrac{\sin\gamma}{\sin\beta} \\ \dfrac{n}{d} = \dfrac{\sin\theta}{\sin\varphi} \\ \dfrac{d}{m} = \dfrac{\sin\alpha}{\sin\omega} \end{cases} \Rightarrow$$

$$\frac{m}{n}\cdot\frac{n}{d}\cdot\frac{d}{m} = \frac{\sin\gamma}{\sin\beta}\cdot\frac{\sin\theta}{\sin\varphi}\cdot\frac{\sin\alpha}{\sin\omega} = 1$$



2.57-rasm

Chevaning to‘rtburchaklar uchun sinuslar teoremasi



2.58-rasm

$$\frac{x}{\sin\alpha_8} = \frac{y}{\sin\alpha_1} \Rightarrow \frac{x}{y} = \frac{\sin\alpha_8}{\sin\alpha_1} \quad (1)$$

$$\frac{x}{\sin\alpha_3} = \frac{m}{\sin\alpha_2} \Rightarrow \frac{m}{x} = \frac{\sin\alpha_2}{\sin\alpha_3} \quad (2)$$

$$\frac{p}{\sin\alpha_4} = \frac{m}{\sin\alpha_5} \Rightarrow \frac{p}{m} = \frac{\sin\alpha_4}{\sin\alpha_5} \quad (3)$$

$$\frac{p}{\sin\alpha_7} = \frac{y}{\sin\alpha_6} \Rightarrow \frac{y}{p} = \frac{\sin\alpha_6}{\sin\alpha_7} \quad (4)$$

(1), (2). (3), (4) tengliklarning o‘ng va chap tomonini ko‘paytirib, quyidagi (5) tenglikni hosil qilamiz. Bu to‘rtburchaklar uchun cheva sinusi teoremasi deyiladi.

$$\frac{x}{y}\cdot\frac{m}{x}\cdot\frac{p}{m}\cdot\frac{y}{p} = \frac{\sin\alpha_8}{\sin\alpha_1}\cdot\frac{\sin\alpha_2}{\sin\alpha_3}\cdot\frac{\sin\alpha_4}{\sin\alpha_5}\cdot\frac{\sin\alpha_6}{\sin\alpha_7} = 1 \qquad (5)$$

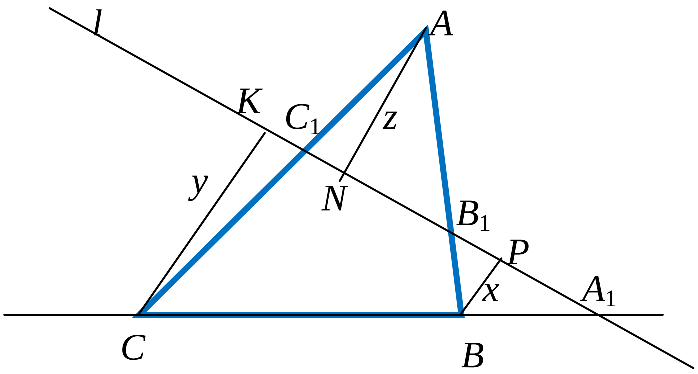
Menelay teoremasi, odatda berilgan uch nuqtaning kollinearligini isbot etishda va uchburchak tomonlarini biror to‘g‘ri chiziq kesganda hosil bo‘lgan kesmalarning nisbatini aniqlashda qo‘llaniladi.

26-teorema. (*Menelay teoremasi*) Agar biror to‘g‘ri chiziq *ABC* uchburchakning *AB*, *BC*, *CA* tomonlarini yoki ularning davomlarini C_1, A_1, B_1 nuqtalarda kesib o‘tsa, u holda

$$\frac{CA_1}{A_1B}\cdot\frac{BB_1}{B_1A}\cdot\frac{AC_1}{C_1C}=-1. \qquad (1)$$

(Agar har bir nisbatni tashkil etuvchi kesmalar bir xil yo‘nalishga ega bo‘lsa, nisbat musbat, qarama-qarshi yo‘nalishga ega bo‘lsa, manfiy hisoblanadi.)

Isbot. *ABC* uchburchakning tomonlarini kesib o‘tuvchi *l* to‘g‘ri chiziqqa *B* uchlaridan tushirilgan AC ga parallel BP kesma o‘tkazamiz. Uchburchaklarnimg o‘xshashligidan foydalanamiz (2.59-rasm).



2.59-rasm

1) $\Delta CA_1K \sim \Delta BA_1P$ dan foydalanamiz $\frac{CA_1}{BA_1}=\frac{y}{x}$;

2) $\Delta ANC_1 \sim \Delta CKC_1$ dan foydalanamiz: $\frac{AC_1}{C_1C}=\frac{z}{y}\Rightarrow\frac{C_1A}{C_1C}=-\frac{z}{y}$;

3) $\Delta B_1NB \sim \Delta B_1PB$ dan foydalanamiz: $\frac{B_1B}{B_1A}=\frac{x}{z}$.

Bu tengliklarni hadma-had ko‘paytirib, quyidagi tenglikni hosil qilamiz:

$$\frac{CA_1}{BA_1}\cdot\frac{C_1A}{C_1C}\cdot\frac{B_1B}{B_1A}=\frac{y}{x}\cdot\left(-\frac{z}{y}\right)\cdot\frac{x}{z}=-1$$

Biz bu teoremani kesuvchi to‘g‘ri chiziq uchburchakning ikki tomonini va uchinchi tomonining davomini kesib o‘tgan hol uchun isbot qildik. To‘g‘ri chiziq uchala tomonining davomini kesgan hol uchun ham shunday isbot qilinadi.

Menelayning teskari teoremasi. ABC uchburchakning AB, BC, AC tomonlarida yoki ularning davomlarida uchala mos A_1, B_1, C_1 nuqtalar olinganda

$$\frac{CA_1}{BA_1}\cdot\frac{C_1A}{C_1C}\cdot\frac{B_1B}{B_1A}=\frac{y}{x}\cdot\left(-\frac{z}{y}\right)\cdot\frac{x}{z}=-1$$

munosabat o‘rinli bo‘lsa, bu olingan uchta nuqta kollinear bo‘ladi yoki ular bir to‘g‘ri chiziqda yotadi.

27-teorema. (*Stuart teoremasi*) Agar ABC uchburchakning BC tomonida ichki D nuqta olingan bo‘lsa,

$$AB^2\cdot DC+AC^2\cdot BD-AD^2\cdot BC=BC\cdot DC\cdot BD$$

tenglik bajariladi (2.60-rasm).

Isbot. ΔABC ning A uchidan $AK\perp BC$ to‘g‘ri chiziq o‘tkazamiz (2.61-rasm). K nuqta D va C nuqtalar orasida yotadi, deb faraz qilamiz. Ikkita to‘g‘ri burchakli ΔAKC va ΔADK ni qaraymiz. ΔAKC da Pifagor teoremasiga ko‘ra

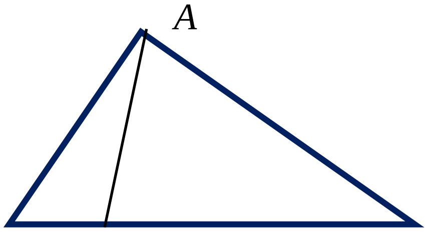
$$AC^2=AK^2+KC^2;\ \Delta ADK \text{ dan } AK^2=AD^2-DK^2$$

munosabatlarni olamiz.

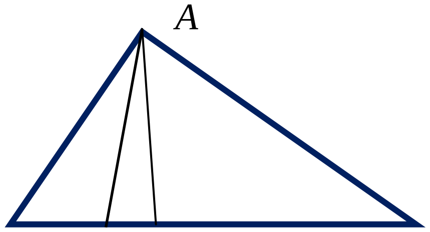
Ulardan $AC^2=AD^2+KC^2-DK^2=AD^2+(KC+DK)(KC-DK)$ yoki

$$AC^2=AD^2+DC(KC-DK=AD^2+DC(DC-2DK)\ ,$$

$$AC^2=AD^2+DC^2-2\,DC\cdot DK \text{ bo‘ladi.}$$



2.60-rasm



2.61-rasm

To‘g‘ri burchakli ΔABK va ΔADK dan $AB^2=AK^2+BK^2$ va $AK^2=AD^2-DK^2$ munosabatlarni olamiz. Ulardan

$$AB^2=AD^2+BK^2-DK^2=AD^2+(BK-DK)(BK+DK)$$

bo‘lishi kelib chiqadi.

$BK-DK=BD$, $BK=BD+DA$ ekanligini hisobga olsak,

$$AB^2=AD^2+BD(BD+DK)=AD^2+BD^2+2BD\cdot DK \quad \text{bo‘ladi.}$$

Endi AC^2 uchun qabul qilingan ifodani BD ga, AB^2 uchun qabul qilingan ifodani DC ga ko‘paytirib, hosil qilingan ifodalarni qo‘shamiz:

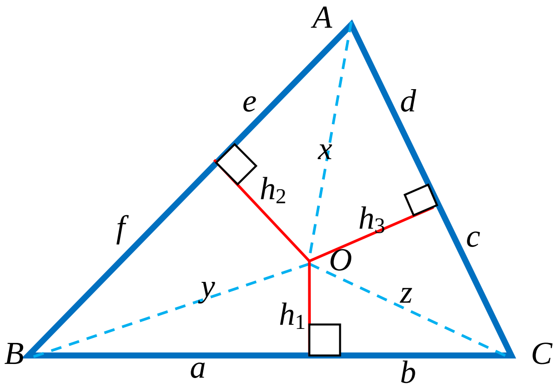
$$AB^2\cdot DC+AC^2\cdot BD=AD^2\cdot\left(BD+DC\right)+DC^2\cdot BD+BD^2\cdot DC=$$
$$=AD^2\cdot BC+DC^2\cdot BD+BD^2\cdot DC=AD^2\cdot BC+DC\cdot BD\left(DC+BD\right)=$$
$$=AD^2\cdot BC+BD\cdot DC\cdot BC$$

ya’ni talab qilingan tenglik olinadi.

Stuart teoremasidan foydalanib, uchburchak medianasini, balandligi, bissektrisasi uzunliklarini topish formulalari topiladi.

28-teorema. (*Karno teoremasi*) Uchburchak ichidan olingan nuqtadan tomonlariga tushirilgan balandliklari asosi uchburchak tomonlarida ajratgan kesmalar uchun quyidagi tenglik o‘rinli (2.62-rasm):

$$a^2+c^2+e^2=b^2+d^2+f^2$$



2.62-rasm

Isbot. 1) ΔBOC dan $h_1^2 = z^2 - b^2 = y^2 - a^2$.

2) ΔBOA dan $h_2^2 = x^2 - e^2 = y^2 - f^2$.

3) ΔAOC dan $h_{23}^2 = x^2 - d^2 = z^2 - c^2$.

Tengliklarni qo‘shib, $+\begin{cases} z^2 - b^2 = y^2 - a^2 \\ y^2 - f^2 = x^2 - e^2 \Rightarrow \\ x^2 - d^2 = x^2 - c^2 \end{cases}$

$$x^2 + z^2 + y^2 - f^2 - b^2 - d^2 = y^2 + x^2 + z^2 - a^2 - e^2 - c^2 \Rightarrow$$

$$-f^2 - b^2 - d^2 = -a^2 - e^2 - c^2 \Rightarrow f^2 + b^2 + d^2 = a^2 + e^2 + c^2$$

Mustaqil ishlash uchun masalalar

2.161.*ABC* uchburchakning *BD* medianasida *E* nuqta shunday olinganki, *BE:ED*=3:2 bo‘lgan, *AE* to‘g‘ri chiziq *BC* tomonni *F* nuqtada kesadi. *B* uchidan hisoblaganda *F* nuqta *BC* ni qanday nisbatda bo‘ladi? javob: 3/4

2.162. *ABC* uchburchakning *AC* tomonida *E* nuqta shunday joylashganki, bunda *AE:EC*=3:4 bo‘lgan.

a) AD mediana *BE* kesmani qanday nisbatda bo‘ladi? javob: 7/4

b) BE kesma *AD* medianani qanday nisbatda bo‘ladi? javob: 3/2

2.163.*ABC* uchburchakning *AC* va *BC* tomonlarida, mos ravishda, *E* va *D* nuqtalar shunday olinganki, bunda *AE:EC*=2:3, *BD:DC*=3:1 nisbatda bo‘lgan.

a) BE kesma *AD* ni qanday nisbatda bo‘ladi? javob: 2

b) AD kesma *BE* ni qanday nisbatda bo‘ladi? javob: 7,5

2.164. *ABC* uchburchakning *AB* tomoni teng 2010 bo‘lakka bo‘lindi. *A* nuqtadan boshlab hisoblaganda 1000 - bo‘linish nuqtasidan *BC* ga parallel to‘g‘ri chiziq o‘tkazilgan. U *AB* ni *E* nuqtada, *AC* ni esa *D* nuqtada kesadi. *ED*=1 dm. *BC* tomonning uzunligini toping. javob: 2,01

2.165. *ABC* uchburchakning *AB* tomoni teng 2010 qismga bo‘lingan va bo‘linish nuqtalarining 99- bo’limidan *BC* tomonga parallel qilib kesma o‘tkazilgan. Agar *BC*=1 m bo‘lsa, bu kesmalarning har birining uzunligini toping.

2.166. To‘g‘ri burchakli uchburchakning bir kateti va gipotenuzasi uzunligi mos ravishda 2,5 va $\frac{\sqrt{281}}{2}$ bo‘lsa, bu uchburchakning yuzini toping. javob: 10

2.167. Teng tomonli uchburchakning balandligi uzunligi $7\sqrt[4]{3}$ ga teng bo‘lsa, bu uchburchakning yuzini toping. javob: 49

2.168.Teng yonli uchburchakning uchidagi burchagi va balandligi mos ravishda 120° va $\frac{5\sqrt[4]{3}}{2}$ berilgan bo‘lsa, bu uchburchakning yuzini toping. javob: 18,75

2.169. Teng yonli to‘g‘ri burchakli uchburchakning gipotenuzasi $3\sqrt{3}$ ga teng bo‘lsa, bu uchburchakning yuzini toping. javob: 6,75

2.170. Teng yonli uchburchakning balandligi 10 ga teng, yon tomoni uzunligi esa $\frac{\sqrt{481}}{2}$ ga teng bo‘lsa, bu uchburchakning yuzini toping. javob: 45

2.171.Uchburchakning ikki tomoni uzunligi 12 va 14 ga, hamda bu tomonlar orasidagi burchak 30° ga teng. Bu uchburchakning yuzini toping. javob: 42

2.172. Teng yonli to‘g‘ri burchakli uchburchakning yuzi 36 ga teng bo‘lsa, gipotenuzasi uzunligini toping. javob: 12

2.173. Teng yonli uchburchakning asosidagi burchagi 30° ga va yon tomoni uzunligi $\sqrt[4]{3}$ ga teng bo‘lsa, bu uchburchakning yuzini toping. javob: 0,75

2.174. Teng yonli to‘g‘ri burchakli uchburchakning gipotenuzasi uzunligi $2\left(\sqrt{2}-1\right)$ ga teng bo‘lsa, bu uchburchakning perimetrini toping. javob:2

2.175. Teng yonli uchburchakning yon tomoni uzunligi 25 ga teng, asosiga tushgan balandligi esa 20 ga teng. Bu uchburchakning perimetrini toping. javob: 80

2.176. To‘g‘ri burchakli uchburchakning bir kateti uzunligi ikkinchi bir kateti uzunligidan 10 ga katta, ammo gipotenuzasi uzunligidan 10 ga kichik. Bu uchburchak gipotenuzasining uzunligini toping. javob: 50

2.177.Teng yonli uchburchakning asosi uzunligi 30 ga teng, balandligi uzunligi esa 20 ga teng. Bu uchburchakning yon tomoni uzunligini toping. javob: 25

2.178. Teng yonli uchburchakning asosi uzunligi 30 ga teng, hamda balandligi uzunligi 20 ga teng. Yon tomoniga tushirilgan balandligi uzunligini toping. javob: 24

2.179. Teng yonli ucburchakning asosidagi burchagi 45° ga teng, hamda asosining uzunligi $9\sqrt{2}$ ga teng. Bu uchburchakning yon tomoni uzunligini toping. javob: 9

2.180.Uchburchakning asosiga tushirilgan balandligi yon tomoni bilan 60° burchak tashkil qiladi. Agar yon tomonning asosidagi proyeksiyasi $11\sqrt{3}$ bo‘lsa, yon tomoni uzunligini toping. javob: 22

2.181. *ABC* uchburchakda, *C* burchagi $\frac{\pi}{6}$ ga teng. Agar *AC*=12,3 va *AB*=61,5 berilgan bo‘lsa, *B* burchakning sinusini toping. javob: 0,1

2.182. *ABC* uchburchakning *AB* tomoniga o‘tkazilgan medianasi *CB* tomoni bilan 60° burchak tashkil qiladi, hamda mediana uzunligi $\frac{\sqrt{6}}{10}$. Agar *ABC* burchakning qiymati 45° bo‘lsa, *AB* tomon uzunligini toping. javob: 0,6

2.183. *ABC* uchburchakda *B* burchak 60° ga teng. Agar $BC = 3\sqrt{3}$ va *AC*=15 bo‘lsa, *A* burchakning sinusini toping. javob: 0,3

2.184. *ABC* uchburchakda $\angle A$=30° va $\angle B$=45° hamda $AC = 10\sqrt{2}$ bo‘lsa, *BC* ni toping. javob: 10

2.185. *ABC* uchburchakda *AB*=20, $AC = 10\sqrt{6}$ va *ABC* burchakning qiymati 120° bo‘lsa, *C* burchakning qimyatini toping. javob: 45

2.186. *ABC* uchburchakda *B* va *C* burchaklari mos ravishda 120° va 45° ga teng. Agar $AC = 15\sqrt{6}$ bo‘lsa, *AB* ning uzunligini toping. javob: 30

2.187. *ABC* uchburchakda *B* va *C* burchaklari mos ravishda 45° va 30° ga teng. Agar $AC = \frac{17}{2}\sqrt{2}$ bo‘lsa, *AB* ning uzunligini toping. javob: 8,5

2.188. *ABC* uchburchakda *B* va *C* burchaklari mos ravishda 30° va 45° ga teng. Agar $AB = 2\sqrt{6}$ bo‘lsa, *B* burchakning bissektrisasining uzunligini toping. javob: $2\sqrt{3}+2$

2.189. *ABC* uchburchakda, *AB*=2,5; *AC*=8 va *A* burchakning kosinus qiymati $\frac{5}{16}$ bo‘lsa, *B* uchidan o‘tkazilgan medianasining uzunligini toping. javob:4

2.190. Teng yonli uchburchakning uchidagi burchagi kosinusi $-\frac{1}{15}$. Agar asosi uzunligi $\sqrt{7{,}5}$ bo‘lsa yon tomonining uzunligini toping. javob: 1,875

2.191. Uchburchakning ikkita tomoni uzunligi 8 va 10 ga teng va bu tomonlar orasidagi burchak kosinusi $\frac{43}{160}$ ga teng bo‘lsa, uchburchakning uchinchi tomonini toping. javob: 11

2.192. Teng yonli uchburchakning yon tomoni uzunligi 19 ga hamda uchidagi burchagi kosinusi $\frac{7}{8}$ ga teng bo‘lsa, bu uchburchak asosining uzunligini toping. javob: 9,5

2.193. Uchburchakda *A* va *B* burchaklari farqi 90° ga teng, hamda bu burchaklar qarshisidagi mos tomonlar esa 10 va 5 ga teng bo‘lsa, *B* burchak tangenisini hisoblang. javob: 0,5

2.194. *ABC* ning *BC* va *AC* tomoni uzunligi 6 va 4 teng. *A* burchak *B* burchakdan ikki marta katta bo‘lsa, B burchakning kosinusini toping. javob: 0,75

2.195. Teng yonli uchburchakning yon tomoni uzunligi 5 ga teng. Agar uchidagi burchagi kosinusi $-\frac{7}{25}$ bo‘lsa, bu uchburchakning balandligini toping. javob:3

2.196. *ABC* uchburchak uchun *AB*=30, *AC*=8 va *CE*=11, bunda *CE* mediana bo‘lsa, *A* burchakning kosinusini toping. javob: 0,7

2.197. *ABC* uchburchakning *B* burchagi 105°, *C* burchagi esa 15°. Agar $AB=\sqrt{3}$ bo‘lsa, *B* uchidan tushirilgan balandlik uzunligini toping. javob: 1,5

2.198. To‘g‘ri burchakli uchburchakning katetlari 5:6 nisbatda, gipotenuzasi uzunligi esa 122 ga teng bo‘lsa, katetlarining gipotenuzadagi proyeksiyalarini toping. javob: 50; 72

2.199. To‘g‘ri burchakli uchburchakning yuzasi 150 ga teng. Bitta kateti uzunligi esa 15 ga teng bo‘lsa, gipotenuzasiga tushirilgan balandligi uzunligini toping. javob: 12

2.200.To‘g‘ri burchakli uchburchakning gipotenuzasiga tushirilgan balandligi uzunligi $2\sqrt{5}$ va bitta kateti uzunligi 6 ga teng bo‘lsa, gipotenuzasining uzunligini toping. javob: 9

2.201. To‘g‘ri burchakli uchburchakning *A* to‘g‘ri burchagi uchidan *AM* mediana va *AK* balandlik o‘tkazilgan. Agar katetlari uzunligi 6 va $3\sqrt{5}$ bo‘lsa, *MK* ning uzunligini toping. javob: 0,5

2.202. Agar to‘g‘ri burchakli uchburchakning gipotenuzasiga tushirilgan balandligi va kichik kateti uzunliklari mos ravishda $2\sqrt{10}$ va $\sqrt{65}$ berilgan bo‘lsa, katta katetining gipotenuzasidagi proyeksiyasi uzunligini toping. javob: 8

2.203. To‘g‘ri burchakli uchburchakning katetlari nisbati 1:3 bo‘lsa, gipotenuzaning uzunligi 40 ga teng bo‘lsa, gipotenuzaga tushirilgan balandlik uzunligini toping. javob: 12

2.204. To‘g‘ri burchakli uchburchakning katetlari $2\sqrt{21}$ va $4\sqrt{7}$ bo‘lsa, gipotenuzaga tushirilgan balandlik, gipotenuzani qanday uzunlikdagi kesmalarga ajratadi? javob: 6; 8

2.205. To‘g‘ri burchakli uchburchakning gipotenuzasi uzunligi va unga tushirilgan balandlikning uzunligi mos ravishda 17 va 4 ga teng. Gipotenuzaga tushirilgan balandlik gipotenuzani qanday uzunlikdagi kesmalarga ajratadi? javob: 1; 16

2.206. To‘g‘ri burchakli uchburchakning gipotenuzasiga tushirilgan balandlik va bir kateti uzunliklari mos ravishda 12 va l5 ga teng bo‘lsa, bu uchburchakning yuzini toping. javob: 150

2.207. To‘g‘ri burchakli uchburchakning katetlari nisbati 3:2 ga teng va gipotenuzasiga tushirilgan balandlik gipotenuzani ikkita biri ikkinchisidan 2 ga ko‘p kesmalarga ajratgan bo‘lsa, gipotenuzaning uzunligini toping. javob: 5,2

2.208. Uchburchakning balandligi uzunligi $2\sqrt{2}$ ga teng. Bu uchburchakning asosiga parallel o‘tkazilgan to‘g‘ri chiziq uchburchakdan ajratgan uchburchak yuzi uchburchak yuzining yarmiga teng bo‘lsa, bu kichik uchburchakning balandligini toping. javob: 2

2.209. *ABC* uchburchakning tomonlari uzunliklari *AB*=3, *AC*=5 va *BC*=6. Uchburchakning *C* uchidan *B* uchidan tuhirilgan balandlikkacha bo‘lgan masofani toping. javob: 5,2

2.210. Uchburchakning asosi uzunligi $\sqrt{98}$ ga teng. Uchburchak asosiga parallel to‘g‘ri chiziq uchburchak yuzini teng ikkiga bo‘lsa, bu chiziqning uchburchak ichidagi kesmasi uzunligini toping. javob: 7

2.211. Teng yonli uchburchakning asosidagi burchagi 30° va asosi uzunligi $\sqrt{24}$ ga teng. Asosiga parallel to‘g‘ri chiziq bu uchburchak yuzini teng ikkiga bo‘ladi. Bu to‘g‘ri chiziq yon tomonini uchidan boshlab hisoblaganda qanday kesmalarga bo‘ladi? javob: $2; 2\sqrt{2}-2$

2.212. Uchburchakning tomonlari uzunliklari 5, $\sqrt{73}$ va 12 ga teng. Uzunligi 12 ga teng tomonga tushirilgan balandlik uni ajratgan kesmalari farqini toping. Javob: 4

2.213. Uchburchak *ABC* da, $\angle A$=48° va *C* burchagining tashqi burchagi 100° ga teng. Burchak *B* ning qiymatini toping. javob: 52°

2.214. Muntazam uchburchakning yuzasi $4\sqrt{3}$ ga teng bo‘lsa, uchburchakning perimetrini toping. javob: 12

2.215. Teng yonli uchburchakning yon tomoni 3 ga va asosi esa 4 ga teng. Uchburchakning yuzini toping. javob: $2\sqrt{5}$

2.216. 21 va 28 uzunliklardagi tomonlari orasidagi burchak bissektrisasi qarshisidagi tomonni ikkita kesmaga ajratadi, bu kesmalardan kichigining uzunligi 12 ga teng bo‘lsa, bu uchburchakning perimetrini toping. javob: 77;70

2.217. Uchburchakning ikkita tomoni uzunligi 15 va 25 ga teng. Uchinchi tomonga o‘tkazilgan mediananing uzunligi 16 ga teng bo‘lsa, uchinchi tomon uzunligini toping. javob: 26

2.218. Uchburchakning uchta tomoni uzunliklari 26, 40 va 42 ga teng. Uchburchakning o‘rtancha tomoniga tushgan balandlikning uzunligini toping. javob: 25,2

2.219. *MNP* uchburchakda, *MN*=9, *NP*=8 va $\cos N = \frac{1}{6}$ lar berilgan bo‘lsa, *MP* tomon uzunligini toping. javob: 11

2.220. Uchburchakning tomonlari uzunliklari 5, 6 va $\sqrt{91}$ ga teng bo‘lsa uchburchakning katta burchagini toping. javob: 120°

2.221. Uchburchak *BDC* da, $CD = \sqrt{2}$, $\angle B$=30°, $\angle D$=45° ga teng. *BD* tomonning uzunligini toping. javob: $\sqrt{3}+1$

2.222. Uchburchak *ABC* ning yuzasi 16 ga teng. *AC*=5, *BC*=8 va *C* burchagi o‘tmas bo‘lsa, *AB* tomonning uzunligini toping. javob: $\sqrt{137}$

2.223. To‘g‘ri burchakli uchburchakning gipotenuzasi 4 ga va bitta o‘tkir burchagi 30° ga teng bo‘lsa, bu uchburchakning yuzini toping. javob: $2\sqrt{3}$

2.224. Teng yonli to‘g‘ri burchakli uchburchakning yuzasi 36 ga teng. Gipotenuza uzunligini toping. javob: 12

2.225. To‘g‘ri burchakli uchburchakning o‘tkir burchaklari bissektrisalari orasidagi burchagini toping. javob: 45°

2.226. To‘g‘ri burchakli uchburchakning katetlari uzunliklari 30 va 40 ga teng. Gipotenuzasiga o‘tkazilgan medianasi uzunligini toping. javob: 25

2.227. To‘g‘ri burchakli uchburchakning gipotenuzasiga o‘tkazilgan balandlik gipotenuzani 1:4 nisbatda bo‘ladi, hamda balandlik uzunligi 4 ga teng. Gipotenuzaning uzunligini toping. javob: 10

2.228. Uchburchak *ABC* ning uchta balandligi *O* nuqtada kesishadi. Agar *OC*=*AB* bo‘lsa, *ACB* burchakni kattaligini toping. javob: 45°

2.229. To‘g‘ri burchakli uchburchakning *B* to‘g‘ri burchagi medianasi va balandligi *ABC* burchakni teng uchga bo‘ladi. *ABC* uchbuchburchakning burchaklari kattaligini toping. javob: 30°, 60° va 90°

2.230. Uchburchakning tomonlari 5:12:13 nisbatda. Uchburchakning tomonlari o‘rtalarini tutashtirishdan hosil qilingan uchburchakning yuzasi 30 ga teng bo‘lsa, dastlabki uchburchakning perimetrini toping. javob: 60

2.231. Uchburchak *ABC* ning *BAC* burchagi 120° ga teng. *AB*=3, *AC*=5 ga bo‘lsa, *BK* bissektrisaning uzunligini toping. javob: $\frac{3\sqrt{7}}{2}$

2.232. Uchburchak *ABC* ning *ABC* burchagi 30° ga teng. *AB*=4, *BC*=6 ga bo‘lsa, *ABC* burchak bissektrisasi *AC* tomonni *K* nuqtada kesadi. *ABK* uchburchakning yuzini toping. javob: 2,4

2.233. Uchburchakning asosi uzunligi 14 ga teng. Yon tomonlariga tushirilgan medianalari $3\sqrt{7}$ va $6\sqrt{7}$ ga teng. Uchburchakning kichik yon tomoni uzunligini toping. javob: $4\sqrt{7}$

2.234. Uchburchakning ikkita tomoni uzunliklari 6 va 8 ga teng. Bu tomonlarga tushirilgan medianalar o‘zaro perpendikular. Uchburchakning uchinchi tomoni uzunligini toping. javob: $2\sqrt{5}$

2.235. Uchburchakning medianalari 3, 4 va 5 ga teng. Bu uchburchakning yuzini toping. javob:8

2.236. Teng yonli uchburchakning yon tomoni uzunligi 20 ga va asosi 24 ga teng. Uchburchakning medianalari va bissektrisalari kesishgan nuqtalari orasidagi masofani toping. javob: $\frac{2}{3}$

2.237. Teng yonli uchburchakning yon tomoniga tushirilgan balandlik uzunligi 12 ga va asosi uzunligi esa 15 ga teng. Bu uchburchakning yuzini toping. javob: 75

2.238. Teng yonli uchburchakning yon tomoni 18 va asosi esa 12 ga teng. Yon tomonlariga balandliklar o‘tkazilgan, bu balandliklar asoslari orasidagi masofani toping. javob: $\frac{28}{3}$

2.239. O‘tmas burchakli uchburchakning asosi uzunligi 6 ga teng. Yon tomonlariga tushirilgan balandliklar uzunligi 2 va $2\sqrt{3}$ ga teng. Asosiga tushirilgan balandlik uzunligini toping. javob: $\sqrt{2}$

2.240. Uchburchakning balandligi 4 ga teng. Balandlik qarshisidagi tomonni 1:8 nisbatda bo‘ladi. Bu balandlikka parallel va uchburchak yuzini teng ikkiga bo‘luvchi chiziqning uzunligini toping. javob: 3

2.241. M nuqta AC tomonning o‘rtasi. AB=2, BC=3 va $\angle ABM=2\angle MBC$ bo‘lsa, ABC uchburchakning yuzini toping. javob: $\frac{15\sqrt{7}}{16}$

2.242. Uchburchak ABC da AB=3, BC=5, AC=6. M nuqta AB tomonda, K nuqta esa BC tomonda olingan. Agar $BM=2AM$, $3BK=2KC$ bo‘lsa, MK ning uzunligini toping. javob: $\frac{8\sqrt{30}}{15}$

2.243. Uchburchak ABC da A burchak B burchakdan ikki marta katta, bu burchaklar qarshisidagi tomonlar 6 va 4 ga teng. Uchinchi tomon uzunligini toping. javob: 4

2.244. Uchburchak ABC ning BM medianasida K nuqta olingan bo‘lib, $BK:KM$=3:1. AK to‘g‘ri chiziq uchburchak yuzini qanday nisbatda bo‘ladi? javob: 3: 2

2.245. Teng yonli uchburchak *ABC* da *AB*=*BC*=12. *BD* balandlikning o‘rtasidan *BC* tomonga parallel *MP* to‘g‘ri chiziq o‘tkazilgan. *MP* ning uzunligini toping. javob: 9

2.246. Uchburchak *ABC* ning yuzasi 24 ga teng. *M* nuqta *AB* tomonda, *N* nuqta esa *BC* tomonda, *K* nuqta *AC* tomonda olingan bo‘lib, *AM:MB*=2:1, *BN=NC*, *AK:KC*=1:3 bo‘lsa, *MNK* uchburchakning yuzini toping. javob:7

2.247. Ikkita o‘xshash to‘g‘ri burchakli uchburchaklarning bir katetlari 12 dan. Agar uchburchaklarning yuzlari nisbati 100:9 bo‘lsa, katta uchburchakning yuzini toping. javob: $24\sqrt{30}$

2.248. To‘g‘ri burchakli uchburchakning gipotenuzasida olingan nuqta katetlaridan teng uzoqlikda joylashgan. Bu nuqta gipotenuzani 30 va 40 uzunlikdagi kesmalarga bo‘ladi. To‘g‘ri burchakli uchburchakning perimetrini toping. javob: 168

2.249. To‘g‘ri burchakli uchburchakning perimetri 60 ga teng. Gipotenuzaga tushirilgan balandlik uzunligi 12 ga teng bo‘lsa, bu uchburchakning yuzini toping. javob: 150

2.250.Teng yonli uchburchak *ABC* da (*AB=AC*) va $\angle BAC$=80° ma’lum. *M* nuqta shunday tanlanganki, bunda, $\angle MBC$=30°, $\angle MCB$=10° bo‘lsa, $\angle AMC$ burchakning kattaligini toping. javob:70°

2.251. Teng yonli uchburchak *ABC* ning *AB* va *BC* yon tomonlari 15 ga teng. Agar cos*A*=0,8 bo‘lsa uchburchakning balandliklar kesishgan nuqtasidan bissektrisalari kesishgan nuqtasigacha bo‘lgan masofani toping. javob: 12

2.252. Uchburchak *ABC* da $AB = 4\sqrt{7}$, $AC = 5\sqrt{7}$, $BC = 6\sqrt{7}$ bo‘lsa, uchburchakning *B* uchidan uchburchakning balandliklar kesishgan nuqtasigacha bo‘lgan masofani toping. javob: 9

2.253. Uchburchak *ABC* ning tomonlarida olingan *M, K, N* nuqtalar uchun, *AM:MB*=1:4, *BK:KC*=2:3 va *AN:NC*=2:3 o‘rinli. *AK* va *MN* chiziqlar *P* nuqtada kesishadi. *PK:AP* ni toping. javob: 3

2.254. Uchburchak *ABC* ning ichidagi nuqtadan tomonlariga parallel chiziqlar o‘tkazilgan, bu chiziqlar va uchburchakning tomonlari orqali hosil bo‘lgan uchburchaklar yuzlari 16, 25 va 49 ga teng bo‘lsa, uchburchak ABC ning yuzini toping. javob:256

2.255. Teng yonli uchburchak *ABC* ning *B* va *C* asosi uchlari va asosiga tushirilgan balandlik o‘rtasi *N* nuqta orqali asosga parallel o‘tkazilgan to‘g‘ri chiziqlari *AB* tomonni *D* nuqtada, *AC* tomonni esa *E* nuqtada kesib o‘tadi. Agar uchburchak *ABC* ning yuzasi 27 ga teng bo‘lsa, uchburchak *CED* ning yuzasini toping. javob: $\frac{27}{4}$

2.256. Uchburchak *ABC* da *AD* va *CF* bissektrisalar o‘tkazilgan. Agar *AB*=21, *AC*=28 va *CB*=20 bo‘lsa, uchburchak *AFD* ning yuzini uchburchak *ABC* ning yuziga nisbatini toping. javob: 1:4

2.257. Uchburchak *ABC* ning yuzasi 12 ga teng. *AK*, *BL*, *CN* medianalarida olingan *P, Q, R* nuqtalar uchun, *AP:PK*=1:1, *BQ:QL*=1:2, *CR:RN*=5:4 bo‘lsa, *PQR* uchburchakning yuzini toping. javob: 1

2.258. Uchburchak *ABC* da *AD* mediana va *BE* bissektrisa o‘zaro perpendikular va *F* nuqtada kesishadi. Uchburchak *DEF* ning yuzasi 6 ga teng bo‘lsa, uchburchak *ABC* ning yuzini toping. javob: 72

2.259. Perimetri 10 ga teng to‘g‘ri burchakli uchburchakning gipotenuzasiga tushirilgan balandlik uchburchakni ikkita uchburchakka ajratadi. Bu uchburchaklardan birining perimetri 6 ga teng bo‘lsa, ikkinchisining perimetrini toping. javob: 8

2.260. To‘g‘ri burchakli uchburchakning medianalari kesishgan nuqtasi katetlaridan 3 va 4 ga teng masofalarda joylashgan, bu nuqtadan gipotenuzagacha bo‘lgan masofani toping. javob: 2,4

2.261. To‘g‘ri burchakli uchburchakning to‘g‘ri burchagi uchidan tushirilgan balandlik va bissiktrisalari uzunliklari mos ravishda 3 va 4 ga teng bo‘lsa uchburchakning yuzini toping. javob: 72

2.262. To‘g‘ri burchakli uchburchakning *A* to‘g‘ri burchagi uchidan *AM* mediana, *AK* bissektrisa, *AH* balandlik o‘tkazilgan. *MK*=4 va *KH*=3 bo‘lsa, uchburchak *ABC* ning yuzini toping. javob: $3\sqrt{118}$

2.263.To‘g‘ri burchakli uchburchakning gipotenuzasiga tushirilgan balandlik gipotenuzani 7,2 va 12,8 ga teng kesmalarga ajratadi. Uchburchakning yuzini toping. javob: 96

2.264.To‘g‘ri burchakli uchburchakning bir o‘tkir burchagi 30° ga va gipotenuzasi uzunligi $5\sqrt[4]{3}$ ga teng bo‘lsa, bu uchburchakning yuzini toping. javob: 9,375

2.265.To‘g‘ri burchakli uchburchakning 60° li o‘tkir burchagi qarshisidagi kateti uzunligi $\frac{5\sqrt[4]{3}}{2}$ bo‘lsa, bu uchburchakning yuzini toping. javob: 3,125

2.266.To‘g‘ri burchakli uchburchakning gipotenuzasiga tushirilgan mediana to‘g‘ri burchakni 1:2 nisbatda bo‘ladi va mediana uzunligi 6 ga teng. Uchburchakning yuzini toping. javob: $18\sqrt{3}$

2.267.To‘g‘ri burchakli uchburchakning katetlari 6 va 18 ga teng, to‘g‘ri burchagi uchidan chiqarilgan bissektrisa uzunligini toping. javob: $\frac{9\sqrt{2}}{2}$

2.268.To‘g‘ri burchakli uchburchakning yuzasi $2\sqrt{2}$ ga teng. Gipotenuzasiga tushirilgan balandlik gipotenuzani 1:2 nisbatda bo‘lsa gipotenuzaning uzunligini toping. javob: $2\sqrt{3}$

2.269.To‘g‘ri burchakli uchburchakning o‘tkir burchagi bissektrisasi qarshisidagi katetni 10 va 26 ga teng kesmalarga ajratadi. Bu uchburchakning yuzini toping. javob: 270