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ODDIY DIFFERENSIAL TENGLAMALARDAN MISOL VA MASALALAR YECHISH(1-qism)

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Oddiy differensial tenglamalardan misol va masalalar yechish.(1-qism) O‘quv qo‘llanma. Guliston, 2024-yil, 168 bet.

Ushbu o‘quv qo‘llanma **Oddiy differensial tenglamalar** fanidan Oliy ta’lim muassasalarining 60540100-matematika ta’lim yo’nalishi kunduzgi va sirtqi ta’lim bakalavr talabalariga mo‘ljallab yozilgan. Unda **Oddiy differensial tenglamalar** fanining asoslarini sodda tilda ifodalashga harakat qilingan. O‘quv qo‘llanmada mavzularni chuqur o‘zlashtirish maqsadida o‘z-o‘zini tekshirish uchun savollar va mustaqil yechishga mo‘ljallangan masollar ham keltirilgan. O‘quv qo‘llanma ko‘proq amaliy tavsifga ega bo‘lib, undan talabalar, tadqiqotchilar va professor-o‘qituvchilar foydalanishi mumkin.

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SO‘ZBOSHI

Ushbu qo‘llanma “**Oddiy differensial tenglamalar**” fanidan o‘quv jarayonining III-IV semestrida ajratilgan soatlarda o‘tiladigan amaliy mashg‘ulot mavzularini qamrab olgan.

Qo‘llanmaning har bir paragrafida mavzuga oid asosiy ta’rif va xossalar (teoremalar) to‘la keltirilgan bo‘lib, amaliy mashg‘ulotda bajariladigan ishning o‘zi uch qismdan iborat. Birinchi qismda mavzuga oid nazariy savollar, ikkinchi qismda nazariy (muammoli) topshiriqlar, uchinchi, oxirgi qismda esa, amaliy topshiriqlar berilgan. Amaliy topshiriqlar bir necha masalalardan iborat va ularning har biridan bittadan namunaviy masala yechib ko‘rsatilgan.

Izoh: Qo’llanmada misol va masalalarni tartiblashda misol tartibi, qavs ichida esa “Филиппов А. Ф. Сборник задач по дифференциальным уранениям” kitobining misollar tartib raqami ko’rsatilgan.

“**Oddiy differensial tenglamalar**” fanidan nazorat materiallari bo‘limida oddiy differensial tenglamalar fanidan o‘tkaziladigan nazorat turlarida foydalanish mumkin bo‘lgan materiallar berilgan. Ushbu bo‘lim “Oddiy differensial tenglamalar” fanini o‘rganish jarayonida talabaning mustaqil ishlashini ta’minlovchi o‘quv-uslubiy materiallarni o‘z ichiga oladi hamda talaba olgan bilimining sifatini doimo nazorat qilishni ta’minlaydi.

§-1.
Differensial tenglamalar

Ta'rif. Erkli o'zgaruvchilar, ularning noma'lum funksiyasi (yoki vector funksiya) va noma'lum funksiyaning hosilasi qatnashgan tenglik **differensial tenglama** deyiladi. Agar differensial tenglamada erkli o'zgaruvchi bitta bo'lsa u oddiy differensial tenglama deyiladi. Erkli o'zgaruvchilar soni ikkita va undan ortiq bo'lsa u **hususiy hosilali differensial tenglama** deyiladi. Differensial tenglamada qatnashgan noma'lum funksiya hosilasining eng yuqori tartibi **tenglama tartibi**ni belgilaydi.

Hosilaga nisbatan yechilgan birirnchi tartibli oddiy differensial tenglama quyidagi korinishga ega:

$$y' = f(x, y) \qquad (1).$$

$f(x, y)$ funksiya $\Gamma \subset R^2$ sohada aniqlangan bo'lsin. Γ sohaning Ox o'qdagi proeksiyasi I intervaldan iborat bo'lsin.

Ta'rif. Agar I intervalda aniqlangan $y = y(x)$ funksiya quyidagi uchta shartni qanoatlantirsin:

1. $(x, y(x)) \in \Gamma,\ x \in I$ (2)

2. $y(x) \in C^1(I)$

3. $y'(x) = f(x, y(x)),\ x \in I$. (3)

U holda $y = y(x)$ funksiya I intervalda (1) **tenglamaning yechimi** deb ataladi. (1) tenglamaning har bir yechimining grafigi bu tebglamaning **integral chizig'i** deyiladi.

(1) tenglamaning

$$y(x_0) = y_0 \qquad (5)$$

boshlang'ich shartni qanoatlantiruvchi yechimini toppish masalasi – **Koshi masalasi** deyiladi. Bunda x_0, y_0 boshlang'ich berilganlar (qiymatlar) deb ataladi. Koshi masalasining geometric ma'nosi – (1) tenglamaning (x_0, y_0) nuqtadan o'tuvchi integral chizig'ini topishdan iborat. (1) tenglamaning faqat bitta integral chizig'i otadigan R^2 tekislikning nuqtalaridan iborat to'plamni D orqali belgilaylik.

Ta'rif. Agar 1) $y=\varphi(x,C)$ (6) bir parametrli chiziqlar oilasining har bir chizig'i (1) tenglamaning integral chizig'idan iborat; 2) ihtiyoriy $(x,y)\in D$ nuqtada (6) tenglamani C ga nisbatan bir qiymatli yechish mumkin bo'lsa, u holda (6) chiziqlar oilasi (1) tenglamaning **umumiy yechimi** deyiladi.

$y=\varphi(x,C)$ chiziqlar oilasining differensial tenglamasini tuzish uchin $\begin{cases} y=\varphi(x,C) \\ y'=\varphi'(x,C) \end{cases}$ sistemadan C ni yo'qotish kerak.

Quyidagi chiziqlar oilasi uchun differesial tenglama tuzilsin.

1.(21)-misol.

$x^2+Cy^2=2y$

Yechilishi .

$x^2+Cy^2-2y=0$

Бerilgan $y=y(x)$ - tenglamaning uzluksiz differensiallanuvchi echimi bo'lsin , bu terda Cx ga bog'liq bo'lmagan parametrdir. U holda o'ziga xoslik to'g'ri kelishi kerak

$$F(x,C)=x^2+Cy^2(x)-2y(x)=0$$

Bu erda $x\in X\subset R, X$ - bazi to'plamdir. F funksiya x ga nisbatan differensiallanadi. $\frac{\partial F(x,C)}{\partial x}=2x+2y(x)y'(x)C-2y'(x)=0$

$C=\frac{y'-x}{yy'}$ (yy'≠ 0)$(yy'\neq 0)$

(2) ni (1) ga almashtiramiz, hosilasini olib differensial tenglamasini olamiz .

2.(22)-misol.

$y^2+Cx=x^3$

Yechilishi.

$$y^2+Cx=x^3$$

$2yy' + C = 3x^2$

$C = 2x^2 - 2yy'$

$y^2 + (3x^2 - 2yy')x = x^3$

$y^2 - 2xyy' + 2x^3 = 0$

3.(23)-misol.

$$y = c(x-c)^2$$

Yechilishi. $y = c(x-c)^2$ $\quad y^2 = 2c(x-c)$ $\quad \frac{y}{y^2} = \frac{x-c}{2}$

$$\frac{2y}{y^2} = x - c$$

$C = x - \frac{2y}{y'}$

Shunday qilib

$y = (x - \frac{2y}{y'})(\frac{2y}{y'})^2$

Ikkala tomonga $(y')^3$ ko'paytiramiz

$y(y')^3 = (xy' - 2y)4y^2$

$$(y')^3 = (xy' - 2y)4y^2$$

4.(24)-misol.

$Cy = \sin C\,x$

Yechilishi.

$Cy = \sin C\,x \quad (1)$

$$Cy' = C\cos C\,x$$

$y' = \cos C\,x \quad 1 - (y')^2 = \sin^2 C\,x$

$$\sqrt{1-(y')^2} = \sin C\,x$$

Bu ifodaga berilganga qo'yamiz

$$C = \frac{\sqrt{1-(y')^2}}{y}$$

Bu ifodani ga almashtiramiz

$$y' = cos\frac{\sqrt{1-(y')^2}x}{y}$$

5.(25)-misol.

$y = ax^2 + be^x$

Yechilishi.

$\begin{cases} y' = 2xa + be^x \\ y'' = 2a + be^x \end{cases}$

$a = \frac{y'-bx}{2x}$

$$y'' = \frac{y'-bx}{2x} + be^x$$

$$xy'' = y' - bx + xbe^x$$

$b = \frac{xy''-y'}{xe^x - e^x}$

$a = \frac{y' - \frac{xy''-y'}{x-1}}{2x} = \frac{y'x - xy''}{2x(x-1)}$

$y = \frac{y'-y''}{2(x-1)}x^2 + \frac{xy''-y'}{x-1}$

$2y(x-1) = (y'-y'')x^2 + 2xy'' - 2y'$

$-y'x^2 + y''x^2 - 2xy'' + 2y' + 2(x-1)y = 0$

$xy''(x-2) - y'(x^2-2) + 2(x-1)y = 0$

6.(26)-misol

$(x-a)^2 + by^2 = 1$

Yechilishi.

$$(x-a)^2 + by^2 = 1$$

$\begin{cases} 2(x-a) + 2byy' = 0 \\ 2 - 2b(y')^2 + 2byy' = 0 \end{cases} \Rightarrow \begin{cases} a = x + byy' \\ b = -\frac{1}{y^2 + yy'} \end{cases}$

$$b^2y^2(y')^2 + by^2 = 1$$

1. $\frac{y^2(y')^2 - y^2((y')^2 + yy'}{(y')^2 + yy'}$

$$-y^3y' = (y')^2 + yy'$$

7.(27)-misol.

Berilishi.

$$ln(y) = ax + by$$

Yechilishi.

$$ln(y) = ax + by$$

$$(ln(y))' = (ax + by)'$$

$$\frac{y'}{y} = a + by' \Rightarrow b = \frac{(y''y - (y')^2}{y''y^2}$$

$$a = \frac{(y' - byy')}{y} = \frac{y'}{y} - \frac{yy'(y''y - (y')^2)}{y''y^3} = \frac{y'}{y} - \frac{y'(y''y - (y')^2)}{y''y^2}$$
$$= \frac{(y'y''y - y'y''y + (y')^3)}{y''y^2} = \frac{(y')^3}{y''y^2}$$

$ln(y) = ax + by = \frac{x(y')^3}{y''y^2} + \frac{y(y''y+(y')^2)}{y''y^2} y''y^2\, ln(y) = x(y')^2 + y''y^2 - y(y')^3 \Rightarrow y''y^2(ln(y) - 1) = (y')^2(xy' - y)$

$$y''y^2(ln(y) - 1) = (y')^2(xy' - y)$$

8.(28)-misol.

Berilishi.

$$y = ax^3 + bx^2 + cx$$

Yechilishi.

$$y = ax^3 + bx^2 + cx$$

$$y' = 3ax^2 + 2bx^2 + c$$

$$y'' = 6ax^2 + 2b$$

$$y''' = 6a$$

$$a = \frac{y'''}{6}$$

$$b = -\frac{xy''' - y''}{2}$$

$$c = \frac{x^2y'''}{2} - xy'' + y''$$

$$x^3y''' - 3x^2y'' + 6xy' - 6y = 0$$

9.(29)-misol.

$$x = ay^2 + by + c$$

Yechilishi.

$$x'''_{yyy} = 0$$

$$x'_y = \frac{1}{y'_x}$$

$$x'''_{yyy} = \frac{1}{y'_x}\left(\frac{1}{y'_x}\left(\frac{1}{y'_x}\right)_x{}'\right)_x{}' = \frac{y'_x y'''_{xxx} - 3(y''_{xx})^2}{(y'_x)^5}, y'_x y'''_x - 3(y''_x)^2 = 0$$

10.(30)-misol.

Markazi y=2x to'g'ri chiziqda yotuvchi radiusi 1 ga teng aylanalarning differentsial tenglamasini tuzing**.**

Yechilishi.

$$y = 2x$$

$$(x-a)^2 + (y-b)^2 = 1$$

$$y = 2x$$

$$b = 2a$$

$$(x-a)^2 + (y-2a)^2 = 1$$

$$2(x-a)^2 + 2(y-2a)^2 y' = 0$$

$$x - a + yy' - 2ay' = 0$$

$$a(2y'+1) = x + yy'$$

$$a = \frac{x + yy'}{2y'+1}$$

$$x - a = x - \frac{x + yy'}{2y'+1} = \frac{y'(2x + y\quad)}{2y'+1}$$

$$y - 2a = y - 2\frac{x + yy'}{2y'+1} = \frac{y - 2x}{2y'+1}$$

$$\frac{y'^2(y-2x)^2}{(2y'+1)^2} + \frac{(y-2x)^2}{(2y'+1)^2} = 1$$

$$(y-2x)^2(y'^2+1) = (2y'+1)^2$$

$$(y-2x)^2(y'^2+1)-(2y'+1)^2=0$$

11(40)-misol.

Berilgan oilaning chiziqlarini ma'lum φ burchak ostida kesishgan traektoriyalar uchun differentsial tenglamalarni tuzing:

$x^2 + y^2 = a^2, \varphi = 45°$

Yechilishi:

$x^2 + y^2 = a^2$ $\qquad tg\varphi = \frac{y_1'-y'}{(1+y_1'y')} = 1$

Berilgan egri chiziqdan hosila olamiz va y'=- $\frac{x}{y}$ ni topamiz.

1. Agar y'=$-\frac{x}{y}$ bo‘lsa buni oxirgi tenglikga qo‘yib y_1' ni topamiz:

$y' = -\frac{x}{y}$, $y_1' + \frac{x}{y}$ = 1-$y_1' * \frac{x}{y}$ yoki $\qquad yy_1' + x = y - y_1'x$

$$y_1' = \frac{y-x}{x+y}$$

2. Agar $y_1' = -\frac{x}{y}$ bo‘lsa, buni $\frac{y_1'-y'}{(1+y_1'y')} = 1$ tenglikga qo‘yib differensial tenglama tuzamiz:

$$y'(x-y) = y + x$$

12(41)-misol. Berilgan oilaning chiziqlarini ma'lum φ burchak ostida kesishgan traektoriyalar uchun differentsial tenglamalarni tuzing: $y = kx\ \varphi = 60°$

Yechilishi:

$$y = kx \quad y' = k$$

$$tg\varphi = \frac{y_1'-y'}{(1+y_1'y')} = tg60° = \sqrt{3}$$

1. $y_1' = k$ $\quad \frac{k-y'}{1+ky'} = \sqrt{3}$ tenglikdan y' ni topamiz,

$$y' = \frac{k-\sqrt{3}}{1+k\sqrt{3}}$$

$y' = \frac{y-\sqrt{3}x}{x+\sqrt{3}y}$

13(42)-misol. Berilgan oilaning chiziqlarini ma'lum φ burchak ostida kesishgan traektoriyalar uchun differentsial tenglamalarni tuzing:

$3x^2 + y^2 = C$, $\varphi = 30°$.

Yechilishi.

$3x^2 + y^2 = C$ dan hosila olamiz

$6x + 2yy' = 0$ dan $y' = -\frac{3x}{y}$

$$tg\varphi = \frac{y_1' - y'}{(1 + y_1'y')} = tg30° = \frac{\sqrt{3}}{3}$$

$y_1' = -\frac{3x}{y}$, $\frac{-\frac{3x}{y}-y'}{1-\frac{3x}{y}y'} = \frac{\sqrt{3}}{3}$ dan differensial tenglama tuzamiz:

$$y' = \frac{y - 3\sqrt{3}x}{\sqrt{3} - 3x}$$

14.(43)-misol.

Berilgan oilaning chiziqlarini ma'lum φ burchak ostida kesishgan traektoriyalar uchun differentsial tenglamalarni tuzing:

$\varphi = 60°$. $y^2 = 2px$

Yechilishi:

$y^2 = 2px$ $\quad 2yy' = 2px$ $\quad 2y\,y' = p$ $\quad y' = \frac{p}{3y}$

$\frac{(y_1'-y')}{(1+y1'y')} = tg60° = \sqrt{3}$

1. $y_1' = \frac{p}{y}$; $\left(\frac{p}{y} - y'\right)\left(1 + \frac{py'}{y}\right) = \sqrt{3}$ $p - yy' = \sqrt{3}$ $y = py'$ $\quad y'\left(y + \sqrt{3}\ p\right)$

$$= p - \sqrt{3}\ y \quad y'\left(\frac{2x}{y} + \frac{\sqrt{3}y^2}{2x}\right) = \frac{y^2}{2x} - \frac{2x}{\sqrt{3}y} \qquad y'\left(2x + \sqrt{3}y\right)$$

$$= y - 2\sqrt{3}x$$

$$2.\ y' = \frac{p}{y}: \quad \frac{\left(y_1' - \frac{p}{y}\right)}{\left(1 + \frac{py_1'}{y}\right)} = \sqrt{3}\ ; \ \ yy_1' - p = \sqrt{3}(y + py_1') \quad y_1'\left(y - \sqrt{3}p\right)$$

$$= p + \sqrt{3}y; \quad y_1'\left(\frac{2x}{y} - \frac{\sqrt{3}\,y^2}{2x}\right) = \frac{y^2}{2x} + \frac{2x}{\sqrt{3}y} \qquad y_1'\left(2x - \sqrt{3}\,y\right)$$

$$= y + \ 2\sqrt{3}\,x$$

15.(44)-misol

vazifa

Berilgan oilaning chiziqlarini ma'lum φ burchak ostida kesishgan traektoriyalar uchun differentsial tenglamalarni tuzing:

$r = a + cos\theta$, $\varphi = 90°$.

Yechilishi:

$$r = a + cos\theta \quad r' = -sin\theta \quad r' = r * ctg\theta = r * ctg\theta\ (\beta + 90^{\circ}) = -r * tg\beta$$

$$= -\frac{r}{ctg\beta} = -\frac{r^2}{r * ctg\beta} = -\frac{r^2}{r_\beta'} \qquad -\frac{r^2}{r_\beta'} = -sin\theta\ =$$

$$> \quad r_\beta' * sin\theta = r^2$$

16.(45)-misol

Berilgan oilaning chiziqlarini ma'lum φ burchak ostida kesishgan traektoriyalar uchun differentsial tenglamalarni tuzing:

$r\ =\ a\,cos2\theta, \varphi = 90°$

Yechilishi

$$r = a * cos^2\theta \qquad r' = \ -2a * sin\theta cos\theta$$

$$\frac{r}{r'} = -\frac{1}{2} * ctg\theta * \ a = \ -\frac{r'}{2sin\theta cos\theta}$$

$$r = \ -\frac{r'}{2ctg\theta}$$

$$r' = -\frac{r'}{rb'}$$

$$r = \frac{r'}{2r * b' * ctg\theta} \Rightarrow rb' = \frac{r}{2ctg\theta}$$

17(47)-misol

Vazifa:

Berilgan oilaning chiziqlarini ma'lum φ burchak ostida kesishgan traektoriyalar uchun differentsial tenglamalarni tuzing:

$y = x\,ln\,x + Cx, \varphi = arctg2$

Yechilishi:

$$y = x * lnx + Cx$$

$$y' = lnx + 1 + C = ln(ex) + C \Rightarrow C = \frac{y - x * lnx}{x}$$

$$\frac{y1' - y'}{1 + y1'y'} = tg(arctg2) = 2$$

$$y' = ln(ex) + \frac{y - x * lnx}{x} = \frac{x * lnx + x + y - x * lnx}{x} = 1 + \frac{y}{x}$$

1. $y' = 1 + \frac{y}{x} : \frac{y_1' - 1 - \frac{y}{x}}{1 + y_1'\left(1 + \frac{y}{x}\right)} = 2 \quad \frac{x\,y_1' - x - y}{x + y_1'(x + y)} = 2$

$$2x + 2\,y_1' * (x + y) = x\,y_1' - x - y$$

$$y_1'(x + 2y) = -3x - y$$

2. $y' = 1 + \frac{y}{x} \quad \frac{1 + \frac{y}{x} - y'}{1 + y'\left(1 + \frac{y}{x}\right)} = 2$

$$\frac{x+y-y'x}{x+y'(x+y)} = 2 \quad 2x+2y'(x+y) = x+y-y'x$$

$$y'(3x+2y) = y-x\,.$$

18.(48)-misol.

Vazifa:

Berilgan oilaning chiziqlarini ma'lum φ burchak ostida kesishgan traektoriyalar uchun differentsial tenglamalarni tuzing:

$x^2 + y^2 = 2ax, \varphi = 45°$.

Yechilishi:

$$x^2 + y^2 = 2ax \quad x^2 + y^2 = 2x^2 + 2xyy'$$

$$2x + 2yy' = 2a \qquad y' = \frac{-x^2+y^2}{2xy}$$

$$a = x + yy' \qquad \frac{y_1' - y'}{1 + y'y_1'} = 1$$

$$1 + y'y_1' = y_1' - y'$$

$$1. y' = \frac{y^2-x^2}{2xy} \qquad 1 + y_1' * \frac{y^2-x^2}{2xy} = y_1' - \frac{y^2-x^2}{2xy}$$

$$\frac{y_1'(y^2-x^2-2xy)}{2xy} = \frac{x^2-y^2-2xy}{2xy}$$

$$y_1'(x^2-y^2+2xy) = 2xy + y^2 - x^2$$

$$2. y_1' = \frac{y^2-x^2}{2xy} \quad 1 + \frac{y'(y^2-x^2)}{2xy} = \frac{y^2-x^2}{2xy} - y'$$

$$y'(y^2-x^2+2xy) = y^2-x^2-2xy\,.$$

19.(49)-misol

Berilgan oilaning chiziqlarini ma'lum φ burchak ostida kesishgan traektoriyalar uchun differentsial tenglamalarni tuzing:

$x^2 + C^2 = 2Cy \;\; \varphi = 90°$.

$Yechilishi$:

$x^2 + C^2 = 2Cy$

$$2x = 2Cy' \;\; => \;\; C = \frac{x}{y'}$$

$$x^2 + \frac{x^2}{y'^2} = \frac{2xy}{y'}$$

$$x(y'^2 + 1) \;\; = \; 2yy'$$

$$T.K\; L = 90 \;\; => \;\; y_1' = -\frac{1}{y'}$$

$$x\left(\left(-\frac{1}{y1'}\right)^2 + 1\right) = -\frac{2y}{y_1'} \;\; x\left[\frac{y_1'^2 + 1}{y_1'^2}\right] = \frac{2y\, y_1'}{y_1'^2} \;\; =>$$

$$x(y_1'^2 + 1) \; = \; -2yy_1'$$

20.(50)-misol.

Vazifa:

Berilgan oilaning chiziqlarini ma'lum φ burchak ostida kesishgan traektoriyalar uchun differentsial tenglamalarni tuzing:

$y \; = \; Cx \; + \; C^3 \,, \varphi = \; 90°$.

$Yechilishi$:

$y = \; Cx + C^3$

$$y' = C \;\; => \; y = y'x \; + \; y'^3$$

$$L = 90° \;\; => \; y_1' = -\frac{1}{y'};$$

$$y = \left(-\frac{1}{y_1'}\right) x \; - \frac{1}{y_1'^3} \; = \frac{-xy_1'^2 - 1}{y_1'^3}$$

$$y * y_1'^3 + xy_1'^2 \; = -1$$

§-2.

***O’zgaruvchilari ajralgan va ajraluvchi differentsial tenglamalar*.**

O’zgaruvchilarini ajratib èki boshqacha qilib aytganda, har ikkala tomonini bir xil funktsiyaga ko’paytirib èki bo’lib, bir tomonida faqat x ikkinchi tomonida y ishtirok etadigan ko’rinishga keltirish mumkin bo’lgan differentsial tenglama o’zgaruvchilari ajraladigan tenglama deyiladi. Bunday tenglamalarni echish uchun o’zgaruvchilarini ajratish va hosil bo’lgan tenglikni integrallash kerak.

21.(60) Misol.

Tenglamani yeching va bir nechta egri chiziqlarni chizing. $z' = 10^{x+z}$

$$\frac{dz}{dx} = 10^{x+z}$$
$$dz \cdot 10^{-z} = dx \cdot 10^x$$
$$\int 10^{-z}\, dz = -\frac{10^{-z}}{ln\, 1\, 0} + C$$
$$\int 10^x dx = \frac{10^x}{ln\, 1\, 0} + C$$
$$10^{-z} + 10x = C$$
$$z = -\, lg(C - 10^x)$$
$$Javob\,; z = -\, lg(C - 10^x)$$

22.(61) Misol

Tenglamani yechish va ko’paytmalarini tuzish integral egri chiziqlar.$x\frac{dx}{dt} + t = 1$

$$x\frac{dx}{dt} = 1 - t$$
$$xdx = (1 - t)dt$$
$$\frac{x^2}{2} = \frac{(1 - t)^2}{2} \; + C$$

$$x^2 + t^2 - 2t + 1 = 2C$$
$$x^2 + t^2 - 2t = C_1$$
$$C_1 = (2C - 1)$$
$$Javob\,; x^2 + t^2 - 2t = C$$

23.(62) Misol

Tenglamani yeching integral egri chiziqlar. $y' = cos(y - x)$

$$y' = cos(y - x)$$
$$y(x) - x = t(x)$$
$$y'(x) - 1 = t'(x)$$
$$y' = t' + 1$$
$$t' + 1 = cos\,t$$
$$cos\,t = 1 \Rightarrow t = 2\pi k \Rightarrow y = x + 2\pi k, k \in Z$$
$$\frac{dt}{cos\,t - 1} = dx$$
$$\int \frac{dt}{cos\,t - 1} = \int dx$$
$$\int \frac{dt}{cos\,t - 1} = \int \frac{dt}{-2\,sin^2\frac{t}{2}} = ctg\frac{t}{2} + C$$
$$\int dx = x + C$$
$$Javob\,; c\,tg\frac{x - y}{2} = x + C, y = x + 2\pi k, k \in Z$$

24.(63) Misol

Tenglama va chizmani yechish bir nechta integral egri chiziqlar.$y' - y = 2x - 3$

$$y' - y = 2x - 3$$
$$y' = y + 2x - 3$$
$$y + 2x = t(x)$$
$$y' = t' - 2$$
$$t' - 2 = t - 3$$
$$\frac{dt}{dx}t - 1$$
$$\frac{dt}{t - 1} = dx$$
$$x = ln\,|\,t - 1| + C$$
$$t = 1 + Ce^x$$
$$y + 2x = 1 + Ce^x$$
$$Javob\,; y + 2x - 1 = Ce^x$$

25(64) Misol

Tuzilgan tenglamani yeching va dastlabki shartlarga mos keladiganini toping.

$(x+2y)y' = 1\,; y\,(0) = -1$
$(x+2y)y' = 1$
$y(0) = -1$
$x + 2y = t$
$1 + 2y' = t' \Rightarrow y' = \frac{t'-1}{2}$
$t\frac{t'-1}{2}$
$t\frac{dt}{dx} = t + 2$
$t = -2 \Rightarrow x + 2y + 2 = 0$
$\frac{tdt}{t+2} = dx$
$\int \frac{tdt}{t+2} = \int \left(1 - \frac{2}{t+2}\right)dt = t - 2\,ln\,|\,t+2|$
$t - 2\,ln\,|\,t+2| = x + C$
$x + 2y - 2\,ln\,|\,x + 2y + 2| = t + C$
$ln\,|\,x + 2y + 2| = y + C_1, \left(C_1 = -\frac{C}{2}\right)$
$x + 2y + 2 = Ce^y, C = 0, y(0) = -1$
$0 - 2 + 2 = Ce^{-1} \Rightarrow C = 0$
$Javob\,; x + 2y + 2 = 0$

26.(65) Misol

Integralni yeching.$y' = \sqrt{(4x+2y-1)}$

$y' = \sqrt{(4x+2y-1)}$
$y' = \sqrt{4x+2y-1}$
$4x + 2y - 1 = z$
$z'4 + 2y'$
$y' = \frac{z'-4}{2}$
$\frac{z'-4}{2} = \sqrt{z}$
$\frac{1}{2}\frac{dt}{dx} - 2 = \sqrt{z}, \sqrt{z} \neq -2$
$\frac{1}{2}dt - (2 + \sqrt{z})dx = 0, \left(2 + \sqrt{z}\right)$
$\frac{1}{2}\int \frac{dt}{\sqrt{z+2}} - \int dx = 0$
$\sqrt{z+2} = t$

$$\frac{1}{2\sqrt{z}}dz = dt$$
$$\int \frac{2\sqrt{z}dt}{t} = \int \frac{2(t-2)}{t}dt = 2\int dt - 2\int \frac{2dt}{t} =$$
$$= 2t - 4\ln t = 2(\sqrt{z}+2) - 4\ln(\sqrt{z}+2) + C$$
$$\sqrt{z} + 2 - 2\ln|\sqrt{z}+2| - x = C$$
$$Javob;\sqrt{4x+2y-1} - 2\ln|\sqrt{4x+2y-1}+2| - x = C$$

27.(66) Misol

Differensial tenglamani yechimini toping $x \to +\infty$ da berilgan shartni qondirsin.$x^2y' - \cos 2y = 1; y(+\infty) = \frac{9\pi}{4}$

$$x^2y' - \cos 2y = 1; y(+\infty) = \frac{9\pi}{4}$$
$$\frac{dy}{2\cos^2 y} = \frac{dx}{x}, x \neq 0, \cos y \neq o$$
$$\frac{1}{2}tgy = C - \frac{1}{x}, y = arctg\left(2C - \frac{2}{x}\right) + 2kx, k \in Z$$
$$\pi = \lim_{x\to+\infty} y = \lim_{x\to+\infty}(arctg\left(2C - \frac{2}{x}\right) + 2k\pi) = arctg2C + 2k\pi$$
$$|arctg2C| \prec \frac{\pi}{2}, k = 1$$
$$2C = \frac{\pi}{4}, C = \frac{1}{2}$$
$$y = arctg\left(1 - \frac{2}{x}\right) + 2\pi$$

28.(67) Misol

Differensial tenglamani yechimini toping $x \to +\infty$ da berilga shartni qondirsin.$3y^2y' + 16x = 2xy^3; y(x)$

$$3y^2y' + 16x = 2xy^3$$
$$3y^2y' + 16xdx - 2xy^3dx = 0$$
$$3y^2dy + 2x(8 - y^3)dx = 0; 8 - y^3 \neq 0$$
$$\frac{dy^3}{8 - y^3}dy + 2xdx = 0$$
$$\frac{dy^3}{8 - y^3} + 2xdx = 0$$
$$-\ln|y^3 - 8| + \ln e^{x^2} = \ln e$$
$$y^3 - 8 = ce^{x^2}$$
$$y \Rightarrow \infty, x \Rightarrow \infty, c = 0, y = 2$$

29.(68) Misol

Quydagi turkumlarning chiziqlariga ortogonal trayektoriyalarini toping.

$a) y = Cx^2\,; b) y = Ce^x\,; d) Cx^2 + y^2 = 1 \quad y = Cx^2 \quad C = \frac{y'}{2x} \quad y' = \frac{2y}{x} \quad y'$
$= \frac{x}{2y}$

$2ydy = -xdx \quad 2\int ydy = -\int xdx, \quad y^2 + \frac{x^2}{2} = C \quad y' - \frac{1}{y}$

$ydy - dx; \quad \int ydy = -\int xdx, \quad \frac{y^2}{2+x} - c \quad y^2 + 2x - C, \qquad Cx^2 + y^2 = 1$

$y' = \frac{y^2-1}{xy}, y' \ \frac{xy}{1-y^2} \quad \left(\frac{1}{y} - y\right) dy0dxy; \quad \int \left(\frac{1}{y} - y\right) dy = 2y^2 - x^2 = C, \ \ y$
$= Ce^x \ \ y' = y \quad \int xdx, ln|y| = \frac{x^2}{2} + \frac{y^2}{2} + C$

$Javob: a) \ \ 2y^2 + x^2 = C; \quad b) \ \ y^2 + 2x = C \quad d) \ y^2 = Ce^{x^2+y^2}$

30.(69) Misol

$y' = \sqrt{\left(\frac{(y^2+1)}{(x^4+1)} : 3\right)}$ tenglamaning har bir integral egri chizig'i ikkita gorizantal asimptotaga ega ekanligini korsating

$$y' = \sqrt{\left(\frac{(y^2+1)}{(x^4+1)} : 3\right)}$$
$$y' = \sqrt[3]{\frac{y^2+1}{x^4+1}}$$
$$\int_{y_0}^{y} \frac{du}{\sqrt[3]{u^2+1}} = \int_{x_0}^{x} \frac{dt}{\sqrt[3]{t^4+1}}$$
$$x \to +\infty$$
$$\int_{x_0}^{x} \frac{dt}{\sqrt[3]{t^4+1}}$$
$$\lim_{x\to+\infty} \int_{y_0}^{y(x)} \frac{du}{\sqrt[3]{u^2+1}} = a$$
$$\int_{y_0}^{+\infty} \frac{du}{\sqrt[3]{u^2+1}}$$
$$\lim_{x\to+\infty} y(x) = y(+\infty), y(+\infty) \succ y_0, a \succ 0$$
$$x \to +\infty,$$
$$\int_{y_0}^{y(x)} \frac{du}{\sqrt[3]{u^2+1}} = b, \int_{x_0}^{+\infty} \frac{dt}{\sqrt[3]{t^4+1}} \prec 0$$
$$-\infty \prec y(-\infty) \prec y_0$$
$$y(-\infty), y(+\infty)$$

31.(70) Misol

$y' = \sqrt{\left(\frac{ln(1+y)}{sin\,x}\right)}$ tenglamasining integral egri chiziqlarining kelib chiqishi yaqindagi harakatni o'rganing. Buni har bir nuqtadan kuzating.

$$y' = \sqrt{\frac{ln(1+y)}{sin\,x}}$$

$$D = \bigcup_{k=-\infty}^{+\infty} M_{k1}\{(x,y) \in R^2 | 2k\pi \prec x \prec (2k+1)\pi, 0 \le y \prec +\infty\} \cup$$

$$\cup \{(x,y) \in R^2 | (2k+1)\pi \prec x \prec 2(k+1)\pi - 1 \prec y \le 0\}$$

$$0 \le y \prec +\infty, 0 \prec x \prec \pi, 0 \prec x_0 \prec \pi$$

$$\int_{x_0}^{x} \frac{dt}{\sqrt{sin\,t}} = \int_{0}^{y} \frac{du}{\sqrt{ln(1+u)}}$$

$$u \to 0\; ln(1+u) \sim u$$

$$\int_{0}^{y} \frac{du}{\sqrt{ln(1+u)}} \succ 0, \int_{x_0}^{x} \frac{dt}{\sqrt{sin\,t}} \succ 0$$

$$\int_{0}^{x} \frac{dt}{\sqrt{sin\,t}} = \int_{y_0}^{y} \frac{du}{\sqrt{ln(1+u)}}, 0 \le y \prec +\infty, 0 \prec x \prec \pi, 0 \le y_0 \prec +\infty$$

$$-1 \prec y \le 0, -\pi \prec x \prec o$$

$$\int_{x_0}^{x} \frac{dt}{\sqrt{-\,sin\,t}} = \int_{0}^{y} \frac{du}{\sqrt{-\,ln(1+u)}}$$

§-3.

Geometrik va fizik masalalar.

79-misol. Berilishi:

Hajmi $200m^3$ bo'lgan xona havosining $0,15$ % is gazidan (CO_2) iborat. Tashqaridagi havoning $0,04$ % is gazidan iborat.Vintilyator har minutda ichkaridagi $20m^3$ havoni tashqaridagi havoga almashtiradi. Qancha vaqtdan keyin honadagi havoda is gazi ikki marta kamayadi ?

Yechilishi:

Faraz qilaylik , $Q(t)$ - t vaqtda is gazi bo'lsin ,u holda t vaqtdagi qqq is gazi kansentratik ga teng bo'ladi (o'lchov m^3 da).

Masala shartiga ko'ra , har minutda CO_2 miqdori $0,1Q(t)m^3$ bo'lgan $20m^3$

Havo chiqib ketadi. Bu vaqtda vintilyator $\frac{0,04\%}{100\%} * 20$ dt $m^3 = 0,008$ dt m^3 CO_2 honaga haydaydi . Shunday qilib dt vaqt oralig'ida CO_2 ning dQ miqdori ($0,08 - 0,1Q$)dt teng bo'ladi.

Yani

dQ=($0,08 - 0,1Q$)dt

differensial tenglamani qanoatlantiradi. Bu tenglamani yechib

Q(t)=($0,08 - Ce^{-0,1t}$)dt m^3 ni topamiz. Masala shartiga ko'ra t=0 bo'lganda

$Q = 200 * 0,15\% \, m^3 = 200 * \frac{0,15}{100} m^3 = 0,3m^3$ tenglikdan $C = -0,22$ ni topamiz.
Shunday qilib $Q(t) = 0,08 + 0,22 * e^{-0,1t}$ ni hosil qilamiz , endi $Q = 0,1$ bo'lganda t=T ni topish zarur. bundan $T = 10 * \ln 11 \approx 24\min$ .Javob: 24min

80-misol. Berilishi:

Jisim 10 minutda 100° dan 60° gacha sovidi. Jism atrofidagi temperatura 20° ga teng. Qachon jism 25° gacha soviydi?

Yechilishi:

T(t)-jismning t vaqtigacha tempuraturasi , T_0- temperaturasi bo'lsin.

U holda jism temperaturasi o'zgarishi

$$\frac{dT}{dt} = k(T - T_0) \qquad (1)$$

kabi bo'ladi, bu yerda k-proporsionallik kayfitsenti . Masala shartiga ko'ra $T_0=20^\circ$.
Endi (1) ni $\frac{dT}{T-20} = kdt$ kabi integrallaymiz va $T(t) = 20 + Ce^{kt}$ ni topamiz .

$T(0) = 100^o$ ekanligidan $C = 80$ ni topamiz. U holda $T(t) = 20 + 80e^{kt}$

$T(10) = 60^o$ shartdan k ni topamiz. $60 = 20 + 80e^{10k}$ bundan $k = -0,1\ln 2$, demak

$$T(t) = 20 + 80e^{-0,1t\ln 2} \quad \text{yoki}$$

$$T(t) = 20 + 80 * 2^{-0,1t}$$

U holda $T = 25^o$ tenglikga ko'ra $t = 40\min$. Javob: $40\min$.

81-misol. Berilishi:

Idishda 20^o tempiraturaga ega 1 kg suv bor. Idishda $0,5kg$ massali solishtirma issiqlik sig'imi 0,2 ga teng va 75^o tempiraturali metal solindi. Qachon suv va metal tempiraturalari bir biridan 1^o farq qiladi. Bunda idishning isishi hisobiga issiqlik yo'qolishi hisobga olinmaydi?

Yechilishi :

$T_j, T_s -$ jism va suvning t-vaqtdagi tempiraturasi bo'lsin, bundan oldingi masalaga ko'ra

$\frac{dT_j}{dt} = k_1(T_j - T_s), \frac{dT_s}{dt} = k_2(T_s - T_j)$ bu tenglikdan birinchisidan ikkinchisini ayirib,

va $T = T_j - T_s$ kabi belgilab $\frac{dT}{dt} = kT, k = k_1 + k_2$ tenglikni hosil qilamiz. Bundan

$T = Ce^{kt}$ ni topamiz. Masala shartiga ko'ra t=0 da T=55 teng ekanligidan C=55 ni topamiz.

Demak , $T = 55e^{kt}$ (1)

k ni topish uchun , issiqlik muozanati tenglamasidan foydalanamiz.

$Q = cm(T_k - T_j),$

Bu yerda c-jismning solishtirma issiqlik zichligi , m-uning massasi ,

$T_k - T_j -$ tempiratura farqi . $Q_1 = 2c_1, Q_2 = 0,2*0,5(75 - T)c_1$

C_1 –suvning solishtirma zichligi. Q_1 - suv yutgan issiqlik miqdori. Q_2 –T tempiraturagacha jism yutgan tempiratura miqdori.Masala shartiga ko'ra .

$Q_1 = Q_2$ yani $2c_1 = 0,1(75 - T)c_1 \Rightarrow T = 55^o$ shunday qilib , $R = 55^o - 22^o = 33^o$.

U xolda (1) ga ko'ra $t = 1^o$ bo'lganda $33 = 55e \Rightarrow k = \ln\frac{3}{5} = \ln 0,6$ shu sababli

$R = 55e^{(\ln 0,6)t} = 55*0.6^t$ bundan R=1 bo'lganda

$A^t = \frac{1}{55} \Rightarrow (\ln 0.6)^t = -\ln 55 \Rightarrow t = \frac{\ln 55}{\ln 5 - \ln 3} \approx 8\min$ .Javob:8 min.

82-misol. Berilishi :

Tempiraturasi a gradus bo'lgan metal parchasi pechkaga joylandi. Bir soat ichida pechkaning tempiraturasi a gradusdan b gradusga oshirildi. Metal va pechka tempiraturasi orasidagi farq T bo'lganda , metal tempiraturasi minutiga kT gradus ko'tariladi. Bir soatdan keyin metalning tempiraturasini toping?

Yechilishi :

Masala shartiga va oldingi masalalarga (80-misol) ko'ra $\frac{dT_m}{dt} = k(T_n - T_m)$

Bu yerda T_n , T_m -mos ravishta pech va metalning tempiratursi. T_n –pech tempiraturasi xar minutda oshib borganligi sababli $T_n = a + \frac{b - a}{60}t$

T_n –vaqt(minutlarda) . U xolda $\frac{dT_m}{dt} = k(a + \frac{b-a}{60}t - T_m)$ (1)

$a + \frac{b-a}{60}t - T_m = z$ belgilash kiritamiz. Bundan $a + \frac{b-a}{60}dt - dT_m = dz$ /:dk

$\frac{b-a}{60} - \frac{dT_m}{dt} = \frac{dz}{dt}$ (1) ga ko'ra $\frac{b-a}{60} - kz = \frac{dz}{dt}$

bundan $\frac{dz}{kz - \frac{b-a}{60}} = -dt$ xosil bo'ladi. Buni integrallaymiz:

$\frac{1}{k}\ln(kz - \frac{b-a}{60}) = -t + \frac{1}{k}\ln C_o$, k≠0 ; $kz - \frac{b-a}{60} = Ce^{-kt}$ $\Longrightarrow$

$$k(a + \frac{b-a}{60}t - T_m) - \frac{b-a}{60} = Ce^{-kt} \Rightarrow T_m = a + \frac{b-a}{60}(t - \frac{1}{k}) + Ce^{-kt}$$

$T_m(0)=a$ bo'lganligi uchun $C = \frac{b-a}{60k}$ demak , $T_m = a + \frac{b-a}{60}(t - \frac{1}{k}(1 - e^{-kt}))$

Metalning 1 soat=60 minutdagi tempiraturasi $T_m(60) = b - \frac{b-a}{60k}(1 - e^{60k})$

83- misol. Berilishi .

Qayiqning tezligi suvning qarshiligi tasirida ,qayiq tezligiga proparsional ravishta sekinlashadi. Boshlang'ich tezlik $1{,}5m/sek$ bo'lgan qayiqning tezligi 4 sekunddan keyin $1m/sek$ ga tushdi. Qachon qayiqning tezligi $1sm/sek$ ga tushadi.qayiq qancha masofani bosib o'tgach to'xtaydi?

Yechilishi:

v(t)-qayiqning t-vaqtdagi tezligi bo'lsin. U holda $\frac{dv}{dt}$ -tezlanish. Nyutenonning

2-qonunuga ko'ra $\frac{dv}{dt} = \frac{F}{m}$ (1)

F-suvning qarshilik kuchi , m-massa . F=kv bo'lganligi uchun

$\frac{dv}{dt} = k_o v$, $k_o = \frac{k}{m} - const$ tenglamani integrallab. $v(t) = Ce^{k_o t}$ ni

topamiz. v(0)=1,5 dan C=1,5 ni topamiz. $v(t) = 1{,}5e^{k_o t}$,t-sekundlarda, v(4)=1 tenglikdan $k_o = 0,25\ln(\frac{2}{3})$ ni topamiz .Shunday qilib qayiqning tezligi

$v(t) = (\frac{2}{3})^{t/4-1}$ qonun bo'yicha o'zgarar ekan . Agar $v = 1\frac{sm}{s} = 0,01\frac{m}{s}$ dan

$t_1 = 4(1 + \frac{\ln 0,01}{\ln(\frac{2}{3})}) \approx 50 sekund$. Masalani yechish uchun $v(t) = (\frac{ds(t)}{dt})$

tenglikdan foydalanishimiz zarur. Javob:50 sekund .

84-misol. Berilishi:

30 kunda radioktiv modda dastlabki miqdorining 50% i yemirildi. Qancha vaqt o'tib modda modda dastlabki miqtorining 1% qoladi?

Yechilishi:

Q(t)-t vaqatdagi radioktiv modda miqdori bo'lsin. U xolda radiaktiv modda miqdorining o'zgarish tezligi modda miqdoriga proporsional bo'lganligi uchun $\frac{dQ}{dt} = kQ$ tenglamada beriladi. Bundan $Q(t) = Ce^{kt}$ ni topamiz .

T=0; da Q(0)=Q_0 dan C= Q_0 ,$Q(t) = Q_o e^{kt}$ masala shartiga ko'ra , t=30 kunda .

$0,5Q_o = Q_o e^{30k} \Rightarrow k = -\frac{\ln 2}{30}$, $Q_1 = 0,01Q_o$ Dan $0,01Q_o = Q_o e^{-\frac{\ln 2}{30}t}$,

t=200 kun . Javob : 200 kun .

85-misol. Berilishi:

Tajribalar shuni ko'rsatadiki bir yilda har bir gram radiyning $0.44mg$ i yemiriladi.

Necha yildan keyin mavjud radiyning yarmi eriydi ?

Yechilishi:

Q(t)-t vaqatdagi radiy miqdori bo'lsin. U xolda 84- masalaga ko'ra

$Q(t) = Q_o e^{kt}$ -radiyning yemirilishi qanday bo'ladi. t-vaqt(yil). Masala shartiga ko'ra $Q(0) = 1gramm$, $Q(1) = 999,56milligramm$, u holda $e^k 1g = 999,56mg \Rightarrow e^k = 0,99956$. Demak $Q(t) = Q(0)(0.99956)^t$

t=T-? $Q(T) = \frac{1}{2}Q(0)$, $\frac{1}{2}Q(0) = Q(0)(0.99956)^T$,

$$T=-\frac{\ln 2}{\ln 0.99956}\approx 1600 yil$$. Javob: 1600 yil.

86-misol. Berilishi:

Tog' jinsini tarkibi o'rganilganda, unda 100 mg uran va 14 mg qo'rg'oshin mavjudligi aniqlandi. Ma'lumki , $4.5*10^9$ yilda uranning yarmi yemiriladi va 238 gr uran to'liq nurlanganda 206 gr qo'rg'oshinni hosil qiladi. Tog' jinsi yoshini aniqlang. Bunda tog' jinsi hosil bo'lgan vaqtda uning tarkibida qo'rg'oshin bo'lmagan deb hisoblang hamda uran va qo'rg'oshinning oraliq radioktiv moddalarini e'tiborga olmang (chunki bunday moddalar urandan ko'ra tezroq yemiriladi) ?

Yechilishi :

y-boshlang'ich paytda tog' jinsida bo'lgan uran miqdori bo'lsin. Unda masala shartiga ko'ra $\frac{y}{14}=\frac{238}{206}\Rightarrow y\approx 46.2mg$. Oldingi masalalar kabi muloxaza yuritib

$Q(t)=46.2e^{kt}$, yechilishidan qolgan uran miqdori va uning yarim yemirilishdan $k=-\frac{\ln 2}{4.5*10^9}$ ni topamiz . Masala shartiga ko'ra ,Q(t)=100mg

$$100=116.2e^{kt}\Rightarrow T=-\frac{1}{k}\ln 1{,}162=\frac{4{,}5*10^9}{\ln 2}\ln 1{,}162\approx 970*10^6\ yil$$.

Javob: $970*10^6$ yil.

87-misol. Berilishi:

Kichik qalinlikdagi suv qatlamida yutilayotgan yorug'lik miqdori suvga tushayotgan yorug'lik miqdoriga va qatlam qalinligiga proporsional. Qalinligi 35 sm bo'lgan suv qatlami unga tushayotgan yorug'likni yarmini yutadi. Qalinligi 2 m bo'lgan suv qatlami yorug;likni qancha qismini yutadi?

Yechilishi:

J(z)-z-qalinlikdagi suvdasn o'tgan yorug'lik miqdori. U holda masala shartiga ko'ra $J(z+\Delta z)-J(z)=kJ(z)\Delta z\,, k-const$ shunday qilib , J(z)-o'tgan yorug'lik miqdoriga nisbatan $\frac{dJ(z)}{dz}=kJ$ differensial tenglama hosil bo'ladi. Bu tenglamani integrallab , $J(z)=J(0)e^{kt}$ hosil qilamiz . $J(35)=\frac{1}{2}J(0)$

ekanligidan $\frac{1}{2}=e^{35k} \Rightarrow k=-\frac{\ln 2}{35}$ ni topamiz . $J(z)=J(0)e^{-\frac{\ln 2}{35}z}=J(0)(\frac{1}{2})^{\frac{z}{35}}$,

z-uzunlik(sm). Agar z=2m=200sm ga yorug'likning $\frac{J(200)}{J(0)}100\% =$

$\left(\frac{1}{2}\right)^{\frac{200}{35}}100\% = \left(\frac{1}{2}\right)^{\frac{1}{2}}100\% \approx 98\%$ Место для формулы. qismi o'tadi. Demak 2% yutilar ekan . Javob: 2% .

88-misol. Berilishi:

Parashutchi 1.5 km balandlikda sakradi va 0.5 km balandlikda parashudini ochdi. U parashudini ochganga qadar qancha vaqt pastga tushdi? Ma'lumki insonning narmal zichlikka ega havodagi tushish tezligi chegarasi 50 m/sek. Havoning qarshilik tezlikning kvadratiga proporsional. Havo zichligining balandlikka bog'lik o'zgarishini hisobga olmang ?

Yechilishi:

Masala shartiga binoan nyutonning 2-qonunuga ko'ra $m\frac{dv}{dt}=mg-kv^2$ (k>0)

m-parashudchi massasi , g≈10 $\frac{m}{sm^2}$, $\frac{dv}{g-k_o v^2}=dt, k_o=\frac{k}{m};$ differensial tenglamani

integrallaymiz. $\frac{1}{2\sqrt{gk_o}}\ln\left|\frac{a-v}{a+v}\right|=t+\ln C$, $a=\sqrt{\frac{g}{k_o}}\approx 50$,

$\frac{1}{2g\sqrt{\frac{k_o}{g}}}\ln\left|\frac{a-v}{a+v}\right|=t+\ln C$, v(0)=0 tenglikdan lnC=0 kelib chiqadi .

$$\frac{1}{2*10*\frac{1}{50}}\ln\left|\frac{50-v}{50+v}\right|=t \Rightarrow \ln\left|\frac{50-v}{50+v}\right|=\frac{2}{5}t \Rightarrow 50-v=e^{0,4t}(50+v) \Rightarrow v=50\frac{1-e^{0,4t}}{1+e^{0,4t}}=$$

$=50\frac{e^{-0,2t}-e^{0,2t}}{e^{-0,2t}+e^{0,2t}}=50th0,2t$, $v=50th\frac{t}{5}$. U holda bosib o'tilgan yo'l

$S=\int vdt=250\ln coh\frac{t}{5}$, S(t)=1000m=1km bo'lganda T=? ,

$$250\ln ch\frac{T}{5}=1000 \Rightarrow ch\frac{T}{5}=e^4 \Rightarrow \frac{e^{-0,2T}+e^{0,2T}}{2}\approx e^4 \Rightarrow \frac{e^{0,2T}}{2}\approx e^4 \Rightarrow$$

$\Rightarrow T=5\ln(2e^4)=5(4+\ln 2)\approx 23sekund$. Javob :23 sekund .

§-4.
Birinchi tartibli chiziqli tenglama

Birinchi tartibli chiziqli differensial tenglama deb

$$y' = a(x)y + b(x) \qquad (1)$$

ko'rinishdagi tenglamani aytamiz.

Teorema. Agar $a(x)$ va $b(x)$ funksiyalar biror I intervalda uzluksiz bo'lsa u holda $\Gamma = \{(x, y): x \in I, -\infty < y < \infty\}$ sohaning ihtiyoriy olingan (x_0, y_0) nuqtasidan (1) tenglamaning faqat bitta integral chizig'i o'tadi va bu chiziq

$$y = \left[y_0 + \int_{x_0}^{x} e^{-A(t)} b(t)dt \right] e^{A(x)}, \quad e^{A(x)} = \int_{x_0}^{x} a(t)dt \qquad (2)$$

formula bilan ifodalanadi.

(2) tenglamaning umumiy yechimini hosil qilishning **o'zgarmasni variatsiyalash** usuli bilan tanishamiz. $y' = a(x)y$ tenglama (1)ga mos bir jinsli tenglama deb ataladi. Bu tenglama o'zgaruvchilari ajraladigan differensial tenglama bo'lib uning umumiy yechini yozaylik: $y = Ce^{A(x)}$. (1) tenglamani umumiy yehimini

$$y = C(x)e^{A(x)} \qquad (3)$$

ko'rinishda qidiramiz. (3) funksiyani va uning hosilasini (1)ga qo'yamiz:

$$C'(x)e^{A(x)} + C(x)e^{A(x)}a(x) = a(x)C(x)e^{A(x)} + b(x).$$

Bundan

$$C'(x) = e^{-A(x)}b(x), \quad C(x) = C + \int_{x_0}^{x} e^{-A(t)} b(t)dt.$$

$C(x)$ ning topilgan ifodasini (3) ga qo'ysak (1) tenglamanaing **umumiy yechimi** hosil bo'ladi:

$$y = \left[C + \int_{x_0}^{x} e^{-A(t)} b(t)dt \right] e^{A(x)}. \qquad (4)$$

Bernulli tenglamasi deb

$$y' = a(x)y + b(x)y^m \qquad (5)$$

ko'rinishdagi tenglamani aytamiz. Agar $m = 0$ bo'lsa bu tenglama (1) ko'rinishni oladi. Agar $m = 1$ bo'lsa (5) tenglama o'zgaruvchilari ajraladigan differensial tenglamadan iborat. Biz $m \neq 0$ va $m \neq 1$ bo'lgan holda (5) tenglamani integrallash ketma-ketligini ko'rib chiqamiz. (5)ni y^m ga bo'lamiz:

$$y^{-m}y' = a(x)y^{1-m} + b(x)$$

No'ma'lum funksiyani $z = y^{1-m}$ formula bilan almashtiramiz ($z' = (1-m)y^{-m}y'$):

$$z' = (1-m)a(x)z + (1-m)b(x).$$

Bu tenlama z ga nisbatan birinchi tartibli chiziqli differensial tenglamadir va biz uni integralalshni yuqorida ko'rib o'tdik.

Ta'kidlash joizki $m > 0$ bo'lgan holda (1) tenglama hamma vaqt $y = 0$ yechimga ega bo'ladi. Agar $m < 1$ bolsa bu yechim mahsus yechimdan, aks holda hususiy yechimdan iborat.

Misol. $y' - \frac{1}{x}y = -\frac{1}{x}y^2$ tenglamani qaraymiz. Uni y^2 ga bo'lamiz:

$$y^{-2}y' - \frac{1}{x}y^{-1} = -\frac{1}{x}$$

Bu yerda $y^{-1} = z$ almashtirish bajaramiz, natijada: $z' + \frac{1}{x}z = \frac{1}{x}$. Bu chiziqli tenglamani umumiy yechimi $z = \frac{1}{x}(C + x)$. Bundan $y = \frac{x}{C + x}$. Berilgan tenglamaning bu umumiy yechimga kirmagan $y = 0$ hususiy yechimi ham mavjud.

Javob: $y = \frac{x}{C + x}$, $y = 0$.

Rikkati tenglamasi deb

$$y' = a(x)y^2 + b(x)y + c(x) \qquad (6)$$

ko'rinishdagi tenglamani aytamiz. Agar $a(x) \equiv 0$ bo'lsa bu tenglama (1) ko'rinishni olai. Agar $c(x) \equiv 0$ bo'lsa (6) tenglama Bernulli tenglamasidan iborat bo'ladi.

Teorema. Agar Rikkati tenglamasining bitta hususiy yechimi ma'lum bo'lsa u holda uni kvadraturalarda integrallash mumkin.

Isbot. $y = y_1(x)$ funksiya (6) tenglamani qanoatlantirsin. U holda

$$y_1'(x) \equiv a(x)y_1^2 + b(x)y_1 + c(x) \qquad (7)$$

ayniyat o'rinli. (6) tenglamada $y = z + y_1(x)$ almashtirish bajaramiz:

$$z' + y_1'(x) = a(x)[z + y_1(x)]^2 + b(x)[z + y_1(x)] + c(x).$$

Bu va (7) tenglikdan $z' = [2a(x)y_1(x) + b(x)]z + a(x)z^2$ Bernulli tenglamasi hosil bo'ladi va uni kvadraturalara integrallanishi bizga ma'lum. Teorema isbotlandi.

Teorema isbotida ko'rdikki Rikkati tenglamasi Bernulli tenglamasining $m = 2$ bo'lgan holiga aylanadi. Misollar yechish vaqtida agar birdan $y = \frac{1}{z} + y_1(x)$ almashtirish bajarilsa Rikkati tenglamasini yechish chiziqli tenglamani integrallashga keladi.

Misollar yechish vaqtida (6) tenglamani hususiy yechimi berilmagan bo'lsa ba'zan uni biror ko'rinishda izlab topish mumkin bo'ladi. Bunda $a(x), b(x), c(x)$ funksiyalarning ko'rinishi hisobga olinadi.

Misol. $y' = xy^2 + x^2y - 2x^3 + 1$ tenglamani qaraymiz. Bu erda $y = x$ hususiy yechim. $y = \frac{1}{z} + x$ almashtirish bajaramiz, u holda $z' + 3x^2z = -x$ bundan

$$z = e^{-x^2}\left(C - \int xe^{x^2}dx\right).$$

Berilgan tenglamaning umumiy yechimini yozamiz:

$$y = x + \frac{e^{x^2}}{C - \int e^{x^2}xdx}.$$

Javob: $y = x + \dfrac{e^{x^2}}{C - \int e^{x^2} x dx}$, $y = x$.

32.(141) Misol

$y = x(y' - x\,cosx)$

$y = xy' - x^2 cosx \;\; /: x \neq o$

$y' - \dfrac{y}{x} = x\,cosx$

1) $y' - \frac{y}{x} = 0$

$\dfrac{dy}{y} = \dfrac{dx}{x}$

$Ln|y| = ln|x| + ln\hat{c}$

$y = c'\,x$

2) $c'x + c' - c' = xcosx$

$\int dc' = \int cosxdx$

$c' = +sinx + c$

$y = x(c + sinx)$

$J: y = x\,(c + sinx)$

41.(142) Misol

$2x(x^2 + y)dx = dy$

$y' = 2x(x^2 + y)$

$y' = 2x^3 + 2xy$

$y' - 2xy = 0$

$\dfrac{dy}{dx} = 2xy \;\; | * \dfrac{dx}{y}$

$\dfrac{dy}{y} = 2d(x)x$

$lny = x^2 + c$

$\frac{Ce^{x^2}}{c} = y$ c=c(x) y=C(x)e^{x^2} y' =C'(x) e^{x^2}+2C(x)xe^{x^2}

$y' = 2x(x^2 + y)$

$C'(x)e^{x^2} + 2C(x)xe^{x^2} = 2x\left(x^2 + C(x)e^{x^2}\right)$

$C'(x)e^{x^2} = 2x^3$

$C'(x) = 2x^3 * e^{-x^2}$ $C(x) = \int 2x^3 * e^{-x^2}\ dx$

$C(x) - e^{-x^2}(x^2 + 1) + C$

$y = e^{x^2}(-\frac{(x^2+1)}{e^{x^2}} + C)$

$y = e^{x^2} - (x^2 + 1)$

$Javob \quad y = Ce^{x^2} - (x^2 + 1)$

42.(143) Misol

$(xy' - 1)lnx \; = \; 2y$

$xlnxy' - lnx \; = 2y \qquad /: x \neq 0$

$lnx \neq 0$

$$y' - 2\frac{1}{xlnx}\, y = \frac{1}{x}$$

1) $y' = \frac{2}{xlnx}\, y$

$\int \frac{dy}{dx} = 2 \quad \frac{dy}{xlnx} \qquad 2\int \frac{dlnx}{lnx}$

lny =2lnlnx+lnĉ

$$y = c' * ln^2x$$

2) 2ĉ lnx + $\frac{2ĉlnx}{x}$ - $\frac{2}{xlnx} *$ $ĉln^2x = \frac{1}{x}$

2ĉ lnx + $\frac{2ĉlnx}{x}$ - $\frac{2ĉlnx}{x} = \frac{1}{x}$

$2\int dĉ = \frac{dx}{xlnx} = \int \frac{dlnx}{lnx}$

$\int dĉ * \int \frac{dlnx}{ln^2x}$

$ĉ = -\frac{1}{lnx} + c$

y =$ln^2x \left(C - \frac{1}{lnx}\right) = cln^2x - ln^2x$ J : y = $cln^2x - ln^2x$

43.(144) Misol

$xy' + (x + 1)y \; = \; 3x^2e^{-x} \quad |: x \neq 0$

$$y' + \frac{x+1}{x}y = 3xe^{-x}$$

1) $y' + \frac{x+1}{x}\, y = 0$

$\frac{dy}{dx} = -\frac{x+1}{x}y$

$$\frac{dy}{y} = -\frac{x+1}{x}\,dx$$

$$\int \frac{dy}{y} = -\int dx - \int \frac{dx}{x}$$

$$lny = -x - lnx + lnC$$

$$y = -x - lnx$$

$$y = e^{-x}\frac{c}{x}$$

$$y' = \frac{\left(e^{-x}c + c'e^{-x}\right)x - e^{-x}c}{x^2} = \frac{-xe^{-y}c + xe^{-x}c' - e^{-x}c}{x^2} =$$

$$= -\frac{e^{-x}c}{x} + \frac{e^{-x}c'}{x} - \frac{e^{-x}c}{x^2} + \frac{x+1}{x}e^{-x}\frac{c}{x} = 3e^{-x}x$$

$$= -\frac{e^{-x}c}{x} + \frac{e^{-x}c\prime}{x} - \frac{e^{-x}c}{x^2} + e^{-x}\frac{c}{x} + \frac{e^{-x}c}{x^2} = 3e^{-x}x$$

$\frac{c\prime}{x} = 3x \quad c' = 3x' \qquad c = x^3 + C'$

$$y = e^{-x}\frac{x^3 + c}{x}$$

$$y = (x^3 + c)\,e^{-x}$$

44.(145) Misol

$(x + y^2)dy = ydx \quad x + y^2 = yx' \quad \frac{}{:} \neq 0 \qquad x' - x\frac{1}{y} = y$

1) $x' = x\frac{1}{y}$

$\int \frac{dx}{x} = \int \frac{dy}{y} \qquad lx|x| = \ln|y| + \ln|c'| \qquad x = c'y$

2) $\hat{c}y + \hat{c} - \frac{1}{y}\,\hat{c}y = y$

$$\int dc' = \int ydy$$

$c' = \frac{y}{2} + c$

$x = (\frac{y^2}{2} + c)$y = $\frac{y^2}{2} + cy$ $y = 0$ $Javob\text{:}\ x = y^2 + cy \quad y = 0$

45.(146) Misol

(2e^y-x)y'=1

2e^y-x=$\frac{1}{y'}$ $\frac{1}{y'_x} = x'_y$

x'+ x =2e^y

1) x'+x =0

$\frac{dx}{dy} = -x \quad \frac{dx}{x} = dy$

$lnx = -$y + c

$x = e^c e^{-y} = ce^{-y}$

2) $c'e^{-y} - c'e^{-y} + c'e^{-y} = 2e^y$

$\int dc = 2\int e^{2y}\, dy$

c = 4$e^{2y}dy$ + č

x=C$e^{-y} + e^y$

x = ($e^{2y} + c)e^{-y}$

x = $e^y + ce^{-y}$ J: x = $e^y + ce^{-y}$

46.(147) Misol

$(sin^2 y + xctgy)y' = 1$

$x' = ln^2 y + x\, ctgy$

$x' - x\, ctgy = sin^2 y$

1) $x' - x\, ctgy = 0$

$\int \frac{dx}{x} = \int ctgy\, dy$

2)$\int \frac{dx}{x} = \int \frac{dsiny}{siny}$

$Ln|x| = ln|siny| + ln$ĉ

$x = $ ĉ$siny$

ĉ $siny$ + ĉ $cosy$ − ĉ $cosy = sin2y$

$$\int d\hat{c} = \int sinydy$$

Ĉ $= -cosy + c$

$X = siny\,(c - cosy)$ $J\text{:}\ X = siny\,(c - cosy)$

47.(148) Misol

$(2x + y)dy = ydx + 4lnydy$
$2x + y = yx' + 4lny$
$2\frac{x}{y} + 1 = x' + 4\frac{lny}{y}$
$X' - \frac{2}{y}x = 1 - 4\frac{lny}{y}$
1)$x' - \frac{2}{y}x = 0$
$\frac{dx}{dy} = \frac{2}{y}x$
$\frac{dx}{x} = 2\frac{dy}{y}$
$Lnx\ = 2lny + lnx$
X=y^2c
2)2yc +c'y^2-$\frac{2}{y} * y^2c = 1 - 4\frac{lny}{y}$
c'y^2=1-4$\frac{lny}{y}$ |:$y^2 \neq 0$
c'=$\frac{1}{y^2} - 4\frac{lny}{y^3}$
dc=($\frac{1}{y^2} - 4\frac{lny}{y^3}$)dy
dc= $\frac{1}{y^2}$dy-4$\frac{lny}{y^3}$dy
lny=4 du=$\frac{1}{y}dy$
$\frac{dy}{y^3}dv \qquad v = -\frac{1}{2y^2}$
$\int \frac{lndy}{y^3} = -\frac{lny}{2y^2} + \int \frac{1}{2y^3} = -\frac{lny}{2y^2} - \frac{1}{2y^2}$
$c = -\frac{1}{y} + \frac{2lny}{y^2} + \frac{1}{y^2} + č$
X=-y+2lny+1+y^2+č
J: x=-y^2c-y+2lny+1

48.(149) Misol
y'=$\frac{y}{3x-y^2}$
$\frac{dy}{dx} = \frac{y}{3x - y^2}$
$\frac{dx}{dy} = \frac{3x - y^2}{y}$
X $\frac{3}{y}x + y = 0$

$x' - \frac{3}{y}x = 0$

$\frac{dy}{dx} = \frac{3x}{y}$

$\frac{dx}{x} = 3\frac{dy}{y}$

x =Cy^3

x'=C'$y^3 + 3y^2C$

y^3 C'+3y^2C-3y^2C =-y

C'= -$\frac{1}{y^2}$

C=$\frac{1}{y} + c_1$

x = $y^2 + y^3C$

J: x = $y^2 + y^3C$ y =0

49.(150) Misol

$(1 - 2xy)y' = y(y - 1)$

$(1 - 2xy)\frac{dy}{dx} = y(y - 1)$

$y' = 0 => y = C => C(C - 1) = 0 => y = 0; y = 1$

$(1 - 2xy) = x'y(y - 1)$

$x'y(y - 1) + 2xy = 1$

$\frac{dy}{dx}$(y-1) =-2x

$\frac{dx}{x} = -\frac{2}{y-1}dy$

$\ln|x| = -2\ln|y - 1| + lnC$

x=$\frac{C}{(y-1)^2}$

x'=$\frac{C'}{(y-1)^2}$-2 $\frac{C}{(y-1)^3}$

$$\frac{yC'}{(y-1)} - \frac{2yC}{(y-1)^2} + \frac{2yC}{(y-1)^2} = 1$$

C'= $\frac{y-1}{y} = 1 - \frac{1}{y}$

C=y-lny+c_1

x=$\frac{y-lny+C}{(y-1)^2}$

x$(y - 1)^2$y-lnCy

J: x$(y - 1)^2$y-lnCy y=0 ; y=1

50.(151) Misol

$y'+2y=y^2e^x$ $|:y^2$

$$\frac{y'}{y^2}+\frac{2}{y}=e^x$$

$$\frac{1}{y}=z$$

z' =- $\frac{1}{y^2}*y'$

$y'=-z'y^2$

$-z'+4z^2=2*z^{\frac{3}{2}}*e^x$

$\frac{-z\prime y^2}{y^2}+2z=e^x$

$-z'+2z=e^x$

$$\frac{dz}{dx}=2z$$

lnz =2x+c z=ce^{2x}

-c'$e^{2x}-2e^{2x}+2e^{2x}=e^x$

-c'$=-e^x$

$$\int dc=\int -e^xdx$$

C =$-e^x+\hat{c}$

Z=($e^{-x}+c)\,e^{2x}$
Z=c$e^{2x}+e^x$

$$\frac{1}{y}=ce^{2x}+e^x$$

$$\begin{cases} y=\dfrac{1}{e^x(ce^x+1)} \\ y=0 \end{cases}$$

51.(153) Misol

$y'=y4cosx+ytgx\ |:y4$

$$\frac{y'}{y^4} = cosx + \frac{1}{y^3} tgx \qquad z = \frac{1}{y^3} \qquad z' = -\frac{3}{y^4} * y' => \frac{y'}{y^4} = \frac{z'}{-3}$$

$$\frac{z'}{3} + z\, tgx = -cosx \qquad z' + 3z(tgx) = -3cosx$$

1) $z' + 3ztgx = 0$ $\qquad \frac{dz}{dx} = -3ztgx \;=> \frac{dz}{z} = -tgxdx$

lnz=3ln|cos(x)|+c

2) $z'(x) = 3cos^2x \,*\, (-sinx) * C(x) + C'(x)cos^3x \qquad z = cos^3x * C(x)$

$$-3cos^2xsinxC(x) + C'(x)cos^3x + 3sinxcos^3x + c(x)tgx = -3cosx$$

$$C'(x) = -\frac{3}{cos^2x} \qquad C(x) = -3tgx + c$$

$$z = cos^3x(-3tgx + C) = -3sinxcos^2x + C\, cos^3x$$

$$\begin{cases} \frac{1}{y^3} = \text{C cos3x} - 3\text{sinx}cos^2\text{x} \\ y = 0 \end{cases}$$

52.(154) Misol

$$xy^2y' = x^{2+}y^3$$

$$xy^2\frac{dy^3}{dx} = x^{2+}y^3$$

$$\frac{1}{3}x\frac{dy^3}{dx} = x^{2+}y^3 \qquad y^3 = t$$

$$\frac{1}{3}xt' - t = 0$$

$$\frac{dt}{t} = 3\frac{dx}{3x} \qquad ln|t| = ln|x^3| + C \qquad t = Cx^3 \quad t'$$

$$= 3x^2C + xC'x^3 \qquad x^3C + \frac{1}{3}C'x^4 - Cx^3 = x^2$$

$$\frac{1}{3}x^4C' = x^2 \quad x = 0 \qquad C' = 3x^{-2} \quad C = \int 3x^2dx = -3x^{-1} + C_1$$

$t = Cx^3 = C_1x^3 - 3x^2 \qquad y^3 = Cx^3 - 3x^2 \qquad Javob\colon\; y^3 = Cx^3 - 3x^2 \quad C \in R$

53.(155) Misol

$$Xydy = (y^2 + x)dx$$

$$Xyy' = y^2 + x \quad |: x \neq 0$$

$$y' = \frac{y}{x} + \frac{1}{y}$$

$$y' - \frac{1}{x}y = y^{-1} \qquad |: y$$

$$yy' - \frac{1}{x}y^2 = 1 \quad z = y^2 \quad z' = 2yy'$$

$y' = \frac{z'}{2y}$

$\frac{z'}{2} - \frac{z}{x} = 1 \quad z' = 2 + \frac{2z}{x} \quad t - \frac{z}{x} \quad t' = x\,t' + t$

$xt' + t \; = 2 + 2t$

$xt' = 2 + t$

$\frac{dt}{t+2} = \frac{dx}{x}$

$ln|t+2| = ln|x| + lnc$

$t + 2 = cx$

$\frac{z}{x} + 2 = c - x$

$z = cx^2 - 2x$

$y^2 = cx^2 - 2x$

$x = 0$

$J\!: y^2 = cx^2 - 2x \quad x = 0$

54.(156) Misol

$xy' - 2x^2\sqrt{y} = 4y$

$x\frac{y'}{y^2} - 2x^2 = 4\sqrt{y} \qquad x(\sqrt{y}) - x^2 = 2\sqrt{y} \qquad \sqrt{y} = t \qquad xt' - 2t = x^2$

$x\frac{dt}{dx} = 2t$

$\frac{dt}{t} = 2\frac{dx}{x} \qquad t = cx^2 \quad 2cx^2 + c'x^3 - 2cx^2 = x^2 \quad C' = \frac{1}{x} \qquad C = lnCx$

$t = x^2 lnCx \quad J\!: y = x^4 ln^2 Cx \qquad y = 0$

55.(157) Misol

$xy' + 2y + x^5y^3e^x = 0 \quad /\!: x \neq 0$

$y' + \frac{2}{x}y = -x^4e^xy^3 \quad |\!: y^3 \neq 0$

$\frac{y'}{y^3} + \frac{2}{xy^2} = -x^4e^x$

$\frac{1}{y^2} = z$

$y' = -\frac{2}{y^2} \qquad y' = -\frac{z' * y^3}{2}$

$-\frac{z'y^3}{2y^3} + \frac{2z}{x} = -x^4e^x$

$-z' + 4\frac{z}{x} = -2x^4e^x$

$z' - 4\frac{z}{x} = 2x^4e^x$

1) $z' - 4\frac{z}{x} = 0$

$\int\frac{dz}{z} = 4\int\frac{dx}{x}$ $\quad lnz = 4lnx + lnC \quad z = x^4C$

2) $4x^3c + x^4c - 4x^3c = 2x^4e^x \quad C' = 2e^x$

$\int d\,ĉ = 2\int e^x dx \quad C' = 2e^x + C \quad z = x^4(2e^x + c) = x^4c + 2x^4e^x$

$\frac{1}{y^2} = x^4c + 2x^4e^x \;\; y = 0 \;\; Javob\!: \; y^2 = x^4(c + 2e^x) \;\; y = 0$

56.(159) Misol

y'x^{3}siny=xy'-2y $\quad y'\,x^3siny = xy' - 2y \quad x^3siny = x - 2y\frac{dx}{dy}$

$\frac{1}{x^2} = z\,; \quad x' = -2z'$

$siny = \frac{1}{x^2} + yz' \quad yz' + z_0 = 0 \quad z_0' = -\frac{z_0}{y}$

$\int\frac{dz_0}{z_0} = -\int\frac{dy}{y} \quad ln|z_0| = -\,ln|y| + \,ln|c|$

$z_0 = \frac{c}{y}$

$z_y = \frac{c(y)}{y}$

Z'=$\frac{c(y)}{y} - \frac{c(y)}{y^2}$

$z'\frac{c'(y)}{y} - \frac{yc(y)}{y^2} + \frac{c(y)}{y} = siny \quad c'(y) = siny$

$\int dC = \int sinydy$

C=-cosy

$z_y = -\frac{cosy}{y}$

$z = z_0 + z_y = \frac{c}{y} - \frac{cosy}{y}$

$\frac{1}{x^2} = z \quad \frac{1}{x^2} = \frac{c}{y} - \frac{cosy}{y}$

J: $\frac{1}{x^2} = \frac{c}{y} - \frac{cosy}{y}$

57.(160) Misol

$(2x^2ylny - x)y' = y$

$2x^2ylny - x = yx'$

$x' + \frac{x}{y} = 2x'lny \qquad |: x^2 \neq 0$

$\frac{x'}{x^2} + \frac{1}{xy} = 2lny$

$\frac{1}{x} = z$

Z'=$-\frac{1}{x^2}x'$

X'=-z'x^2

-z'+$\frac{z}{y} = 2lny$

1) +z'=$\frac{z}{y}$

$$\frac{dz}{z} = \frac{dy}{y} \qquad lnz = lny + lnc \qquad z = cy$$

2)

$-cy = c - \frac{\hat{c}y}{y} - 2lny$

$-\frac{d\hat{c}}{dy} = 2\frac{lny}{y}$

$-dc = 2\frac{lny\,dy}{y}$

$-\int d\,\hat{c} = 2(\int lny\;d\;lny) \qquad -c = ln^2y + c \qquad z = -y(ln^2y + c\;\;)$

$$\frac{1}{x} = -y(ln^2y + c\;\;)$$

J: $\frac{1}{x} = -y(ln^2y + c\;\;)$

58.(161) MIsol

$xdx = (x^2 - 2y + 1)dy$

$x^2 + 1 = x_2 \quad dx_1 = 2xdx \quad dx_2 = (x_2 - y_2)dy_2$

$2y = y_2 \qquad dy_1 = 2dy \qquad x'_2 - x_2 = -y_2$

1. $\frac{dx_2}{x_2} = dy_2$

$lnx_2 = y_2 \Rightarrow x\ \frac{x}{2} = e^{y_2} - C(y_2)$

$x'_2 = C'(y_2)e^{y_2} + C(y_2)e^{y_2}$

$\Rightarrow C'(y_2)e^{y_2} + C(y_2)e^{y_2} - e^{y_2}C(y_2) = -y_2$

$\int dc = \int -y_2\, e^{-y_2} dy_2$

$C(y_2) = (1 + y_2)e^{-y_2} + C_2$

$x_2 = (1 + y_2) + C_2 e^{y_2}$

$(x^2 + 1) = (1 + 2y) + C_2 e^{2y}$

$Javob: x^2 = 2y + C_2 e^{2y}$

59. (162) MIsol

$$(x + 1)(yy' - 1) = y^2$$
$$(y^2 = z)(2yy' = z')$$
$$(x + 1)\left(\frac{z'}{z} - 1\right) = z$$
$$x - \frac{z'}{z} - x + \frac{z'}{z} - 1 - z = 0$$
$$z'\left(\frac{x}{2} + \frac{1}{2}\right) - z = x + 1$$

$1. z'\left(\frac{x}{2} + \frac{1}{2}\right) - z = 0$

$\frac{dz}{z} = 2 - \frac{dx}{x + 1}$

$ln\, z = ln(x + 1)^2 + C = ln(C(x + 1)^2)\, z = C(x + 1)^2$

$z' = C'(x + 1)^2 + 2(x + e)C$

$2. \frac{1}{2}(C'(x + 1)^2 + 2(x + 1)C)(x + 1) - C(x + 1)^2 = x + 1$

$\frac{C'(x + 1)^2}{2} = 1$

$e' = 2(x + 1)^{-2}$

$C = -2(x + 1)^{-1} + C_1$

$z - (-2(x + 1)^{-1} + C_1)(x + 1)^2 = -2(x + 1) + C(x + 1)^2$

$Javob: C\,(x + 1)^2 - 2(x + 1) = y^2$

60. (**163**)**MIsol**

$x(e^y - y') = 2$
$(e^y - y')x = 2$
$e^y = z(x): \quad b\, iz\, Bernulli \quad tenglama\, sin\, i\, olamiz$
$z' + \frac{2}{x} z = z^2$
$Uning\ umumiy\ yechimi : z\,(x) = \frac{1}{x(1 + Cx)}$
$Javob : y = -\,ln(x + Cx^2)$

61.(164) MIsol
$(x^2 - 1)y' \sin y + 2x \cos y = 2x - 2x^3$
$\frac{(x^2 - 1)dy}{dx} \sin y + 2x \cos y = 2x(1 - x^2)$
$-(x^2 - 1)\frac{d(\cos y)}{dx} + 2x \cos y = 2x(1 - x^2)$
$\frac{d(\cos y)}{dx} - \frac{2x \cos y}{x^2 - 1} = 2x$
$\cos y = u$
$\frac{du}{dx} - \frac{2xu}{x^2 - 1} = 2x$
$(x^2 - 1)$
$(x = \pm 1)$
$\frac{du}{u} = \frac{2xdx}{x^2 - 1}$
$ln(u) = ln(x^2 - 1) + C(x)$
$\frac{u = (x^2 - 1)C(x)\left(x^{\,2} - 1\right)}{u' = 2xC(x) + C'(x)}$
$2xC(x) + C'(x)(x^2 - 1) - \frac{2x}{x^2 - 1}(x^2 - 1)C(x) = 2x$
$C'(x) = \frac{2x}{x^2 - 1}$
$C(x) = ln(x^2 - 1)\, C_1$
$Javob : \cos y = (x^2 - 1)(ln(x^2 - 1)\, C_1)$

62.(165) Misol

$y(x) = \int_0^x y(t)dt + x + 1$
$y(x) = \int_0^x y(t)dt + x + 1$

$y' = y + 1$
$y = Ce^{x} - 1$
$y(0) = 1$
$C = 2$
$Javob: y = 2e^{x} - 1$

63.(166) Misol

$$\int_0^x (x-t)y(t)dt = 2x + \int_0^x y(t)dt$$
$$\int_0^x (x-t)y(t)dt = 2x + \int_0^x y(t)dt$$
$$\int_0^x y(t)dt = 2 + y(x)$$
$y(x) = y'(x)$
$y(0) = -2$
$y(x) = Ce^{x}$
$C = 2$
$Javob: y\,(x) = -2e^{x}$

167-171 misollarni Rikkati tenglamasini xususiy yechimini tanlash usulida toping.Bernulli tenglamasiga keltiring va integrallang.

64.(167) MIsol

$x^2y + xy + x^2y^2 = 4$
$y_1(x) = \frac{a}{x}$
$a = const$
$-a + a + a^2 = 4$
$a = \pm 2$
$a = 2$
$y = \frac{2}{x} + \frac{1}{2}$
$x^2z' - 5xz - x^2 = 0$
$z = Cx^5 - \frac{x}{4}$
$y = \frac{2}{x} + \frac{4}{C5x - x}$
$y_1 = \frac{2}{x}$
$x^2y + xy + x^2y^2 = 4$
$ax^2mx^{m-1} + axx^m + a^2x^2x^{2m} = 4$
$-a + a + a^2 = 4$

$a = 2$
$y = \frac{2}{x}$
$y = z + \frac{2}{x}$
$y' = z' + \frac{2}{x^2}$
$x^2\left(z' - \frac{2}{x^2}\right) + x\left(x + \frac{2}{x}\right) + x^2\left(x + \frac{2}{x}\right)^2 = 4$
$x^2z' - 2 + xz + 2 + x^2z^2 + 4 + 4xz = 4$
$x^2 + 5xz + x^2z^2 = 0$
$xz' + 5z + xz^2 = 0$
$x\frac{dt}{dx} = 5t$
$\frac{dt}{t} = 5\frac{dx}{x}$
$t = Cx^5$
$t' = 5Cx^4 + C'x^5$
$-5Cx^5 - C'x^6 + 5Cx^5 + x = 0$
$C'\frac{1}{x^5}$
$C = -\frac{x-4}{-4} + C_1$
$t = \left(-\frac{1}{4}x + Cx^5\right)$
$z = \frac{1}{t} = \frac{4}{-x + 4Cx^5}$
$y = z + \frac{2}{x} = \frac{4}{Cx^5 - x} + \frac{2}{x}$

65.(168) MIsol

$3y' + y^2 + \frac{2}{x^2}$
$y = -\frac{a}{x}$
$y' = \frac{a}{x^2}$
$\frac{3a}{x^2} + \frac{a^2}{x^2} + \frac{2}{x^2} = 0 \Rightarrow a^2 + 3a + 2 = 0 \Rightarrow a_{1,2} = \frac{-3 \pm 1}{2} \Rightarrow a_1 = -2, a_2 = -1$
$a = 1, y = \frac{1}{x} \Rightarrow (y = \frac{1}{x} + z\,; y' = z' - \frac{1}{x^2})$
$3\left(z' - \frac{1}{x^2}\right) + \left(\frac{1}{x} + 2\right)^2 + \frac{2}{x^2} = 0$
$3z' - \frac{3}{x^2} + \frac{1}{x^2} + \frac{2z}{x} + z^2 + \frac{2}{x^2} = 0$

$$3z' + \frac{2z}{x} + z^2 = 0/\frac{1}{z^2} \Rightarrow 3\left(\frac{z'}{z^2}\right) + \frac{2}{zx} + 1 = 0 \Rightarrow -3\left(\frac{1}{z}\right)' + \frac{2}{x}\left(\frac{1}{z}\right) + 1 = 0|\frac{1}{z}$$
$$= t$$
$$-3t' + \frac{2}{x}t + 1 = 0 \Rightarrow -3\frac{dt}{dx} + \frac{2}{x}t = 0 \to -3\frac{dt}{dx} = -\frac{2}{x}t \to 3\int\frac{dt}{t} = 2\int\frac{dx}{x}$$
$$3\,ln\,|\,t| = 2\,ln\,|\,x| + C \to ln\,|\,t|^3 = ln\,|\,x|^2 + C \to t^3 = C_1 x^{\frac{2}{3}} \to C_1 = const$$
$$C_1 = D(x) \to' = t' = C_1 x^{\frac{2}{3}} + C_1\frac{2}{3}x^{-\frac{1}{3}}$$
$$-3\left(C' x^{\frac{2}{3}} + \frac{2}{3}C_1 x^{-\frac{1}{3}}\right) + \frac{2}{x}\left(C_1 x^{\frac{2}{3}}\right) + 1 = 0$$
$$-3C' x^{\frac{2}{3}} - 2C_1 x^{-\frac{1}{3}} + 2C_1 x^{-\frac{1}{3}} + 1 = 0 \to -3C_1 x^{\frac{2}{3}} = -1 \to 3\frac{dC_1}{dx}x^{\frac{2}{3}} = 1$$
$$\Rightarrow 3\int(dC_1) = \int\frac{dx}{x^{\frac{2}{3}}}$$
$$3C_1 = 3x^{\frac{1}{3}} + 3C \to C_1 = x^{\frac{1}{3}} + C \Rightarrow t = x^{\frac{1}{3}}x^{\frac{2}{3}} + Cx^{\frac{2}{3}} = \underline{x + Cx^{\frac{2}{3}}}$$
$$t = \frac{1}{z}, z = \frac{1}{x + Cx^{\frac{2}{3}}}; y = \frac{1}{x} + z \Rightarrow$$

$Javob: y = \frac{1}{x} + \frac{1}{x + Cx^{\frac{2}{3}}}; y = \frac{1}{x}.$

66.(169) Misol

$xy' - (2x+1)y + y^2 = -x^2$

$y_{1(x)} = ax + b$

$ax - (2x+1)(ax+b) + (ax+b)^2 = -x^2$

$2ab - 2b = 0; a = 1; -b + b^2 = 0$

$a = b = 1, a = 1, b = 0.$

$$x\left(1 - \frac{z'}{z}\right) - (2x+1)\left(x + \frac{1}{x}\right) + \left(x + \frac{1}{x}\right)^2 = -x^2$$

$xz' + z - 1 = 0$

$z = 1 + \frac{C}{x}$

$javob: y = x + \frac{x}{x+C}$

67.(170) Misol

$y' - 2xy + y^2 = 5 - x^2$

$y = ax + b \Rightarrow a - 2x(ax+b) + a^2x^2 + 2ab + b^2 = 5 - x$

$a - 2ax^2 - 2bx + a^2x^2 + 2abx + b^2 - 5 - x = 0$

$$\begin{Bmatrix} -2a + a^2 + 1 = 0 \Rightarrow (a-1)^2 = 0 \Rightarrow a = 1 \\ -2b + 2ab = 0 \Rightarrow -2b + 2b = 0 \Rightarrow b = ? \\ a + b^2 - 5 = 0 \Rightarrow 1 + b^2 - 5 = 0 \Rightarrow b = \pm 2 \end{Bmatrix}$$

$a = 1, b = 2$
$y = x + 2$

almashtirish qilamiz: $y = x + 2 + \frac{1}{z}$

$$1 - \frac{z'}{z^2} - 2x\left(x + 2 + \frac{1}{z}\right) + \left(x^2 + 4 + \frac{4}{z} + \frac{1}{z^2}\right) + 2x\left(2 + \frac{1}{z}\right) = 5 - x^2$$

$$1 - \frac{z'}{z^2} - 2x^2 - 4x - \frac{2x}{z} + x^2 + 4 + \frac{4}{z} + \frac{4}{z^2} + 4x + \frac{2x}{z} + x^2 - 5 = 0$$

$$-\frac{z'}{z^2} + \frac{4}{z} + \frac{1}{z^2} = 0$$

$$-z' + 4z + 1 = 0 \Rightarrow z' - 4z - 1 = 0$$

$$\frac{dz}{dt} = 4z \Rightarrow \frac{dz}{dt} = 4dx \Rightarrow 4x + C$$

$$z = e^{4x}C_1 - \frac{1}{4}$$

$$y = x + 2 + \frac{1}{C_1 e^{4x} - \frac{1}{4}} = x + 2 + \frac{4}{Ce^{4x} - 1}$$

$javob: y = x + 2, y = x + 2 + \frac{4}{Ce^{4x}-1}.$

§-5.
To'liq differensialli tenglama

Agar

$$M(x,y)dx + N(x,y)dy = 0 \qquad (1)$$

tenglamaning chap tomoni Γ sohada biror $U(x,y)$ funksiyaning to'liq differensialidan iborat bo'lsa, y'ani

$$dU(x,y) = \frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial y}dy = M(x,y)dx + N(x,y)dy \qquad (2)$$

tenglik o'rinli bo'lsa (1) tenglama Γ sohada **to'liq differensialli** deyiladi. To'liq differensialli tenglamani $dU(x,y) = 0$ ko'rinishda yozish mumkin. Bunga ko'ra uning **umumiy yechimi** $U(x,y) = C$ ko'rinihga ega.

Misol. Ushbu $(x^3 + y)dx + (x - y)dy = 0$ tenglamani to'liq differensialli bo'lishini tekshiramiz va umumiy yechimini topamiz. Buning uchun uning chap tomonini differensial ostiga kiritishga harakat qilamiz:

$$x^3dx + ydx + xdy - ydy = 0; \qquad d\left(\frac{x^4}{4}\right) + d(xy) - d\left(\frac{y^2}{2}\right) = 0;$$

$$d\left(\frac{x^4}{4} + xy - \frac{y^2}{2}\right) = 0.$$

Demak berilgan tenglama to'liq differensialli ekan va uning umumiy yechimi:

$$\frac{x^4}{4} + xy - \frac{y^2}{2} = C$$

Javob: $\frac{x^4}{4}+xy-\frac{y^2}{2}=C$.

Har doim ham berilgan tenglamani to'liq differensialli bo'lishini to'g'ridan to'g'ri tekshirish oson kechmaydi. Bizga quyidagi teorema bu ishda qo'l keladi.

Teorema. (1) tenglama Γ sohada to'liq differensialli bo'lishi uchun

$$\frac{\partial M}{\partial y}\equiv\frac{\partial N}{\partial x} \qquad (3)$$

ayniyat Γ sohada o'rinli bo'lishi zarur va yetarli.

Isbot. Zarurligi. (1) tenglama to'liq differensialli bo'lsin. U holda (2) tenglik o'rinli. Ushbu

$$\frac{\partial U}{\partial x}=M(x,y),\ \frac{\partial U}{\partial y}=N(x,y) \qquad (4)$$

ayniyatlardan birinchisini y bo'yicha ikkinchisini x bo'yicha differensiallaymiz: $\frac{\partial^2 U}{\partial x\partial y}=\frac{\partial M}{\partial y},\ \frac{\partial^2 U}{\partial y\partial x}=\frac{\partial N}{\partial x}$. Bu tengliklarning chap qismlari aynan tengligidan (3) ayniyat o'rinli bo'lishi kelib chiqadi.

Yetarliligi. (3) ayniyat o'rinli bo'lsin. (2) tenglikni qanoatlantiruvchi $U(x,y)$ funksiya mavjudligini ko'rsatamiz, yanada aniqrog'i bu funksiyani quramiz. Uni quyidagi ko'rinishda qidiraylik:

$$U(x,y)=\int_{x_0}^{x}M(x,y)dx+\varphi(y), \qquad (5)$$

bunda $\varphi(y)$ ihtiyoriy differensiallanuvchi funksiya, $(x_0,y_0)\in\Gamma$. Bu funksiya (4) tenkliklardan birinchisini qanoatlantirishi ravshan. $\varphi(y)$ funksiyani shunday tanlaylikki (4)ning ikkinchi tengligi ham o'rinli bo'lsin:

$$N(x,y)=\frac{\partial U}{\partial y}=\frac{\partial}{\partial y}\int_{x_0}^{x}M(x,y)dx+\varphi'(y)=\int_{x_0}^{x}\frac{\partial M}{\partial y}dx+\varphi'(y)$$

Bu erda (3) ayniyatdan foydalanamiz:

$$\int_{x_0}^{x}\frac{\partial N}{\partial x}dx+\varphi'(y)=N(x,y)-N(x_0,y)+\varphi'(y)=N(x,y)$$

Bunga ko'ra: $\varphi(y)=\int_{y_0}^{y}N(x_0,y)dy+C$. Buni (5)ga olib borib qo'ysak izlanayotgan $U(x,y)$ funksiya hosil bo'ladi: $U(x,y)=\int_{x_0}^{x}M(x,y)dx+\int_{y_0}^{y}N(x_0,y)dy+C$. Teorema isbotlandi.

Isbotlangan teoremaga ko'ra (3) tenglik o'rinli bo'lsa (1) tenglamaning **umumiy yechimi**

$$\int_{x_0}^{x}M(x,y)dx+\int_{y_0}^{y}N(x_0,y)dy=C$$

formula bilan ifodalanadi. Agar teorema isbotida $U(x,y)$ funksiyani

$U(x,y)=\int\limits_{y_0}^{y} N(x_0,y)dy+\phi(x)$ ko'rinishda qidirganimizda (1) tenglamaning umumiy yechimini

$$\int\limits_{x_0}^{x} M(x,y_0)dx+\int\limits_{y_0}^{y} N(x,y)dy=C$$

formulasiga ega bo'lar edik.

Misol. Yana $(x^3+y)dx+(x-y)dy=0$ tnglamani qaraymiz. Bu erda

$$M=x^3+y,\ N=x-y,\ \frac{\partial M}{\partial y}=1,\ \frac{\partial N}{\partial x}=1;$$

(3) shart o'rinli. Umumiy integralni

$$\int\limits_{0}^{x}(x^3+y)dx+\int\limits_{0}^{y}(-y)dy=C$$

formuladan foydalanib hosil qilamiz. **Javob:** $\frac{x^4}{4}+xy-\frac{y^2}{2}=C$

2-reja. Yuqorida ko'rdikki to'liq differensialli tenglamani integrallash juda oson. Bu erda shunday savol tug'iladi: to'liq differensialli bo'lmagan tnglamani to'liq differensialli tenglamaga keltirish mumkinmi?

Agar (1) tenglamani $\mu(x,y)$ funksiyaga ko'paytirsak hosil bo'lgan

$$\mu(x,y)M(x,y)dx+\mu(x,y)N(x,y)dy=0 \qquad (6)$$

tenglama to'liq differensialli bo'lsa $\mu(x,y)$ni (1) tenglamaning integrallovchi ko'paytuvchisi deb ataymiz. (6) tenglamaniny umumiy yechimi (1) tenglama uchun ham **umumiy yechim** bo'ladi. Demak to'liq differensialli bo'lmagan tenglamani integrallovchi ko'paytuvchisini topa olsak uni integrallay olamiz. Endi (1) tenlamani faqat x ga bog'liq integrallovchi ko'paytuvchisini qidiramiz.

$$\mu(x)M(x,y)dx+\mu(x)N(x,y)dy=0$$

tenglama to'liq differensialli bo'lishi uchun $\frac{\partial(\mu M)}{\partial y}=\frac{\partial(\mu N)}{\partial x}$ tenglik o'rinli bo'lishi zarur va yetarli. Bunga ko'ra:

$$\mu\frac{\partial M}{\partial y}=\mu\frac{\partial N}{\partial x}+\frac{d\mu}{dx}N;$$

$$\frac{d\mu}{\mu}=\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}\cdot dx$$

Bu tenglikni chap tomoni faqat x ga bog'liq. Demak yuqoridagi tenglik ma'noga ega bo'lishi, ya'ni (1) tenglama $\mu(x)$ ko'rinishdagi integraloovchi ko'paytuvchiga ega bo'lishi uchun $\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}=p(x)$ kasr faqat x ga bog'liq bo'lishi zarur. Bu holda integrallovchi ko'paytuvchi $\mu(x) = e^{\int p(x)dx}$ formula bilan aniqlanadi.

Yuqoridagiga o'xshash mulohazalar yuritib (1) tenglama $\mu(y)$ ko'rinishdagi integraloovchi ko'paytuvchiga ega bo'lishi uchun $\dfrac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = q(y)$ kasr faqat y ga bog'liq bo'lishi zarurligini va integrallovchi ko'paytuvchi $\mu(y) = e^{\int q(y)dy}$ formula bilan topilishini aniqlash mumkin.

Takidlash joizki (1) tenglama $\mu(x, y)$ integrallovchi ko'paytuvchiga ega bo'lsa uning **mahsus yechimi** $\dfrac{1}{\mu(x, y)} = 0$ tenglikni qanoatlantiruvchi $y(x)$ funksiyalar orasidan qidiriladi.

Misol. $(xy^2 - y)dx + xdy = 0$ tenglamani qaraylik. Bu yerda

$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{2(xy-1)}{xy^2 - y} = \frac{2}{y}$$

Demak berilgan tenglama $\mu(y) = e^{\int \frac{2dy}{y}} = y^{-2}$ integrallovchi ko'paytuvchiga ega. Berilgan tenglamani y^{-2} ga ko'paytiramiz:

$$(x - \frac{1}{y})dx + \frac{x}{y^2}dy = 0$$

Bu tenglamaning umumiy yechimini yozamiz: $\dfrac{x^2}{2} - \dfrac{x}{y} = C$. Berilgan tenglama mahsus yechimga ega, chunki $\dfrac{1}{\mu(x, y)} = y^2 = 0$ tenglikni va tenglamani o'zini $y = 0$ funksiya qanoatlantiradi. **Javob:** $\dfrac{x^2}{2} - \dfrac{x}{y} = C$, $y = 0$.

68.(186) Misol

$2xydx = (x^2 - y^2)dy = 0$

$(2xy)'_y = 2x \qquad (x^2 - y^2)'_x = 2x$

$(F)'_x dx + (F)'_y dy = 0 \qquad (F)'_x = 2xy$

$\int dF = \int 2xydx \qquad F = x^2y + \varphi(y)$

$(F)'_x = x^2 + \varphi'(y) = x^2 - y^2$

$\varphi'(y) = -y^2 \quad d\,\varphi(y) = -y^2dy$

$\varphi(y) = -\frac{y^3}{3} + C \qquad F = x^2y - -\frac{y^3}{3} = C$

69.(187) MIsol

$(2 - 9xy^2)xdx + (4y^2 - 6x^3)ydy = 0$
Yechish

$$\frac{\partial M}{\partial y} = -18x^2y$$
$$\frac{\partial N}{\partial x} = -18x^2y$$
$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$
$$f_x'(x, y) = M$$
$$f(x, y) = \int Mdx = \int (2 - 9xy^2)\, xdx = x^2 - 3x^3y^2 + C(y)$$
$$f_x'(x, y) = N = -6x^3y + C'(y) = 4y^3 - 6x^3y$$
$$C'(y) = 4y^3$$
$$C(y) = y^4 + C_1$$
$$x^2 - 3x^3y^2 + y^4 = C$$

70.(188)-Misol
$e^{-y}dx - (2y + xe^{-y})dy = 0$
Yechish
$e^{-y}dx - (2y + xe^{-y})dy = 0$
$M = e^{-y}$
$N = -2y - xe^{-y}$
$\frac{\partial M}{\partial y} = -e^{-y}$
$\frac{\partial N}{\partial x} = -e^{-y}$
$f_x' = e^{-y}$
$df = e^{-y}dx$
$f = xe^{-y} + u(y)$
$f_y' = xe^{-y} + u'(y) = -2y - e^{-y}$
$u'(y) = -2y$
$\int du(y) = \int -2ydy$
$u(y) = y^2$
$f = xe^{-y} - y^2$
$xe^{-y} - y^2 = c$

71.(189-Misol)

$$\frac{y}{x}dx + (y^3 + ln\,x)dy = 0$$

Yechish

$$\frac{\partial M}{\partial y} = \frac{1}{x}$$
$$\frac{\partial N}{\partial x} = \frac{1}{x}$$
$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$
$$f(x,y) = \int \frac{y}{x}dx = y\,ln\,x + C(y)$$
$$f_y'(x,y) = ln\,x + C'(y) = y^3 + ln\,x$$
$$C'(y) = y^3$$
$$C(y) = \frac{1}{4}y^4 + C_1$$
$$y\,ln\,x + \frac{1}{4}y^4 = C_1$$
$$4y\,ln\,x + y^4 = C_2$$

72.(190)-Misol

$$\frac{3x^2 + y^2}{y^2}dx - \frac{2x^3 + 5y}{y^3}dy = 0$$

Yechish

$$\frac{\partial M}{\partial y} = -\frac{6x^2}{y^3}$$
$$\frac{\partial N}{\partial x} = -\frac{6x^2}{y^3}$$
$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$
$$f_x'(x,y) = M$$
$$f(x,y) = \int Mdx = \int \frac{3x^2 + y^2}{y^2}dx = x + \frac{x^3}{y^2} + C(y)$$
$$f_y'(x,y) = N = -\frac{2x^3}{y^3} + C'(y) = -\frac{2x^3}{y^3} - \frac{5}{y^2}$$
$$C'(y) = -\frac{5}{y^2}$$
$$C(y) = \frac{5}{y^2} + C_1$$

$$x+\frac{x^3}{y^2}+\frac{5}{y^2}=C$$

73.(191)-Misol

$$2x\,(1+\sqrt{x^2-y}\quad)dx-\sqrt{x^2ydy}=0$$

Yechish

$$\frac{\partial N}{\partial y}=\frac{\partial}{\partial y}2x(1+\sqrt{x^2-y})=-\frac{x}{\sqrt{x^2-y}}$$

$$\frac{\partial M}{\partial x}=\frac{\partial}{\partial x}(-\sqrt{x^2-y})=-\frac{2x}{2\sqrt{x^2-y}}=-\frac{x}{\sqrt{x^2-y}}$$

$$\frac{\partial N}{\partial y}=\frac{\partial M}{\partial x}$$

$$f(x,y)=-\int\sqrt{x^2-ydy}=\frac{\left(\sqrt{x^2-y}\right)^{1.5}}{1.5}+C(x)=\frac{2}{3}(x^2-y)^{1.5}+C(x)$$

$$\frac{\partial F}{\partial x}=2x\sqrt{x^2-y}+C'(x)=N(x,y)=2x\sqrt{x^2-y}$$

$$C'(x)=2x$$

$$C(x)=x^2$$

$$f(x,y)=\frac{2}{3}(x^2-y)^{1.5}+x^2=C$$

$$Javob\,;\frac{2}{3}(x^2-y)^{1.5}+x^2=C$$

74.(192)-Misol

$$(1+y^2\sin 2\,x)dx-2y\cos^2 x\,dy=o$$

Yechish

$$(1+y^2\sin 2\,x)dx-2y\cos^2 x\,dy=o$$

$$M=1+y^2\sin 2\,x$$

$$N=-2y\cos^2 x$$

$$\frac{\partial M}{\partial y}=2y\sin 2\,x$$

$$\frac{\partial N}{\partial x}=2y2\cos x\sin x=2y\sin 2\,x$$

$$f_x'=1+y^2\sin 2\,x$$

$$\int df=\int dx+y^2\int\sin 2\,xdx$$

$$f=x-\frac{1}{2}y^2\cos 2\,x+u(y)$$

$$f_y'=-y\cos 2\,x+u'(y)=-2y\cos^2 x$$

$-y\cos^2 x + y\sin^2 x + u'(y) = -2y\cos^2 x$
$u'(y) = -y(\sin^2 x + \cos^2 x)$
$du(y) = -ydy$

$$u(y) = -\frac{y^2}{2}$$

$$f = x - \frac{1}{2}y^2\cos 2x - \frac{y^2}{2}$$

$$Javob;\, x - \frac{1}{2}y^2\cos 2x - \frac{y^2}{2} = c$$

75.(193)-Misol

$$3x^2(1+\ln y)dx = \left(2y - \frac{x^3}{y}\right)dy$$

Yechish

$$3x^2(1+\ln y)dx = \left(2y - \frac{x^3}{y}\right)dy$$

$$3x^2(1+\ln y)dx = \left(\frac{x^3}{y} - 2y\right)dy = 0$$

$M(x,y) = 3x^2 + 3x^2\ln y$

$$N(x,y) = \frac{x^3}{y} - 2y$$

$$\frac{\partial M}{\partial y} = \frac{3x^2}{y}$$

$$\frac{\partial N}{\partial x} = \frac{3x^2}{y}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$f_x' = 3x^2(1+\ln y)$
$f = x^3(1+\ln y) + u(y)$

$$f_y' = \frac{x^3}{y} + u'(y) = \frac{x^3}{y} - 2y$$

$du = -2ydy$
$u(y) = -y^2$
$x^3(1+\ln y) - y^2 = c$
$Javob;\, x^3(1+\ln y) - y^2 = c$

76.(194)-Misol

$$\left(\frac{x}{\sin y} + 2\right)dx + \frac{(x^2+2)\cos y}{\cos 2y - 1}dy = 0$$

Yechim;

$$\left(\frac{x}{\sin y} + 2\right)dx + \frac{(x^2+1)\cos y}{-2\sin^2 y}dy = 0$$

$$M(x,y) = \frac{x}{\sin y} + 2$$
$$N(x,y) = \frac{(x^2+1)\cos y}{-2\sin^2 y}$$
$$\frac{\partial M}{\partial y} = -\frac{x\cos y}{\sin^2 y}$$
$$\frac{\partial N}{\partial x} = -\frac{2x\cos y}{2\sin^2 y} = -\frac{x\cos y}{\sin^2 y}$$
$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$
$$f_x' \frac{x}{\sin y} + 2$$
$$df = \frac{x + 2\sin y}{\sin y} dx$$
$$\int df = \frac{1}{\sin y}\int xdx + 2\int dx$$
$$f = \frac{x^2}{2\sin y} + 2x + u(y)$$
$$f_y' = -\frac{x^2\cos y}{2\cos^2 y} + u'(y) = \frac{x^2\cos y}{-2\cos^2 y} - \frac{\cos y}{2\sin^2 y}$$
$$\int du = -\frac{1}{2}\frac{\cos y\, dy}{\sin^2 y} = -\frac{1}{2}\int \frac{d\sin y}{\sin^2 y}$$
$$u = \frac{1}{2}\frac{1}{\sin y}$$
$$f = \frac{x^2}{2\sin y} + 2x + \frac{1}{2\sin y} = c$$
$$\frac{x^2+1}{2\sin y} = c - 2x$$
$$x^2 + 1 = 2(c-2x)\ln y$$
$$Javob; x^2 + 1 = 2(c-2x)\ln y$$

77.(195)-Misol

$$(x^2 + y^2 + x)dx + ydy = 0$$
Yechim;
$$(x^2 + y^2 + x)dx + ydy = 0$$
$$M_y' = 2y$$
$$N_x' = 0$$
$$M_y' \neq N_x'$$
$$\frac{d\mu}{d\omega} = -\frac{2y - 0}{(x^2+y^2+x)\omega_y' - y\omega_x'} - \mu$$

$\omega = xy$
$\omega = x$
$\mu'_{\omega} = -\frac{2y}{-y}\mu$
$\mu'_{x} = 2\mu$
$\frac{d\mu}{\mu} = 2d\omega$
$ln\,\mu = 2\omega + \mu$
$\mu = e^{2\omega}\mu,$
$\omega = x$
$\mu = e^{2x}$
$(e^{2x}x^2 + e^{2x}y^2 + e^{2x}x)dx + e^{2x}ydy = 0$
$\mu'_{y} = 2ye^{2x}$
$\mu'_{x} = 2ye^{2x}$
$f'_{y} = e^{2x}y$
$df = e^{2x}ydy$
$f = e^{2x}\frac{y^2}{2} + u(x)$
$f'_{x} = 2e^{2x}\frac{y^2}{2} + u'(x) = e^{2x}x^2 + e^{2x}y^2 + e^{2x}x$
$u'(x) = e^{2x}x^2 + e^{2x}x$
$du(x) = e^{2x}x^2dx + e^{2x}xdx$
$Javob\,; d\,u(x) = e^{2x}x^2dx + e^{2x}xdx$

78.(196)-Misol

$(x^2 + y^2 + y)dx - xdy = 0$
Yechim;
$(x^2 + y^2 + y)dx - xdy = 0$
$M = x^2 + y^2 + y$
$N = -x$
$M'_{y} = 2y + 1$
$N'_{x} = -1$
$\frac{d\mu}{d\omega} = -\frac{2y + 1 + 1}{(x^2 + y^2 + y)\omega'_{y} + x\omega'_{x}} = -\frac{2(y + 1)}{(x^2 + y^2 + y)\omega'_{y} + x\omega'_{x}}$
$\omega = x^2 + y^2$
$\omega'_{y} = 2y$
$\omega'_{x} = 2x$
$\frac{d\mu}{d\omega} = \frac{2(y + 1)}{2x^2y + 2y^3 + 2y^2 + 2x^2}\mu = -\frac{y + 1}{x^2(y + 1) + y^2(y + 1)} = \frac{d\mu}{\omega}$
$\frac{d\mu}{\mu} = -\frac{d\omega}{\omega}$

$$ln\,\mu = ln\,\omega$$
$$\mu = \frac{1}{\omega}$$
$$\mu = \frac{1}{x^2 + y^2}$$
$$\left(1 + \frac{y}{x^2 + y^2}\right)dx - \frac{x}{x^2 + y^2}dy = 0$$
$$f_x^{'} = 1 + \frac{y}{x^2 + y^2}$$
$$df = dx + 1 + \frac{y}{x^2 + y^2}dx$$
$$f = x + y\frac{1}{y}arctg\frac{x}{y} + u(y) = x + arctg\frac{x}{y} + u(y)$$
$$f_y^{'} = -\frac{y^2}{x^2 + y^2}\frac{x}{y^2} + u^{'}(y) = -\frac{x}{x^2 + y^2}$$
$$u^{'}(y) = 0$$
$$u(y) = 0$$
$$f = x + arctg\frac{x}{y}$$
$$f_x^{'} = 1 + \frac{y^2}{x^2 + y^2}\frac{1}{y} = \frac{x^2 + y^2 + y}{x^2 + y^2}$$
$$f_y^{'} = -\frac{y^2}{x^2 + y^2}\frac{x}{y} = -\frac{x}{x^2 + y^2}$$
$$Javob\,; f = x + arctg\frac{x}{y}$$

79.(197)-Misol

$$ydy = (xdy + ydx)\sqrt{1 + y^2}$$

Yechish;

$$ydy = (xdy + ydx)\sqrt{1 + y^2}$$

Tenglikning ikkala tomonini $\sqrt{1 + y^2}$bo’lamiz

$\frac{y}{\sqrt{1+y^2}}dy = xdy + ydx$ chap tomondan to’liq differensial olamiz

$$c_1 = 2c$$

$y(x^2 - c_1) = 2x$javobi aniq bo’lgan oddiy tenglik paydo bo’ldi

$$xy + C = \sqrt{1 + y^2}$$

80.(198)-Misol

$$xy^2(xy^{'} + y) = 1$$

Yechim;

$$xy^2(xy^{'} + y) = 1$$
$$xy^2\left(x\frac{dy}{dx} + y\right) = 1$$
$$xy^2(xdy + ydx) = dx$$

$xy^2d(xy) = dx$
$(xy)^2d(xy) = xdx$
Almashtirish kiritamiz
$xy = t$
$t^2dt = xdx$
$\frac{1}{3}t^3 = \frac{1}{2}x^2 + c$
Qayta almashtirishdan olamiz
$\frac{1}{3}x^3y^3 = \frac{1}{2}x^2 + c$
$2x^3y^3 - 3x^2 = c_1$

81.(199)-Misol

$y^2dx - (xy + x^3)dy = 0$
Yechim;
Kompanentlarni guruhlaymiz
$y^2dx - xydy - x^3dy = 0$
$y(ydx - xdy) - x^3dy = 0$
$y = 0$yechim bo’lmagligi sababli ikkala tomonni y^3ga bo’lamiz
$\frac{ydx - xdy}{y^2} - \frac{x^3}{y^3}dy = 0$
$d\left(\frac{x}{y}\right) - \frac{x^3}{y^3}dy = 0$
Belgilash kiritamiz
$\frac{x}{y} = t$
$dt - t^3dy = 0$
$t = 0 \Rightarrow x = 0$tenglamaning yechimi bo’lib chiqdi.
Bu yechimni eslab qolamiz va ikkala bo’lakni t^3ga bo’lamiz
$\frac{dt}{t^3} = dy$
$y = \frac{1}{2}t^{-2} + c$
$y = -\frac{1}{2}\frac{y^2}{x^2} + c$
$2y = -\frac{y^2}{x^2} + c$
$2yx^2 = -y^2 + cx^2$
$x^2(c - 2y) = y^2$

82.(200)-Misol

$$\left(y-\frac{1}{x}\right)dx+\frac{dy}{y}=0$$

Yechom;

$$\left(y-\frac{1}{x}\right)dx+\frac{dy}{y}=0$$

$$ydx+d\,ln\,y-d\,ln\,x=0$$

$$ydx+d\,ln\frac{y}{x}=0$$

Almashtirish kiritamiz

$$ln\frac{y}{x}=t$$

Almashtirishni$y$ni x va $t$da ifodalaylik

$$y=xe^t$$

$$xe^tdx+dt=0$$

$$xdx=-e^{-t}dt$$

$$\frac{x^2}{2}=e^{-t}+c$$

Almashtirishni ortga qaytaramiz

$$e^{-t}=e^{-ln\frac{y}{x}}=e^{ln\frac{x}{y}}=\frac{x}{y}$$

$$\frac{x^2}{2}-\frac{x}{y}=c$$

$$x^2y+2x=2cy$$

$$y(x^2-2c)=2x$$

$$c_1=2c$$

$$y(x^2-c_1)=2x$$

83.(201) Misol

$$(x^2+3lny)ydx=xdy$$

Yechilishi Tenglamaning ikkala qismini y ga bo'lamiz

$$(x^2+3lny)dx=x\frac{dx}{y}$$

$$(x^2+3lny)dx=xd(lnt)$$

$lnt=t$ belgilash kiritamiz

$$(x^2+3t)dx=xdt$$

Ushbu tenglamani osongina chiziqli tengkamaga keltirish mumkin $dx=0$ ildiz ekanini tekshiramiz $dx=0$ $x=C$ => $x=0$ tenglamaning yechimi.Oxirgi o'tish dastlabki

$x=C$ tenglamani almashtirish orqali olinadi ikkala tomonini dx ga bo'lib, biz odatiy chiziq tenglamani olamiz

$xt = 3t + x^2$

$xt - 3t = x^2$bu tenglamaga mos bir jinsli tenglamani yechamiz

$x\frac{dt}{dx} = 3t$

$t = 0\ lny = 0\ \ y = 1$ bu yechim emas

$\frac{dt}{t} = \frac{3dx}{x}$

$lnt = 3lnx + lnC \quad t = Cx^3 \quad t = Cx^3 + 3x^2C \qquad Cx^4 + 3Cx^3 - 3Cx^3 = x^2 \qquad C = x^{-2} \quad C = -x^{-1} + C_1 \quad T = x^3(C - x^{-1})$

Teskari almashtirishni amalga oshiramiz
$lny = Cx^3 - x^2$
$lny + x^2 = Cx^3$

84.(202) Misol

$y^2dx + (xy + tgxy)dy = 0$

$ydx + xydy + tg(xy)dy = 0$

$y(ydx + xdy) + tg(xy)dy = 0$

$yd(xy) + tg(xy)dy = 0 \quad xy = t \qquad ydt + tg(t)dy = 0$

$\frac{dt}{tg(t)} = -\frac{dy}{y} \qquad \frac{cosdt}{sint} = -\frac{dy}{y}$

$\ln(sint) = -lny + lnC$

$\ln(sint) + lmy = lnC$

$\ln(ysint) = lnC$

$ysint = C \quad Javob\ \ ysin(xy)$

85.(203) Misol

$y(x + y)dx + (xy + 1)dy = 0$

$xydx + y^2dx + xdy + dy = 0$

$y(ydx + xdy) + xydx + dy = 0$

$ydx + xydx + dy = 0$

$y(dxy + xdy) + dy = 0 \qquad \left(dxy + d\left(\frac{1}{2}x^2\right)\right) + dy = 0 \quad yd\left(\frac{1}{2}x^2 + xy\right) + dy$

$= 0 \quad \frac{1}{2}x^2 + xy = t \quad ydt + dy = 0 \quad ydt = -dy \quad y = 0 \quad dt = -\frac{dy}{y} \quad t$

$= -ln|y| + C \quad \frac{1}{2}x^2 + xy + ln|y| = C \qquad y = 0$

86.(204) Misol

$y(y^2 + 1)dx + x(y^2 - x + 1)dy = 0$

$y(y^2 + 1)dx + x(y^2 + 1)dy - x^2dy = 0$

$(y^2 + 1)(ydx + xdy) - x^2dy = 0 \qquad (y^2 + 1)dxy - x^2dy = 0$

$(y^2 + 1)dxy = x^2dy \quad x = 0 \;\; y = 0 \qquad \frac{dxy}{x^2y^2} \; = \frac{dy}{y^2(y^2+1)}$

$\int \frac{dxy}{(xy)^2} = - \; \frac{1}{xy}$

$\int \frac{dy}{y^2(y^2+1)} = \int (\frac{1}{y^2} - \frac{1}{y^2+1})$ -$dy = -\frac{1}{y} arctg(y) + C$

-$\frac{1}{xy} = \frac{1}{y} = arctg(y) + C \qquad 1 = x + xyarctg(y) + xyC$

87.205) Misol

$(x^2 + 2x + y)dx = (x - 3x^2)dy$

$(x^2 + 2x)dx + ydx - xdy + 3x^2ydy = 0$

$x = 0$tenglamaning ikkala qisminix^2 ga bo’lamiz

$\left(1 + 2\frac{1}{x}\right)dx + \frac{ydx - xdy}{x^2} + 3ydy = 0$

$\left(1 + 2\frac{1}{x}\right)dx - d\left(\frac{y}{x}\right) + 3ydy = 0 \quad d(x + 2\ln|x| - d\left(\frac{y}{x}\right) + d\left(\frac{3}{2}y^2\right) = 0$

$d\left(x + 2\ln|x| - \frac{y}{x} + \frac{3}{2}y^2\right) = 0$

$x + 2\ln|x| - \frac{y}{x} + \frac{3}{2}y^2 = C$

88.(206) Misol

$ydx - xdy = 2x^3 tg\frac{y}{x}dx$

Tenglamaning ikkala qismini x^2 ga bo’lamiz

$\frac{y\,dx - x\,dy}{x^2} = 2xtg\frac{y}{x}dx \quad -d\left(\frac{y}{x}\right) = 2xtg\frac{y}{x} - dx^2$

$d\left(\frac{y}{x}\right) + tg\frac{y}{x} - dx^2 = 0 \quad \frac{y}{x} = t \quad \frac{dt}{tg(t)} = -dx^2$

$\frac{costdt}{sint} = -dx^2 \quad \ln|sint| = -x^2 + C \quad sint = Ce^{-x^2}$

Javob $sin\frac{y}{x} = Ce^{-x^2}$

89.(207) Misol

$y^2dx + (e^x - y)dy = 0$

$y^2e^xdx + dy - ye^xdy = 0 \quad -y^2de^x + dy - ye^{-x}dy = 0$

$e^{-x} = \mathrm{t} \quad -y^2dt_{dy} - tydy = 0 \quad dy = 0 \quad y^2t' + ty = 0 \quad yt' + t = 0$

$\frac{dt}{dy} = -t$

$\frac{dt}{t} = -\frac{dy}{y} \quad \ln|t| = -\ln|y| + \ln\mathrm{C}$

$t = \frac{C(y)}{y}$

$$t = \frac{C'y - C}{y^2} \quad C'y - C + C = 1$$

$C' = \frac{1}{y} \quad C = \ln|y| + C_1 \quad Ty = \ln|y| + C \quad e^{-x}y = \ln|y| + C$

90.(208) Misol

$xydx = (y^3 + x^2y + x^2)dy$

$\frac{\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial Y}}{P}\frac{x + 2xy + 2x}{xy} = \frac{2x + 2xy}{xy} = \frac{2y + 3}{y} = 2 + \frac{2}{y}$

$$\mu = e^{-\int\left(2+\frac{3}{y}\right)dy} = e^{-2y-3lny} = y^{-3}e^{-2y}$$

$\mathrm{x}y^{-2}e^{-2y}dx - (1 + x^2y^{-2}x^2y^{-3})e^{-2y}dy = 0$

$\frac{\partial P}{\partial y} = -2xy^{-3}e^{-2y} - 2xy^{-2}e^{-2y}$

$\frac{\partial Q}{\partial x} = -2xy^{-2}e^{-2y} - 2xy^{-3}e^{-2y}$

$\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$

$$\int xy^{-2}e^{-2y}dx = \frac{1}{2}x^2y^{-2}e^{-2y} + C(y) - x^2y^{-3}e^{-2y} - x^2y^{-2}e^{-2y} + C'(y)$$
$$= -e^{-2y} - x^2y^{-3}e^{-2y} - x^2y^{-2}e^{-2y}$$

$C'(y) = e^{-2y}$

$C'(y) = \frac{1}{2}e^{-2y}$

$\frac{1}{2}x^2y^{-2}e^{-2y} + \frac{1}{2}e^{-2y} = C$

$e^{-2y}\left(\frac{x^2}{y^2} + 1\right) = C$

$\frac{x^2}{y^2} + 1 = Ce^{2y}$

$ln(\frac{x^2}{y^2} + 1) = C + 2y$

91.(209) Misol

__$x^2y(ydx + xdy) = 2ydx + xdy$ $x^2y(ydx + xdy) = 2(ydx + xdy) - xdy$

$x^2ydxy = 2ydxy - xydy$ $xy = t$ $t^2dt = 2ydt - tdy$ $dt = 0$

$\underline{dt = 0 \Rightarrow t = C \Rightarrow t = 0 \Rightarrow xy = 0 \Rightarrow x = 0; y = 0}$

$t = C \Rightarrow t = 0$ $ty' - 2y = -t^2$ $ty' - 2y = 0$

$t\frac{dy}{dt} = 2y$

$\frac{dy}{y} = 2\frac{dt}{t}$ $lny = 2lnt + lnC$ $y = Ct^2$

'$2Ct^2 + C't^3 - 2Ct^2 = -t^2$

$C't^3 = -t^2$ $C' = -\frac{1}{t}$ $C = -lnt + lnC_1$ $y = t^2lnCt$ $y = -x^2y^2lnCxy$

$-x^2$ylnC=1

92.(210) Misol

___$(x^2 - y^2 + y)dx + x(2y - 1)dy = 0$ $\left(x^2 - (y^2 + y)\right)dx + xd(y^2 - y) = 0$ $y^2 - y = t$

$\underline{xdt + (x^2 - t)dx = 0 \quad xdt - tdx + x^2dx = 0 \qquad x = 0}$

tenglamaning ikkala qismini x^2 ga bo’lamiz

$\frac{xdt - tdx}{x^2} + dx = 0$

$d(\frac{t}{x}) + dx = 0$

$\frac{t}{x} + x = C$

$\frac{y^2 - y}{x} + x = C$ $y^2 - y + x^2 = Cx$ $y^2 + x^2 = y + Cx$

93.(211) Misol

$(2x^2y^2+y)dx+(x^3y-x)dy=0$

$2x^2y^2dx+x^3ydy)+(ydx-xdy)=0$

$2x^2y^2dx+x^3ydy$

$$\frac{\partial P}{\partial y}=4x^2$$

$$\frac{\partial Q}{\partial x}=3x^2y$$

$\frac{\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}}{Q}=\frac{1}{x}$

$\mu=e^{\int\frac{1}{x}dx}=x$

$\frac{1}{x}(2x^3y^2dx+x^4ydy)+ydx-xdy=0$

$\frac{1}{x}d\left(\frac{1}{x}x^4y^2\right)+y^2\frac{ydx-xdy}{y^2}=0$

$\frac{1}{2x}d(x^4y^2)+y^2d\left(\frac{x}{y}\right)=0$

$\frac{1}{2}d(x^4y^2)+y^2xd\left(\frac{x}{y}\right)=0$

$\begin{cases}x^4y^2=t\\ \frac{x}{y}=m\end{cases}$

$\frac{x^2}{y^2}=m^2$

$\frac{x^2}{y^2}=\frac{1}{m^2}$

$y^4x^2=\frac{t}{m^2}$

$y^2x=\frac{\sqrt{t}}{m}$

$\frac{1}{2}dt+\frac{\sqrt{t}}{m}dm$

$\frac{dt}{2\sqrt{t}}=-\frac{dm}{m}$

$\sqrt{t}=-\ln|m|+C$

$\sqrt{x^4y^2}+\ln\left|\frac{x}{y}\right|=C \quad Javob \quad x^2y+\ln\left|\frac{x}{y}\right|=C$

94(212) MIsol

$(2x^2y^3-1)ydx+(4x^2y^3-1)xdy=0$

$2x^2y^4dx+4x^3y^3dy-ydx-xdy=0 \qquad 2x^2y^2(y^2\,dx+2xydy)-d(xy)=0$

$2x^2y^2d(xy^2)-d(xy)=0$

$$\begin{cases} xy = t \\ xy^2 = m \end{cases} \quad x^2y^2 = t^2 \qquad 2t^2dm = dt \qquad 1)\ \ t = 0 => x = 0;\ \ y = 0$$

tenglamani t^2 ga bo'lamiz

$$2dm = \frac{dt}{t^2}$$

$-\frac{1}{t} + C = 2m \quad 2m + \frac{1}{t} = C$

$2m+\frac{1}{t}=C$

$$2xy^2 + \frac{1}{xy} = C$$

Javob; $2xy^2 + \frac{1}{xy} = C\ \ x = 0; y = 0$

95.(213) Misol

$y(x + y^2)dx + x^2(y - 1)dy = 0 \qquad xydx + y^3dx + x^2ydy - x^2dy = 0$

$:/y^2 \neq 0$

$$x\frac{ydx - xdy}{y^2} + ydx + \frac{x^2}{y}dy = 0$$

$$xd(\frac{x}{y}) + \frac{x^2y^2}{y}(\frac{dx}{x^2} + \frac{dy}{y^2}) = 0.\ /: x \neq 0$$

$$d(\frac{x}{y}) + \frac{x}{y}d\left(-\frac{1}{x} - \frac{1}{y}\right) = 0$$

$\frac{x}{y} = u$

$$\frac{1}{x} + \frac{1}{y} = \text{v} \quad \text{dU + UdV} \qquad \frac{dU}{U} = dV \quad lnU = V + C \quad dU + UdV$$

Javob: $ln\frac{x}{y} = \frac{1}{x} + \frac{1}{y} + C \quad x = 0,\ y = 0$

96.(214) Misol

$(x^2 - sin^2y)dx + xsin^2ydy = 0$

$x^2dx - sin^2ydx + xsin^2ydy$

$$x^2dx - sin^4y\frac{sin^2ydx - xsin^2ydy}{sin^4y} = 0$$

$$x^2dx - sin^4y\frac{sin^2ydx - xdsin^2y}{sin^4y} = 0$$

$$x^2dx - sin^4yd\frac{x}{sin^2y} = 0$$

$$\frac{x^2}{sin^4y}dx - d\frac{x}{sin^2y} = 0$$

$$(\frac{x}{sin^2y})^2 d\frac{x}{sin^2y} = 0$$

$\frac{x}{sin^2y} = t$ $\quad t^2dx - dt = 0 \quad dx = \frac{dt}{t^2}$

$$x - \frac{1}{t} + C$$

$x - \frac{sin^2y}{x} + C \quad sin^2y - Cx - x^2 \qquad Javob: \ sin^2y = Cx - \ x^2$

97.(215) Misol

$x(lny + 2lnx - 1)dy = 2ydx \quad :/x \neq 0$

$-2y^2d\left(\frac{lnx}{y}\right) + (ln - 1)dy = 0 \quad lnydy + 2lnxdy - dy = 2yd(lnx)$

$2(lnxdy - ydlnx) + lnydy - lnydy = 0$

$-2y^2d(\frac{lny}{y}) - y^2d\left(\frac{lny}{y}\right) = 0 \ \frac{1}{:y^2} \neq 0$

$$2d\left(\frac{lnx}{y}\right) + d\left(\frac{lny}{x}\right) = 0, \quad \frac{lnx}{y} = u, \quad \frac{lny}{y} = v$$

$2du + dv = 0$

$u = -\frac{v}{2} + c$

$\frac{lnx}{y} = -\frac{lny}{2y} + c$

$\frac{lnx^2+lny}{2y} = C_2 \quad lnx^2 = yC_1 \qquad Javob\text{:} \ \ yClnx^2y \ \ x = 0 \ \ y = 0$

98.(217) Misol

$(2x^3y^2 - y)dx + (2x^2y^3 - x)dy = 0$

$2x^2y^2(xdx + ydy) - (ydy + xdy) = 0$

$2x^2y^2(x^2 + y^2) - d(xy) = 0$

$$\begin{cases} xy = m \\ x^2 + y^2 = t \end{cases} \quad m^2dt = dm$$

$$1) \ x = 0 \ \ y = 0 \quad 2) \ dt = \frac{dm}{m^2}$$

$$t = \frac{1}{m} + C \quad x^2 + y^2 = -\frac{1}{xy} + C \qquad xy(x^2 + y^2 - C) = -1$$

$Javob$: $xy(x^2+y^2-C=-1 \quad x=0 \;\; y=0$

99.(218) Misol

$x^2y^3dx+y+(x^3y^2-x)y'=0$

$x^2y^3+y+(x^3y^2-x)y'$=0

$x^2y^3dx+ydx+(x^3y^2-x)dy=0$

$x^2y^2(ydx+xdy)+ydx-xdy=0$

$x^2y^2d(xy)+y^2d\left(\frac{x}{y}\right)=0$

$\begin{cases} \frac{x}{y}=0 \\ xy=m \end{cases}$

$y^2=\frac{m}{t}$

$$m=0 \Rightarrow y=0$$

$$mdm=-\frac{dt}{t}$$

$\frac{m^2}{2}=-C\ln|t| \quad m^2=-C\ln|t| \quad x^2y^2=C\ln\frac{y^2}{x^2} \quad Javob: y^2=Cx^2e^{x^2y^2}$

100.(219) Misol

$(x^2-y)dx+x(y-1)dy=0 \qquad x^2dx-ydx+xydy+xdy=0$

$$x(xdx+ydy)-ydx+xdy=0$$

$$\frac{1}{2}d(x^2+y^2)+x^2d\left(\frac{y}{x}\right)=0$$

$\frac{1}{2}d(x^2+y^2)+xd\left(\frac{y}{x}\right)=0$

$\begin{cases} x^2+y^2=t \\ \frac{y}{x}=m \end{cases} \quad y=mx \quad x^2(m^2+1)=t$

$$x=\sqrt{\frac{t}{m^2+1}}$$

$\frac{1}{2}\frac{dt}{\sqrt{t}}=-\frac{dm}{\sqrt{m^2+1}}$

$\sqrt{t}=-\ln(m+\sqrt{m^2+1}+C$

Javob; $\sqrt{t}+\ln(\frac{y}{x}+\sqrt{1+\frac{y^2}{x^2}}=C \;\; x=9$

101.(220) Misol

$$y^2(ydx-2xdy)=x^3(xy-2ydx) \quad :\frac{1}{x^2y} \quad x,y\neq 0$$

$$\frac{y^2dx-2xdy}{x^2}=\frac{x^2dy-2xydx}{y^2}$$

$$-\mathrm{d}\frac{y^2}{x}=-yd\frac{\mathrm{x}^2}{\mathrm{y}}$$

$$\frac{y^2}{x}=u\Rightarrow \mathrm{x}=\frac{y^2}{u}\quad \frac{y^2}{x}=v => y=\frac{x^2}{v}=\frac{y^4}{u^2v} => y^3\ \mathrm{u}^{\frac{2}{3}}\mathrm{v}^{\frac{1}{3}}$$

$$du=ydv\quad du=\mathrm{u}^{\frac{2}{3}}\mathrm{v}^{\frac{1}{3}}dv\quad \frac{\mathrm{du}}{\mathrm{u}^{\frac{2}{3}}}=\mathrm{v}^{\frac{1}{3}}dv\quad 3\mathrm{u}^{\frac{1}{3}}=\frac{3}{4}\mathrm{v}^{\frac{4}{3}}+C$$

$$3(\frac{y^2}{x})^{\frac{1}{3}}=\frac{3}{4}(\frac{x^2}{y})^{\frac{4}{3}}+C\quad /*\ x^{\frac{1}{3}}y^{\frac{4}{3}}$$

$$3y^2-\frac{3}{4}x^3=Cy\sqrt[3]{xy}\ \frac{}{*}-\frac{4}{3}\qquad Javob\text{: } x=0\ \ y=0\ \ x^3-4y^2=Dy\sqrt[3]{xy}$$

§-6.
Hosilaga nisbatan yechilmagan birinchi tartibli oddiy differensial tenglamalar

Hosilaga nisbatan yechilmagan birinchi tartibli oddiy diffensial tenglamalar

$$F(x,y,y')=0 \qquad (1)$$

ko'rinishda yoziladi. Agar (a,b) intervalda uzluksiz differensiallanuvchi $y=y(x)$ funksiya (1) tenglamani shu intervalda ayniyatga aylantirsa, yani $F(x,y(x),y'(x))=0$ tenglik barcha $x\in(a,b)$lar uchun bajarilsa $y=y(x)$ funksiya (1) tenglamaning (a,b) intervaldagi **yechimi** deyiladi.

Agar parametrik ko’rinishda berilgan $x=\varphi(t)$, $y=\psi(t)$ funksiya uchun (t_0,t_1) intervalda $F(\varphi(t),\psi(t),\frac{\psi'(t)}{\varphi'(t)})\equiv 0$ ayniyat o’rinli bo’lsa bu funksiya (1) tenglamaning (t_0,t_1) intervaldagi **parametrik yechimi** deyiladi. (1) tenglamani yechimi **oshkormas** ko’rinishda aniqlanishi ham mumkin.

(1) tenglama har bir (x,y) nuqtada y'ning bitta yoki bir nechta qiymatini aniqlaydi. Har bir (x,y) nuqtada har bir y'ga mos Ox o’qinini musbat yo’nlishi bilan α ($tg\alpha=y'$) burchak tashkil etuvchi birlik vector chizamiz. Hatijada **yo’nalishlar maydoni** hosil bo’ladi.

2-reja. (1) differensial tenglamani $y(x_0)=y_0$ boshlang’ich shartni qanoatlantiruvchi yechimini topish masalasi – **Koshi masalasi** deyiladi. Agar (1) tenglamani $y(x_0)=y_0$ shartni qanoatlantiruvchi har qanday ikkita yechimi (x_0,y_0) nuqtada umumiy urinmaga ega bo’lmasa (x_0,y_0) nuqtada **Koshi masalasi yagona yechimga ega** deyiladi. Agar (1) tenglamani $y(x_0)=y_0$ shartni qanoatlantiruvchi yechimi mavjud bo’lmasa yoki shu shartni qanoatlantiruvchi har qanday ikkita yechimi (x_0,y_0) nuqtada umumiy urinmaga ega bo’lsa (x_0,y_0) nuqtada **Koshi masalasi yechimi yagonaligi busiladi** deymiz.

Teorema. Agar $F(x,y,y')$ funksiya quyidagi uchta shartni qanoatlantirsa:

1) $F(x,y,y')$ funksiya (x_0,y_0,y'_0) nuqtaning biror atrofida o’zinin birinchi tartibli hususiy hoslilari bilan uzluksiz;

2) $F(x_0, y_0, y_0') = 0$;

3) $F'_{y'}(x_0, y_0, y_0') \neq 0$,

u holda (1) tenglamaning $y(x_0) = y_0$, $y'(x_0) = y_0'$ tengliklarni qanoatlantiruvchi $x = x_0$ nuqtaning biror atrofida aniqlangan $y = y(x)$ yechimi mavjud va yagona.

Isbot. Oshkormas funksiyalar haqidagi teoremaga ko'ra (x_0, y_0, y_0') nuqtaning atrofida (1) tenglamani y' ga nisbatan bir qiymatli yechish mumkin: $y' = f(x, y)$, bu erda $f(x, y)$ funksiya (x_0, y_0) nuqtaning atrofida o'zining birinchi tartibli hususiy hosilalari bilan uzluksiz va $y_0' = f(x_0, y_0)$. U holda Koshi teoremasiga ko'ra $y' = f(x, y)$, $y(x_0) = y_0$ Koshi masalasi $x = x_0$ nuqtaning biror atrofida yagona $y = y(x)$ yechimga ega. Bu funksiya (1) tenglamani ham echimidir. Bu yechim $y'(x_0) = f(x_0, y(x_0)) = y_0'$ tenglikni ham qanoatlantiradi.

3-reja. (1) differensial tenglama y' ga nisbatan yechilsin:

$$y' = f_k(x, y),\ k = 1.2....,m \qquad (2)$$

(2) tenglamalarnin umumiy yechimlari to'pami (1) tenglamaning **umumiy yechimi** deyiladi. Kiritilgan ta'rif (2) tenglamalar soni cheksiz bo'lgan hol uchun ham o'rinli.

Misol. $y'^2 + (y^2 - 1)y' - y^2 = 0$ (3)

tenglamani qaaymiz. U ikkita tenglamaga ajraladi: $y' = 1$, $y' = -y^2$. Bu tenglamalarning umumiy yechimini mos ravishda yozamiz:

$$y = x + C,\ y = \frac{1}{x + C}$$

Bu yechimlar to'plami (3) tenglamaning umumiy yechimini ifodalaydi. Umumiy yechimni bitta munosabat bilan quyidagicha yozish mumkin:

$$(y - x - C)\left(y - \frac{1}{x + C}\right) = 0.$$

Javob: $(y - x - C)\left(y - \frac{1}{x + C}\right) = 0$

$y = y(x)$ yechimning har bir nuqtasida Koshi masalasi yagona yechimga ega bo'lsa u (1) tenglamaning **hususiy yechimi** deyiladi. $y = y(x)$ yechimning har bir nuqtasida Koshi masalasi yechimi yagonaligi buzilsa u (1) tenglamaning **mahsus yechimi** deyiladi.

Endi (1) tenglamani mahsus yechimini toppish masalasi bilan shug'ulanamiz.

$$\begin{cases} F(x, y, y') = 0 \\ F'_{y'}(x, y, y') = 0 \end{cases}$$

sistemadan y' ni yo'qotib biror $y = y(x)$ funksiyaga ega bo'lamiz. Bu funksiya (1) tenglamaning **diskriminant chizig'i** deyiladi. (1) tenglama mahsus yechimga ega bo'lsa u diskriminant tenglamaning chizig'idan iborat bo'ladi.

Misol. $xy'^2 - 2yy' + 4x = 0$ (4)

tenglamani qaraymiz. Diskriminant chiziqni topamiz:

$$\begin{cases} xy'^2 - 2yy' + 4x = 0 \\ 2xy' - 2y = 0 \end{cases} \qquad y' = \frac{y}{x} \qquad \frac{xy^2}{x^2} - \frac{2y^2}{x} + 4x = 0 \qquad y^2 = 4x^2.$$

Bu tenglam ikkita $y = \pm 2x$ to’g’ri chiziqni ifodalaydi va ular (4) tenglamaning mahsus yechimidan iborat.

4-reja. $F(y') = 0$ (5)

ko’rinishdagi faqat hosila qatnashgan tenglamalarni o’rganamiz. (5) tenglamani y' ga nisbatan haqiqiy yechimlari $y' = k_i$, $(i = 1,...,m)$ deylik. U holda $y = k_i x + C$ yoki $k_i = \frac{y - C}{x}$ kelib chiqadi. $y' = k_i$ ni hisobga olib buni (5) ga qo’ysak $F\left(\frac{y-C}{x}\right) = 0$ bu tenglamaning **umumiy yechimi** bo’ladi.

Noma’lum funksiya qatnashmagan tenglamani o’rganamiz:

$$F(x, y') = 0 \qquad (6)$$

Bu tenglamani y' ga nisbatan yechish mumkin bo’lsin: $y' = f_k(x)$, $k = 1,..,m$ u holda uning **umumiy yechimi** $y = \int f_k(x)dx + C$, $k = 1,..,m$ funksiyalar to’plamidan iborat.

(6) tenglamani x ga nisbatan yechish mumkin bo’lsin: $x = \varphi(y')$. Bu tenglamani integrallash uchun $y' = p$ parametr kiritamiz. U holda $x = \varphi(p)$, $dy = pdx$ tengliklardan $dy = p\varphi'(p)dp$ yoki $y = \int p\varphi'(p)dp + C$ kelib chiqadi. Natijada (6) tenglamaning umumiy yechimi parametrik formada yoziladi:

$$x = \varphi(p), \; y = \int p\varphi'(p)dp + C.$$

Erkli o’zgaruvchi qatnashmagan tenglamani o’rganamiz:

$$F(y, y') = 0 \qquad (7)$$

Bu tenglamani y' ga nisbatan yechish mumkin bo’lsin: $y' = g_k(y)$, $k = 1,..,m$ u holda uning **umumiy yechimi** $\int \frac{dy}{g_k(y)} = x + C$, $k = 1,..,m$ funksiyalar to’plamidan iborat.

(6) tenglamani y ga nisbatan yechish mumkin bo’lsin: $y = \varphi(y')$. Bu tenglamani integrallash uchun ham $y' = p$ parametr kiritamiz. U holda $y = \varphi(p)$, $dx = \frac{dy}{p}$ tengliklardan $dx = \frac{1}{p}\varphi'(p)dp$ yoki $x = \int \frac{1}{p}\varphi'(p)dp + C$ kelib chiqadi. Natijada (6) tenglamaning **umumiy yechimi** parametrik formada yoziladi:

$$y = \varphi(p), \; x = \int \frac{1}{p}\varphi'(p)dp + C$$

Parametr kiritish usuli

$$F(x, y, y') = 0 \qquad (1)$$

tenglamani noma’lum funksiyaga nisbatan yechish mumkin bo’lsin, ya’ni

$$y = f(x, y') \qquad (2)$$

ko'rinishda yozish mumkin bo’lsin. $y' = p$ deb belgilaymiz. Natijada (2) tenglama

$$y = f(x, p), \; y' = p$$

ko’rinishni oladi. Bundan

$$dy = f'_x dx + f'_p dp$$

Bu erda $dy = pdx$ o'rniga qo'yishni bajaramiz:

$$pdx = f'_x dx + f'_p dp \qquad\qquad p = f'_x + f'_p \frac{dp}{dx} \qquad (3)$$

Bu hosilaga nisbatan yechilgan tenglamadir. Uning umumiy yechimi $p = \omega(x, C)$ bo'lsa (2) tenglamaning **umumiy yechimi** $y = f(x, \omega(x, C))$ formula bilan aniqlanadi. Agar (3) tenglama $p = \gamma(x)$ mahsus yechimga ega bo'lsa (2) tenglamaning $y = f(x, \gamma(x))$ mahsus yechimga ega bo'lishi mumkin.

(1) tenglamani erkli o'zgaruvchiga nisbatan yechish mumkin bo'lsin:

$$x = f(y, y') \qquad (4)$$

$y' = p$ deb belgilaymiz. Natijada (4) tenglama quyidagi ko'rinishni oladi:

$$x = f(y, p),\ y' = p;$$

$$dx = f'_y dy + f'_p dp; \qquad \frac{1}{p} dy = f'_y dy + f'_p dp;$$

$$\frac{1}{p} = f'_y + f'_p \frac{dp}{dy}. \qquad (5)$$

Ohirgi tenglama hosilaga nisbatan yechilgan differensial tenglamadir. Uning umumiy yechimi $p = \omega(y, C)$ bo'lsa (4) tenglamaning **umumiy yechimi** $x = f(y, \omega(y, C))$ formula bilan ifodalanadi. (5) tenglama $p = \gamma(y)$ mahsus yechimga ega bo'lsa (4) tenglama $x = f(y, \gamma(y))$ mahsus yechimga ega bo'lishi mumkin.

. Quyidagi ko'rinishdagi tenglama **Lagranj tenglamasi** deyiladi:

$$y = \varphi(y')x + \psi(y'). \qquad (6)$$

Lagranj tenglamasini hamma vaqt kvadraturalarda integrallash mumkin. Haqiqatdan ham $y' = p$ parametr kiritsak

$$y = \varphi(p)x + \psi(p),\ y' = p$$

$$pdx = \varphi(p)dx + [\varphi'(p)x + \psi'(p)]dp$$

$$[\varphi(p) - p]dx + [\varphi'(p)x + \psi'(p)]dp = 0 \qquad (7)$$

$$\frac{dx}{dp} + \frac{\varphi'(p}{\varphi(p) - p} x = \frac{\psi'(p)}{p - \varphi(p)}$$

Bu – erkli o'zgaruvchisi p dan nomalum funksiyasi x dan iborat chiziqli differensial tenglamadir. Uning umumiy yechimi $x = \omega(p, C)$ bo'lsa (6) tenglamaning **umumiy yechimi**

$$y = \varphi(p)\omega(p, C) + \psi(p),\ \ x = \omega(p, C)$$

parametrik ko'rinishda ifodalanadi.

Yuqorida (7) tenglamani $\varphi(p) - p$ ifodaga bo'lishni amalgam oshirdik. Agar p_i ($i = 1, \ldots, m$) sonlar $\varphi(p) = p$ tenglamaning ildizlari bo'lsa Lagranj tenglamasining quyidagi yechimlari ham kelib chiqadi:

$$y = p_i x + \psi(p_i),\ (i = 1, \ldots, m)$$

Bu yechimlar mahsus bo'lishi ham hususiy bo'lishi ham mumkin. Demak Lagranj tenglamasining **mahsus yechim**lari faqt to'g'ri chiziq bo'lishi mumkin.

Misol. Ushbu $y = xy'^2 + y'^2$ tenglamani qaraymiz. $y' = p$ parametr kiritamiz. Natijada: $y = xp^2 + p^2$

$$pdx = p^2dx + (2px + 2p)dp$$

$$(p^2 - p)dx + 2p(x+1)dp = 0$$

$$\frac{dx}{dp} + \frac{2}{p-1}x = \frac{2}{1-p}$$

Bu chiziqli tenglamani umumiy yechimi: $x = \frac{C}{(p-1)^2} - 1$. Bu ifodani $y = xp^2 + p^2$ ga qo'yamiz: $y = \frac{Cp^2}{(p-1)^2}$. Demak berilgan tenglamaning umumiy yechimi quyidagicha parametrik ko'rinishda yoziladi

$$x = \frac{C}{(p-1)^2} - 1, \quad y = \frac{Cp^2}{(p-1)^2}.$$

Bu erda p ni yo'qotsak umumiy yechim oshkor ko'rinishni oladi:

$$y = \left(\sqrt{x+1} + C\right)^2$$

Tenglamani yechish jarayonida $p^2 - p$ ifodaga bo'lish bajarildi. Bu ifoda $p = 0$ va $p = 1$da nolga aylanadi. Bularni $y = xp^2 + p^2$ga qo'yib berilgan tenglamaning ikkita yechimini topamiz:

$$y = 0 \text{ va } y = x + 1.$$

Ulardan birinchisi mahsus yechim ikkinchisi hususiy yechimdir.

Javob: $y = \left(\sqrt{x+1} + C\right)^2$, $y = 0$, $y = x + 1$

Agar Lagranj tenglamasida $\varphi(y') \equiv y'$ bo'lsa u

$$y = y'x + \psi(y') \qquad (8)$$

ko'rinishni oladi. (8) tenglama **Klero tenglamasi** deb ataladi. Bu erda ham $y' = p$ parametr kiritamiz. Natijada

$$y = px + \psi(p) \qquad (9)$$

$$dy = pdx + [x + \psi'(p)]dp$$

$$pdx = pdx + [x + \psi'(p)]dp$$

$$[x + \psi'(p)]dp = 0$$

Ohirgi tenglama ikkita tenglamaga ajraladi:

$$dp = 0 \text{ va } x = -\psi'(p) \qquad (10)$$

Ularning birinchsidan $p = C$ kelib chiqadi va buni (9)ga qo'ysak (8) tenglamaning **umumiy echimi**ni hosil qilamiz: $y = Cx + \psi(C)$. Berilgan tenglama va umumiy yechim ko'rinishlarini taqqoslab shunday hulosaga kelamiz: Klero tenglamasining umumiy yechimini yozish uchun tenglamda $y' = C$ o'rniga qo'yish bajarish kifoya. (10)ning ikkinchi tenglamasidan Klero tenglamasining yana bir yechimi paydo bo'ladi:

$$y = -p\psi'(p) + \psi(p), \qquad x = -\psi'(p) \quad (11)$$

Parametrik ko'rinishdagi bu yechim **mahsus yechim** bo'lishini isbotlaymiz. Avvalgi darsda ta'kidlanganidek (8) tenglamaning mahsus yechimi diskriminant chiziqlar orasida bo'ladi. Bu chiziq

$$\begin{cases} y = y'x + \psi(y') \\ 0 = x + \psi'(y') \end{cases}$$

sistemadan y' ni yo'qotib aniqlanadi. Bu sistema va (11) tengliklarni solishtirsak, (11)dan p ni yo'qotsak ham ayni diskriminant chiziq hosil bo'lishini ko'rish mumkin. Qolaversa (11) funksiya Klero tenglamasining yechimidan iborat. Demak u mahsus yechimdir.

Misol. Ushbu $y = y'x - \frac{1}{4}y'^2$ tenglamani qaraymiz. $y' = C$ o'rniga qo'yishni bajarib umumiy yechimni aniqlaymiz: $y = Cx - \frac{1}{4}C^2$.

Bu tenglamaning diskriminant chizigini topaylik:

$$\begin{cases} y = Cx - \frac{1}{4}C^2 \\ 0 = x - \frac{1}{2}C \end{cases}$$

Bu sistemadan $y = x^2$ fuksiyani aniqlaymiz. Bu funksiya berilgan tenglamaning mahsus yechimidir. **Jabob:** $y = Cx - \frac{1}{4}C^2$, $y = x^2$

102.(231) Misol

$y^{(n)} = x + y^2$ tenglamaning bir vaqtda ikkita y(0)=1 ,$\frac{dy}{dx}(0) = 2$ shartni qanoatlantiruvchi yechimlari nechta? n=1,2,3 holatlar alohida qaralsin.

Yechim:

n=1 uchun $\frac{dy}{dx}(0) = (x + y^2)|_{x=0} = y^2(0) = 1 \neq 2$ bor.Demak,agar n=1 bo'lsa, u holda tenglama boshlang'ich shartlarga ega bo'lgan yagona yechimga ega emas.

f(x,y,$\frac{dy}{dx}) = x + y^2$ funksiyasi hosilalari bilan birga uzluksiz bo'lgani uchun (0,1,2,) nuqtalarda

$\frac{\partial f}{\partial y} = 2y, \frac{\partial f}{\partial y'} = 0$, u holda mavjudlik teoremasi bo'yicha $y'' = x + y^2$, y(0)=1, y'(0)=2 muammosi yetarlicha kichik nuqtalar((0,1,2)) da yagona yechimga ega. Yagonalik teoremasi tufayli $y''' = x + y^2$, y(0)=1, $y'(0) = 2, y''(0) = 0$ ixtiyoriy nuqtada yagona yechimga ega (0,1,2,$y''(0)$).shuning uchun (0,1,2) nuqtalarning yetarlicha kichik qiymatlarida bir parametrli yechimlar to'plami mavjud.

$$y''' = x + y^2, \;\; y(0) = 1, \;\; y'(0) = 2 \; .$$

103(232) Misol

$y^{(n)} = f(x, y)$ *tenglamaning* (f *va* f_y funksiyalar butun (x,y) nuqtalar tekisligida uzluksiz)

(x_0, y_0) nuqtadan belgilangan yo'nalishda o'tuvchi , ya'ni shu nuqtadagi urinmasi Ox o'qi bilan α burchak tashkil qiluvchi yechimlari nechta?

n=1, n=2 va n≥3 holatlar alohida qaralsin.

Yechim:

n = 1 bo'lsin. U holda f va $\frac{\partial f}{\partial y}$ funksiyalarning uzluksizligi tufayli y'= f(x, y), y(x_0) =y_0 masala bo'lsin.Nuqtaning yetarlicha kichik mahallasida (x_0, y_0) yagona yechimga ega. Bundan tashqari, y' (x_0) = tga bo'lsa, tga = f(x_0, y_0), u holda muammoning yagona yechimi bor. Agar tgα ≠f(x_0, y_0), u holda muammoning yechimlari yo'q.

n = 2 bo'lsin. U holda uzluksizlik tufayli y'' = f(x,y), y(x_0) = y_0, y' (x_0) = tgα masala bo'lsin. f , $\frac{\partial f}{\partial y}, \frac{\partial f}{\partial y\prime}$funktsiyalari nuqtaning etarlicha kichik mahallasida yagona yechimga ega (x_0, yo, tga).

Nihoyat, agar m≥3 bo'lsa, u holda . $y^{(n)}$= f(x, y), y(x_0)= y_0, y' (x_0)=tgα masala son-sanoqsizdir. $y^{(n)}$ (x, y), y(x_0)= y_0, y' (x_0)=tg$\alpha, y''(x_0) = y_0'', \dots, y^{(n-1)}(x_0) = y_0^{(n-1)}$ bo'lishi sababli yechimlar to'plami $(x_0, y_0, tg\alpha, y_0'', \dots, y_0^{(n-1)})$ nuqtaning etarlicha kichik mahallasida noyob yechimga ega. Boshqacha aytganda, raqamlarning o'zboshimchaligi tufayli $y_0'', y_0''', \dots, y_0^{(n-1)}$

muammo $y^{(n)}$= f(x, y), y(x_0)= y_0, y' (x_0)=tgα $(n \geq 3)$ $(n-2)$ parametrlar majmuasiga ega yechimlar.Oxirgi xulosa n > 3 bo'lgan holat uchun oldingi misolga ham qo'llanilishi mumkin.

104.(233) Misol

n ning qanday qiymatlarida $y^{(n)}$= $f(x, y)$ tenglamaning (f va $f_y$uzluksiz) yechimlari orasida $y_1 = x$, $y_2 = x + x^4$ ikkita funksiya ham bo'lishi mumkin?

Yechim;

Buning uchun (0, 0) nuqtada bizda y_1(0) =y_2 (0), $y_1{}'$(0) =$y_2{}'$(0), y_1'' (0) = y_2'' (0), $y_1{}'''$ (0) =$y_2{}'''$ (0), y_1^{IV}(0) = 0,

y_2^{IV} (0) = 24, ya'ni y_1^{IV}(0)$\neq$ y_2^{IV}(0) . Ko'ramizki, n birga teng bo'la olmaydi, chunki orqali (0, 0) nuqta ikki xil yechimdan o'tadi va yagonalik teoremasi, x0y tekislikning har bir nuqtasidan faqat bitta yechim o'tadi. Keyinchalik, n≠2, beri aks holda, ko'rsatilgan teorema bo'yicha, faqat

bitta integral egri chiziq. Xuddi shu sababga ko'ra n$\neq 3$, n$\neq 4$.

n = 5 bo'lsin. U holda masala y^V= f(x, y), y(0)= 0, y'(0) = 1, y''(0) = y''(0) =$y'''(0) = y^{IV}(0) = 0$ bo'lsin. y_1= x yechimga ega bo'lishi mumkin va muammo y^V= f(x, y), y(0)= 0, y'(0) = 1, y''(0) = y''(0) =$y'''(0) = 0$ y^{IV}(0)= 24 - y=x+x^4 yechimdir. Buning sababi o'zgaruvchilar fazosida $(x, y, y', y'', y''', y^{IV})$ y_1 va y_2 egri chiziqlar bo'lib, hech qanday nuqtada mos kelmaydi, ya'ni. yechimlari bo'lishi mumkin bir xil tenglamaning y^V= f(x, y) ($f, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial y'}, \frac{\partial f}{\partial y''}, \frac{\partial f}{\partial y'''}, \frac{\partial f}{\partial y^{IV}}$ funksiyalariga e'tibor bering,bu erda uzluksiz). Bu egri chiziqlar hech bir nuqtada va fazoda bir-biriga to'g'ri kelmaydi o'zgaruvchilar $(x, y, y', y'', y''', y^{IV}$ ...), shuning uchun $y^{(n)}$= f(x, y) tenglamada ular quyidagicha bo'lishi mumkin.Yechimlar va n > 5 uchun. Arzimas misollar: y^V =0, $y^{VI} = 0$.

105.(234) Misol

n ning qanday qiymatlarida $y^{(n)} = f(x, y, y', \dots, y^{(n-1)})$ tenglamaning (f funksiya uzluksiz differensiallanuvchi) yechimlari orasida $y_1 = x$, $y_2 = x + x^4$ ikkita funksiya ham bo'lishi mumkin?

Yechim;

Muammoning yechimi:

X va sin(x) funksiyalarini nolga tenglashtirib oling:

1) va mavjudligi teoremasi tufayli tenglama birinchi tartibli bo'lishi mumkin emas yagonalik: x va sin(x) funksiyalarning grafiklari nol nuqtasida kesishadi.

2) Tenglama ham ikkinchi tartibli bo'lishi mumkin emas, chunki bu funksiyalarning hosilalari nolga teng 1 va grafiklar tegib turadi.

3) Xuddi shunday, tenglama uchinchi tartibli bo'lishi mumkin emas, chunki ikkalasining ikkinchi hosilalari noldagi funktsiyalar 0 ga teng.

4) Tenglama 4 tartibli bo'lishi mumkin, chunki noldagi uchinchi hosilalar boshqacha (funktsiya x uchinchi hosilasi xuddi shunday 0 ga teng, sin(x) funksiyasi esa uchinchi hosilaga teng. -cos(x), nolda - (-1)).

5) 4-darajali tenglamaga misol: $y'''' + y'' = 0$,umumiy yechim y=A + Bx + Csin(x) + Dcos (x), bu erda A,B, C, D ixtiyoriy konstantalardir. X va sin(x) berilgan differensial tenglamaning yechimlari ekanligi aniq.

106.(235) Misol

$f(x, y)$ funksiya x, y bo'yicha uzluksiz va har bir x uchun y ning o'sishida bu funksiya o'smaydi. Agar y'= $f(x, y)$ tenglamaning ikkita yechimi y(x_0)= $y_0$boshlang'ich shartni qanoatlantirsa, u holda bu yechimlar x $\geq x_0$ da ustma-ust tushishini isbotlang.

Yechiim;

Muammoning yechimi:

$y_1{}'= f\ (x, y_1{}'(\ ))$ birlikdan $y_2{}'= (x, y_2{}'(\ ))$ birlikni had bo‘yicha hadni ayirish va u funksiyani hisobga olish, bunda u(x) = $y_1{}'(\ x) - y_2{}'(\ x)$ ($x \geq x_0$), muammoni olamiz

$$u'(x) = f(x, y_2(\ x) + u(x)) - f(x, y_2(\ x))\ ,\ u(x_0) = 0, x \geq x_0 \qquad (1)$$

aniq yechimga ega u(x)≡0. Boshqa yechimlar yo'qligini isbotlaylik. Keling,teskarisini faraz qilish usulini qo’llaymiz. Shunday x$> x_0$ bo'lsin, buning uchun u>0. Keyin, u funksiyaning uzluksizligi tufayli ikkita $\varepsilon_1 > 0\ va\ \varepsilon_2 > \varepsilon_1$ soni mavjud bo'lib, ular uchun u(x)> 0 bo'ladi.

$x_0 \leq x_0 + \varepsilon_1 < x \leq \varepsilon_2$ va ε_1 ni kamaytirish orqali har doim u($x_0 + \varepsilon_1 = 0$bo'lishi mumkin. Integratsiya

(1) da biz olamiz

$$u(x)=\int_{x_0+\varepsilon_1}^{x} f\big(t, y_2(\ t) +\ u(t)\big) -\ f\big(t, y_2(\ t)\big)dt,\ \ x\in [x_0 + \varepsilon_1, x_0 + \varepsilon_2].$$

(2)

t∈ $[x_0 + \varepsilon_1, x]$ uchun u(t)> 0 bo‘lganligi sababli, $f(x, y)$ funksiya y ga nisbatan o‘smasligi tufayli tengsizlik yuzaga keladi.

$$(t, y_2(\ t) + u(t)) - (t, y_2(\ t))\leq 0. \qquad (3)$$

(3) ni hisobga olsak, (2) dan u(x)≤0 ni uchun topamiz $x \in (x_0 + \varepsilon_1, x_0 + \varepsilon_2]$.

Shunday qilib, biz ziddiyatga keldik, bundan kelib chiqadiki, u funksiya hech qanday $x \geq x_0$ uchun musbat bo‘la olmaydi. Xuddi shunday, biz buni aniqlaymiz va salbiy bo'lishi mumkin emas.

Demak, barcha $x \geq x_0$ uchun u(x)≡0.

107.(236) Misol

Quyida berilgan tenglamalar va sistemalarning yechimlari koordinatalar boshi atrofida qaysi tartibli hosilallarga ega?

a) $y' = x + y^{\frac{7}{3}}$, b) $y' = x|x| - y^2$,

c) $y'' = |x^3| + y^{\frac{5}{3}}$, d) $y''' = y - x\sqrt[3]{x}$,

e) $\frac{dx}{dt} = t + y, \frac{dy}{dt} = x + t^3|t|$, f) $\frac{dx}{dt} = y^2 + \sqrt[3]{t^4}, \frac{dy}{dt} = \sqrt[3]{x}$.

Yechim:

$ab = f(x, y)$ tenglama yechimlarining differensialligi haqidagi teoremani $(c0, V0$nuqta qo'shnisida qo'llaymiz, bu esa f funktsiya shu qo'shnilikda bo'lishini bildiradi.

k-tartibgacha uzluksiz qisman hosilalar, u holda $y(t_0) = \%$ boshlang‘ich shartli ko‘rsatilgan tenglamaning yechimi$(k + 1) -$ tartibgacha uzluksiz hosilalarga ega bo‘ladi.

inklyuziv. Xuddi shunday bayonot F vektor funksiya bo'lgan holat uchun ham to'g'ri keladi.

a) $f = (x, y) = x + yi$ ga egamiz.F funktsiya koordinata boshining qo'shnisida ikki marta uzluksiz differentsiallanishini ko'ramiz. Demak, yuqoridagi teorema tufayli$y = x + yi$ masala, $y(0) = 0$ koordinatalarning koordinatalarida uch marta differensiallanuvchi $y = y'$yechimga ega.

6) $f(x, y) = z|x|$ funksiyasi bo'lgani uchun $-y$ uzluksiz qisman hosilalari $2(x)$ $2 = -2y$ va kelib chiqishi qo'shnisida ikki marta farqlanmaydi (chunki funktsiya $2x = 0$uchun differentsiallanmaydi), u holda masalaning yechimi $y = y(x)$ $y' = xxl - y'$ $y(0) = 0$

ikki marta uzluksiz differensiallanadi.

c) f(x, y, y) funktsiya = || bo'lgani uchun + uzluksiz hosilalari A = 0 va I = 0 koordinata boshiga yaqin bo'lsa, u holda § 8.4 ning mavjudlik teoremasiga ko'ra masala.

y'' = '1+ yi, y(0) = 0, bu mahallada yagona uzluksiz y = y(x) yechimga ega.

Ushbu tenglamaga (x) ni qo'yib, biz o'ziga xoslikni olamiz y''(*) = le?] + uz(),

Bundan y'' funksiya uzluksiz ekanligi kelib chiqadi. Bu o'ziga xoslikni farqlab, biz quyidagilarni topamiz:

y'' (@) = 3'sgnz + yi (2)y' (*),

(bir)

(1) tenglamaning o'ng tomoni koordinata qo'shnisida uzluksiz bo'lganligi uchun va o'ng tomoni

(2) tenglamaning bir qismi y = 0 da uzluksiz bo'lsa, biz uzluksiz borligini kafolatlay olamiz.

ko'rib chiqilayotgan muammoning yechimining uchinchi hosilasi.

D) Chunki

Bilan

y'' = y – tx, y(0) = y'0) = y'' (0) = 0 muammosi to'rt marta uzluksiz differentsiallanuvchiga ega

y(x) yechim koordinata koordinatasida.

e) funksiyalar mos, z, y) = 1 + y, 12, 1, y) = 1 + t?lt| uzluksiz va uzluksizdir qisman hosilalar

afi afi afi afi

nuqtaga yaqin joyda (to, Fo, Vo), bu erda to = zo = 0 = 0, shuning uchun bu tenglamalar tizimi

bu mahallada uch marta uzluksiz differensiallanuvchi yechim (x(t), y(t)).

f) Bunda 1, 2, y) = y* + yn, 120, 1, y) = Vz funksiyalar qo‘shnilikda uzluksiz bo‘ladi.

nuqtalar (0, 0, 0), ammo, lotin beri

bu nuqtada uzluksiz bo'lsa, biz faqat berilgan tizimning x(t), y) yechimlarining uzluksiz differentsialligini kafolatlay olamiz.

§-7.
Aralash misollar.

108.(241) Misol

$y'^2 - y^2 = 0 \quad y'^2 = y^2 \quad y' = y^2 \quad \int \frac{(dy)}{y} = \pm \int dx \quad lny = \pm\text{x} + \text{lnC}$

$y = Ce^{\pm\text{x}} \quad y_1 = Ce^{\text{x}} \quad y_2 = Ce^{-\text{x}}$

109.(242) Misol

8y'³=27y $\;8y'^3 = 27y \quad 2^3y'^3 = 3^3y \quad 2y' = 3\sqrt[3]{y} \quad y' = \frac{3}{2}\sqrt[3]{y}$

$\frac{(dy)}{(dx)} = \frac{3}{2}\sqrt[3]{y} \quad \int \frac{(dy)}{\sqrt[3]{y}} = \frac{3}{2}\int dx \quad \left(\frac{3}{2}\right)\sqrt[3]{y^2} = \left(\frac{3}{2}\right)x + C \quad \sqrt[3]{y^2} = x + C \quad y^2 = (x + C)^2$

110.(243) Misol

$(y' + 1)^3 = 27(x + y)^2 \qquad z = x + y \quad z' = y' + 1 \quad z'^3 = 3^3 z^2$

$z' = 3z^{\frac{2}{3}} \quad \int \frac{(dz)}{z^{\frac{2}{3}}} = 3\int dx \quad 3z^{\frac{1}{3}} = 3x + C \quad (x + y)^{\frac{1}{3}} = x + C$

111.(244) Misol

$y^2(y'^2 + 1) = 1 \quad y'^2y^2 + y^2 = 1 \quad y'^2 = \frac{(1-y^2)}{y} \quad y' = \pm\frac{\sqrt{(1-y)}}{y} \quad \int \frac{(ydy)}{\sqrt{(1-y)}} = \pm\int dx \quad -\sqrt{(1-y)} = \pm\text{x} + \text{C} \quad y^2 + (\pm\text{x} + \text{C})^2 = 1$

112.(245) Misol

y'²-4y³=0 $\;y'^2 - 4y^3 = 0 \quad y'^2 = 4y^3 \quad y' = 2y^{\frac{3}{2}} \quad \int \frac{dy}{y^{\frac{3}{2}}} = 2\int dx \quad -\frac{2}{y^{\frac{1}{2}}} = 2\int dx \quad -\frac{1}{y^{\frac{1}{2}}} = 2x + C \quad y^{\frac{1}{2}}(x + C) = -1 \quad y(x + C)^2 = 1$

113.(246) Misol

$y'^2 = 4y^3(1 - y) \quad y' + \pm 2\left(y^3(1 - y)\right)^{\frac{1}{2}} \quad \pm\int \frac{dy}{(2y^3(1 - y))} = x + C$

$\pm\int \frac{dt}{sint^2} = x + C \quad \left\{y = sint \quad \left(0 < t < \frac{\pi}{2}\right)\right\} \quad y = \frac{1}{(1 + (x + C))} \quad y = 1$

114.(247) Misol

$xy'^2 = y \quad y' = p \quad dy = pdx \quad y = p^2x \quad dy = p^2dx + 2pxdp \quad pdx = p^2dx + 2pxdp \quad dx = pdx + 2xdp \quad (1 - p)dx = 2xdp \quad \int \frac{dx}{x} = 2\int \frac{dp}{1-p} \qquad \int \frac{dx}{x} =$

$-2\int \frac{d(p-1)}{p-1}$ $\quad lnCx = -2\ln(p-1) \quad Cx = (p-1)^2 \quad x = \frac{C}{(p-1)^2} \quad (p-1)^2 = \frac{C}{x} \quad p-1 = \pm\left(\frac{C}{x}\right)^{\frac{1}{2}} \quad p = 1 \pm \left(\frac{C}{x}\right)^{\frac{1}{2}} \quad y = \left(1+\frac{C}{x}\right)^2 x = x\left(1 \pm 2\left(\frac{C}{x}\right)^{\frac{1}{2}} + \frac{C}{x}\right)$

$x\left(1+\frac{C}{x}\right) \pm 2(\mathrm{Cx})^{\frac{1}{2}} = y \quad \left(y - x\left(1+\frac{C}{x}\right)\right)^2 = 4Cx \quad y^2 - 2x\left(1+\frac{C}{x}\right)y + x^2\left(1+\frac{C}{x}\right)^{\frac{1}{2}} = 4xC \quad (y-x)^2 = 2C(x-y) - C^2$

115.(248) Misol

$yy'^3 + x = 1 \quad yy'^3 = 1 - x \quad \int \sqrt[3]{y}dy = \int \sqrt[3]{(1-x)}dx \quad \sqrt[3]{y^4} = \sqrt[3]{(1-x)^4} + C$

116.(249) Misol

$y'^3 + y^2 = yy'(y'+1) \quad y' = y \quad y' = \pm\sqrt{y} \quad y = C_1e^x \quad y = \frac{1}{4}(x+C_2)^2 \quad y = 0$

117.(250) Misol

$4(1-y) = (3y-2)^2y'^2 \quad y' = \pm\frac{2(1-y)^{\frac{1}{2}}}{|3y-2|} \quad y \neq \frac{2}{3} \quad y \leq 1$

$\pm\frac{1}{2}\int \frac{|3y-2|}{(1-y)^{\frac{1}{2}}}dy = x + C \quad y^2\ (1-y) - (x+C) = 0$

118.(271) Misol

$y = y'^2 + 2y'^3$ Parametrni kiritib tenglamani yeching.

$y = a$ ni almashtiramiz

$dy = pdx \quad y = p^2 + 2p^3 \quad dy = (2p + 6p^2)dp \quad pdx = (2p + 6p^2)dp$

$dx = (2 + 6p)dp \quad \int dx = \int (2+6p)dp \quad x = 2p + 3p^3$

119.(272) Misol

$y = \ln(1 + y'^2) \quad y' = p \rightarrow \frac{dy}{dx} = p \rightarrow dx = \frac{dy}{p} \quad y = \ln(1+p^2)$

$dy = \frac{1}{(1+p^2)} * 2pdp \quad dx = \frac{dy}{p} = \frac{2pdp}{(1+p^2)p} \quad dx = \frac{2dp}{(1+p^2)} \quad \int dx = 2\int \frac{dp}{(1+p^2)} + C \quad x = 2arctgp + C$

$$Javob: \{ \quad x = 2arctgp + C \; y = \ln(1+p^2) \}$$

120.(273) Misol

$$(y'+1)^3 = (y'-y)^2 \quad x' = \frac{1}{p} \qquad y' = p \quad dy = pdx \quad (p+1)^3 = (p-y)^2$$

$$(p+1)^3 = p^2 - 2py + y^2 \qquad y^2 - 2py + p^2 - (p+1)^3 = 0$$

$$y = \frac{2p \pm \sqrt{4p^2 - 4p^2 + 4(p+1)^3}}{2} = \frac{2p \pm 2(p+1)^{\frac{3}{2}}}{2} = p \pm (p+1)^{\frac{3}{2}}$$

$(y' + 1)^3 = (y' - y)^2.$

$$x' = \frac{1}{p}$$

$y' = p$

$dy = pdx$

$(p+1)^3 = (p-y)^2$

$(p+1)^3 = p^2 - 2py + y^2$

$y^2 - 2py + p^2 - (p+1)^3 = 0$

$$y = \frac{2p \pm \sqrt{4p^2 - 4p^2 + 4(p+1)^3}}{2} = \frac{2p \pm 2(p+1)^{\frac{3}{2}}}{2} = p \pm (p+1)^{\frac{3}{2}}$$

1)

$y_1 = p + (p+1)^{\frac{3}{2}}$

$$dx = \frac{1}{p}dy$$

$$dx = \frac{dp}{p} + \frac{3}{2} \cdot \frac{\sqrt{p+1}}{p}dp$$

$$\frac{\sqrt{p+1}}{p}dp$$

$$2\int \frac{t^2 - 1 + 1}{t^2 - 1}dt = 2\int dt - 2\int \frac{dt}{1-t^2} = 2t - \frac{21}{2}ln\left|\frac{1+t}{1-t}\right|$$

$$x = ln\,p \pm (3t - \frac{3}{2}ln\frac{1+t}{1-t}) + C$$

$$x = ln\,p \pm (3\sqrt{p+1} - \frac{3}{2}ln\frac{1+\sqrt{p+1}}{1-\sqrt{p+1}}$$

121.(274) Misol

$y = (y' - 1)e^{y'}.$

$y' = p$

$$dx = \frac{1}{p}dy$$

$y = (p-1)e^p$

$$dx = \frac{1}{p}(e^p + pe^p - e^p)dp$$
$$dx = e^p dp$$
$$x = e^p + C$$
$$javob: \begin{cases} x = e^p + e \\ y = (p-1)e^p \\ y = -1 \end{cases}$$

122.(275) MIsol

$$y'^4 - y'^2 = y^2.$$
$$y' = p \rightarrow \frac{dy}{dx} = p$$
$$p^4 - p^2 = y^2$$
$$y = \pm\sqrt{p^4 - p^2}$$
$$\frac{dy}{dx} = \pm\frac{4p^4 - 2p}{2\sqrt{p^4 - p^2}} * \frac{dp}{dx}$$
$$p = \pm\frac{2p^3 - p}{p\sqrt{p^2 - 1}} * \frac{dp}{dx}$$
$$p = \pm\frac{2p^2 - 1}{\sqrt{p^2 - 1}} * \frac{dp}{dx}$$
$$dx = \pm\frac{2p^2 - 1}{p\sqrt{p^2 - 1}} * \frac{dp}{dx}$$
$$x = \pm\int\frac{2p^2 - 1}{p\sqrt{p^2 - 1}}dp = \pm(2\sqrt{p^2 - 1} - \int\frac{dp}{p\sqrt{p^2 - 1}}) + C$$
$$\int\frac{dp}{p\sqrt{p^2 - 1}} = \int\frac{dp}{p\sqrt{1 - \frac{1}{p^2}}} = \begin{bmatrix} \frac{1}{p} = t \\ dt = -\frac{1}{p^2}dp \\ dp = -p^2 dt \end{bmatrix} = -\int\frac{dt}{\sqrt{1 - t^2}} = -\arcsin t$$
$$= -\arcsin\frac{1}{p}$$
$$javob: \begin{cases} x = \pm(2\sqrt{p^2 - 1} + \arcsin\frac{1}{p}) + C \\ y = \pm p\sqrt{p^2 - 1} \end{cases}$$

$$y'^2 - y' = y^2.$$

$y' = p \to \frac{dy}{dx} = p \to dx = \frac{dy}{p}$

$p^2 - p^3 = y^2$

$y = \pm\sqrt{p^2 - p^3}$

$dy = \pm\frac{1}{2\sqrt{p^2 - p^3 * p}} * (2p - 3p)dp$

$dx = \frac{dy}{p} = \pm\frac{p(2-3p)}{2\sqrt{p^2 - p^3 * p}}dp$

$dx = \pm\frac{2-3p}{2\sqrt{p^2 - p^3}}dp$

$dx = \pm\left(\frac{1}{p\sqrt{1-p}} - \frac{3}{2\sqrt{1-p}}\right)dp$

123.(276) Misol

$\int dx = \pm\int\left(\frac{1}{p\sqrt{1-p}} - \frac{3}{2\sqrt{1-p}}\right)dp + C$

$\int\frac{1}{n\sqrt{a-bn}}dn = \frac{1}{\sqrt{a}}ln\left|\frac{\sqrt{a}-\sqrt{a-bn}}{\sqrt{a}+\sqrt{a-bn}}\right|$

$\int\frac{1}{p\sqrt{1-p}}dp = ln\left|\frac{\sqrt{1}-\sqrt{1-p}}{\sqrt{1}+\sqrt{1-p}}\right|$

$\int\frac{3}{2\sqrt{1-p}}dp = \frac{3}{2} * 2\sqrt{1-p} = 3\sqrt{1-p}$

$x = \pm\left(ln\left|\frac{1-\sqrt{1-p}}{1+\sqrt{1-p}}\right| - 3\sqrt{1-p}\right) + C$

$javob: \begin{cases} x = \pm\left(ln\left|\frac{1-\sqrt{1-p}}{1+\sqrt{1-p}}\right| - 3\sqrt{1-p}\right) + C \\ y = \pm\sqrt{p^2 - pp^3} \end{cases}$

124.(277) Misol

$$y'^4 = 2yy' + y^2.$$

$y' = p \to \frac{dy}{dx} = p \to dx = \frac{dx}{p}$

$y^2 = p^4 - 2yp$

$y^2 + 2py - p^4 = 0$

$D = 4p^2 + 4p^4 = 4p^2(1 - p^2)$

$\sqrt{D} = 2p\sqrt{1 + p^2}$

$y = -p \pm p\sqrt{1 + p^2}$

$dy = \left(-1 \pm \frac{1}{2\sqrt{p^2 + p^4}} * (2p + 4p^3)\right)dp$

$$dx = \frac{dy}{p} = \frac{\left(-1 \pm \frac{1}{2\sqrt{p^2+p^4}} * (2p+4p^3)\right)dp}{p}$$

$$x = -\int \frac{dp}{p} \pm \int \frac{2p(1+2p^2)}{2p\sqrt{p^2+p^4}}dp + C = \int \frac{dp}{p} \pm \int \frac{1+2p^2}{\sqrt{p^2+p^4}}dp + C$$

$$\int \frac{1+2p^2}{\sqrt{p^2+p^4}}dp = \int \frac{dp}{\sqrt{p^2+p^4}} + \int \frac{2p^2}{\sqrt{p^2+p^4}}dp$$

$$= \int \frac{dp}{p\sqrt{1+p^2}} + \int \frac{2p^2}{p\sqrt{1+p^2}}dp = \int \frac{dp}{p\sqrt{1+p^2}} + \int \frac{2p^2}{p\sqrt{1+p^2}}dp =$$

$$\int \frac{dp}{p\sqrt{1+p^2}} = \int \frac{dp}{p^2\sqrt{\frac{1}{p^2}+1}} \begin{bmatrix} \frac{1}{p} = t \\ dt = -\frac{1}{p^2}dp \\ dp = -p^2 dt \end{bmatrix} = \int \frac{-p^2 dt}{p^2\sqrt{t^2+1}} = -\int \frac{dt}{\sqrt{t^2+1}}$$

$$= -\ln\left|t + \sqrt{t^2+1}\right| = -\ln\left|\frac{1}{p} + \sqrt{\frac{1}{p^2}+1}\right| = -\ln\left|1 + \sqrt{1+p^2}\right|$$

$$\int \frac{2p}{\sqrt{1+p^2}}dp = \begin{bmatrix} 1+p^2 = t \\ dt = 2pdp \\ dp = \frac{dt}{2p} \end{bmatrix} = \int \frac{2p}{\sqrt{t}} * \frac{dt}{2p} = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} = 2\sqrt{1+p^2}$$

$$x = -\ln|p| \pm \left(\ln\left|\frac{1-\sqrt{1+p^2}}{-p}\right| + 2\sqrt{1+p}\right) + C$$

$$javob: \begin{cases} \left\{x = -\ln|p| \pm \left(\ln\left|1+\sqrt{1+p^2} + 2\sqrt{1+p^2}\right|\right) + C \right. \\ y = -p \pm p\sqrt{1+p^2} \end{cases}$$

125.(278) Misol

$$y'^2 - 2xyy' = x^2 - 4y$$

$$y' = p$$

$$x' = \frac{1}{p}$$

$$p^2 2xp = x^2 - 4y$$

$$y = \frac{x^2 + 2xp - p^2}{4}$$

$$dx = \frac{1}{4p}[(2x+2p)dx + (2x-2p)dp]$$

$$dx = \frac{x}{2p}dx + \frac{1}{2}dx + \frac{x}{2p}dp - \frac{1}{2}dp$$

$$\frac{x}{2p}dx + \frac{x}{2p}dp = \frac{1}{2}dx + \frac{1}{2}dp$$
$$\frac{x}{2p}(dp + dx) = \frac{1}{2}(dp + dx)$$
$$\frac{x}{p} = 1$$
$$x = p$$
$$y = \frac{x^2}{2}$$
$d)$
$$x = -p + C$$
$$p = C - x$$
$$y = \frac{1}{4}[x^2 - (C - x)^2 + 2x(C - x)]$$

126.(279) Misol

$$5y + y'^2 = x(x + y').$$
$$y' = p$$
$$dx = \frac{1}{p}dy$$
$$5y + \mathrm{p}^2 = x^2 + xp$$
$$y = \frac{x^2 - p^2 xp}{5}$$
$$dx = \frac{1}{5p}[(2x + p)dx + (-2p + x)dp]$$
$$5pdx = 2xdx + pdx - 2pdp + xdp$$
$$x \neq 0$$
$$5\frac{p}{x}dx - 2dz - \frac{p}{x}dx + 2\frac{p}{x}dp - dp = 0$$
$$4\frac{p}{x} - 2 + 2\frac{p}{x}\frac{dp}{dx} - \frac{dp}{dx} = 0$$
$$\frac{p}{x} = z$$
$$p' = z'x + z$$
$$4z - 2 + 2zp\frac{1}{x} - p\frac{1}{x} = 0$$
$$4z - 2 + 2z(z + z'x) - z'x - z = 0$$
$$3z - 2 + 2z^2 + 2zx \cdot z' - z'x = 0$$
$$2z^2 + 3z - 2 = x(1 - 2z)z'$$
$$\frac{2z^2 + 3z - 2}{1 - 2z} = x\frac{dz}{dx}$$
$$\frac{dx}{x} = \frac{(1 - 2z)dz}{2z^2 + 3z - 2}$$

$$2z^2+3z-2=0$$
$$D=9+16=25$$
$$z=\frac{-3\pm 5}{4}$$
$$z_1=-2$$
$$z_2=\frac{1}{2}$$
$$2z^2+3z-2=2(z+2)\left(z-\frac{1}{2}\right)=(z+2)(2z-1)$$
$$\frac{dx}{x}=-\frac{dz}{z+2}$$
$$ln\,x=-ln|z+2|+ln\,C$$
$$x=\frac{C}{z+2}=\frac{C}{\frac{p}{x}+2x}$$
$$x\left(\frac{p}{x}+2\right)=C$$
$$p+2x=C$$
$$5y=\left(C-\frac{p}{2}\right)^2-p^2+\left(c-\frac{p}{2}\right)p$$
$$5y=C^2-Cp+\frac{p^2}{4}-p^2+Cp-\frac{p^2}{2}$$
$$5y=-\frac{5}{4}p^2+C^3$$
$$javob:\begin{cases}x=-\frac{p}{2}+C\\5y=C^2-\frac{5}{4}p^2\end{cases}$$

127.(280) Misol

$$x^2y'^2=xyy'+1$$
$$y'=p\rightarrow\frac{dy}{dx}=p\rightarrow dx=\frac{dy}{p}$$
$$x^2p^2=xyp+1$$
$$xyp=x^2p^2-1$$
$$y=xp-\frac{1}{xp}$$
$$p=p+xp'+\frac{1}{x^2p}+\frac{p'}{xp^2}$$
$$0=p'\left(x+\frac{1}{xp^2}\right)+\frac{1}{x^2p}$$
$$-\frac{1}{x^2p}=p'\left(x+\frac{1}{xp^2}\right)$$
$$p'=-\frac{1}{x^2p\left(x+\frac{1}{xp^2}\right)}$$

Teskari funksiyaga o'tamiz

$(p(x) \to x(p), p' = 1|x')$

$$\frac{1}{x'} = -\frac{1}{x^2 p\left(x + \frac{1}{xp^2}\right)}$$

$$x' = -x^3 p - \frac{x}{p}$$

$$\frac{x'}{x^3} + \frac{1}{px^2} = -p$$

$z = \frac{1}{x^2}$

Biz Bernulli tenglamasini ishlab chiqamiz

$$z' - \frac{2}{p} z = 2p$$

$$\frac{dz}{dp} - \frac{2}{p} z = 0$$

$$\frac{dz}{z} = \frac{2dp}{p}$$

$$ln|z| = 2\, ln|p| + ln\, C$$

$$z = Cp^2$$

$$2pC + C' p^2 - \frac{2}{p} * Cp^2 = 2p$$

$$C' p^2 = 2p$$

$$C' = \frac{2}{p}$$

$$C = 2\, ln|p| + ln\, C_1$$

$$z = p^2 (2\, ln|p| + ln\, C_1) = 2p^2\, ln\, C\, p$$

$$\frac{1}{x^2} = 2p^2\, ln\, C\, p$$

$$x^2 = \frac{1}{2p^2\, ln\, C\, p}$$

$$x = \pm \frac{1}{p\sqrt{2p^2\, ln\, C\, p}}$$

Topilgan x ni tenglamaga almashtiring

$$y = xp - \frac{1}{xp} = p * \left(\pm\frac{1}{p\sqrt{2\ln Cp}}\right) - \frac{1}{p\left(\pm\frac{1}{p\sqrt{2\ln Cp}}\right)} =$$

$$= \pm\left(\frac{1}{p\sqrt{2\ln C\,p}} - \sqrt{2\ln C\,p}\right)$$

$javob$:

$$\begin{cases} x = \pm\frac{1}{p\sqrt{2\ln Cp}} \\ y = \pm\left(\frac{1}{p\sqrt{2\ln Cp}} - \sqrt{2\ln C\,p}\right) \end{cases}$$

128.(281) Misol

Tenglamalarni parametr kiritish usulida yeching

$y'^3 + y^2 = xyy'$

Yechilishi:

$$y'^3 + y^2 = xyy'$$

$$y' = p \to \frac{dy}{dx} = p \to dy = pdx$$

$$p^3 + y^2 = xyp$$

$$x = \frac{p^2}{y} + \frac{y}{p}$$

$$dx = \frac{2p}{y}dp - \frac{p^2}{y^2}dy - \frac{y}{p^2}dp + \frac{1}{p}dy$$

$$dy = pdx = p\left(\frac{2p}{y}dp - \frac{p^2}{y^2}dy - \frac{y}{p^2}dp + \frac{1}{p}dy\right)$$

$$dy = \frac{2p^2}{y}dp - \frac{p^3}{y^2}dy - \frac{y}{p}dp + dy$$

$$\frac{2p^2}{y}dp - \frac{p^3}{y^2}dy - \frac{y}{p}dp = 0$$

$$\left(\frac{2p^2}{y} - \frac{p^3}{y^2}\right)\frac{dy}{dp} - \frac{y}{p} = 0$$

$$\frac{p^3}{y^2}y' = \frac{2p^3 - y^2}{yp}$$

$$y' = \frac{2p^3y^2 - y^4}{yp^4}$$

$$y' - \frac{2y}{p} = -\frac{y^3}{p^4}$$

$$\frac{y'}{y^3} - \frac{2}{py^2} = -\frac{1}{p^4}$$

$$z = \frac{1}{y^2}$$

$$z' + \frac{4}{p} z = \frac{2}{p^4}$$
$$z' + \frac{4}{p} z = 0$$
$$z' = -\frac{4}{p} z$$
$$\frac{dz}{dp} = -\frac{4z}{p}$$
$$\frac{dz}{z} = -\frac{4dp}{p}$$
$$ln|z| = ln|p| + ln\, C$$
$$z = \frac{C}{p^4}$$
$$-4\frac{C}{p^5} + \frac{C'}{p^4} + \frac{4}{p} \cdot \frac{C}{p^4} = \frac{2}{p^4}$$
$$\frac{C'}{p^4} = \frac{2}{p^4}$$
$$C' = \frac{2p^4}{p^4} = 2$$
$$C = 2p + C_1$$
$$z = \frac{2p + C_1}{p^4}$$
$$\frac{2p + C_1}{p^4} = \frac{1}{y^2}$$
$$y^2 = \frac{p^4}{2p + C_1}$$
$$y^2 = \pm \frac{p^4}{2p + C_1}$$
$$y = \pm \frac{p^4}{2p + C_1}$$
$$Javob:$$
$$y = \pm \frac{p^4}{2p + C_1}$$
$$x = \frac{p^2}{y} + \frac{y}{p}$$

129.(282) Misol

Tenglamalarni parametr kiritish usulida yeching

$$2xy' - y = y'\, ln\, y\, y'$$

Yechilishi:

$$2xy' - y = y'\, ln\, y\, y'$$

$$y' = p \to \frac{dy}{dx} = p \to \frac{dx}{dy} = \frac{1}{p}$$

$$2xp - y = p\, ln\, y\, p$$

$$2xp = y + p\, ln\, y\, p$$

$$x = \frac{y}{2p} + \frac{1}{2} ln\, y\, p$$

$$x = \frac{y}{2p} + \frac{1}{2}(ln\, y + ln\, p)$$

$$\frac{dx}{dy} = \frac{1}{2p} - \frac{y}{2p^2}\frac{dp}{dy} + \frac{1}{2}\left(\frac{1}{y} + \frac{1}{p}\frac{dp}{dy}\right)$$

$$\frac{1}{p} = \frac{1}{2p} - \frac{yp'}{2p^2} + \frac{1}{2y} + \frac{p'}{2p}$$

$$\frac{p'(y-p)}{2p^2} = \frac{p-y}{2yp}$$

$$a)\, y = p$$

$$x = \frac{y}{2p} + \frac{1}{2} ln\, y\, p = \frac{1}{2} + ln|y|$$

$$2x = 1 + 2\, ln|y|$$

$$b)$$

$$\frac{p'}{p} = -\frac{1}{y}$$

$$\frac{dp}{p} = -\frac{dy}{y}$$

$$\int \frac{dp}{p} = \int \frac{dy}{y}$$

$$ln|p| = -\, ln|y| + ln\, C$$

$$p = \frac{C}{y}$$

$$x = \frac{y^2}{2C} + \frac{1}{2} ln\, C$$

$$2Cx = y^2 + C\, ln\, C$$

$Javob$:

$$2x = 1 + 2\, ln|y|$$

$2Cx = y^2 + C\, ln\, C$

130.(283) Misol

Tenglamalarni parametr kiritish usulida yeching

$y' = e^{\frac{xy'\prime}{y}}$

Yechilishi:

$$y' = e^{\frac{xy'}{y}}$$

$$y' = p$$

$$p = e^{\frac{xp}{y}}$$

$$\frac{xp}{y} = ln\,p$$
$$x = \frac{y\,ln\,p}{p} = u$$
$$dx = [u'_y dy + u'_p dp]$$
$$dy = pdx$$
$$d\left(\frac{u}{v}\right) = \frac{udv - vdu}{v^2}$$
$$p^{-1}\,ln\,p = \left[-\frac{ln\,p}{p^2} + \frac{1}{p}\right]$$
$$dy = ln\,p\,dy + p\,\frac{1 - ln\,p}{p}\,dp$$
$$\int \frac{dy}{y} = \int \frac{dp}{p}$$
$$ln\,y = ln\,C\,p$$
$$y = Cp$$
$$p = \frac{y}{C}$$
$$x = C\,ln\,p$$
$$x = C\,ln\frac{y}{C}$$
$Javob$:
$$xC = ln\,C\,y$$
$y = ex$

131.(284) Misol

Tenglamalarni parametr kiritish usulida yeching

$y = xy' - x^2y'^3$

Yechilishi:

$$y = xy' - x^2y'^3$$
$$y' = p \rightarrow \frac{dy}{dx} = p$$
$$y = xp - x^2p^3$$
$$\frac{dy}{dx} = p + x\frac{dp}{dx} - 2xp^3 - 3x^2p^2\frac{dp}{dx}$$
$$p = p + xp' - 2xp^3 - 3x^2p^2p'$$
$$p' = \frac{2xp^3}{x - 3x^2p^2} = \frac{2p^3}{1 - 3xp^2} =$$
$$a)p = 0$$
$$y = xp - x^2p^3 = 0$$
$$b)p' = \frac{1}{x'}$$
$$\frac{1}{x'} = \frac{2p^3}{1 - 3xp^2}$$

$$x' = \frac{1-3xp^2}{2p^3}$$
$$x' = \frac{1}{2p^3} - \frac{3x}{2p}$$
$$x' + \frac{3x}{2p} = \frac{1}{2p^3}$$
$$\frac{dx}{dp} = -\frac{3x}{2p}$$
$$\frac{dx}{x} = -\frac{3}{2}\frac{dp}{p}$$
$$ln|x| = -\frac{3}{2}ln|p| + ln\,C$$
$$x = Cp^{-\frac{3}{2}}$$
$$-\frac{3}{2}Cp^{-\frac{5}{2}} + C'p^{-\frac{3}{2}} + \frac{3}{2p}Cp^{-\frac{3}{2}} = \frac{1}{2p^3}$$
$$C'p^{-\frac{3}{2}} = \frac{1}{2p^3}$$
$$C' = \frac{1}{2p^{\frac{3}{2}}}$$
$$C = \frac{1}{2}\cdot\left(p^{-\frac{1}{2}}\cdot(-2)\right) = -\frac{1}{\sqrt{p}} + C_1$$
$$x = -\frac{1}{p^2} + \frac{C_1}{\sqrt{p^3}}$$
$$xp^2 = C_1\sqrt{p} - 1$$
$Javob$:
$$y = 0$$
$xp^2 = C_1\sqrt{p} - 1$

132.(285) Misol

Tenglamalarni parametr kiritish usulida yeching

$$y = 2xy' + y^2y'^3$$

Yechilishi:

$$y = 2xy' + y^2y'^3$$
$$y' = p \to \frac{dy}{dx} = p \to \frac{dx}{dy} = \frac{1}{p}$$
$$2xp = y - y^2p^3$$
$$x = \frac{y}{2p} - \frac{y^2p^2}{2}$$
$$\frac{dx}{dy} = \frac{1}{2p} - \frac{y}{2p^2}\frac{dp}{dy} - yp^2 - y^2p\frac{dp}{dy}$$

$$\frac{1}{p}=\frac{1}{2p}-\frac{yp'}{2p^2}-yp^2-y^2pp'$$
$$\frac{2-1+2p^3y}{2p}=-\left(\frac{yp'+2y^2p^3p'}{2p^2}\right)$$
$$\frac{1+2p^3y}{2p}=-\left(\frac{p'y(1+2p^3y)}{2p^2}\right)$$
$a)1+2p^3y=0$
$$x=\frac{y}{2p}-\frac{y^2p^2}{2}=\left(-\frac{1}{2p^3}\right)\cdot\frac{1}{2p}-\left(-\frac{1}{2p^3}\right)\frac{p^2}{2}=\frac{1}{4p^4}-\frac{1}{8p^4}=-\frac{3}{8p^4}$$
$$y=-\frac{1}{2p^3},y^4=\frac{1}{2^4y^{12}},p^{12}=\frac{1}{2^4y^4}$$
$$x=-\frac{3}{8p^4},x^3=-\frac{27}{2^9p^{12}},x^3=-\frac{27}{2^9}\cdot 2^4y^4$$
$-32x^3=27y^4$
$b)$
$$\frac{1}{2p}=-\frac{p'y}{2p^2}$$
$$\frac{1}{2p}=-\frac{y}{2p^2}\frac{dp}{dy}$$
$$\frac{dy}{y}=-\frac{dp}{p}$$
$ln|y|=-ln|p|+ln\,C$
$$y=\frac{C}{p}$$
$$x=\frac{1}{2p}\cdot\frac{C}{p}-\frac{p^2}{2}\cdot\left(\frac{C}{p}\right)^2=\frac{C-C^2p^2}{2p^2}$$
Javob:
$-32x^3=27y^4$
$$y=\frac{C}{p}$$
$x=\frac{C-C^2p^2}{2p^2}$

133.(286) Misol

Tenglamalarni parametr kiritish usulida yeching

$y(y-2xy')^3=y'^2$

Yechilishi:

$$y(y-2xy')^3=y'^2$$
$$y'=p\to\frac{dy}{dx}=p\to\frac{dx}{dy}=\frac{1}{p}$$
$$y(y-2xp)^3=p^2$$
$$\sqrt[3]{y(y-2xp)^3}=\sqrt[3]{p^2}$$

$$(y-2xp)y^{\frac{1}{3}}=p^{\frac{2}{3}}$$
$$y^{\frac{4}{3}}-2xpy^{\frac{1}{3}}=p^{\frac{2}{3}}$$
$$2xpy^{\frac{1}{3}}=y^{\frac{4}{3}}-p^{\frac{2}{3}}$$
$$x=\frac{y^{\frac{4}{3}}-p^{\frac{2}{3}}}{2py^{\frac{1}{3}}}$$

$x=\frac{y}{2p}-\frac{1}{2(py)^{\frac{1}{3}}}$

$$\frac{dx}{dy}=\frac{1}{2p}-\frac{y}{2p^2}\frac{dp}{dy}+\frac{p}{6(py)^{\frac{4}{3}}}+\frac{y}{6(py)^{\frac{4}{3}}}\frac{dp}{dy}$$

$\frac{1}{p}=\frac{1}{2p}-\frac{yp\prime}{2p^2}+\frac{p+yp\prime}{6(py)^{\frac{4}{3}}}$

$$\frac{1}{2p}=\frac{(p+yp')p^{\frac{2}{3}}-3y^{\frac{4}{2}}\cdot yp'}{6p^2y^{\frac{4}{3}}}$$
$$\frac{1}{2p}-\frac{1}{6p^{\frac{1}{3}}y^{\frac{4}{3}}}=\frac{yp'\left(p^{\frac{2}{3}}-3y^{\frac{4}{3}}\right)}{6p^2y^{\frac{4}{3}}}$$
$$\frac{p^{\frac{1}{3}}\left(3y^{\frac{4}{3}}-p^{\frac{2}{3}}\right)}{6p^{\frac{4}{3}}y^{\frac{4}{3}}}=\frac{yp'\left(p^{\frac{2}{3}}-3y^{\frac{4}{3}}\right)}{6p^2y^{\frac{4}{3}}}$$

$a)$

$$3y^{\frac{4}{3}}-p^{\frac{2}{3}}=0$$
$$p^2=27y^4$$
$$p=\pm y^2\sqrt{27}$$
$$x=\frac{y}{2p}-\frac{1}{2(py)^{\frac{1}{3}}}=\pm\left(\frac{y}{2y^2\sqrt{27}}-\frac{1}{2\left(yy^2\sqrt{27}\right)^{\frac{1}{3}}}\right)=\pm\left(\frac{1}{y\sqrt{27}}\right)$$

$x^2=\frac{1}{27y^2}\rightarrow 27x^2y^2=1$

$b)$

$$\frac{p^{\frac{1}{3}}}{6p^{\frac{4}{3}}y^{\frac{4}{3}}}=-\frac{yp'}{6p^2y^{\frac{4}{3}}}$$
$$p^{\frac{1}{3}}=-\frac{yp'}{p^{\frac{2}{3}}}$$
$$p=-y\frac{dp}{dy}$$

$$\frac{dp}{p} = -\frac{dy}{y}$$
$$\int \frac{dp}{p} = -\int \frac{dy}{y}$$
$$ln|p| = -ln|y| + ln|C|$$
$$p = \frac{C}{y}$$
$$x = \frac{y}{2p} - \frac{1}{2(py)^{\frac{1}{3}}} = \frac{y^2}{2C} - \frac{1}{2C^{\frac{1}{3}}} = \frac{y^2 - C^{\frac{2}{3}}}{2C}$$
$$2Cx = y^2 - C^{\frac{2}{3}}$$
$$C = C^{\frac{2}{3}}$$
$$2C^3x = y^2 - C^2$$
$Javob$:
$$27x^2y^2 = 1$$
$2C^3x = y^2 - C^2$

134.(287) Misol

Lagranj va Klero tenglamasini yeching
$y = xy' - y'^2$
Yechilishi:

$$y = xy' - y'^2 = 0$$
$$y = xy' - y'^2$$
1)
$$y = Cx + C^2$$
2)
$$y - Cx + C^2 = 0$$
$$\frac{\partial F}{\partial \mathrm{C}} = -x + 2C$$
$$c = \frac{x}{2}$$
$$y - \frac{x^2}{2} + \frac{x^2}{4} = 0$$
$$y = \frac{x^2}{4}$$

135.(288) Misol
Lagranj va Klero tenglamasini yeching
$y + xy' = 4\sqrt{y'}$
Yechilishi:
$$y + xy' = 4\sqrt{y'}$$

$$y = -xy' + 4(y')^{\frac{1}{4}}$$
$$y' = p$$
$$y = -xp + 4p^{\frac{1}{4}}$$
$$y' = p = -p - xp' + p^{-\frac{3}{4}}p'$$
$$p = \left(\frac{1}{p^{\frac{3}{4}}} - x\right)\frac{p'}{2}$$
1)
$$p' \neq 0$$
$$x_2'2p = \frac{1}{p^{\frac{3}{4}}} - x$$
$$x' + \frac{x}{2p} = \frac{1}{2p^{\frac{7}{4}}}$$
$a)$
$$x'_p + \frac{x}{2p} = 0$$
$$\frac{dx}{x} = -\frac{dp}{2p}$$
$$\ln x = -\frac{1}{2}\ln p + \ln C$$
$$x = \frac{C}{\sqrt{p}}$$
$b)$
$$\frac{C'_p}{\sqrt{p}} - \frac{C}{2p^{\frac{3}{2}}} + \frac{C}{2p^{\frac{3}{2}}} = \frac{1}{2p^{\frac{7}{4}}}$$
$$C'_p = \frac{1}{2}\frac{p^{\frac{2}{4}}}{p^{\frac{7}{4}}} = \frac{1}{2}\frac{1}{p^{\frac{5}{4}}}$$
$$dC = \frac{1}{2}\frac{dp}{p^{\frac{5}{4}}}$$
$$C = \frac{1}{2}\frac{p^{-\frac{5}{4}+1}}{-\frac{5}{4}+1} + C_1$$
$$C = -\frac{1}{2}4p^{-\frac{1}{4}} + C_1 = -2p^{-\frac{1}{4}} + C_1$$

136.(289) Misol

Lagranj va Klero tenglamasini yeching

$y = 2xy' - 4y'^3$

Yechiliishi:

$$y = 2xy' - 4y'^3$$
$$x'_p - \frac{2x}{p - 2p} = \frac{(-4p^3)'}{p - 2p}$$
$$x'_p + \frac{2x}{p} = \frac{12p^2}{p}$$
1)
$$x'_p = -\frac{2x}{p}$$
$$\frac{dx}{2x} = -\frac{dp}{p}$$
$$\frac{1}{2} ln\, x = -ln\, C\, p$$
$$x = (Cp)^{-2}$$
2)
$$-2C^{-3}p^{-2}C'_p = 12p$$
$$-C^{-3}dC = Cp^3dp$$
$$C^{-2} = 3p^4 + C_1$$
$$x = (3p^4 + C_1)p^{-2} = 3p^2 + C_1p^{-2}$$
$$dy = pdx$$
$$dy = p(6p - 2Cp^{-3})dp$$
$$dy = 6p^2dp - 2Cp^{-2}dp$$
$$y = 2p^3 + 2C\frac{1}{p}$$
$$p - 2p = 0$$
$$p = 0$$
$$y = x \cdot 2 \cdot 0 + 4 \cdot 0$$
$$y = 0$$
$Javob$:
$$x = 3p^2 + Cp^{-2}$$
$$y = 2p^3 + 2Cp^{-1}$$
$y = 0$

137.(290) Misol

Lagranj va Klero tenglamasini yeching

$$y = xy' - (2 + y')$$
$$y' = p \to \frac{dy}{dx} = p$$
$$y = xp - 2 - p$$
$$\frac{dy}{dx} = p + x\frac{dp}{dx} - \frac{dp}{dx}$$
$$p = p + x\frac{dp}{dx} - \frac{dp}{dx}$$

$$\frac{dp}{dx}(x-1)=0$$
$$x-1=0\rightarrow x=1$$
$$\frac{dp}{dx}=0\rightarrow p=const$$
$$y=Cx-2-C$$
$Javob$:

$$y=Cx-2-C$$

$$y'^3+y^2=xyy'$$
$$y'=p\rightarrow\frac{dy}{dx}=p\rightarrow dy=pdx$$
$$p^3+y^2=xyp$$
$$x=\frac{p^2}{y}+\frac{y}{p}$$
$$dx=\frac{2p}{y}dp-\frac{p^2}{y^2}dy-\frac{y}{p^2}dp+\frac{1}{p}dy$$
$$dy=pdx=p\left(\frac{2p}{y}dp-\frac{p^2}{y^2}dy-\frac{y}{p^2}dp+\frac{1}{p}dy\right)$$
$$dy=\frac{2p^2}{y}dp-\frac{p^3}{y^2}dy-\frac{y}{p}dp+dy$$
$$\frac{2p^2}{y}dp-\frac{p^3}{y^2}dy-\frac{y}{p}dp=0$$
$$\left(\frac{2p^2}{y}-\frac{p^3}{y^2}\right)\frac{dy}{dp}-\frac{y}{p}=0$$
$$\frac{p^3}{y^2}y'=\frac{2p^3-y^2}{yp}$$
$$y'=\frac{2p^3y^2-y^4}{yp^4}$$
$$y'-\frac{2y}{p}=-\frac{y^3}{p^4}$$
$$\frac{y'}{y^3}-\frac{2}{py^2}=-\frac{1}{p^4}$$
$$z=\frac{1}{y^2}$$
$$z'+\frac{4}{p}z=\frac{2}{p^4}$$
$$z'+\frac{4}{p}z=0$$

$$z' = -\frac{4}{p}z$$
$$\frac{dz}{dp} = -\frac{4z}{p}$$
$$\frac{dz}{z} = -\frac{4dp}{p}$$
$$ln|z| = -4\,ln|p| + ln\,C$$
$$z = \frac{C}{p^4}$$
$$-4\frac{C}{p^5} + \frac{C'}{p^4} + \frac{4}{p}\cdot\frac{C}{p^4} = \frac{2}{p^4}$$
$$\frac{C'}{p^4} = \frac{2}{p^4}$$
$$C' = \frac{2p^4}{p^4} = 2$$
$$C = 2p + C_1$$
$$z = \frac{2p + C_1}{p^4}$$
$$\frac{2p + C_1}{p^4} = \frac{1}{y^2}$$
$$y^2 = \frac{p^4}{2p + C_1}$$
$$y = \pm\frac{p^4}{2p + C_1}$$
$Javob$:
$$y = \frac{p^4}{2p + C_1}$$
$$x = \frac{p^2}{y} + \frac{y}{p}$$

138.(291) Misol

Lagranj va Klero tenglamasini yeching

$y'^3 = 3(xy' - y)$

Yechilishi:

$$y'^3 = 3(xy' - y)$$
$$\frac{y'}{3} = xy' - y$$
$$y - xy' = -\frac{1}{3}y'^3$$
$$y = xy' - \frac{1}{3}y'^3 \rightarrow Klerotenglamasi$$

$$y = cx + \frac{1}{3}c^3 \Rightarrow c^3 = 3(cx - y)$$
$$x + c^2 = 0$$
$$x = -c^2$$
$$y = -cc^2 + \frac{1}{3}c^3$$
$$y = -\frac{2}{3}c^3$$
$$y = \frac{2}{3}c^3 \to c^3 = \frac{3}{2}y \to c = \sqrt[3]{\frac{3y}{2}}$$
$$x = -\left(\frac{3y}{2}\right)^{\frac{2}{3}}$$
$$x^3 = \frac{9y^2}{4}$$
$$4x^3 = 9y^2$$
$Javob$:
$$c^3 = 3(cx - y)$$
$4x^3 = 9y^2$

139.(292) Misol

Lagranj va Klero tenglamasini yeching

$y = xy'^2 - 2y'^3$

Yechilishi:

$$y = xy'^2 - 2y'^3$$

$$y' = p$$
$$y = xp^2 - 2p^3$$
$$pdx = d(xp^2 - 2p^3)$$
$$p\big((p-1)dx + 2(x - 3p)dp\big) = 0$$
$$p = 1, p = 0$$
$$(p-1)\frac{dx}{dp} + 2x = 3p$$
$$x = \frac{c}{(p-1)^2} + 2p + 1$$
$$y = 0, y = x - 2,$$
$$x = \frac{c}{(p-1)^2} + 2p + 1$$
$y = \frac{cp^2}{(p-1)^2} + p^2$

140.(293) Misol

Lagranj va Klero tenglamasini yeching

$xy' - y = ln\, y'$

Yechilishi:

$$xy' - y = \ln y'$$
$$y = xy' - \ln y' \rightarrow Klerotenglamasi$$
$$y = cx - \ln c$$
$$x - \frac{1}{c} = 0$$
$$x = \frac{1}{c} \Rightarrow c = \frac{1}{x}$$
$$y = -\ln\frac{1}{x} + 1 = 1 + \ln x$$

$Javob$:

$$y = cx - \ln c$$

$y = 1 + \ln x$

141.(294) Misol

Lagranj va Klero tenglamasini yeching

$y = xy'(y' + 2)$

Yechilishi:

$$y = xy'(y' + 2)$$
$$y = xy'^2 + 2xy'$$
$$y' = y'^2 + 2xy'y'' + 2y' + 2xy''$$
$$y' = p$$
$$p = p^2 + 2xpp' + 2p + 2xp'$$
$$2xp'(p + 1) + p(p + 1) = 0$$
$$(2xp' + p)(p + 1) = 0 \rightarrow 2xp' + p = 0, p + 1 = 0$$
$$p + 1 = 0$$
$$y' = -1$$
$$y = -x$$
$$2xp' + p = 0$$
$$2p' = -\frac{p}{x}$$
$$2\frac{dp}{p} = -\frac{dx}{x}$$
$$2\int\frac{dp}{p} = -\int\frac{dx}{x}$$
$$2\ln|p| = -\ln|x| + \ln c$$
$$\ln p^2 = -\ln|x| + \ln c$$
$$\ln p^2 = \ln\frac{c}{x}$$
$$p^2 = \frac{c}{x}$$
$$p = \pm\sqrt{\frac{c}{x}}$$

$$y' = \pm\sqrt{\frac{c}{x}}$$
$$\frac{dy}{dx} = \pm\sqrt{\frac{c_1}{x}}$$
$$dy = \pm\sqrt{\frac{c_1}{x}}dx$$
$$y = \pm 2\sqrt{c_1 x} + c_2$$

$$\pm 2\sqrt{c_1 x} + c_2 = x * \left(\pm\sqrt{\frac{c_1}{x}}\right)\left(\pm\sqrt{\frac{c_1}{x}} + 2\right)$$
$$\pm 2\sqrt{c_1} + c_2 = x * \left(\frac{c_1}{x} \pm 2\sqrt{\frac{c_1}{x}}\right)$$
$$\pm 2\sqrt{c_1} + c_2 = c_1 \pm 2\sqrt{c_1}$$
$$c_1 = c_2$$
$$Javob:$$
$$y = -x$$
$$y = \pm 2\sqrt{cx} + c$$

142.(295) Misol

Lagranj va Klero tenglamasini yeching
$2y'^2(y - xy') = 1$
Yechilishi:

$$2y'^2(y - xy') = 1$$
$$y - xy' = \frac{1}{2y'^2}$$
$$y = xy' + \frac{1}{2y'^2} \rightarrow Klerotenglamasi$$
$$1) y = xc + \frac{1}{2c^2}$$
$$2) x - \frac{1}{c^3} = 0$$
$$x = \frac{1}{c^3}$$
$$c = \sqrt[3]{\frac{1}{x}}$$
$$y = x^{\frac{2}{3}} + \frac{1}{2x^{-\frac{2}{3}}} = x^{\frac{2}{3}} + \frac{1}{2}x^{\frac{2}{3}} = \frac{3}{2}x^{\frac{2}{3}}$$
$$2y = 3x^{\frac{2}{3}}$$
$$8y^3 = 27x^2$$

$$Javob:$$
$$y = xc + \frac{1}{2c^2}$$
$$8y^3 = 27x^2$$

143.(296) Misol

Lagranj va Klero tenglamasini yeching

$2xy' - y = ln\, y'$

Yechilishi:

$$y' = p \Rightarrow 2xp - y = ln\, p$$
$$2p + 2xp' - p = p'$$
$$p = \frac{p'(1 - 2px)}{p}$$
$$p^2 = p'(1 - 2px)$$
$$p(x) \to x(p), \left(p' = \frac{1}{x'}\right)$$
$$x'p^2 = 1 - 2px$$
$$x' + \frac{2}{p}x = \frac{1}{p^2}$$
$a)$
$$x' + \frac{2}{p}x = 0$$
$$\frac{dx}{x} = -\frac{2dp}{p}$$
$$ln|x| = -2\, ln|p| + const; x = \frac{c}{p^2}$$
$b)$
$$x = \frac{c(p)}{p^2} \Rightarrow \frac{c'(p)}{p^2} - \frac{2c(p)}{p^3} + \frac{2c(p)}{p^3} = \frac{1}{p^2}$$
$$c'(p) = 1; c(p) = p + c$$
$$x = \frac{1}{p} + \frac{c}{p^2}$$
$$y = 2xp - ln\, p = 2\left(\frac{1}{p} + \frac{c}{p^2}\right)p - ln\, p = 2 + \frac{2c}{p} - ln\, p$$
$$Javob:$$
$$x = \frac{1}{p} + \frac{c}{p^2}$$
$$y = 2 + \frac{2c}{p} - ln\, p$$

144.(297) Misol

Agar differinsial tenglamaning yechimlari oilasi berilgan bo’lsa bu tenglamaning mahsus yechimini toping

$$a) y = cx^2 - c^2$$

$$b)cy = (x - c)^2$$
$$c)y = c(x - c)^2$$
$d)cy = cy - c^2$

Yechilishi:

$$a)y = cx^2 - c^2$$
$$cx^2 - c^2 - y = 0$$
$$x^2 - c = 0$$
$$x = \sqrt{2c}$$
$$2c^2 - c^2 - y = 0$$
$$y = c^2; c = \frac{x^2}{2}$$
$$c^2 = \frac{x^4}{4}$$
$$4y = x^4$$
$$b)y = c(x - c)^2$$
$$c(x - c^2) - y = 0$$
$$(x - c) - 2c(x - c) = 0$$
$$x - c - 2xc + 2c^2 = 0$$
$$x = \frac{2c^2 - c}{2c - 1} = c$$
$$1)y = x(x - x)^2$$
$$y = 0$$
$$2)xy = cy - c^2$$
$$\phi = cy - c^2 - xy = 0$$
$$\frac{\partial\phi}{\partial c} = y - 2c = 0$$
$$c = \frac{y}{2}$$
$$\frac{y^2}{2} - \frac{y^2}{4} - xy = 0$$
$$\frac{y^2}{4} - xy = 0$$
$$y\left(\frac{y}{4} - x\right) = 0$$
$$y = 0$$

$y = 4x$

145.(291) Misol.

Lagranj va Klero tenglamalarini yeching.

Yechish:

$$y'^3 = 3(xy' - y)$$
$$\frac{y'}{3} = xy' - y$$
$$y - xy' = -\frac{1}{3}y'^3$$

$$y = xy' - \frac{1}{3}y'^3 \rightarrow Klerotenglamasi$$
$$y = cx + \frac{1}{3}c^3 \Rightarrow c^3 = 3(cx - y)$$
$$x + c^2 = 0$$
$$x = -c^2$$
$$y = -cc^2 + \frac{1}{3}c^3$$
$$y = -\frac{2}{3}c^3$$
$$y = \frac{2}{3}c^3 \rightarrow c^3 = \frac{3}{2}y \rightarrow c = \sqrt[3]{\frac{3y}{2}}$$
$$x = -\left(\frac{3y}{2}\right)^{\frac{2}{3}}$$
$$x^3 = \frac{9y^2}{4}$$
$$4x^3 = 9y^2$$
$Javob$:
$$c^3 = 3(cx - y)$$
$$4x^3 = 9y^2$$

146.(292) Misol.

Lagranj va Klero tenglamalarini yeching.

Yechish:

$$y = xy'^2 - 2y'^3$$

$$y' = p$$
$$y = xp^2 - 2p^3$$
$$pdx = d(xp^2 - 2p^3)$$
$$p\big((p-1)dx + 2(x - 3p)dp\big) = 0$$
$$p = 1, p = 0$$
$$(p-1)\frac{dx}{dp} + 2x = 3p$$
$$x = \frac{c}{(p-1)^2} + 2p + 1$$
$$y = 0, y = x - 2,$$
$$x = \frac{c}{(p-1)^2} + 2p + 1$$
$$y = \frac{cp^2}{(p-1)^2} + p^2$$

147.(293) Misol

Lagranj va Klero tenglamalarini yeching.

Yechish:

$$xy' - y = \ln y'$$
$$y = xy' - \ln y' \rightarrow Klerotenglamasi$$
$$y = cx - \ln c$$
$$x - \frac{1}{c} = 0$$
$$x = \frac{1}{c} \Rightarrow c = \frac{1}{x}$$
$$y = -\ln\frac{1}{x} + 1 = 1 + \ln x$$
$Javob$:
$$y = cx - \ln c$$
$$y = 1 + \ln x$$

148.(294) Misol.

Lagranj va Klero tenglamalarini yeching.

Yechish:

$$y = xy'(y' + 2)$$
$$y = xy'^2 + 2xy'$$
$$y' = y'^2 + 2xy'y'' + 2y' + 2xy''$$
$$y' = p$$
$$p = p^2 + 2xpp' + 2p + 2xp'$$
$$2xp'(p + 1) + p(p + 1) = 0$$
$$(2xp' + p)(p + 1) = 0 \rightarrow 2xp' + p = 0, p + 1 = 0$$
$$p + 1 = 0$$
$$y' = -1$$
$$y = -x$$
$$2xp' + p = 0$$
$$2p' = -\frac{p}{x}$$
$$2\frac{dp}{p} = -\frac{dx}{x}$$
$$2\int\frac{dp}{p} = -\int\frac{dx}{x}$$
$$2\ln|p| = -\ln|x| + \ln c$$
$$\ln p^2 = -\ln|x| + \ln c$$
$$\ln p^2 = \ln\frac{c}{x}$$
$$p^2 = \frac{c}{x}$$
$$p = \pm\sqrt{\frac{c}{x}}$$
$$y' = \pm\sqrt{\frac{c}{x}}$$
$$\frac{dy}{dx} = \pm\sqrt{\frac{c_1}{x}}$$

$$dy = \pm\sqrt{\frac{c_1}{x}}dx$$
$$y = \pm 2\sqrt{c_1 x} + c_2$$

$$\pm 2\sqrt{c_1 x} + c_2 = x * \left(\pm\sqrt{\frac{c_1}{x}}\right)\left(\pm\sqrt{\frac{c_1}{x}} + 2\right)$$
$$\pm 2\sqrt{c_1} + c_2 = x * \left(\frac{c_1}{x} \pm 2\sqrt{\frac{c_1}{x}}\right)$$
$$\pm 2\sqrt{c_1} + c_2 = c_1 \pm 2\sqrt{c_1}$$
$$c_1 = c_2$$
$Javob$:
$$y = -x$$
$$y = \pm 2\sqrt{cx} + c$$

149.(295) Misol

Lagranj va Klero tenglamalarini yeching.

Yechish:

$$2y'^2(y - xy') = 1$$
$$y - xy' = \frac{1}{2y'^2}$$
$$y = xy' + \frac{1}{2y'^2} \rightarrow Klerotenglamasi$$
$$1) y = xc + \frac{1}{2c^2}$$
$$2) x - \frac{1}{c^3} = 0$$
$$x = \frac{1}{c^3}$$
$$c = \sqrt[3]{\frac{1}{x}}$$
$$y = x^{\frac{2}{3}} + \frac{1}{2x^{-\frac{2}{3}}} = x^{\frac{2}{3}} + \frac{1}{2}x^{\frac{2}{3}} = \frac{3}{2}x^{\frac{2}{3}}$$
$$2y = 3x^{\frac{2}{3}}$$
$$8y^3 = 27x^2$$
$Javob$:
$$y = xc + \frac{1}{2c^2}$$
$$8y^3 = 27x^2$$

150.(296) Misol

Lagranj va Klero tenglamalarini yeching.

Yechish:

$$y' = p \Rightarrow 2xp - y = \ln p$$

$$2p + 2xp' - p = p'$$
$$p = \frac{p'(1-2px)}{p}$$
$$p^2 = p'(1-2px)$$
$$p(x) \to x(p), \left(p' = \frac{1}{x'}\right)$$
$$x'p^2 = 1 - 2px$$
$$p' + \frac{2}{p}x = \frac{1}{p^2}$$
$a)$
$$x' + \frac{2}{p}x = 0$$
$$\frac{dx}{x} = -\frac{2dp}{p}$$
$$ln|x| = -2\,ln|p| + const; x = \frac{c}{p^2}$$
$b)$
$$x = \frac{c(p)}{p^2} \Rightarrow \frac{c'(p)}{p^2} - \frac{2c(p)}{p^3} + \frac{2c(p)}{p^3} = \frac{1}{p^2}$$
$$c'(p) = 1; c(p) = p + c$$
$$x = \frac{1}{p} + \frac{c}{p^2}$$
$$y = 2xp - ln\,p = 2\left(\frac{1}{p} + \frac{c}{p^2}\right)p - ln\,p = 2 + \frac{2c}{p} - ln\,p$$
$Javob$:
$$x = \frac{1}{p} + \frac{c}{p^2}$$
$$y = 2 + \frac{2c}{p} - ln\,p$$

151.(297) Misol

Differensial tenglamaning maxsus yechimini toping. Agar bu tenglamaning old tomoni ma’lum bo’lsa:

$a) y = cx^2 - c^2$
$b) cy = (x-c)^2$
$c) y = c(x-c)^2$
$d) cy = cy - c^2$

Yechish:

$$a) y = cx^2 - c^2$$
$$cx^2 - c^2 - y = 0$$
$$x^2 - c = 0$$
$$x = \sqrt{2c}$$

$$2c^2 - c^2 - y = 0$$
$$y = c^2; c = \frac{x^2}{2}$$
$$c^2 = \frac{x^4}{4}$$
$$4y = x^4$$
$$b) y = c(x-c)^2$$
$$c(x-c^2) - y = 0$$
$$(x-c) - 2c(x-c) = 0$$
$$x - c - 2xc + 2c^2 = 0$$
$$x = \frac{2c^2 - c}{2c-1} = c$$
$$1) y = x(x-x)^2$$
$$y = 0$$
$$2) xy = cy - c^2$$
$$\phi = cy - c^2 - xy = 0$$
$$\frac{\partial\phi}{\partial c} = y - 2c = 0$$
$$c = \frac{y}{2}$$
$$\frac{y^2}{2} - \frac{y^2}{4} - xy = 0$$
$$\frac{y^2}{4} - xy = 0$$
$$y\left(\frac{y}{4} - x\right) = 0$$
$$y = 0$$
$$y = 4x$$

152.(298) Misol

Har bir tegishli egri chiziqni toping. Kordinata o'qlari bilan maydon uchburchagini hosil qiladi. $2a^2$

$Y - y = y'(X - x)$ urinma tenglamasi

1) $y = 0, \quad -y = y'(X - x) \to X = x - \frac{Y}{y'},$

2) $X = 0, \quad Y - y = xy' \Rightarrow Y = y - xy'$

$S = \frac{|xy|}{2} = 2a^2$

$(x - \frac{y}{y'})(y - xy') = 2a^2$

$\frac{(y'x-y)^2}{y'} = \pm 4a^2 \Rightarrow (xy' - y)^2 \pm 4a^2 y' = 0$ **(*)**

$y' = p$ belgilash kiritib tenglamadan hosila olamiz:

$2(xp - y)(2p' + p - p) \pm 4a^2 p' = 0$ bundan

$(xp-y)xp' \pm 2a^2p' = 0$ yoki

$$(x^2p - yx \pm 2a^2)p' = 0$$

$$\left.\begin{cases} p' = 0 \\ x^2p - yx \pm 2a^2 = 0 \end{cases}\right\} \Rightarrow \left\{\begin{matrix} p = c \\ p = \frac{y}{x} \pm \frac{2a^2}{x^2} \end{matrix}\right\} \quad p = c\ bo'lsin$$

1) $y' = c, y' = c$ ni (*) ga qo'yamiz.

$(xc - y)^2 \pm 4a^2c = 0$

2) $p = \frac{y}{x} \pm \frac{2a^2}{x^2} \Rightarrow$ (*) ga qo'yamiz. $\left(x\left(\frac{y}{x} \pm \frac{2a^2}{x^2}\right) - y\right)^2 \pm 4a^2\left(\frac{y}{x} \pm \frac{2a^2}{x^2}\right) = 0$

$$\frac{4a^2}{x^2} \pm \frac{4a^2y}{x} \pm \frac{8a^4}{x^2} = 0 \quad \frac{4a^2y}{x} \pm \frac{4a^2}{x^2} = 0 \ \rightarrow \ y = \pm\frac{a^2}{x^2}$$

Javob: $y = \pm\frac{a^2}{x}$

158.(299) Misol

Har bir tangensi kordinata o'qlarida shunday segmentlarni kesib tashlaydigan kesadigan egri chiziqni topingki, bu segmentlar uzunliklari kvadratlarining yig'indisi o'zaro 1 ga teng bo'lsin**.**

Yechish:

Masala shartiga ko'ra

$\frac{1}{x^2} + \frac{1}{y^2} = 1$ bundan $\frac{y'^2}{(xy'-y)^2} + \frac{1}{(xy'-y)^2} = 1$ tenglamani hosil qilamiz bundan

$(xy' - y)^2 = 1 + y'^2, y' = p$ belgilash kiritsak

$y = xp \pm \sqrt{1+p^2}$ (*) ni hosil qilamiz. Bu tenglamadan hosila olamiz:

$$y' = p + xp' \pm \frac{pp'}{\sqrt{1+p^2}}, y' = p$$

$\left[x \pm \frac{p}{\sqrt{1+p^2}}\right]p' = 0$ bundan

$p = c, x \pm \frac{p}{\sqrt{1+p^2}}$ larni topib (*) ga qo'yamiz.

$$y = \pm\frac{1}{\sqrt{1+p^2}} \pm \sqrt{1+p^2} = \frac{\pm p^2 \pm (1+p^2)}{\sqrt{1+p^2}} = \frac{\pm 1}{\sqrt{1+p^2}}$$

$$x^2 + y^2 = \frac{p^2}{1+p^2} + \frac{1}{1+p^2} = 1$$

$Javob: x^2 + y^2$

$Javob: x^2 + y^2 = 1$

159.(300)Misol

Kordinata boshidan o'tuvchi shunday egri chiziqni topingki, unga nisbata birinchi kordinata burchagi tomonlari bilan kesilgan normal kesimi ga teng o'zgarmas uzunlikka ega bo'lsin.

Masala shartiga ko'ra $|AB| = 2 \ M(x,y)$ nuqtada o'tkazilgan urinma tenglamasi.

$Y-y=-\frac{1}{y'}(X-x)$ bundan $X=0$ deb $Y=OB$= ni $Y=0$ deb $Y=OA$ ni topamiz.

$|OB|=y+\frac{x}{y'}, |OA|=x+yy'$ u holda $(AB)^2=(OB)^2=(OA)^2=4$

$$(y+\frac{x}{y'})^2+(x+yy')^2=4$$

ni hosil qilamiz va $y'=p$ belgilsh kiritib y ni topamiz.

$$(y^2+\frac{2xy}{p'})+\frac{x^2}{p^2}+x^2+2xyp+y^2p^2=4$$

$$y^2(1+p^2)+2y(\frac{x}{p}+xp)+\frac{x^2(1+p^2)}{p^2}-4=0$$

$$\frac{y^2(1+p^2)+\frac{2xy}{p}(1+p^2)+\frac{x^2(1+p^2)}{p^2}-4=0}{(1+p^2)}$$

$$y^2+\frac{2xy}{p}+\frac{x^2}{p^2}-\frac{4}{1+p^2}=0$$

$$y=-\frac{x}{y}\pm\sqrt{\frac{x^2}{y^2}-\frac{x^2}{p^2}+\frac{4}{1+p^2}}$$

$y=-\frac{x}{p}\pm\frac{2}{\sqrt{1+p^2}}$ (1) bundan

$$y'=-\frac{p-xp'}{p^2}+\frac{2pp'}{(1+p^2)\sqrt{1+p^2}}$$

$p=-\frac{1}{p}+\frac{xp'}{p^2}\pm 2p(1+p^2)^{-\frac{3}{2}}p'$

$$p'=\frac{dp}{dx}\Rightarrow\frac{dx}{dp}=\frac{1}{p'}$$

$(p+\frac{1}{p})\frac{dx}{dp}-\frac{x}{p^2}=\pm 2p(1+p^2)^{-\frac{3}{2}}$

X ga nisbatan chiziqli differensial tenglamani hosil qilamiz va x ni topamiz.

$x=\frac{cp}{\sqrt{1+p^2}}\pm\frac{p}{(1+p^2)^{\frac{3}{2}}}$ (2)

Egri chiziq $O(0.0)$ nuqtadan o'tkazilgan $p=\infty$ ni topamiz.

U holda $\begin{cases} x=\pm\frac{p}{(1+p^2)^{\frac{3}{2}}} \\ y=\frac{1+2p^2}{(1+p^2)^{\frac{3}{2}}} \end{cases}$

Parametrik yechim hosil qilamiz.

301-310 Tenglamalarni yeching va ularning yechimlarining grafigini yasang.

160.(301)Misol

$xy'+x^2+xy-y=0$

Yechilishi:

$$x + x^2 + xy - y = 0$$
$$y' + x + y - \frac{y}{x} = 0$$
$$y' + y\left(1 - \frac{1}{x}\right) = -x$$
$$1) y' + y\left(1 - \frac{1}{x}\right) = -x$$
$$\frac{dy}{dx} = y\left(\frac{1}{x} - 1\right)$$
$$\frac{dy}{y} = \frac{dx}{x} - dx$$
$$\ln y = \ln x - x + \ln C$$
$$y = Cxe^{-x}$$
$$2) C'xe^{-x} + \boxed{Ce^{-x}} - \boxed{Cxe^{-x}} + \boxed{Cxe^{-x}} - \boxed{Ce^{-x}} = -x$$
$$C' = -e^x$$
$$dC = -e^x dx$$
$$C = -e^x + c$$
$$y = (-e^x + c)xe^{-x} = -x + cxe^{-x}$$

$Javob: y = x(ce^x - 1)$

161.(302) Misol

$2xy' + y^2 = 1$

Yechilishi:

$$2xy' + y^2 = 1$$
$$2xy' = 1 - y^2$$
$$\frac{dy}{1 - y^2} = \frac{1}{2}\frac{dx}{x}$$
$$\frac{1}{2}\ln\left|\frac{1 + y}{1 - y}\right| = \frac{1}{2}\ln x + \frac{1}{2}\ln c$$
$$\frac{1 + y}{1 - y} = cx$$
$$1 + y = cx - ycx$$

$Javob: (cx + 1)y = cx - 1$

162.(303) Misol

$(2xy^2 - y)dx + xdy = 0$

Yechilishi:

$$(2xy^2 - y)dx + xdy = 0$$
$$2xy^2 - y + xy' = 0$$
$$y = z^m$$

$$2xz^{2m} - z^m + xmz^{m-1}z' = 0$$
$$\begin{bmatrix} x = k\bar{x}; z = k\bar{z} \Rightarrow k^{1+2m}(2\bar{x}\bar{z}^{2m}) + k^{1+m-1}(m\bar{x}\bar{z}^{m-1}z') = 0 \\ 1 + 2 = m = 1 + m - 1 \Rightarrow m = -1; t.e. y = z^{-1} \end{bmatrix}$$
$$2xz^{-2} - z^{-1} - xz^{-2}z' = 0/* z^2$$
$$2x - z - xz' = 0/: x \neq 0$$
$$2 - \frac{z}{x} - z' = 0$$
$$\frac{z}{x} = t \Rightarrow z' = t + t'x$$
$$2 - 2t - t'x = 0$$
$$2(1 - t) = t'x$$
$$\int \frac{dt}{2(1-t)} = \int \frac{dx}{x}$$
$$\frac{1}{2} ln|1 - t| = ln|x| + C \Leftrightarrow \sqrt{1-t} = Cx \Leftrightarrow \sqrt{1 - \frac{1}{xy}} = Cx$$
$$1 - \frac{1}{xy} = C_2x^2$$
$$xy - 1 = C_2x^3y$$
$$-\frac{1}{C_2x^3 - x} = y; y = 0$$
$Javob: -\frac{1}{C_2x^3-x} = y; y = 0$

163.(304) Misol

$$(xy' + y)^2 = x^2y'$$

Yechilishi:

$$(xy' + y)^2 = x^2y'$$
$$\left(y' + \frac{y}{x}\right)^2 = y'$$
$$z = \frac{y}{x}$$
$$zy' = xz' + z$$
$$(xz' + z + z)^2 = xz' + z$$
$$(xz' + 2z)^2 = xz' + z$$
$$x^2z'^2 + 4zxz' + 4z^2 = xz' + z$$
$$x^2z'^2 + 4zxz' - xz' + 4z^2 - z = 0$$
$$x^2z'^2 + xz'(4z - 1) + z(4z - 1) = 0$$
$$D = x^2(4z-1)^2 - 4x^2z(4z-1) = x^2(16z^2 - 8z + 1) - 16x^2z^2 + 4x^2z =$$
$$= 16x^2z^2 - 8x^2z + x^2 - 16x^2z^2 + 4x^2z = -4x^2z + x^2 = x^2(1 - 4z)$$
$$z' = \frac{-x(4z-1) \pm \sqrt{1-4z}}{2x^2}$$
$z' = \frac{-(4z-1)-\sqrt{1-4z}}{2x} = \frac{(4z-1)-\sqrt{1-4z}}{2x}$

164.(305) Misol

$$y - y' = y^2 + xy'$$

Yechilishi:

$$y - y' = y^2 + xy'$$
$$y = 0$$
$$y'(x+1) = y - y^2$$
$$y' = \frac{y - y^2}{x+1}$$
$$x \neq -1$$
$$y' = \frac{dy}{dx}$$
$$\frac{dy}{dx} = \frac{y - y^2}{x+1}$$
$$\int \frac{dx}{x+1} = \int \frac{dy}{y - y^2}$$
$$ln|x+1| = \int \frac{dy}{y(1-y)}$$
$$\int \frac{dy}{y(1-y)} = \int \frac{1}{y} + \frac{1}{1-y} dy = ln|y| - ln|1-y| + C$$
$$x + 1 = \frac{y}{1-y} C$$
$$(x+1)(1-y) = Cy$$
$$x - xy + 1 - y = Cy$$
$$x + 1 = y(C + y)$$

$Javob: y = 0; x + 1 = y(C + y)$

165.(306) Misol

$(z + 2y^3)y' = y$

Yechilishi:

$$(z + 2y^3)y' = y$$
$$xy' + 2y^3y' - y = 0$$
$$y = z^m$$
$$y' = mz^{m-1}z'$$
$$xmz^{m-1}z' + 2z^{3m}mz^{m-1}z' - z^m = 0$$
$$1 + m - 1 = 3m + m - 1 = m$$
$$4m - 1 = m$$
$$3m = 1$$
$$m = \frac{1}{3}.$$
$$y = z^{\frac{1}{3}}$$

$$\frac{1}{3}xz^{\frac{-2}{3}}z' + \frac{2}{3}zz^{\frac{-2}{3}}z' - z^{\frac{1}{3}} = 0$$
$$\frac{1}{3}xz^{\frac{-2}{3}}z' + \frac{2}{3}z^{\frac{1}{3}}z' - z^{\frac{1}{3}} = 0 | : x^{\frac{1}{3}} \neq 0$$
$$\frac{1}{3}\left(\frac{z}{x}\right)^{\frac{-2}{3}}z' + \frac{2}{3}\left(\frac{z}{x}\right)^{\frac{1}{3}}z' - \left(\frac{z}{x}\right)^{\frac{1}{3}} = 0$$
$$t = \frac{z}{x}$$
$$z' = t + t'x$$
$$\frac{1}{3}t^{\frac{-2}{3}}(t + t'x) + \frac{2}{3}t^{\frac{1}{3}}(t + t'x) - t^{\frac{1}{3}} = 0$$
$$\frac{1}{3}t^{\frac{1}{3}} + \frac{1}{3}t^{\frac{-2}{3}}xt' + \frac{2}{3}t^{\frac{4}{3}} + \frac{2}{3}t^{\frac{1}{3}}xt' - t^{\frac{1}{3}} = 0$$
$$\frac{2}{3}t^{\frac{1}{3}} + \frac{2}{3}t^{\frac{4}{3}} + \frac{1}{3}xt'\left(t^{\frac{-2}{3}} + 2t^{\frac{1}{3}}\right) = 0$$
$$2t^{\frac{1}{3}} + 2t^{\frac{4}{3}} + xt't^{\frac{-2}{3}}(1 + 2t) = 0 | : t^{\frac{1}{3}} \neq 0$$

$$2 + 2t + \frac{xt'}{t}(1 + 2t) = 0$$
$$\frac{xt'}{t}(1 + 2t) = -(2 + 2t)$$
$$-\frac{(1 + 2t)dt}{(2 + 2t)t} = \frac{dx}{x}$$
$$-\frac{1}{2}\left(\frac{dt}{t(t + 1)}\right) - \frac{dt}{1 + t} = \frac{dx}{x}$$
$$-\frac{1}{2}\frac{dt}{t} + \frac{1}{2}\frac{dt}{t + 1} - \frac{dt}{1 + t} = \frac{dx}{x}$$
$$-\frac{dt}{t} - \frac{dt}{t + 1} = 2\frac{dx}{x}$$
$$ln\, t^{-1} + ln(t + 1)^{-1} = ln\, x^2 + ln\, c$$
$$\frac{1}{t(t + 1)} = cx^2$$
$$\frac{x^2}{z(z + x)} = cx^2$$
$$\frac{1}{z(z + x)} = c$$
$$\frac{1}{y^3(y^3 + x)} = c$$
$$cy^6 + cy^3x = 1$$
$$cy^3x = 1 - cy^6$$
$$x = \frac{1}{cy^3} - y^3$$

$y = 0$

166.(307) Misol

$y'^3 - y'e^{2x} = 0$

Yechilishi:

$$y'^3 - y'e^{2x} = 0$$
$$y' = p$$
$$p^3 - pe^{2x} = 0$$
$$pe^{2x} = p^3$$
$$e^{2x} = p^2$$
$$2x = \ln p^2$$
$$x = \ln p$$
$$dy = pdx$$
$$dy = \cancel{p}\frac{1}{\cancel{p}}dp$$
$$y = p + c$$
$$x = \ln(y + c)$$

$Javob: y = e^x + c$

167.(308) Misol

$x^2y' = y(x + y)$

Yechilishi:

$$x^2y' = y(x + y) |: x^2$$
$$y' = \frac{y}{x}\left(1 + \frac{y}{x}\right)$$
$$z = \frac{y}{x}$$
$$y' = z + z'x$$
$$z + z'x = z(1 + z)$$
$$\cancel{z} + z'x = \cancel{z} + z^2$$
$$z'x = z^2$$
$$\frac{dz}{z^2} = \frac{dx}{x}$$
$$-\frac{1}{z} = \ln x + c$$
$$-\frac{x}{y} = \ln x + \ln c\,x$$
$$-x = y \ln c\,x$$

$Javob: y \ln c\,x = -x$

168.(309) Misol

$(1 - x^2)dy + xydx = 0$

Yechilishi:

$$(1-x^2)dy + xydx = 0$$
$$(1-x^2)y' = -xy$$
$$(x^2-1)y' = xy$$
$$\frac{dy}{y} = \frac{xdx}{x^2-1}; x \neq \pm 1$$
$$\frac{dy}{y} = \frac{1}{2} ln|x^2-1| + ln\, c$$
$$y = c\sqrt{x^2-1}$$
$$y^2 = c(x^2-1)$$

$x = \pm 1$

169.(310) Misol

$y'^2 + 2(x-1)y' - 2y = 0$

Yechilishi:

$$y'^2 + 2(x-1)y' - 2y = 0$$
$$2y = 2(x-1)y' + y'^2$$
$$y = xy' - y' + \frac{y'^2}{2}$$
$$y = xy' - y'\left(1 + \frac{y'}{2}\right) \rightarrow$$
$$y = cx - c\left(1 + \frac{c}{2}\right)$$
$$2y = 2cx - c + c^2$$
$$Maxsusyechimi$$
$$x - \left[1 + \frac{c}{2} + c\frac{1}{2}\right] = 0$$
$$x - 1 - c = 0$$
$$c = x - 1$$
$$2y = 2(x-1)^2 + (x-1)^2$$

$2y = 3(x-1)^2$

170.(312) Misol

$x^2y' - 2xy = 3y.$

$$y' = \frac{dy}{dx},$$
$$x^2\frac{dy}{dx} - 2xy = 3y$$
$$x^2dy - 2xydx = 3ydx/x^2y$$
$$x^2\frac{dy}{x^2y} - 2xy\frac{dx}{x^2y} = \frac{3ydx}{x^2y}$$
$$\frac{dy}{y} - \frac{2dx}{x} = \frac{3dx}{x^2}$$

$$\int \frac{1}{y}dy - \int \frac{2}{x}dx = \int \frac{3}{x^2}dx$$
$$ln(|y|) - 2\,ln(|x|) + ln(|C|) = \frac{-3}{x}$$
$$ln\left(\left|\frac{Cy}{x^2}\right|\right) = \frac{-3}{x}$$
$$x^2 > 0$$
$$e^{\frac{-3}{x}} = \frac{Cy}{x^2}$$
$$y = C_1 x^2 e^{\frac{-3}{x}}$$
$$C_1 = \frac{1}{C}$$

171.(313) Misol

$$x + yy' = y^2(1 + y'^2).$$
$$x + yy' = y^2 + (y\,y')^2$$
$$y^2 = t$$
$$2yy' = t'$$
$$x + \frac{t'}{2} = t + \left(\frac{t'}{2}\right)^2$$
$$\frac{1}{2} = \frac{t'}{2}$$
$$t' = 1$$
$$t = x + C$$
$$y^2 = x + C$$
$$x + \frac{1}{2} = x + C + \frac{1}{4} \Rightarrow C = \frac{1}{4}$$
$$y^2 = x + \frac{1}{4} \Rightarrow 4(y^2 - x) = 1$$
$$x + \frac{t'}{2} = t + \left(\frac{t'}{2}\right)^2$$
$$t' = p \Rightarrow dt = pdx$$
$$t = x + \frac{p}{2} - \frac{p^2}{4} \Rightarrow dt = dx + \frac{dp}{2} - \frac{pdp}{2}$$
$$p = 1 + \frac{p'}{2} - \frac{pp'}{2} \Rightarrow (p - 1) = \frac{1}{2}(1 - p)p'$$
$$p' = -2 \Rightarrow p = -2x + C_1$$
$$t = -2x + C_1$$
$$2yy' = -2x + C_1 \Rightarrow y^2 = -x^2 + C_{\,1}x + C_2$$
$$y^2 + x^2 + C_{\,1}x + C_2 = 0 \Rightarrow (x - C)^2 + y^2 = C$$

172.(314) Misol

$$y = (xy' + 2y)^2.$$
$$y = x^2y'^2 + 4xyy' + 4y^2$$
$$x^2y'^2 + 4xyy' + 4y^2 - y = 0$$
$$y' = \frac{-4xy \pm \sqrt{16x^2y^2 - 4x^2(4y^2 - y)}}{2x^2}$$
$$y' = \frac{-4xy \pm \sqrt{4x^2y}}{2x^2}$$
$$y' = \frac{-4xy \pm 2x\sqrt{y}}{2x^2}$$
$$y' = \frac{-2y \pm \sqrt{y}}{x}$$
$$\left[\begin{array}{l} y' = \dfrac{-2y + \sqrt{y}}{x} \\ y' = \dfrac{-2y - \sqrt{y}}{x} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \dfrac{dy}{-2y + \sqrt{y}} = \dfrac{dx}{x} \\ \dfrac{dy}{-2y - \sqrt{y}} = \dfrac{dx}{x} \end{array}\right. \Leftrightarrow$$
$$\left[\begin{array}{l} -\ln(1 - 2\sqrt{y}) = \ln x + \ln C \\ -\ln(1 - 2\sqrt{y}) = \ln x + \ln C \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \dfrac{1}{-2y + \sqrt{y}} = Cx \\ \dfrac{1}{-2y - \sqrt{y}} = Cx \end{array}\right. \Leftrightarrow$$
$$\left[\begin{array}{l} 2\sqrt{y} = 1 - \dfrac{1}{Cx} \\ 2\sqrt{y} = \dfrac{1}{Cx} - 1 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} y = \left(\dfrac{1}{2} - \dfrac{1}{2Cx}\right)^2 \\ y = \left(\dfrac{1}{2Cx} - \dfrac{1}{2}\right)^2 \end{array}\right.$$

173.(315) Misol

$$y' = 1/(x - y^2).$$
$$\frac{dy}{dx} = \frac{1}{x - y^2} \Rightarrow$$
$$\frac{dx}{dy} = y^2 + x \Rightarrow x' - x = -y^2$$
$$x = uv \Rightarrow$$
$$x' = u'v + uv'$$
$$u'v + uv' - uv = -y^2$$
$$u'v + u(v' - v) = -y^2$$
$$\frac{dv}{dy} = v \Rightarrow \frac{dv}{v} = dy \Rightarrow \ln v = y \Rightarrow v = e^y$$

$$u'e^y = -y^2 \Rightarrow u = \int e^y\,(-y^2)dy = \begin{vmatrix} t = -y^2 dt = -2ydy \\ z = \int e^{-y}dy = -e^{-y} \end{vmatrix} =$$

$$y^2e^{-y} - \int 2y\,e^{-y}dy = \begin{vmatrix} t = 2ydt = 2dy \\ z = \int e^{-y}dy = -e^{-y} \end{vmatrix} =$$

$$y^2e^{-y} - (-e^{-y}2y + 2\int e^{-y}dy) = y^2e^{-y} + 2ye^{-y} - 2\int e^{-y}dy =$$

$$y^2e^{-y} + 2ye^{-y} + 2e^{-y} + C$$

$$x = uv = e^y(y^2e^{-y} + 2ye^{-y} + 2e^{-y} + C) =$$

$$y^2 + 2y + 2 + Ce^{-y}$$

174.(316) Misol

$$y'^3 \; + \; (3x \; - \; 6)y' \; = \; 3y.$$

$$y = \frac{1}{3}y'^3 + (x-2)y'$$

$$p = y' = \frac{dy}{dx}$$

$$y = \frac{1}{3}p^3 + (x-2)p(*)$$

$$dy = p^2dp + pdx + (x-2)dp$$

$$dp(p^2 + x - 2) = 0$$

$$1)dp = 0$$

$$p = C$$

$$y = \frac{1}{3}C^3 + (x-2)C$$

$$3y = 3(x-2)C + C^3$$

$$2)p = \pm\frac{2}{3}(x-2)^{\frac{3}{2}}$$

$$y = \pm(\frac{1}{3}(2-x)\sqrt{2-x} + (2-x)\sqrt{2-x})$$

$$y = \pm\frac{2}{3}(x-2)^{\frac{3}{2}}$$

$$9y^2 = 4(2-x)^3$$

$$9y^2 = 4(2-x)^3,$$

$$3y = 3(x-2)C + C^3$$

175.(317) Misol

$$x \; - \frac{y}{y'} \; = \frac{2}{y} \; .$$

$$x - \frac{y}{y'} = \frac{2}{y}$$

$$x - yx' = \frac{2}{y}$$

$$x' - \frac{x}{y} = -\frac{2}{y^2} /* \, e^{\int \frac{-1}{y} dy} = e^{-\ln|y|} = \frac{1}{|y|}$$

$$\left(\frac{x}{y}\right)' = -\frac{2}{y^3} = -2y^{-3}$$

$$\frac{x}{y} = \frac{-2y^{-2}}{-2} + C = y^{-2} + C = \frac{1}{y^2} + C$$

$$x = \frac{1}{y} + Cy$$

$$2y'^3 - 3y'^2 + x = y.$$

$y' = p; dy = pdx$

$2p^3 - 3p^2 + x = y$

$dy = 6p^2 dp - 6pdp + dx$

$pdx - dx = 6p(p-1)dp$

$(p-1)dx = 6p(p-1)dp$

$\int 6pdp = \int dx$

$3p^2 = x + C_1$

$3y'^2 = x + C_1$

$y' = \sqrt{\frac{x+C_1}{3}} dx$

176.(318) Misol $\int dy = \int \sqrt{\frac{x+C_1}{3}} dx$

$y = \int \sqrt{\frac{x+C_1}{3}} dx = \frac{1}{\sqrt{3}} \frac{2}{3} \sqrt{(x+C_1)^3} + C_2$

$3\sqrt{3}(y - C_2) = 2(x + C_1)^{\frac{2}{3}} /* \, 2$

$1) 27(y - C_2)^2 = 4(x + C_1)^3$

$6y'^2 - 6y = 0 \Rightarrow y' = 0; y' = 1$

$x = y; x - 1 = y$

$x = y; x - 1 = x$

$x - 1 = y; -1 + x = x - 1$

$y' = 1$

$27(y - C_2)^2 = 4(x + C_1)^3;$

$y = x - 1.$

177.(319) Misol

$(x + y)^2 y' = 1.$

$x + y = t,$

$y' = t' - 1$

$t^2(t' - 1) = 1$

$$t^2t' = t^2 + 1$$
$$\int \frac{t^2dt}{t^2+1} = \int dx$$
$$t - arctgt = x + C$$
$$x + y - arctg(x + y) = x + C$$
$$y - C = arctg(x + y)$$
$$tg(y - C) = tg(arctg(x + y))$$
$$tg(y - C) = x + y$$

178.(320) Misol

$$2x^3yy' + 3x^2y^2 + 7 = 0.$$
$$2x^3ydy + (3x^2y^2 + 7)dx = 0$$
$$6x^2ydydx + 6x^2ydxdy = 0$$
$$F(y) = \int 2x^3ydy =$$
$$\frac{1}{2}2x^3y + C(x) =$$
$$x^3y + C(x)$$
$$F_x'(y) = 3x^2y^2 + C'(x) =$$
$$3x^2y^2 + 7 \Rightarrow C'(x) = 7$$
$$C(x) = 7x + C$$
$$F(x,y): x^3y^2 + 7x = C_3$$

179.(321) Misol

Tenglamani yeching va uning yechimini grafigini yasang

$$\frac{dx}{x} = (\frac{1}{y} - 2x)dy$$

Yechilishi;

$$\frac{dx}{x} = (\frac{1}{y} - 2x)dy$$
$$\frac{x'}{x^2} + \frac{1}{yx} = -2$$
$$z = \frac{1}{x}$$
$$x = \frac{1}{z}$$
$$x' = -\frac{z'}{z^2}$$

$$z' + \frac{1}{y}z = 2$$

$$z' + \frac{1}{y}z = 0$$

$$z' = -\frac{1}{y}z$$

$$\int \frac{dz}{z} = -\int \frac{dy}{y}$$

ln|z|$ln|z| = -ln|y| + ln|u|$

$$z = \frac{u}{y}$$

$$z' = \frac{u'}{y} - \frac{u}{y^2}$$

$$\frac{u'}{y} - \frac{u}{y^2} + \frac{1}{y} \cdot \frac{u}{y} = 2$$

$$\frac{u'}{y} = 2$$

$$\int du = 2\int ydy$$

$$u = y^2 - c$$

$$z = \frac{u}{y} = y - \frac{C}{y}$$

$$\frac{1}{x} = y - \frac{C}{y}$$

$$xy^2 - y = Cx$$

$y(xy - 1) = Cx$

180.(322) misol;

Tenglamani yeching va uning yechimini grafigini yasang

$$xy' = e^y + 2y'$$

Yechilishi;

$$xy' - 2y' = e^y$$

$$y'(x - 2) = e^y$$

$$\frac{dy}{dx} = \frac{e^y}{x - 2}$$

$$\int \frac{dy}{e^y} = \int \frac{dx}{x - 2}$$

$$-e^{-y} = \frac{d(x - 2)}{x - 2} + C$$

$$-e^{-y} = ln(x - 2) + ln\, C$$

$-e^{-y} = ln\, C\,(x - 2)$

181.(323) Misol

Tenglamani yeching va uning yechimining grafigini yasang.

$2(x-y^2)dy = ydx$

Yechilishi;

$$2\left(\frac{x}{y}-y\right)=\frac{dx}{dy}$$
$$y=0$$
$$\frac{x}{y}=\xi \Rightarrow x'=\xi' y+\xi$$
$$2\xi-2y=\xi' y+\xi$$
$$\frac{d\xi}{dy}=-2y+\xi \Rightarrow \frac{d\xi}{dy}=\frac{\xi}{y}-2$$
$$\frac{d\xi}{dy}=\frac{\xi}{y} \Rightarrow \xi = Cy$$
$$\xi = C(y)\cdot y \Rightarrow c'\cdot y=-2$$
$$dc=-2\frac{dy}{y}$$
$$C(y)=ln\frac{c}{y^2} \Rightarrow \xi = ln\frac{c}{y^2}\cdot y$$

$\frac{x}{y}=ln\frac{c}{y^2}\cdot y \Rightarrow y=0 \Rightarrow x=y^2[C-2\,ln|y|]$

182.(324) Misol

Tenglamani yeching va uning yechimini grafigini yasang.

$x^2y'^2+y^2=2x(2-yy')$

Yechilishi;

$$x^2y'^2+y^2=4x-2xyy'$$
$$(xy')^2 2xyy'+y^2=4x$$
$$(xy'+y)^2=4x \to xy'+y=\pm 2\sqrt{x}$$
$$xy'+y=0 \Rightarrow \frac{xdy}{dx}=-y$$
$$\int\frac{dy}{y}=-\int\frac{dx}{x} \Rightarrow ln\,y=-ln\,x+C^* \Rightarrow y=C^*x^{-1}$$
$$y=C(x)x^{-1} \Rightarrow y'=C'(x)x^{-1}-C(x)x^{-2}$$
$$x(C'(x)x^{-1}-C(x)x^2)+C(x)x^{-1}=\pm 2\sqrt{x}$$
$$C'(x)-C(x)x^{-1}+C(x)x^{-1}=\pm 2\sqrt{x}$$
$$C(x)=\pm 2\int x^{0.5}dx=\pm 2\frac{x^{1.5}}{1.5}+C=C\pm\frac{4}{3}\sqrt{x^3}$$
$$y=(C\pm\frac{4}{3}\sqrt{x^3})x^{-1}$$
$$xy=C\pm\frac{4}{3}\sqrt{x^3}$$

$3xy=C\pm 4\sqrt{x^3}$

183.(325) Misol

Tenglamani yeching va uning yechimini grafigini yasang.

$dy + (xy - xy^3)dx = 0$

Yechilishi;

$$\frac{dy}{y^3} + (\frac{x}{y^2} - x)dx = 0$$

$$y = 0$$

$$-\frac{1}{2}d(\frac{1}{y^2}) + (x\frac{1}{y^2} - x)dx = 0$$

$$\frac{1}{y^2} = z$$

$$-\frac{1}{2}dz + (xz - x)dx = 0$$

$$-\frac{1}{2}z_x{}' = -xz + x$$

$$\frac{1}{2}\frac{dz}{dx} = xz - x$$

$$\frac{dz}{z} = 2xdx$$

$$z = Ce^{x^2}$$

$$z_n = C(x)e^{x^2}$$

$$\frac{1}{2}C'e^{x^2} = -x$$

$$dc = \frac{-2x}{e^{x^2}}dx$$

$$dc = e^{-x^2}d(-x^2)$$

$$C(x) = e^{-x^2} + C_2 \Rightarrow z = (e^{-x^2} + C_2)e^{x^2} = 1 + C_2e^{x^2}$$

$\frac{1}{y^2} = \left[1 + C_2e^{x^2}\right], y = 0$

184.(326) Misol

Tenglamani yeching va uning yechimini grafigini yasang.

$2x^2y' = y^2(2xy' - y)$

Yechilishi;

$$2x^2y' - y^2 2xy' = -y^3$$

$$y'(2x^2 - 2xy^2) = -y^3$$

$$y' = \frac{-y^3}{2x^2 - 2xy^2} = \frac{y^3}{2xy^2 - 2x^2} \to x' = \frac{2xy^2 - 2x^2}{y^3} = \frac{2x}{y} - \frac{2x^2}{y^3}$$

$$x' - \frac{2x}{y} = -\frac{2x^2}{y^3} \to : x^2$$

$x^{-2} \cdot x' - \frac{2x^{-1}}{y} = -\frac{2}{y^3}$

Bernulli tenglamasiga qo’yamiz

$$z = x^{-1}$$
$$-x^{-2} \cdot x' = z' \to -z' - \frac{2z}{y} = -\frac{2}{y^3}$$
$$z' + \frac{2z}{y} = \frac{2}{y^3}$$
$$z = uv \to z' = u'v + v'u$$
$$u'v + v'u + \frac{2uv}{y} = \frac{2}{y^3}$$
$$u'v + u(v' + \frac{2v}{y}) = \frac{2}{y^3}$$
$$\frac{dv}{dy} = -\frac{2v}{y} \to \frac{dv}{v} = -\frac{2dy}{y} \to ln\, v = -2\, ln\, y \to v = \frac{1}{y^2}$$
$$u' \cdot \frac{1}{y^2} = \frac{2}{y^3} \to u' = \frac{2}{y}$$
$$u = \int \frac{2dy}{y} = 2\, ln\, y + 2\, ln\, C = 2\, ln\, C\, y$$
$$z = uv = \frac{1}{y^2} \cdot 2\, ln\, C\, y$$
$$z = \frac{1}{x} \Rightarrow \frac{1}{x} = \frac{2\, ln\, C\, y}{y^2}$$
$y^2 = 2x\, ln\, C\, y$

Y=0

185.(327) Misol

Tenglamani yeching va uning yechimini grafigini yasang;

$\frac{y-xy'}{x+yy'} = 2$

Yechilishi;
$$y - xy' = 2x + 2yy'$$
$$y'(2y + x) = y - 2x$$
$$y' = \frac{y - 2x}{2y + x}$$
$$y = kx \to y' = k'x + k$$
$$k'x + k = \frac{k - 2}{2k + 1}$$
$$k'x = \frac{k - 2 - 2k^2 - k}{2k - 1}$$
$$k' = \frac{1}{x} \cdot (-2\frac{1 + k^2}{2k - 1})$$
$$-\frac{(2k + 1)dk}{2(k^2 + 1)} = \frac{dx}{x}$$
$$\int \frac{2k}{k^2 + 1} dk + \int \frac{dk}{k^2 + 1} = -\, ln|x| + C$$
$$ln(\, k^2 + 1) + arctgk = -2\, ln|x| + C$$

$javob; ln(x^2+y^2)+arctg(\frac{y}{x})=C$

186.(328) Misol

Tenglamani yeching va uning yechimini grafigini yasang.

$x(x-1)y'+2xy=1$

Yechilishi;

$$\frac{dy}{dx}=\frac{1-2xy}{x(x-1)}=\frac{\frac{1}{x}-2y}{x-1}$$

$$\frac{dy}{dx}=-2\frac{y}{x-1}+\frac{1}{x(x-1)}$$

$$\frac{dy}{y}=-\frac{2d(x-1)}{x-1}\Rightarrow y=\frac{C}{(x-1)^2}$$

$$y_u=\frac{C(x)}{(x-1)^2}$$

$$\frac{C'}{(x-1)^2}=\frac{1}{(x-1)x}\Rightarrow dc=\frac{(x-1)dx}{x}$$

$$C(x)=x-ln|x|+C_2$$

$y=\frac{x-ln|x|+C_2}{(x-1)^2}$

187.(330) Misol

tenglamani yeching va uning yechimini grafigini yasang.

$(1-x^2)y'-2xy^2=xy$

Yechilishi;

$$(1-x^2)y'=xy[1+2y]\to y=0$$

$$\frac{dy}{y[1+2y]}=\frac{xdx}{1-x^2}=-\frac{1}{2}\frac{d(1-x^2)}{1-x^2}$$

$$\int(\frac{1}{y}-\frac{2}{2y+1})dy=ln\,y-ln(2y+1)=-\frac{1}{2}ln(1-x^2)$$

$$\frac{y}{2y+1}=\frac{1}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}=2+\frac{1}{y}$$

$$y(\sqrt{1-x^2}-2)=1$$

$y=0$

188.(351) Misol

$$y^2(y-xy')=x^3y' \quad \frac{y^3}{x^3}-\frac{y^2}{x^2}y'=y' \quad \frac{y^3}{x^3}=y'\left[1+\frac{y^2}{x^2}\right] \quad \frac{y}{x}=t \quad => y'=t'x+t$$

$$t'=(t'x+t)(1+t^2)=t'x+t^2xt'+t+t^3 \qquad t'[x(1+t^2)]=-t$$

$$t'(1+t^2)=-\frac{t}{x} \qquad \frac{(1+t^2)}{t}dt=-\frac{dx}{x} \qquad lnt+t^2+c_2=-lnx$$

$$lny - lnx + \frac{1}{2}\left(\frac{y}{x}\right)^2 + c_2 = -lnx \qquad \begin{cases} 2\ln(yc)\,x^2 + y^2 = 0 \\ y = 0 \end{cases}$$

189.(352) Misol

$$y' = (4x + y - 3)^2 \qquad 4x + y - 3 = p \qquad 4 + y' = p' \qquad y' = p' + 4$$

$$p' = p^2 + 4 \qquad \int \frac{dp}{p^2 + 2^2} = \int dx \qquad \frac{1}{2}arctg\frac{p}{2} = x + c \qquad \frac{p}{2} = tg(2x + c) \quad 4x + y - 3 = 2tg(2x + c)$$

190.(353) Misol

$(cosx - xsinx)ydx + (xcosx - 2y)dy = 0$ (1)

Yechim:

Bu tenglama $P(x,y) = (cosx - xsinx)y \ \ va\ Q(x,y) = (xcosx - 2y)$lar uchun biror $U(x,y)$ ning to'la differensiali bo'ladi

Mos ravishda quyidagi $\frac{\partial P(x,y)}{\partial y} = cos - xsin \quad \frac{\partial Q(x,y)}{\partial x} = cos - xsin$ xususiy hosilalarga ega bo'lamiz bu yerda $\frac{\partial P}{\partial y} = \frac{QP}{\partial x}$ tenglik o'rinli natijada (1) ning chap qismi qandaydir $U(x,y)$ funksiyaning to'la differensiali bo'ladi

$$dU(x,y) = \frac{\partial U(x,y)}{\partial x}dx + \frac{\partial U(x,y)}{\partial y}dy \quad yoki \quad dU = \frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial y}dy = (ycosx - xysin)dx + (xcos - 2y)dy \ \ ya'ni$$

$$\frac{\partial U}{\partial x} = ycosx - yxsin \qquad \frac{\partial U}{\partial y} = xcos - 2y \quad (A)$$

A ning birinchi tenglamada y ni o'zgarmas deb x bo'yicha integrallaymiz natijada bu integralning o''zgarmasi y ning uzluksiz $\varphi(y)$ bo'ladi

$$u(x,y) = \int (ycosx - yxsinx)dx = xycosx + \varphi(y)$$

So'ngra oxirgi tenglikdan y o'zgaruvchi bo'yicha hosila olsak , buning qiymati A ning 2 tenglamasiga teng bo'ladi ya'ni

$$xcosx + \varphi'(y) = xcosx - 2y$$

$$xcosx + \varphi'(y) = xcosx - 2y \qquad \varphi'(y) = -2y \qquad \varphi(y) = \int (-2y)\,dy = -y^2 + c_1 \qquad \varphi(y) = -y^2 + c_1$$

$$Natijada, \quad u(x,y) = xycosx - -y^2 + c_1 \quad (c_1 = -c) \qquad xycosx - -y^2 = c$$

191.(354) Misol

$$x^2y'^2 - 2xyy' = x^2 + 3y^2 \qquad (xy')^2 - 2xyy' + y^2 = x^2 + 4y^2$$

$$(xy'-y)^2=x^2+4y^2 \quad xy'-y=\pm\sqrt{x^2+4y^2} \quad y=xt$$

$$y'=t'+t \quad t=\frac{y}{x} \quad x^2t'+xt-xt=\pm\sqrt{x^2+4x^2}t^2 \quad \frac{dt}{dx}=\pm\frac{\sqrt{1+4t^2}}{x}$$

$$\int\frac{dt}{\sqrt{1+4t^2}}=\pm\int\frac{dx}{x} \;=>\; \frac{1}{2}\ln\left|2t+\sqrt{1+4t^2}\right|=ln|x|+C$$

$$2t+\sqrt{1+4t^2}=Cx^2 \quad \sqrt{1+4t^2}=\;Cx^2-2t \quad 1+4t^2$$
$$=C^2x^4-4Ctx^2+4t^2 \quad 4Ctx^2=C^2x^4-1=> \;4Cxy$$
$$=C^2x^4-1$$

192.(355) Misol

$$\frac{xy'}{y}+2xylnx+1=0 \quad \frac{y'}{y}+\frac{1}{y}=-2lnx \quad z=\frac{1}{y} \;\; y=\frac{1}{z} \;\; y'=-\frac{z'}{z^2}$$

$$z'-\frac{1}{x}z=2lnx \quad z'=\frac{1}{x}z \quad \int\frac{dz}{z}=\int\frac{dx}{x} \quad ln|z|=\;ln|x|=\;ln|u|$$

$$z=ux \quad z'=u'x+u \quad u'x+u-\frac{ux}{x}=2lnx \quad u'x$$
$$=2lnx \quad \int du=2\int\frac{1}{x}lnxdx \quad u=ln^2x+C \quad z=ux$$
$$=\frac{1}{x}(ln^2x+C\quad) \quad \frac{1}{y}=(ln^2x+C)=1 \;\; xy(ln^2x+C)=1$$

193.(356) Misol

$$x\,y'=x\sqrt{y-x^2}+2y \quad y=\;t'x^2+t2x \quad x(t'x^2+t2x)$$
$$=x\sqrt{t'x^2+t2x}+2tx^2 \quad x^3t'+2tx^2=x^2\sqrt{t-1}+2tx^2$$

$$x\,t'=\sqrt{t-1} \quad \frac{dt}{\sqrt{t-1}}=\frac{dx}{x} \quad (t-1)^{-\frac{1}{2}}d(t-1)=\frac{dx}{x}$$

$$-2\sqrt{t-1}=lnx+lnc \quad \sqrt{\frac{y}{x^2}-1}=-\frac{1}{2}lncx \quad \sqrt{y-x^2}=--\frac{x}{2}lncx$$

194.(357) Misol

$$(1-x^2y)dx+x^2(y-x)dy=0 \quad dx-x^2ydx+x^2ydy-x^3dy=0\;/\;x^2$$

$$\frac{dx}{x^2}-ydx+ydy-xdy=0 \quad \frac{dx}{x^2}-ydy-(ydx+xdy)=0$$

$$d\left(-\frac{1}{x}\right)+d\left(-\frac{y^2}{2}\right)-d(xy)=dC \quad -\frac{1}{x}+\frac{y^2}{2}-xy=C$$

195.(358) Misol

$$(2xe^y+y^4)y'=ye^y \quad \frac{2xe^y+y^4}{x'}=ye^y \quad x'=\frac{2xe^y+y^4}{xe^y}$$
$$=\frac{2x}{y}+\frac{y^3}{e^y} \quad x'-x\frac{2}{y}=\frac{y^3}{e^y}*e^{-\int\frac{2}{y}dy}=e^{-lny}=\frac{1}{y^2}$$

$$\left(\frac{x}{y^2}\right)'=\frac{y}{e^y} \quad \frac{x}{y^2}=\int\frac{y}{e^y}dy=\int e^{-y}ydy=\begin{pmatrix}u=y\\du=dy\\dv=e^{-y}dy\\v=-e^{-y}\end{pmatrix}$$

$x = y^2(-ye^{-y} - e^{-y} + c) = -y^3e^{-y} - y^2e^{-y} + c$

196.(359) Misol

$xy'(lny - lnx) = y$ Tenglamaning ikkala tomonini x bo'lamiz

$y'ln\frac{y}{x} = \frac{y}{x}$ $y = tx$ $y' = t + t'x$ $(t + t'x)lnt = t$

$\frac{dt}{dx}x = \frac{t(1 - lnt)}{lnt}$

$$\int \frac{lntdt}{t(1 - lnt)} = \int \frac{lntd(lnt)}{1 - lnt} = -\int \frac{\ln(t + 1 - 1)\,dlnt}{lnt - 1}$$
$$= -\int \left(1 + \frac{1}{lnt - 1}\right) dlnt = -lnt - ln|lnt - 1| - lnt - ln|lnt - 1| + lnC = lnx$$

$x = \frac{C}{t(lnt - 1)}$

Endi almashtirishni o'rniga qo'yamiz $t = \frac{y}{x}$

$$x = \frac{C}{\frac{y}{x}\left(ln\frac{y}{x} - 1\right)} \qquad Javob\text{:}\ \ y(lny - lnx - 1) = C$$

197.(360) Misol

$2y' = x + lny'$ $x = 2y' - lny'$ $y' = p$ $dy = pdx$ $dx = 2pd - \frac{dp}{p}$

$\frac{dx}{dy} = 2\frac{dp}{dy} - \frac{dp}{pdy}$ $\frac{dx}{pdx} = 2\frac{dp}{dy} - \frac{dp}{pdy}$ $\int dy = 2\int pdp - \int dp$

$y = p^2 - p + C$ $Javob\text{:}\ x = 2p - lnp,\ \ y = p^2 - p + P$

198.(361) Misol

$(x^2y - 3y^2)y' = 6x^2 - 2xy^2 + 1$

Yechilishi:

$(2x^2y - 3y^2)dy - 6x^2 - 2xy^2 + 1)dx = 0$

$(2x^2y - 3y^2)'_x = 4xy$ $F = \int (2x^2y - 3y^2)dy = x^2y^2 - y^3 + \int (x)$

$F'_x = 2y^2x + f'(x) = -6x^2 + 2xy^2 - 1$

$f'(x) = -6x^2 - 1$ $f(x) = -2x^3 - x + c_1$

$F = x^2y^2 - y^3 + c_1 - 2x^3 - x + c_2$ $c = x^2y^2 - y^3 - 2x^3 - x$

199.(363) Misol

$y^2y' + x^2sin^3x = y^3ctgx$

Yechish:

$z = y^3$ $z' = 3y^2y'$ $\frac{z'}{3} - zctgx = -3x^2sin^3x$

$z' - 3zctgx = -3x^2sin^3x \qquad e^{\int -3ctgxdx} = e^{-3ln|sinx|} = \frac{1}{|sinx|^3}$

$\left(\frac{z'}{sin^3x}\right)' = -3x^2 \qquad \frac{z'}{sin^3x} = -x^3 + c \qquad y^3 = (c - x^3)sin^3x$

200.(364) Misol

$2xy' - y = siny' \qquad y' = p \quad 2xp - y = sinp \qquad y = sinp - 2xp$

$y = 2p + 2xp' - cospp' \qquad - p = p'(2n - cosp) \quad p' = -\frac{p}{2n - cosp}$

$\frac{1}{x'} = -\frac{p}{2n - cosp} \qquad x' + \frac{2x}{p} = \frac{cosp}{p} \quad x' = -\frac{2x}{p} \qquad ln|x|$
$= -2ln|p| + lnc$

$x = Cp^{-2} \qquad x' = C'p^{-2} + C(-2p^{-1}) \qquad \frac{C'}{p^2} = \frac{cosp}{p} \quad dC = pcospdp$

$C = \int (sinp)dp = psinp - \int sinpdp = psinp = psinp + cosp + \bar{C}$

$x = \frac{(psinp + cosp + \bar{C})}{p^2}$

$xp^2 = psinp + cosp + \bar{C}, \quad y = 0$

$yp = 2psinp + 2cosp + 2\bar{C} - psinp \qquad py = 2cosp + 2\bar{C} + psinp$

$xp^2 = psinp + cosp + \bar{C}, \quad py = psinp + 2cosp + 2\bar{C} \qquad y = 0$

201.(365) Misol

$(x^2y^2 + 1)y + (xy - 1)^2 \quad xy' = 0$

Yechilishi:

$x^2y^3 + y + (x^3y^2 - 2x^2y + x)y' = 0, \qquad y = z^m \qquad y' = mz^{m-1}z'$

$x^2z^{3m} + z^m + (x^2z^{2m} - 2x^2z^m + x)mz^{m-1}z' = 0 \qquad y = \frac{1}{z}, \quad y' = -\frac{1}{z^2}$

$z = \frac{1}{y} \; y = 0 \; y' = 0{,}0 + 0 = 0$

$\frac{x^2}{z^2} + 1 - \left(\frac{x^2}{z^3} - 2\frac{x^2}{z^2} + \frac{x}{z}\right)z' = 0$

$z = tx, \;\; z' = \; t', \;\; t = \frac{x}{z}, \;\; \frac{1}{t^2} + 1 - \left(\frac{1}{t^3} - \frac{2}{t^2} + \frac{1}{t}\right)(t'x + t) = 0$

$\frac{1}{t^2} + 1 - \left(\frac{1}{t^3} - \frac{2}{t^2} + \frac{1}{t}\right)t'x - \frac{1}{t^2} + \frac{2}{t} - 1 = 0, \;\; \left(\frac{1}{t^2} + \frac{2}{t} + 1\right)t'x = 0$

$\int \frac{dt}{t^2} - 2\int \frac{dt}{t^2} + \int dt = 2\int \frac{dx}{x} \qquad x^2y^2 - xylny^2 - 1 = xylnC,$
$x^2y^2 - 1 = xylnCy^2 \quad y = 0$

201.(366) Misol

$ysinx + y'cosx = 1$

Yechilishi:

$$y\frac{sinx}{cosx}+y'=\frac{1}{cosx}, \qquad y'+ytgx=\frac{1}{cosx} \quad e^{\int tgxdx}=e^{-ln|cosx|}=\frac{1}{|cosx|}$$

$$\left(\frac{y}{cosx}\right)'=\frac{1}{cos^2x} \qquad \frac{y}{cosx}=tgx+c \quad y=cosx(tgx+c)=sinx+ccosx$$

367 Misol

$$xdy-ydy=x\sqrt{x^2+y^2}dx$$

Yechilishi:

$$x^2d\left(\frac{y}{x}\right)=xx\sqrt{1+\left(\frac{y}{x}\right)^2}dx \qquad d\left(\frac{y}{x}\right)=\sqrt{1+\left(\frac{y}{x}\right)^2}dx \qquad \frac{y}{x}=9$$

$$ln\left|y+\sqrt{1+y^2}\right|=x+e \quad \frac{y}{x}+\sqrt{1+\left(\frac{y}{x}\right)^2}=Ce^x$$

$$y+\sqrt{x^2+y^2}=xce^x \qquad x=0$$

202.(368) Misol

$$y^2-xy(y'^2+y'^3+x^2y'^5=0$$

Yechilishi:

$$D=\left(-x(y'^2+y'^3)\right)^2-4x^2y'^5=x^2(y'^2+y'^3)^2-4\,x^2y'^5=x^2(y'^2+y'^3)^2$$

$$y=\frac{x(y'^2+y'^3)\pm\sqrt{x^2(y'^2+y'^3)^2}}{2}=\frac{x(y'^2+y'^3)\pm x(y'^2+y'^3)}{2}$$

1) $y=xy'^3$ 2) $y=xy'^2$

1) $y=xy'^3\ \ ,x\neq 0 \quad y'^3=\frac{y}{x}=>\ y'=\sqrt{\frac{y}{x}}=>\frac{\partial y}{\partial x}=\frac{\partial y}{\sqrt{y}}=\frac{dx}{\sqrt[3]{x}}$

$$\int\frac{\partial y}{\sqrt{y}}=\int\frac{dx}{\sqrt[3]{x}}=>\int\frac{1}{\sqrt[3]{y}}\partial y=\int\frac{dx}{\sqrt[3]{x}}=>\frac{3\sqrt[3]{y^2}}{2}+C$$

$$\sqrt[3]{y^2}-\sqrt[3]{x^2}=C$$

2) $y=xy'^2,\ x\pm 0 \quad y'^2=\frac{y}{x}=>\ y'=\pm\sqrt{\frac{y}{x}}$

$a)$ $x>0\ \ \frac{dy}{\sqrt{y}}=\pm\frac{dx}{\sqrt{x}}=>\int\frac{dy}{\sqrt{y}}=\pm\int\frac{dx}{\sqrt{x}}=>2\sqrt{y}=\pm\sqrt{x}+C_1=>\frac{\sqrt{y}}{2}=\pm\frac{\sqrt{x}}{2}+\frac{C_1}{2}$

$y=\frac{C_1}{4}\pm 2C_1\sqrt{x}+x \quad y=C+2\sqrt{x}+x,\ \ (\frac{C_1}{4}=C=>\ C_1=>\ \sqrt{2C},C>0$

$y-x-C=\pm 2\sqrt{Cx},(y-x-C)^2=4Cx$

$(y-x)^2-2C(y-x)+C^2=4Cx, \quad (y-x)^2=2C(y+x)-C^2$

$b)$ $x<0 \qquad (y-x)^2=2C(y+x)-C^2$

3) $y=0$

Javob: $(y-x)^2=2C(y+x)-C^2, \quad (y-x)^2=2C(y+x)-C^2 \quad y=0$

203.(369) Misol

$y' = \sqrt[3]{2x-y}+2$

Yechilishi:

$2x - y = t, \qquad y' = 2 - t' \qquad 2 - t' = \sqrt[3]{t} + 2, \qquad t' = -\sqrt[3]{t}$

$\int \frac{dt}{\sqrt[3]{t}} = -\int dx, \quad \frac{3}{2}\sqrt[3]{t^2} = -x + C$

$3\sqrt[3]{(2x-y)^2} = C - 2x \qquad 27(2x-y)^2 = (C-2x)^2 \;\; y = 2x$

204.(370) Misol

$(x - y\cos\left(\frac{y}{x}\right)dx + x\cos\left(\frac{y}{x}\right)dy = 0$

Yechilishi:

$x = 0 \qquad \left(1 + \frac{y}{x}x\cos\left(\frac{y}{x}\right)\right)dx + x\cos\left(\frac{y}{x}\right)dy = 0 \qquad \frac{y}{x} = t \to dy = tdx + xdt$

$1 - t\cos t)dx + \cos t(tdx + xdt) = 0$

$dx = -\cos t x dt \qquad \frac{dx}{x} = -d(\sin t) \quad \ln|x| = -\sin t + C \qquad \ln x = -\sin\left(\frac{y}{x}\right) + C \qquad x = 0$

205.(413) Misol

$xyy' - x^2\sqrt{y^2+1} = (x+1)(y^2+1)$

Yechilishi: $\quad xyy' - x^2\sqrt{y^2+1} = (x+1)(y^2+1)$

$t^2 = y^2 + 1$

$y = \sqrt{t^2-1}$

$y' = \frac{t}{\sqrt{t^2-1}}$

$x\sqrt{t^2-1}\frac{tt\prime}{\sqrt{t^2-1}} - x^2t = (x+1)t^2$

$xtt' - x^2t = (x+1)t^2$

$xt' - x^2 = (x+1)t$

$t' - \frac{x+1}{x}t - x = 0$

$t' - \frac{x+1}{x}t = 0$

$\frac{dt}{t} = \left(1 + \frac{1}{x}\right)dx$

$\ln(t) = x + \ln(x) + C_0$

$t = xe^xC$

$t' = e^x(C'x + Cx + C)$

$e^x(C'x + Cx + C) - \left(\frac{x+1}{x}\right)xe^xC = x$

$e^xC'x + e^xCx + e^xC - xe^xC - e^xC = x$

$e^xC' = 1$

$C' = e^{-x}$

$$dc = e^{-x}dx$$
$$C = -e^{-x} + C_1$$

206.(414) Misol

$$(x^2-1)y' + y^2 - 2xy + 1 = 0$$

Yechilishi: $(x^2-1)y' + y^2 - 2xy + 1 = 0$

$$(x^2-1)dy = (2xy - y^2 - 1)dx$$
$$x^2dy - dy = 2xydx - y^2dx - dx$$
$$x^2dy - dy = x^2dx - x^2dx + 2xydx - y^2dx - dx$$
$$dx - dy - dx(x-y)^2 = x^2dx - x^2dy$$
$$d(x-y) - dx(x-y)^2 = x^2d(x-y)$$
$$d(x-y)(1-x^2) = dx(x-y)^2$$
$$\int \frac{d(x-y)}{(x-y)^2} = \int \frac{dx}{1-x^2}$$
$$\frac{1}{x-y} = \frac{1}{2}ln\left|C_1\frac{1+x}{1-x}\right|$$
$$2 = (x-y)ln\left|C_1\frac{1+x}{1-x}\right|$$
$$2 = (y-x)ln\left|C_2\frac{1-x}{1+x}\right|$$

207.(415) Misol

$$y'tgy + 4x^3cosy = 2x$$

Yechilishi: $y'tgy + 4x^3cosy = 2x$

$$\cos y = u$$
$$du = -\sin y\, dy$$
$$u'_x = -y_x{}'\sin y$$
$$-\frac{u_x{}'}{u} + 4x^3u = 2x$$
$$\frac{u_x{}'}{u^2} + \frac{2x}{u} = 4x^3$$
$$z = \frac{1}{u}$$
$$dz = -\frac{du}{u^2}$$
$$-z'_x + 2xz = 4x^3$$
$$\frac{dz}{dx} = 2zx$$
$$lnz = x^2 + C_1$$
$$z = C_2e^{x^2}$$
$$z' = C_2{}'e^{x^2} + 2xC_2e^{x^2}$$
$$-\left(C'_2e^{x^2} + 2xC_2e^{x^2}\right) + 2xC_2e^{x^2} = 4x^3$$
$$C'_2 = -\frac{4x^3}{e^{x^2}}$$
$$dC_2 = -\frac{4x^3dx}{e^{x^2}}$$
$$x^2 = p$$
$$4x^3dx = 2pdp$$

$$C_2 = -\int \frac{2pdp}{e^p} = \frac{2(p+1)}{e^p} + C$$
$$-\int \frac{2pdp}{e^p} = -\left[-\frac{2p}{e^p} + 2\int e^{-p}dp\right] = -\left[-\frac{2p}{e^p} - \frac{2}{e^p}\right] = \frac{2(p+1)}{e^p}$$
$$du = 2dp$$
$$u = -e^{-p}$$
$$z = \left[\frac{2(x^2+1)}{e^{x^2}} + C\right]e^{x^2}$$
$$\frac{1}{\cos y} = 2(x^2+1) + Ce^{x^2}$$

208.(417) Misol

$$(x+y)(1-xy)dx + (x+2y)dy = 0$$

Yechilishi: $(x+2y)dy = (-x + x^2y - y + xy^2)dx$
$$xdy + 2ydy + ydx = (xy^2 + x^2y - x)dx$$
$$2ydy + ydx + xdy = x(y^2 + xy - 1)dx$$
$$\frac{d(y^2)+d(xy)}{y^2+xy-1} = xdx$$
$$d\big(ln(y^2+xy-1)\big) = d\left(\frac{x^2}{2}\right)$$
$$ln(y^2+xy-1) = \frac{x^2}{2} + C$$
$$y^2 + xy - 1 = Ce^{\frac{x^2}{2}}$$

209.(418) Misol

$$(3xy + x + y)ydx + (4xy + x + 2y)xdy = 0$$

Yechilishi:
$$3xy^2dx + xydx + y^2dx + 4x^2ydy + x^2dy + 2xydy = 0 \qquad |\cdot y^2x$$
$$3x^2y^4dx + x^2y^3dx + xy^4dx + 4x^3y^3dy + x^3y^2dy + 2x^2y^3dy = 0$$
$$d(x^3y^4) + \frac{1}{3}d(x^3y^3) + \frac{1}{2}d(x^2y^4) + d(x^3y^4) + \frac{1}{3}d(x^3y^3) + \frac{1}{2}d(x^2y^4) = 0$$
$$d(x^3y^4) + \frac{1}{3}d(x^3y^3) + \frac{1}{2}d(x^2y^4) = 0$$
$$6x^3y^4 + 2x^3y^3 + 3x^2y^4 = const$$

§-8.
n-tartibli oddiy differensial tenglamalar

Ushbu

$$F(x, y, y', ..., y^{(n)}) = 0 \qquad (1)$$

ko'rinishdagi tenglama n**- tartibli oddiy differensial tenglama deyiladi**. Faraz qilaylik (1) tenglamani $y^{(n)}$ga nisbatan yechish mumkin:

$$y^{(n)} = f(x, y, y', ..., y^{(n-1)}) \qquad (2).$$

Bu tenglama **yuqori tartibli hosilaga nisbatan yechilgan** n **- tartibli oddiy differensial tenglama** deyiladi.

Agar I intervalda uzluksiz n marta differensiallanuvchi $y = y(x)$ funksiya uchun shu intervalda $F(x, y, y'(x), ..., y^{(n)}(x)) \equiv 0$ ayniyat o'rinli bo'lsa, u holda $y = y(x)$ funksiyani (1) tenglamaning I intervaldagi **yechimi** deb ataymiz.

(2) tenglama uchun **Koshi masalasi.** (2) tenglamaning barcha $y = y(x)$ yechimlari orasidan

$$y(x_0) = y_0,\ y'(x_0) = y'_0,\ \ldots,\ y_0^{(n-1)}(x_0) = y_0^{(n-1)} \qquad (3)$$

tengliklarni qanoatlantiruvchi yechimni toping, bu erda (3) tengliklar boshlang'ich shart, $x_0, y_0, y'_0, ..., y_0^{(n-1)}$ sonlar esa boshlang'ich qiymatlar deyiladi.

(1) tenglama uchun **Koshi masalasi** ham (1) tenglamaga qo'yilganidek keltiriladi. Lekin (1) tenglamani (3) boshlang'ich shartni qanoatlantiruvchi har qanday ikkita $y_1(x)$ va $y_2(x)$ yechimlari uchun $y_1^{(n)}(x_0) \neq y_2^{(n)}(x_0)$ munosabat o'rinli bo'lsa Koshi masalasining yechimi mavjud va yagona hisoblanadi. Agar (1) tenglamaning (3) shartni qanoatlantiruvchi yechimi topilmasa yoki ikkita $y_1(x)$ va $y_2(x)$ yechimlari (3) shartni va $y_1^{(n)}(x_0) = y_2^{(n)}(x_0)$ tenglikni qanoatlantirsa Koshi masalasi yechimining mavjudligi va yagonaligi buziladi deb aytamiz.

D orqali shunday $(x, y, y', ..., y^{(n-1)})$ nuqtalar to'plamini belgilaylikki bu nuqtada (2) tenglama uchun qo'yilgan Koshi masalasi yagona yechimga ega bo'lsin. Agar 1) $C_1, C_2, ..., C_n$ parametrlarning ihtiyoriy qiymatida ham $y = \varphi(x, C_1, C_2, ..., C_n)$ funksiya (2) tenglamani qanoatlantirsa; 2) D to'plamdan olingan har bir $(x, y, y', ..., y^{(n-1)})$ nuqta uchun

$$\begin{cases} y = \varphi(x, C_1, C_2, ..., C_n) \\ y' = \varphi'(x, C_1, C_2, ..., C_n) \\ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \\ y^{(n-1)} = \varphi^{(n-1)}(x, C_1, C_2, ..., C_n) \end{cases} \qquad (4)$$

sistemanini $C_1, C_2, ..., C_n$ larga nisbatan bir qiymatli yechish mumkin bo'lsa u holda $y = \varphi(x, C_1, C_2, ..., C_n)$ funksiyani (2) differensial tenglamaning D to'plamdagi **umumiy yechimi** deb ataymiz.

Umumiy yechimning bitta muhim hossasini aytib o'tamiz. (4) sistemadan aniqlangan $C_1, C_2, ..., C_n$ parametrlarning qiymatlarini $y^{(n)} = \varphi^{(n)}(x, C_1, C_2, ..., C_n)$ tenglikka qo'ysak (2) tenglama hosil bo'ladi. Bu n parametrli chiziqlar oilasining differensial tenglamasini tuzish qoidasi hamdir.

Misol. $y'' + y = 0$ tenglamani $y(0) = 1$, $y'(0) = 0$ boshlang'ich shartni qanoatlantiruvchi yechimini topaylik. Berilgan tenglamaning umumiy yechimi $y = C_1 \cos x + C_2 \sin x$ formula bilan ifodalanadi. Bundan Koshi maslasining yechimini aniqlash mumkin: $y = \cos x$.

(1) tenglama $y^{(n)} = f_k(x, y, y', ..., y^{(n-1)})$, $k = 1, ..., m$ tenglamalarga ajratilishi mumkin bo'lsin. Bu yuqori tartibli hosilaga nisbatan yechilgan tenglamalarning umumiy yechimlari to'plami (1) tenglamaning **umumiy yechimi** deyiladi.

Misol. $(y'')^2 = x^4$ tenglamani qaraylik. Bu tenglama ikkita $y'' = x^2$ va $y'' = -x^2$ differensial tenglamaga ajraladi. Ularning umumiy yechimlarini mos ravishda yozamiz: $y = \frac{x^4}{12} + C_1 x + C_2$, $y = -\frac{x^4}{12} + C_1 x + C_2$. Ular birgalikda berilgan tenglamaning umumiy yechimini beradi.

Bizga

$$\varphi(x, y, y', ..., y^{(k)}, C_1, C_2, ..., C_{n-k}) = 0 \qquad (5)$$

tenglik berilgan bo'lsin. Bu tenglikni x bo'yicha $n-k$ marta differensiallab hosil bo'lgan $n-k$ ta tenglikdan $C_1, C_2, ..., C_{n-k}$ parametrlarni yo'qotsak natijada (1) tenglama hosil bo'lsa (5)ni (1) differensial tenglamaning **oraliq integrali** deb ataymiz. Hususan (5) tenglikda faqat bitta o'zgarmas parametr qatnashsa u (1) differensial tenglamaning **birinchi integrali** deyiladi.

Misol. $y'' = 2\sqrt{y'}$ ikkinchi tartibli differensial tenglamaning birinchi integrali $y' - (x + C_1)^2 = 0$ tenglik bo'lishini tekshiramiz. Bu tenglikni x bo'yicha differensiallaylik: $y'' - 2(x + C_1) = 0$. Bundan: $y'' = 2(x + C_1) = 2\sqrt{y'}$, berilgan tenglama hosil bo'ldi.

§-9.

n -tartibli differensial tenglamalarning kvadraturalarda integrallanuvchi ba'zi turlari

Dastlab

$$y^{(n)} = f(x) \qquad (1)$$

ko'rinishdagi tenglamani o'rganaylik. Agar $f(x)$ funksiya I intervalda uzluksiz bo'lsa bu tenglamani kvadraturalarda integrallash mumkin. (1) tenglamani n marta ketma-ket integrallab, umumiy yechimini topamiz:

$$y(x) = \int_{x_0}^{x}\int_{x_0}^{x}...\int_{x_0}^{x} f(x)dxdx...dx + \frac{C_1(x-x_0)^{n-1}}{(n-1)!} + ... + C_{n-1}(x-x_0) + C_n \qquad (2)$$

Quydagi Direhle formulasini isbotlaymiz:

$$\underbrace{\int_{x_0}^{x}\int_{x_0}^{x}...\int_{x_0}^{x}}_{n\ ta} f(x)\underbrace{dxdx...dx}_{n\ ta} = \frac{1}{(n-1)!}\int_{x_0}^{x} f(z)(x-z)^{n-1}dx$$

Isbotni matematik induksiya usulida olib boramiz.

2. Endi

$$F(x, y^{(n)}) = 0 \qquad (3)$$

ko'rinishdagi tenglamani qaraymiz. Agar $F(\varphi(t), \psi(t)) \equiv 0$ ayniyat o'rinli bo'lsa, u holda (3) tenglamani kvadraturalarda integrallash mumkin. Bu erda $x = \varphi(t)$, $y^{(n)} = \psi(t)$ tengliklarga egamiz. Bundan

$$dy^{(n-1)} = y^{(n)}dx = \psi(t)\varphi'(t)dt$$

$$y^{(n-1)} = \int \psi(t)\varphi'(t)dt + C_1 = \psi_1(t, C_1)$$

Bu erdan esa

$$dy^{(n-2)} = y^{(n-1)}dx = \psi_1(t, C_1)\varphi'(t)dt$$

$$y^{(n-2)} = \int \psi_1(t, C_1)\varphi'(t)dt + C_2 = \psi_2(t, C_1, C_2)$$

Shunday mulohazalar yuritib (3) tenglamaning umumiy yechimini parametrik ko'rinishda hosil qilamiz:

$$x = \varphi(t), \qquad y = \psi_n(t, C_1, C_2, ..., C_n).$$

Misol. $e^{y''}+y''=x$ bu tenglamada $y''=t$, $x=e^t+t$ almashtirsak ayniyat hosil bo'ladi. Bu tengliklarga ko'ra

$$dy'=y''dx=t(e^t+1)dt \qquad y'=(t-1)e^t+\frac{t^2}{2}+C_1$$

$$dy=y'dx=\left((t-1)e^t+\frac{t^2}{2}+C_1\right)(e^t+1)$$

$$y=\left(\frac{t}{2}-\frac{3}{4}\right)e^{2t}+\left(\frac{t^2}{2}+C_1-1\right)e^t+\frac{t^3}{6}+C_1t+C_2.$$

Javob: $x=e^t+t$, $y=\left(\frac{t}{2}-\frac{3}{4}\right)e^{2t}+\left(\frac{t^2}{2}+C_1-1\right)e^t+\frac{t^3}{6}+C_1t+C_2$

3. $F(y^{(n-1)},y^{(n)})=0$ (4) ko'rinishdagi tenglamani qaraylik. Bu tenglamani $y^{(n)}$ga nisbatan yechish mumkin bo'lsin: $y^{(n)}=f(y^{(n-1)})$. Agar $z=y^{(n-1)}$ desak $z'=f(z)$ - o'zgaruvchilari ajraladigan differensial tenglamaga kelamiz. Uning umumiy yechimi $z=\omega(x,C_1)$ bo'lsa belgilashimiz bo'yicha

$$y^{(n-1)}=\omega(x,C_1)$$

tenglamaga ega bo'lamiz. Bu tenglama (1) ko'rinishga ega va uni integrallay olamiz.

Agar (4) tenglamani $y^{(n)}$ga nisbatan yechish mumkin bo'lmasa, lekin $F(\varphi(t),\psi(t))\equiv 0$ ayniyat o'rinli bo'lsa, u holda ham (4)ni kvadraturalarda integrallay olamiz. Bu erda $y^{(n-1)}=\varphi(t)$, $y^{(n)}=\psi(t)$ tengliklarga egamiz. Bundan

$$dy^{(n-1)}=y^{(n)}dx \qquad dx=\frac{dy^{(n-1)}}{y^{(n)}}=\frac{\varphi'(t)dt}{\psi(t)}$$

Bu erdan $x=\int\frac{\varphi'(t)dt}{\psi(t)}+C_1=\psi_1(t,C_1)$, $y^{(n-1)}=\varphi(t)$ tengliklarga egamiz. 2-punktda aynan shunday parametrik tengliklarga ega bo'lgan holda (3) tenglamani integrallashni ko'rgan edik. Ana shu mulohazalarni takrorlab (4) tenglamani umumiy yechimini hosil qilish mumkin.

4. $F(y^{(n-2)},y^{(n)})=0$ (5) ko'rinishdagi tenglamani o'rganamiz. Faraz qilaylik bu tenglamani $y^{(n)}$ga nisbatan yechish mumkin bo'lsin: $y^{(n)}=f(y^{(n-2)})$. Bu erda $y^{(n-2)}=z$ deb olsak $z''=f(z)$ tenglamaga kelamiz. Uni $2z'dx$ ga ko'paytiramiz:

$$2z'z''dx = 2f(z)zdx \qquad d(z'^2) = 2f(z)dz$$

$$z'^2 = 2\int f(z)dz + C_1 \qquad z' = \sqrt{2\int f(z)dz + C_1}$$

Ohirgi o'zgaruvchilari ajraladigan tenglamani yechimi $z = \varphi(x, C_1, C_2)$ bo'lsin. Belgilashimizga ko'ra $y^{(n-2)} = \varphi(x, C_1, C_2)$ (1) k'rinishdagi tenglamaga kelamiz.

Agar (5) tenglamani $y^{(n)}$ga nisbatan yechish mumkin bo'lmasa, lekin $F(\varphi(t), \psi(t)) \equiv 0$ ayniyat o'rinli bo'lsa, u holda ham (5)ni kvadraturalarda integrallay olamiz. Bu erda $y^{(n-2)} = \varphi(t)$, $y^{(n)} = \psi(t)$ tengliklarga egamiz. Bundan

$$dy^{(n-1)} = y^{(n)}dx \qquad dy^{(n-2)} = y^{(n-1)}dx$$

$$y^{(n-1)}dy^{(n-1)} = y^{(n)}dy^{(n-2)} \qquad d\left[y^{(n-1)}\right]^2 = 2\psi(t)\varphi'(t)dt$$

$$y^{(n-1)} = \sqrt{2\int \psi(t)\varphi'(t)dt + C_1} = \psi_1(t, C_1)$$

Demak biz $y^{(n-1)} = \sqrt{2\int \psi(t)\varphi'(t)dt + C_1} = \psi_1(t, C_1)$ va $y^{(n-2)} = \varphi(t)$ tengliklarga egamiz. 3-punktda bunday tengliklarga ega bo'lgan holda (4) tenglamani integrallashni ko'rganmiz. Ana shu mulohazalarni yuritib (5) tenglamani integrallash mumkin.

2-reja. A. $F(x, y^{(k)}, y^{(k+1)}, ..., y^{(n)}) = 0$ ko'rinishdagi tenglamalarda $z = y^{(k)}$ almashtirish yordamida yangi z funksiya kiritsak tenglama tartibi k ga kamayadi, ya'ni $F(x, z', z'', ..., z^{(n-k)}) = 0$ tenglamaga kelamiz.

B. $F(y, y', ..., y^{(n)}) = 0$ ko'rinishdagi tenglamalarda y ni erkli o'zgaruvchi deb hisoblab, $y' = z$ almashtirish bilan yangi z funksiyani kiritsak berilgan tenglamaning tartibi bittaga kamayadi.

Misol. $(1 + y^2)yy'' = (3y^2 - 1)y'^2$ tenglamada $z = y'$ desak $y'' = z'y' = z'z$. Bundan $(1 + y^2)yz'z = (3y^2 - 1)z^2$ o'zgaruvchilari ajraladigan tenglama.

C. Agar $F(x, y, y', ..., y^{(n)}) = 0$ tenglamada F funksiya $y, y', ..., y^{(n)}$larga nisbatan bir jinsli bo'lsa, ya'ni $F(x, ty, ty', ..., ty^{(n)}) = t^m F(x, y, y', ..., y^{(n)})$ tenglik o'rinli bo'lsa, u holda $z = \dfrac{y'}{y}$ almashtirish bilan yangi funksiya tenglama tartibini bittaga kamaytirish mumkin.

Misol. $xyy'' + xy'^2 - yy' = 0$ tenglama y, y', y'' larga nisbatan bir jinslidir. $y' = yz$

$y'' = y'z + yz' = y(z^2 + z')$. Bundan $xy^2(z^2 + z') + xy^2z^2 - y^2z = 0$ yoki $xz' + 2xz^2 - z = 0$. Bernulli tenglamasi hosil bo'ldi.

D. Agar $F(tx, t^k y, t^{k-1}y', ..., t^{k-n}y^{(n)}) = t^m F(x, y, y', ..., y^{(n)})$ tenglik o'rinli bo'lsa $F(x, y, y', ..., y^{(n)}) = 0$ tenglama umumlashgan bir jinsi deyladi. Bu tenglamada $x = e^t$, $y = ze^{kt}$ almashtirish bajarsak erkli o'zgaruvchi t, noma'lum funksiya z dan iborat tartibi $n-1$ ga teng differensial tenglama hosil bo'ladi.

E. Agar $F(x, y, y', ..., y^{(n)}) = 0$ tenglamaning chap tomoni biror $\Phi(x, y, y', ..., y^{(n-1)})$ funksiyadan x bo'yicha olingan hosilaga teng bo'lsa, ya'ni $F(x, y, y', ..., y^{(n)}) = \frac{d}{dx}\Phi(x, y, y', ..., y^{(n-1)})$ tenglik o'rinli bo'lsa u holda qaralayotgan tenglamaning birinchi integrali $\Phi(x, y, y', ..., y^{(n-1)}) = C_1$ dan iborat. Demak bu holda englama tartibi bittaga kamayadi.

Misol. $\frac{y''}{(1+y'^2)^{\frac{3}{2}}} = 0$ tenglamaning chap tomoni $\frac{y'}{\sqrt{1+y'^2}}$ ifodaning to'liq differensialidan iborat. Demak $\frac{y'}{\sqrt{1+y'^2}} = C_1$ tenglamaning birinchi integralidan iborat.

210.(431) Misol

$2yy' = y^2 + y'^2 \quad y' = p \qquad 2ypp' = y^2 + p^2 \quad y'' = pp'$

$2p' = \frac{y}{p} + \frac{p}{y} \quad p = uy \quad 2u'y + 2u = \frac{1}{u} + u \quad p' = u'y + u$

$$2u'y = \frac{1}{u} - u \qquad \int \frac{udu}{1-u^2} = \int \frac{dy}{2y} \qquad -\frac{1}{2}ln|1-u^2| = \frac{1}{2}lnyc$$

$$yc = \frac{1}{1-u^2} \quad 1-u^2 = \frac{1}{cy} \quad u = \sqrt{1-\frac{1}{cy}} \quad u^2 = 1-\frac{1}{cy}$$

$p = \sqrt{y^2 - \frac{y}{c}} \quad y' = \sqrt{y^2 - \frac{y}{c}}$

$$\int \frac{cdy}{\sqrt{cy^2 - y}} = \int dx$$

211.(432) Misol

$.y''^3 + xy'' = 2y \qquad z = y' \quad z'^3 + xz' = 2z \quad p = z' = \frac{dz}{dx}$

$dz = pdx \quad p^3 + xp = 2z \qquad 3p^2dp + xdp + pdx = 2pdx$

$3p^2dp = pdx - xdp \qquad 3p^2dp = p^2d\left(\frac{x}{p}\right)$

$p = 0 => \; z' = 0 => y'' = 0 => y' = c => y = c$

$\int \; 3dp = \int d\left(\frac{x}{p}\right); \; 3p + c = \frac{x}{p}; \; x = p^2 + cp$

p^3 +p$(3p^2 + cp) = 2z \quad 4p^3$+$cp^2 = 2y' = \frac{dy}{dx}$;

$2dy = (4p^3 + cp^2)dx \quad dx = (6p + cdp)$;

$2dy = (4p^3 + cp^2)(6p + c)dp$; $2dy = (24p^4 + 4cp^3 + 6cp^3 + c^2p^2)$dp;

$\int dy = \int 12p^4 + 5$ +cp^3+$\frac{1}{2}c_1p^2dp$

$y = \frac{12}{5}p^5 + \frac{5}{4}cp^4$+$\frac{1}{6}c^2p^3 + c_2$;

$x = 3p^2 + c_1p \;\; y = \frac{12}{5}p^5 + \frac{5}{4}c_1p^4 + \frac{1}{6}c_1 + c_2 \quad y = c$

212.(433) Misol

$y''^2 + y' = xy'' \qquad y' = z \;\; y'' = z'$

$(z')^2 + z = xz' \qquad z' = pdzpdx$

$$p^2 + px$$

$$x = \frac{p^2 + z}{p}$$

$$dx = p\left[\frac{2p^2 - p^2z}{p^2}dp + \frac{1}{p}dz\right]$$

$$dz = \frac{p^2 - z}{p}dp + dz$$

$$p^2 - z = 0$$

$z = 2p^2 \; x = \frac{2p^2}{p} = 2p$

$z = \frac{x^2}{4} \quad y' = \frac{x^2}{4}$

$$\int dy = \frac{1}{4}\int x^2dx$$

$$y = \frac{x^3}{12} + c$$

213.(434) Misol

$y'' + (y')^2$ =$2e^{-y} \qquad y' = p \quad y'' = pp'$

$pp' + p^2$ +$2e^{-y}$

$$\frac{1}{2}(p^2) + p^2 = e^{-p}$$

$$p^2 = t$$

$$t' + 2t = 4e^{-y} \qquad t' + 2t = 0$$
$$\frac{dt}{dy} = 2t$$
$$ln|t| = 2y + c$$
$$t = c(y)e^{-2y}$$
$Ce^{-2y} - 2Ce^{-2y} + 2Ce^{-2y} = 4e^{-y}$
$C = 4e^{y}$
$$C = e^{y} + C_1$$
$$t = 3e^{-y} + C_1 e^{-2y}$$
$(p^2) = 4e^{-y} + 4ce^{-2y}$
$(y')^2 = 4e^{-y} + 4ce^{-2y}$
$$\frac{dy}{2\sqrt{e^{-y+Ce^{-2y}}}} = dx$$
$$\frac{1}{2}\int \frac{dy}{\sqrt{e^{-y} + ce^{-2y}}} = x + c_1$$
214.(435) Misol
$xy''' = y'' - xy'' \quad y'' = z(x), xz'(x) = (1-x)z(x)$
$\frac{dx}{z} = (\frac{1}{x} - 1 - 1)dx \ ln|x| = ln|x| - x + lnc_1$
$z = c_{1xe^{-x}}, y'' = c_1 xe^{-x}$
$y = c_1 e^{-x}(x+2) + c_2 x + c_3$

215.(436) Misol
$y''^2 = y'^2 + 1$
$$p(y) = y'$$
$$y'' = \frac{d(y')}{dx} = \frac{dp}{dx} = \frac{dp}{dy}\frac{dy}{dx} = p'p$$
$$p'p = p^2 + 1$$
$$p' = \mp\frac{\sqrt{p^2+1}}{p}$$
$$\int dy = \int \frac{pdp}{\sqrt{p^2+1}}$$
$$\int \frac{pdp}{\sqrt{p^2+1}}$$
$u = p^2 + 1 \quad du = 2pdp \ \ du = 2pdp => pdp = \frac{du}{2}$
$$\int \frac{pdp}{\sqrt{p^2+1}} = \frac{1}{2}\int \frac{du}{\sqrt{u}} = \sqrt{u} + c$$
$$\sqrt{u} + c = \sqrt{p^2+1} + c$$
$$y = -\sqrt{p^2+1} + c$$
$$y + c = \sqrt{p^2+1}$$

$$y + c = -\sqrt{p^2 + 1}$$
$$(y + c)^2 = p^2 + 1$$
$$p = \sqrt{(y + c)^2 - 1}$$
$$p = -\sqrt{(y + c)^2 - 1}$$
$$p = \frac{dy}{dx} = y'$$
$$\frac{dy}{dx} = \sqrt{(y + c)^2 - 1}$$
$$\frac{dy}{dx} = -\sqrt{(y + c)^2 - 1}$$
$$\int dx = \int \frac{dy}{\sqrt{(y + c)^2 - 1}}$$
$$\int dx = -\int \frac{dy}{\sqrt{(y + c)^2 - 1}}$$
$$y = c_1 + ch(x + c_2)$$
$$y = -c_1 + ch(x + c_2)$$

216.(438) Misol

$y'' - xy'' + y''^3 = 0$ $\quad y' = p \quad y''' = p' \quad p = p(x) \quad p - xp + p'^3 = 0$

$p' = t \quad p - xt + t^3 = 0 \quad p = xt - t^3 \quad dp = tdx$

$xdt - 3t^2dt = 0dy \quad (x - 3t^2)dt = 0dy \quad x = 3t^2 \quad t' = 0$

$$t = \sqrt{\frac{x}{3}}$$
$$t = -\sqrt{\frac{x}{3}}$$

$$y'' = xy''' - y'''^3 = \pm\left(\frac{x^{\frac{3}{2}}}{\sqrt{3}} - \frac{x^{\frac{3}{2}}}{3\sqrt{3}}\right) = \pm\frac{2x^{\frac{3}{2}}}{3\sqrt{3}}$$

$y = \frac{c_{1x^3}}{6} - \frac{{c_1}^3x^3}{2} + c_2x + c_3$

217.(439) Misol

$2y'(y'' + 2xz'^2) \quad z' = p \quad 2z(p + 2) = xp \quad z = \frac{xp^2}{2(p + 2)}$

$dz = pdx = \frac{p^2dx}{2(p + 2)} + \frac{x(p^2 + 4p)dp}{2(p + 2)} \quad \frac{dx}{x} = \frac{dp}{p + 2} \quad p \neq 0 \quad p \neq 2 \quad p \neq -4$

$x = C_1(p + 2) \quad z = \frac{c_1p^2}{2} \quad z(x) = \frac{(x - c_1)}{c_1} \quad p = 0 => z = 0 \quad p = -4 \quad z = -4x \quad y = \int zdx \quad 3c_1y = (x - c_1)^2 + c_2 \quad y = c \quad y = -2x^2$

218.(491) Misol

$yy'' = y'^2 = 2xy^2$

$\frac{y''}{y} = \frac{y'^2}{y^2} + 2x \quad \frac{y'}{y} = p \quad \frac{y''y - y'^2}{y^2} = p'$

$\frac{y''}{y} - \frac{y'^2}{y^2} = p' \quad \frac{y''}{y} - p' + p^2$

$p' + p^2 = p^2 + 2x \qquad p' = 2x \quad p = x^2 + C$

$\frac{y'}{y} = x^2 + C$

Javob: $\frac{y'}{y} = x^2 + C$

219.(492) Misol

$$y''^4 = y'^5 - yy'^3y''$$

$y' = p \quad y'' = p\,p'$

$p\,p'^4 = p^5 - yp^3\,p\,p'$

$p'^4 = p^4 - yp^3p'$

Javob: $p'^4 + yp^3p' = p^4$

220.(493) Misol

$2yy''' = y'$

$y' = p \quad y'' = p\,p' \quad y''' = p'^2 + pp'^2 = p'^2(1+p)$

$2pp'^2(1+p) = p^3 \quad p'^2 = \frac{p^2}{2(1+p)}$

Javob: $p' = \frac{p^2}{2(1+p)}$

221.(494) Misol

$y'''y'^2 = 1$

$y' = p \quad y'' = p\,p' \quad y''' = p'^2 + pp'^2 = p'^2(1+p)$

$p'^2(1+p)p^2 = 1 \quad p'^2 = \frac{1}{p^2(1+p)}$

Javob: $p'^2 = \frac{1}{p^2(1+p)}$

222.(495) Misol

$y^2y''' = y'^3$

$y' = p \quad y'' = p\,p' \quad y''' = p'^2 + pp'^2 = p'^2(1+p)$

$y^2p'^2(1+p) = p^3 \quad p'^2 = \frac{p^3}{y^2(1+p)}$

Javob: $p'^2 = \frac{p^3}{y^2(1+p)}$

223.(496) Misol

$x^2yy'' + 1 = (1-y)xy'$

$y' = p \quad y'' = p'$

$x^2yp' + 1 = (1-y)xp$

$x^2yp' - (1-y)xp + 1 = 0$

Javob: $x^2yp' - (1-y)xp + 1 = 0$

224.(497) Misol

$$yy'y'' + 2y'^2y'' = 3yy''^2$$

$yy' + 2y'^2 = 3yy''$

$y' = p \quad y'' = p\,p'$

$yp\,p\,p' + 2p\,p'p^2 = 3yp\,p'^2$

$yp + 2p^2 = 3yp' \quad p' = \frac{yp+2p^2}{3y}$

Javob: $p' = \frac{yp+2p^2}{3y}$

225.(498) Misol

$$(y'y'' - 3y''^2)y = y'^5$$

$y' = p \quad y'' = p\,p'$

(p $p\,p' - 3p\,p'^2$) $\quad y = p^5$

Javob: ($pp' - 3\,p'^2$) $\quad y = p^4$

226.(499) Misol

$$y^2(y'y'' - y''^3) = y'^5$$

$y' = p \quad y'' = p\,p'$

$y^3(ppp' - p\,p'^3) = p^5$

$y^3(pp' - p'^3) = p^4$

Javob: $y^3(pp' - p'^3) = p^4$

227.(500) Misol

$x^2(yy''' - y'^3) = 2y^2y' - 3xyy'^2$

$y' = p \quad y'' = p' \quad y''' = p''$

$x^2(yp'' - p^3) = 2y^2p - 3xyp^2$

$$p'' = \frac{2y^2p - 3xyp^2 + x^2p^3}{yx^2}$$

$p' = t \quad p'' = t'$

$$t' = \frac{2y^2p - 3xyp^2 + x^2p^3}{yx^2}$$

Javob:

$$t' = \frac{2y^2p - 3xyp^2 + x^2p^3}{yx^2}$$

227(501).

$yy'' = 2xy'^2;\ y(2) = 2,\ \ y'(2) = 0.5$

$y' = yz(x) \quad z' = z^2(2x-1) \quad z = \frac{1}{c_1 + x - x^2} = \frac{y'}{y} \quad c_1 = 6$

$\int \frac{dx}{(x+2)(3-x)} = \ln y - \ln c_2$

$y = c_2 \sqrt[5]{\frac{2+x}{3-x}} \quad x = 2\ y = 2 \quad c_2 = \sqrt[5]{8}$

$y = \sqrt[5]{8\frac{2+x}{3-x}}$

228(502).

$y''' - 3y'^2 = 0 \quad y(0) = -3\ y'(0) = 1\ y''(0) = -1$

$y'' - 3y' = 0 \qquad z = -\sqrt[2]{p^3}$

$y' = p \quad y'' = p'\ y''' = p''$

$p' = -\sqrt[2]{p^3}$

$2p'' - 3p^2 = 0$

$\frac{dp}{dx} = -\sqrt[2]{p^3}$

$p' = z(p) \quad p' = z'z \qquad \frac{dp}{\sqrt{p3}} = -dx \qquad -2p^{-\frac{1}{2}} = -x +$

$2z'z - 3p^2 = 0$

c_1

$2\frac{dz}{dp}z = 3p^2 \quad 2zdz = 3p^2dp \quad p(0) = y'(0) = 1 \quad c_2 = -2$

$-2p^{-\frac{1}{2}} = -x - 2 \quad z^2 = p^3 + c$

$\frac{-2}{\sqrt{p}} = -x - 2 \qquad z(p) = p' = y'' => p'(0) = y''(0) = -1$

$\sqrt{p} = \frac{2}{x+2} \quad p = \frac{4}{(x+2)^2} \quad y' = \frac{4}{(x+2)^2} \quad z(1) = -1 \quad 1 = 1 + c \ => c =$
0

$z^2 = p^3$

$\frac{dy}{dx} = \frac{4}{(x+2)^2} \quad y = -\frac{4}{x+2} + c_1 \quad -3 = -2 + c_2 \ \ c_2 = -1 \quad => \ ; y = -\frac{4}{x+2} -$
1

229(503).

$x^2y'' - 3xy' = \frac{6y^2}{x^2} - 4y \quad y(1) = 1\ y'(1) = 4$

Bu umumlashtirilgan bir xil tenglama va $m = 2$, shuning uchun biz $x = e^t$ o'zgarishni qilamiz.

$y = e^{2t}u(t)$. Biz tenglamani olamiz.

$u'' - 6u^2 = 0$

uning ikkala qismini ham u′ ga ko'paytiramiz

$u'^2 = 4u^3 + c_1$ ga ega bo'lamiz.

$y = 2$ va $x = 1$ bo'lganligi uchun$u(0) = 1$ almashtirishdan $y' = e^t(u' + 2u)$ va $y'(1) = 4$ kelib chiqadi.

$u'(0) + 2u(0) = 4$ $\quad u'(0) = 2$ $t = 0$ $\quad c_1 = 0$

$$u'^2 = 4u^3$$

Bizda $u' = u^{\frac{3}{2}}\, du = 2dt$ $,\frac{1}{\sqrt{u}} = t + c_2$ va $u = \frac{1}{(t+c_2)}$

C_2 doimiyni aniqlash uchun *u(0=1)*shartidan foydalanamiz. Natijada $c_2=\pm1$ kelib chiqadi .Demak

$u = \frac{1}{(t\pm1)^2}$ lekin $u'(0) = 2$ bo'lganligi uchun $u = \frac{1}{(t-1)^2}$ni olamiz .

$y = \frac{x^2}{(lnx-1)^2}$ javobni olamiz .

230(504).

$y''' = 3yy';$ $y(0) = -2$ $y'(0) = 0$ $y''(0) = 4'5$

$y' = p$ $y'' = pp'$ $y''' = p(pp'' + p'^2)$ $\qquad p(pp'' + p'^2)$

$= 3yp$ $p(pp'' + p'^2 - 3y) = 0$

$p = 0$ $y' = 0$ $y = c_1x + c_2$

$\int(-3y + p'^2 + pp')dy = \int 0\, dy \quad -1{,}5y^2 + pp' = c_3$

$p = \pm\sqrt{y^3 + 2c_3} + c_4$

$\int \frac{dy}{\pm\sqrt{y^3+2c_3}+c_4} = \int dx$

$x + c_5 = \int \frac{dy}{\pm\sqrt{y^3+2c_3}+c_4}$

231(505). $y'' = cosy + y'^2 siny = y'$

$y(-1) = \frac{\pi}{6}$ $y'(-1) = 2$ $\quad y' = p(y)$

deb belgilash kiritsak

$p'cosy + psiny = 1$ tenglama kelib chiqadi .

Umumiy natija

$p = siny + c_1 siny \quad yoki \quad y' = siny + c_1 cosy$

c_1 ni topish uchun $y'(-1) = 2$ ni $y = \frac{\pi}{6}$ da topamiz .Natijada $c_1 = \sqrt{3}$.Keyin tenglamani integrallash orqali

$\frac{1}{2}\int \frac{dy}{\cos(y-\frac{\pi}{6})} = x + c_2$ yoki $\frac{1}{2}lntg\left(\frac{y}{2} + \frac{\pi}{6}\right) = x + c_2$ ni topamiz.

$x = -1\ y = \frac{\pi}{6}$ ligidan $c_2 = 1$. shartga ko'ra

$$x = -1 + \frac{1}{2} lntg\left(\frac{y}{2} + \frac{\pi}{6}\right)$$ bo'ladi.

232(507).

Shunday egri chiziqlarni topingki,egrilik radiusi urinma va abscissa o'qi orasidagi burchak kosinusiga teskari proporsional bo'lsin.

Topshiriq shartiga ko'ra $R = \frac{k}{cos\alpha}$ bo'lsa R –egri chizig'i ,α –burchak ,k-proporsionallik koiffitsenti.

$R = \frac{(1+y'^2)^{\frac{3}{2}}}{|y''|}$, $\alpha = \text{arctg}y'$ y' ligidan

Oxirgi tenglamani integrallab

$$arctgy' = \frac{x}{k} + c_1$$

yoki $y' = tg(\frac{x}{k} + c_1\ )$ topamiz.

Yana bir marta integrallab yakuniy natija olamiz .

$$y = -klncos\left(\frac{x}{k} + c_1\right) + c_2$$

508.

Cho'zilmas ipning chetlari mahkamlangan bo'lib,ipning har bir uzuznlik birligiga gorizantal tashkil etuvchisi bir hil bo'lgan yuk tasir qiladi(osma ko'prik).Ipning muvozanat holatini aniqlang.Ipning og'irligi hisobga olinmasin.

ΔS elementning erkin muvozanatini ko'rib chiqamiz .mentning erkin muvozanatini ko'rib chiqamiz .Jism harakatini Ox va Oy o'qlarida ko'rib chiqamiz .

$$-T(x)cosa(x) + T(x + \Delta x)cosa(x + \Delta x) = 0$$

$$-T(x)sina(x) + T(x + \Delta x)sina(x + \Delta x) - \Delta P = 0$$

Bu yerda $T(x)$ipning X kordinatadagi taranglik qiymatlari,α(x) oX va ip orasidagi burchak ,

ΔP
$-\Delta S\ elementning\ og'irligi\ (yoki\ har\ qanday\ taqsimlangan\ yukning\ qiymati).$
Birinchi tenglamadan kelib chiqadiki $T(x)\cos\alpha(x) = T_0$=const yani ip tarangligining gorizantal tashkil etuvchisi doim o'zgarmas bo'ladi .Ikkinchi tenglamadan kelib chiqadiki

$$d\big(T(x)sina(x)\big) = dP(x) \quad yoki \quad T_0 d\big(tga(x)\big) = dP(x), \qquad T_0 dy' = dP(x)$$

Quyidagi masalada $dP(x) = kdx$, k-proporsionallik koiffitsenti .(1)tenglamadan), T_0dy'=kdx kelib chiqadi. Bu tenglikni ikki marta integrallab ipning muvozanat holat tenglamasini olamiz

$$y = \frac{k}{2T_0}x^2 + c_1x + c_2$$

233(509).
chetlari mahkamlab qo'yilgan bir jinsli cho'zilmagan ipning o'z o'irligi ta'sirida muvozanat holatini aniqlang
dP(x)=ρgdS
ρg −ip uzunligining birligi dS=$\sqrt{1+y'^2}$dx shu ipning diferensial tenglamasini tuzamiz

$T_0 y'' = \rho g\sqrt{1+y'^2}$ yoki $\frac{y''}{\sqrt{1+y'^2}} = a^2 \quad a^2 = \frac{\rho g}{T_0}$

$$\frac{y''}{\sqrt{1+y'^2}}\left(= \ln\left(y' + \sqrt{1+y'^2}\right)\right)$$

$$y' + \sqrt{1+y'^2} = e^{a^{2(x+c_1)}}$$
$$y' = 0{,}5(e^{a^{2(x+c_1)}} - e^{a^{-2(x+c_1)}})$$

Tenglamani yana bir marta integrallab quyidagi tenglamani olamiz

$$y = \frac{1}{2a^2}\left(e^{a^{2(x+c_1)}} + e^{a^{-2(x+c_1)}}\right) + c_2 \quad \text{yoki} \quad y = \frac{1}{a^2}ch(a^2x + c_1) + c_2$$

234(510).

$y'' + siny = 0$ mayatnik harakat tenglamasi. $x > +\infty$ $y > \pi$, shartni qanoatlantiruvchi hususiy yechimga egaligini isbotlang **.**

Tenglamani ikki tomonini y' ga ko'paytirsak va natijani integrallasak $y' = 2C_1 + 2cosy$

Quyidagi tenglamada $y'(x) > 0$ va $x > +\infty$ ligidan$y'(x) > \pi$ va $x > +\infty$ $C_1 = 1$

$$y'^2 = 2(cosy + 1)$$

$$\pm \int \frac{dy}{\sqrt{2(1+cosy)}} = \mathrm{x} + C_1$$

$$0.5 \int \frac{dt}{cos\frac{t}{2}} = x + lnc_2 \quad (0 < y < \pi$$

Bu tenglamaning yechimi mavjud .chap tomonni integrallagandan so'ng bizda quyidagi tenglik hosil bo'ladi.

$\ln\big(tg0.25(\pi + y)\big) = x + lnC_2$ $yoki$ $y = arctg(C_2 e^x) - \pi$

Demak $C_2 > 0$ bo'lsa $y(x) > \pi$ $x > \infty$ manashu shartni qanoatlantiruvchi hususiy yechimga ega bo'ladi.

n-tartibli chiziqli differensial tenglamalar quyidagi ko'rinishda yoziladi:

$$y^{(n)} + p_1(x)y^{(n-1)} + ... + p_{n-1}(x)y' + p_n(x)y = g(x) \qquad (1)$$

Agar bu tenglamaning $p_1(x),\ p_2(x),\ ...,\ p_n(x),\ g(x)$ koeffisientlari I intervalda uzluksiz bo'lsa, u holda (1) tenglamaning $y(x_0) = y_0, y'(x_0) = y'_0, ..., y^{(n-1)}(x_0) = y_0^{(n-1)}$ boshlangich shartni qanoatlantiruvchi $y = y(x)$ yechimi mavjud va yagona, bu erda $x_0 \in I$.

Ta'rif. n-tartibli chiziqli bir jinsli differensial tenglamaning n ta $y_1, y_2, ..., y_n$ yechimi I intervalada chiziqli erkli bo'lsa ular **fundamental yechimlar sistemasi** deb ataladi.

§-10.
n-tartibli chiziqli o'zgarmas koeffisientli differensial tenglamalar

Reja

Ushbu

$$L(y) \equiv y^{(n)} + a_1 y^{(n-1)} + a_2 y^{(n-2)} + ... + a_{n-1} y' + a_n y = 0 \qquad (2)$$

tenglama **n-tartibli chiziqli bir jinsli o'zgarmas koeffisientli differensial tenglama** deyiladi. (2) tenglamaning hususiy yechimini $y = e^{\lambda x}$ ko'rinishda qidiraylik, bu erda λ - biror o'zgarmas (haqiqiy yoki kompleks) son. Buni (2)ning chap qismiga qo'yamiz:

$$L(e^{\lambda x}) = [\lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + ... + a_{n-1}\lambda + a_n]e^{\lambda x} = P(\lambda)e^{\lambda x}.$$

Bundan ko'rinadiki $y = e^{\lambda x}$ funksiya (2) tenglamaning yechimi bo'lishi uchun λ son

$$P(\lambda) = \lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + ... + a_{n-1}\lambda + a_n = 0 \quad (3)$$

tenglamaning ildizi bo'lishi zarur va yetarli. (3) tenglama (2) bir jinsli chiziqli tenglamaning **harakteristik tenglamasi**, uning ildizlari esa **harakteristik sonlari** deyiladi.

agar (2) tenglamaning $\lambda_1, \lambda_2, ..., \lambda_n$ **harakteristik sonlari haqiqiy va turlicha** bo'lsa, u holda (4) yechimlar haqiqiy funksiyalar bo'lib, (2) bir jinsli chiziqli tenglamaning **umumiy yechimi** $y = C_1e^{\lambda_1 x} + C_2e^{\lambda_2 x} + ... + C_ne^{\lambda_n x}$ formula bilan ifodalanadi.

harakteristik sonlar jufti ikkita $y_1 = e^{ax}\cos bx$, $y_2 = e^{ax}\sin bx$ hususiy haqiqiy yechimini aniqlaydi. Hullas, (2) tenglamaning harakteristik sonlari turlicha bo'lganda biz hamma vaqt n ta haqiqiqy yechimga ega bo'lamiz va ularning ihtiyoriy chiziqli kombinatsiyasi tenglamaning umumiy yechimini aniqlaydi.

Misol. $y''' - 3y'' + 9y' + 13y = 0$ tenglamani qaraylik. Uning harakteristik tenglamasi $\lambda^3 - 3\lambda^2 + 9\lambda + 13 = 0$. Harakteristik sonlar $\lambda_1 = -1$, $\lambda_2 = 2 + 3i$, $\lambda_2 = 2 - 3i$. Demak e^{-x}, $e^{2x}\cos 3x$, $e^{2x}\sin 3x$ funksiyalar berilgan tenglamaning chiziqli fundamental yechimlar sistemasini tashkil etadi. Umumiy yechim:

$$y = C_1e^{-x} + C_2e^{2x}\cos 3x + C_3e^{2x}\sin 3x$$

Endi harakteristik tenglama ildizlari orasida karralilari ham bor deb faraz qilaylik.

Algebra kursidan ma'lumki n-tartibli algebraik chiziqli tenglama hamma vaqt n ta ildizga ega, ya'ni $\lambda_1, \lambda_2, ..., \lambda_r$ sonlar (3) tenglamaning mos ravishda $k_1, k_2, ..., k_r$ karrali ildizlari bo'lsa u holda $k_1 + k_2 + ... + k_r = n$ tenglik o'rinli bo'ladi. Hulosa qilib aytganda, (2) tenglamaning harakteristik sonlari qanday bo'lmasin biz hamma vaqt n ta haqiqiy yechimga ega bo'lamiz va ularning ihtiyoriy chiziqli kombinatsiyasi korinishida tenglamaning umumiy yechimini aniqlaymiz.

Misol. $y^{(5)} - y^{(4)} + 8y''' - 8y'' + 16y' - 16y = 0$ tenglamani qaraymiz. Harakteristik tenglama $\lambda^5 - \lambda^4 + 8\lambda^3 - 8\lambda^2 + 16\lambda - 16 = 0$. Uning ildizlari: $\lambda_1 = 1$, $\lambda_2 = \lambda_3 = 2i$, $\lambda_4 = \lambda_5 = -2i$. Bu harakteristik sonlarga mos hususiy yechimlar:

$$e^x,\ \cos 2x,\ \sin 2x,\ x\cos 2x,\ x\sin 2x$$

Berilgan tenglamaning umumiy yechimi:

$$y = C_1e^x + (C_2 + C_3x)\cos 2x + (C_4 + C_5x)\sin 2x.$$

Chiziqli bir jinsli bo'lmagan o'zgarmas koeffisientli tenglamalar

n-tartibli chiziqli bir jinsli bo'lmagan ozgarmas koeffisientli tenglama

$$L(y) \equiv y^{(n)} + a_1y^{(n-1)} + a_2y^{(n-2)} + ... + a_{n-1}y' + a_ny = f(x) \quad (1)$$

korinishga ega, bu yerda $a_1, a_2, ..., a_n$ - o'zgarmas haqiqiqy sonlar. Oldingi darsda biz har qanday n-tartibli chiziqli bir jinsli o'zgarmas koeffisientli tenglamani umumiy yechimini qurishni o'rgandik. U holda (1) tenglamaning umumiy yechimini o'zgarmasni variatsiyalash usulida kvadraturalarda topa olamiz. Lekin $f(x)$ funksiyaning ayrim hususiy ko'rinishlarida (1)tenglamaning hususiy yechimi kvadraturalarsiz aniqlanadi. Bunday holatlarda, bir jinsli tenglamaning umumiy yechimiga bu hususiy yechimni qo'shib (1) tenglamaning umumiy yechimini kvaraturalarsiz hosil qilamiz.

(1) tenglamada $f(x)$ funksiya ko'phad va ko'rsatkichli funksiyaning ko'paytmasidan iborat bo'lsin, ya'ni

$$f(x) = (p_0x^m + p_1x^{m-1} + ... + p_{m-1}x + p_m)e^{\alpha x} = P(x)e^{\alpha x}.$$

bu yerda $p_0, p_1, ..., p_m, \alpha$ - o'zgarmas sonlar (ular nolga teng bo'lishi ham mumkin). (1) tenglamaning hususiy yechimini qidirishni 2ta holatga ajratib olib boramiz.

1-holat. α - tenglamaning harakteristik soni emas, ya'ni $P(\alpha) \neq 0$. Bu holatda y_1 hususiy yechimni

$$y_1 = (q_0x^m + q_1x^{m-1} + ... + q_{m-1}x + q_m)e^{\alpha x} \qquad (2)$$

ko'rinishda qidiramiz, bu yerda $q_0, q_1, ..., q_m$ - noma'lum o'zgarmas sonlar. (2) funksiyani (1) tenglamaga qo'yamiz:

$$L[(q_0x^m + q_1x^{m-1} + ... + q_{m-1}x + q_m)e^{\alpha x}] = q_0L(x^me^{\alpha x}) + q_1L(x^{m-1}e^{\alpha x}) + ... +$$
$$+ q_{m-1}L(xe^{\alpha x}) + q_mL(e^{\alpha x}) = (p_0x^m + p_1x^{m-1} + ... + p_{m-1}x + p_m)e^{\alpha x}.$$

Bu yerda $L(e^{\alpha x}) = P(\alpha)e^{\alpha x}$ va $L(x^se^{\alpha x}) = \sum_{i=0}^{s} C_s^i P^{(i)}(\alpha)x^{s-i}e^{\alpha x}$ formulalardan foydalanaylik:

$$q_0\sum_{i=0}^{m} C_m^i P^{(i)}(\alpha)x^{m-i}e^{\alpha x} + q_1\sum_{i=0}^{m-1} C_{m-1}^i P^{(i)}(\alpha)x^{m-1-i}e^{\alpha x} + ... + q_{m-1}\sum_{i=0}^{1} C_1^i P^{(i)}(\alpha)x^{1-i}e^{\alpha x} +$$

$$+ q_mP(\alpha)e^{\alpha x} = (p_0x^m + p_1x^{m-1} + ... + p_{m-1}x + p_m)e^{\alpha x}$$

Ohirgi tenglikni $e^{\alpha x}$ ga bo'lamiz va x ning bir hil darajalari oldidagi koefisientlarni tenglashtiramiz:

$$x^m: \quad q_0P(\alpha) = p_0$$
$$x^{m-1}: \quad q_0C_m^1P'(\alpha) + q_1P(\alpha) = p_1$$
$$.. \qquad (3)$$
$$x^1: \quad q_0C_m^{m-1}P^{(m-1)}(\alpha) + q_1C_{m-1}^{m-2}P^{(m-2)}(\alpha) + ... + q_{m-1}P(\alpha) = p_{m-1}$$
$$x^0: \quad q_0C_m^mP^{(m)}(\alpha) + q_1C_{m-1}^{m-1}P^{(m-1)}(\alpha) + ... + q_{m-1}P'(\alpha) + q_mP(\alpha) = p_m$$

$P(\alpha) \neq 0$ bo'lgani uchun (3) tengliklardan $q_0, q_1, ..., q_m$ koeffisientlarning barchasi ketma-ket va bir qiymatli aniqlanadi.

Misol. $y'' - 5y' + 6y = 6x^2 - 10x + 2$ tenglamani umumiy yechimini topaylik.

Bir jinsli tenglama: $z'' - 5z' + 6z = 0$. Uni integrallaymiz; $\lambda^2 - 5\lambda + 6 = 0$, $\lambda_1 = 2$, $\lambda_2 = 3$; $z = C_1e^{2x} + C_2e^{3x}$. Berilgan tenglamaning o'ng tomoni $P(x)e^{0x}$ ko'rinishida. $\alpha = 0$ harakteristik tenglamaning ildizi emas. Tenglamaning hususiy yechimini $y_1 = ax^2 + bx + c$

ko'rinishda qidiramiz: $y_1' = 2ax + b,\ y_1'' = 2a$. Bularni tenglamaga qo'yamiz:

$$6ax^2 + (6b - 10a)x + 6c - 5b + 2a = 6x^2 - 10x + 2$$

x ning bir hil darajalari oldidagi koeffisientlarni tenglasak quydagi sistema hosil bo'ladi:

$$\begin{cases} 6a = 6 \\ 6b - 10a = -10 \\ 6c - 5b + 2a = 2 \end{cases}$$

Bu yerdan $a = 1,\ b = 0,\ c = 0$. Demak $y_1 = x^2$ va berilgan tenglamaning umumiy yechimi $y = C_1 e^{2x} + C_2 e^{3x} + x^2$.

2-holat. α - harakteristik tenglamaning k ($k \ge 1$) karrali ildizi bo'lsin, ya'ni

$$P(\alpha) = P'(\alpha) = ... = P^{(k-1)}(\alpha) = 0, \qquad P^{(k)}(\alpha) \neq 0.$$

Bu holatda y_1 hususisy yechimni (2) ko'rinishda qurib bo'lmaydi, chunki $P(\alpha) = 0$. Husussiy yechimni

$$y_1 = x^k (q_0 x^m + q_1 x^{m-1} + ... + q_{m-1} x + q_m) e^{\alpha x} \qquad (4)$$

ko'rinishda qidiramiz. (4)funksiyani (1) tenglamaga qo'yamiz:

$$L\left(\sum_{s=0}^{m} q_s x^{k+m-s} e^{\alpha x}\right) = \sum_{s=0}^{m} q_s \sum_{i=k}^{k+m-s} C_{k+m-s}^{i} P^{(i)}(\alpha) x^{k+m-s-i} e^{\alpha x} = \sum_{s=0}^{m} p_s x^{m-s} e^{\alpha x}$$

Bundan $\sum_{s=0}^{m} q_s \sum_{i=k}^{k+m-s} C_{k+m-s}^{i} P^{(i)}(\alpha) x^{k+m-s-i} = \sum_{s=0}^{m} p_s x^{m-s}$. Bu yerda x ning bir hil darajalari oldidagi koefisientlarni tenglashtiramiz:

$$\begin{aligned}
x^m &: \quad q_0 C_{k+m}^{k} P^{(k)}(\alpha) = p_0 \\
x^{m-1} &: \quad q_0 C_{k+m}^{k+1} P^{(k+1)}(\alpha) + q_1 C_{k+m-1}^{k} P^{(k)}(\alpha) = p_1 \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (5) \\
x^1 &: \quad q_0 C_{k+m}^{k+m-1} P^{(k+m-1)}(\alpha) + q_1 C_{k+m-1}^{k+m-2} P^{(k+m-2)}(\alpha) + ... + q_{m-1} C_{k+1}^{k} P^{(k)}(\alpha) = p_{m-1} \\
x^0 &: \quad q_0 P^{(k+m)}(\alpha) + q_1 P^{(k+m-1)}(\alpha) + ... + q_{m-1} P^{(k+1)}(\alpha) + q_m P^{(k)}(\alpha) = p_m
\end{aligned}$$

Bu tenglilardan izlanayotgan barcha $q_0, q_1, ..., q_m$ koeffisientlar bir qiymatli aniqlnadi, chunki $P^{(k)}(\alpha) \neq 0$.

Misol. $y'' - 5y' = -5x^2 + 2x$ tenglamani qaraymiz. Unga mos bir jinsli tenglama: $z'' - 5z' = 0$. Bir jinsli tenglamani yechamiz: $\lambda^2 - 5\lambda = 0$, $\lambda_1 = 0$, $\lambda_2 = 5$. $z = C_1 + C_2 e^{5x}$. Berilgan tenglamaning o'ng tomoni $P(x)e^{0x}$ ko'rinishga ega va $\alpha = 0$ harakteristik tenglamaning oddiy (k=1 karrali) ildizi. Shuning uchun tenglamaning hususiy yechimini $y_1 = x(ax^2 + bx + c)$ ko'rinishda qidiramiz: $y_1' = 3ax^2 + 2bx + c$, $y_1'' = 6ax + 2b$. Bularni berilgan tenglamaga qo'yamiz:

$$-15ax^2 + (6a - 10b)x + 2b - 5c = -5x^2 + 2x$$

Bundan $a = \frac{1}{3}$, $b = 0$, $c = 0$. Berilgan tenglamaning umumiy yechimi:

$$y = C_1 + C_2 e^{5x} + \frac{x^3}{3}.$$

2-reja. Endi (1) tenglamaning o'ng tomoni

$$f(x) = e^{ax}[P_m^{(1)}(x)\cos bx + P_m^{(2)}(x)\sin bx]$$

ko'rinishga ega bo'lganda uning hususiy yechimini qidirish usulini ko'rib chiqamiz, bu yerda $P_m^{(1)}(x)$ va $P_m^{(2)}(x)$ – m-darajali berilgan ko'phadlar. Buning uchun 2ta holatni ajratib olamiz.

1-holat. $\alpha = a + ib$ - harakteristik tenglamaning ildizi emas. Bu holatda hususiy yechimni $y_1 = e^{ax}[Q_m^{(1)}(x)\cos bx + Q_m^{(2)}(x)\sin bx]$ ko'rinishda qidiramiz, bu yerda $Q_m^{(1)}(x)$ va $Q_m^{(2)}(x)$ - m darajali koeffisentlari noma'lum ko'phadlar.

Misol. $y'' + y' - 2y = e^x(\cos x - 7\sin x)$ tenglamani umumiy yechimini topamiz (bu yerda $a = 1$, $b = 1$). Avval chiziqli bir jinsli tenglamani yozaylik: $z'' + z' - 2z = 0$. Uni integrallaymiz:

$$\lambda^2 + \lambda - 2 = 0,\ \lambda_1 = 1,\ \lambda_2 = -2,\ z = C_1 e^x + C_2 e^{-2x}.$$

$a + ib = 1 + i$ son harakteristik tenglamaning ildizi eamas. Shuning uchun Berilgan tenglamaning hususiy yechimini $y_1 = e^x(A\cos x + B\sin x)$ ko'rinishda qidiramiz:

$$y_1'' + y_1' - 2y_1 = e^x[(-A + 3B)\cos x - (B + 3A)\sin x] = e^x(\cos x - 7\sin x)$$

$$\begin{cases} -A + 3B = 1 \\ B + 3A = 7 \end{cases}$$

Bundan $A = 2$, $B = 1$. $y_1 = e^x(2\cos x + \sin x)$. Berilgan tenglamaning umumiy yechimi: $y_1 = e^x(2\cos x + \sin x) + C_1 e^x + C_2 e^{-2x}$.

2-holat. $\alpha = a + ib$ - harakteristik tenglamaning k $(k \geq 1)$ karrali ildizi. Bu holatda hususiy yechimni $y_1 = x^k e^{ax}[Q_m^{(1)}(x)\cos bx + Q_m^{(2)}(x)\sin bx]$ ko'rinishda qidiramiz, bu yerda $Q_m^{(1)}(x)$ va $Q_m^{(2)}(x)$ -- m-darajali koeffisentlari noma'lum ko'phadlar.

Misol. $y'' + y = 2\sin x$ tenglamani qaraymiz (bu erda $a = 0$, $b = 1$).

$$z'' + z = 0,\ \lambda^2 + 1 = 0,\ \lambda_{1,2} = \pm i,\ z = C_1\cos x + C_2\sin x.$$

$a + ib = i$ son harakteristik tenglamaning oddiy ildizi (bir karrali) bo'lgani uchun berilgan tenglamaning hususiy yechimini $y_1 = x(A\cos x + B\sin x)$ ko'rinishda qidiramiz: $A = -1$, $B = 0$, $y_1 = -x\cos x$ hosil bo'ladi. Berilgan tenglamanig umumiy yechimi: $y = -x\cos x + C_1\cos x + C_2\sin x$.

235(511)-misol

O'zgarmas koefitsentli chiziqli tenglamalarni yeching.

$y'' + y' - 2y = 0$

Yechilishi:

$y'' + y' - 2y = 0$

$y' = ʎ \quad ʎ^2 + ʎ - 2 = 0$

$\lambda_1 = 1 \quad \lambda_2 = -2 \qquad\qquad Javob: y = C_1 e^x + C_2 e^{-2x}$

236(512)-misol
O'zgarmas koeffitsentli chiziqli tenglamani yeching
$y'' + 4y' + 3y = 0$

Yechilishi:
$y'' + 4y' + 3y = 0 \quad y' = \lambda \qquad \lambda^2 + 4\lambda + 3 = 0$

$\lambda_1 = -1 \quad \lambda_2 = -3$
$Javob: y = C_1 e^{-x} + C_2 e^{-3x}$
237(513)-misol
O'zgarmas koefitsentli chiziqli tenglamani yeching.
$y'' - 2y' = 0$

Yechilishi:
$y'' - 2y' = 0 \qquad y(0) = 0 \quad y'(0) = 2 \quad y' = \lambda \quad \lambda^2 - 2\lambda = 0$
λ^2-2λ=0
$\lambda_1 = 0 \quad \lambda_2 = 2 \qquad javob: \ y = C_1 + C_2 e^{2x}$

238(514)-misol. O'zgarmas koefitsentli chiziqli tenglamani yeching
$y'' - 5y' + 2y = 0$

Yechilishi:
$y'' - 5y' + 2y = 0 \qquad 2\lambda^2 - 5\lambda + 2 = 0 \quad \lambda_1 = \frac{1}{2} \quad \lambda_2 = 2 \qquad y = C_1 e^{\frac{1}{2}} + C_2 e^{2x}$

239(515)-misol

O'zgarmas koefitsentli chiziqli tenglamani yeching
$y'' - 4y' + 5y = 0$

Yechilishi:
$y'' - 4y' + 5y = 0 \qquad \lambda^2 - 4\lambda + 5 = 0$

$\lambda_1 = 2 + \sqrt{4 - 5} = 2 + i$
$\lambda_2 = 2 - \sqrt{4 - 5} = 2 - i$

$$y = C_1e^{(2+i)x} + C_2e^{(2+i)x} = C_1e^{2x}e^{ix} + C_2e^{2x}e^{-ix}$$
$$= C_1e^{2x}(cosx + isinx) + C_2e^{2x}(cosx - isinx)$$
$$= e^{2x}(C_1cosx + C_2sinx$$

240(516)-misol
O’zgarmas koefitsentli chiziqli tenglamani yeching
$y'' + 2y' + 10y = 0$

Yechilishi:
$$y'' + 2y' + 10y = 0$$
$$ʎ_1 = -1 + 3i \quad ʎ_2 = -1 - 3i$$
$y = C_1e^{-x}\cos 3x + C_2e^{-x}\sin 3x = e^{-x}(C_1cos3x + C_2\sin 3x$

241(517)-misol
O’zgarmas koefitsentli chiziqli tenglamani yeching
$y'' + 4y = 0$

Yechilishi:
$y'' + 4y = 0$
$$ʎ^2 + 4 = 0$$
$$ʎ^2 = -4$$
$$ʎ_1 = 2i \quad ʎ_2 = -2i$$
$$y = C_1e^{2ix} + C_2e^{-2ix} = y = C_1cos2x + C_2sin2x$$
$$Javob\text{: } y = C_1cos2x + C_2sin2x$$
242(518)-misol. O’zgarmas koefitsentli chiziqli tenglamani yeching
$y'' - 8y = 0$

Yechilishi:
$y'' - 8y = 0$
$ʎ^3 - 8 = 0 \qquad ʎ_1 = 2 \quad ʎ_2 = -1 + i\sqrt{3} \quad ʎ_3 = -1 - i\sqrt{3}$
$cos\varphi = \frac{1}{2}\left(e^{-i\varphi} + e^{-\sqrt{3x}}\right) \qquad sin\varphi = \frac{1}{2i}\left(e^{i\varphi} - e^{-i\varphi}\right)$
$y = C_1e^{2x} + \left(C_2cos\sqrt{3x} + C_3sin\sqrt{3x}\right)$

243(519)-misol. O’zgarmas koefitsentli chiziqli tenglamani yeching
$$y'''' - y = 0$$

Yechilishi:
$$y'''' - y = 0$$
$ʎ^4 - 1 = 0 \qquad ʎ_1 = 1 \qquad ʎ_2 = -1 \quad ʎ_3 = i \quad ʎ_4 = -i$
$y = C_1e^{x} + C_2e^{-x} + C_3sinx + C_4cosx$

520-misol

O’zgarmas koefitsentli chiziqli tenglamani yeching

$y'''' + 4y = 0$

Yechilishi:

$y'''' - y = 0$

$\lambda^4 + 4 = 0 \qquad \lambda_1 = 1 + i \qquad \lambda_2 = 1 - i \quad \lambda_3 = -1 - i \quad \lambda_4 = -1 + i$

$$y = \left(C_1 e^{ix} + C_2 e^{-ix}\right) e^{x} + \left(C_3 e^{ix} + C_4 e^{-ix}\right) e^{-x}$$

$y = (\,C_1 cosx + C_2 sinx)e^{x} + (\,C_3 cosx + C_4 sinx)$

244(541)-misol.

$$y'' + 3y' - 4y = e^{-4x} + xe^{-x}$$

$\lambda^2 + 3\lambda - 4 = 0$

$\lambda_1 = 4 \quad \lambda_2 = -1$

$y = C_1 e^{-4x} + C_2 e^{x}$

$y = axe^{-4x} + be^{-4x}$

$$y' = -4axe^{-4x} + ae^{-4x} - 4be^{-4x}$$

$$y'' = e^{-4x}(16ax - 8a + 16b)$$

$e^{-4x}(16ax - 8a + 16b - 12ax + 3a - 12b - 4ax - 4b) = e^{-4x}$

$$-5a = 1$$

$$a = -\frac{1}{5}$$

$$y = -\frac{1}{5}e - 4x$$

$$y = (ax + b)e^{-x}$$

$$y' = e^{-x}(a - ax - b)$$

$$y'' = e^{-x}(-6ax + a - 6b)$$

$$e^{-x}(-6ax + a - 6b) = xe^{-x}$$

$\begin{cases} -6a = 1 \\ a - 6b = 0 \end{cases} \quad a = -\frac{1}{6} \quad b = -\frac{1}{36}$

$$y = e^{-x}\left(-\frac{1}{6}x - \frac{1}{36}\right)$$

$y = C_1 e^{-4x} + C_2 e^{x} - \frac{1}{5}e^{-4x} - e^{-x}\left(\frac{1}{6}x + \frac{1}{36}\right)$

245(542)-misol.

$$y'' + 2y - 3y = x^2 e^{x}$$

$$\lambda^2 + 2\lambda - 3 = 0$$

$$\lambda_1 = -3 \quad \lambda_2 = 1$$

$$y = C_1 e^{-3x} + C_2 e^{x}$$

$$y = x(ax^2 + bx + c)e^{x} = (ax^3 +$$

$$+bx^2 + cx)e^{x}$$

$$y' = (ax^3 + (3a + b)x^2 + (2b + c)x)e^{x}$$

$$y'' = (3ax^2 + (6a + 2b)x + 2b + c$$

$$+ax^3 + (3a + b)x^2 + (2b + c)x +$$

$$+c)e^{x} = (ax^3 + (6a + b)x^2 +$$

$$+(6a + 4b + c)x + 2b + 2c)e^{x}$$

$$x^3 + (6a + 2b)x^2 + (4b + 2c)x +$$

$$+2c - 3ax^3 - 3bx^2 - 3cx = x^2 e^{x}$$

$$12ax^2+(6a+8b)x+2b+4c=x^2$$

$$\begin{cases}12a=1\\6a+8b=0\\2b+4c=0\end{cases}$$

$a=\frac{1}{12}$ $b=-\frac{1}{16}$ $c=\frac{1}{32}$

$$y=(\frac{x^3}{12}-\frac{x^2}{16}+\frac{x}{32})e^x$$
$$y=C_1e^{-3x}+C_2e^x+$$
$$+(\frac{x^3}{12}-\frac{x^2}{16}+\frac{x}{32})e^x$$

246(543)-misol.

$$y''-4y'+8y=e^{2x}+\sin 2x$$
$$y''-4y'+8y=$$
$$=e^{2x}+\frac{1}{2i}e^{2ix}-\frac{1}{2i}e^{-2ix}$$

$\lambda^2-4\lambda+8=0$ $\lambda_1=2+2i$

$$\lambda_2=2-2i$$
$$y=e^{(2+2i)x}C_1+e^{(2-2i)x}C_2$$

$y_1=e^{2x}a+e^{2xi}b+e^{-2ix}c$

$$y'=2ae^{2x}+2ibe^{2ix}-2ice^{-2ix}$$
$$y''=4ae^{2x}-4be^{2ix}-4ce^{-2ix}$$
$$4ae^{2x}-4be^{2ix}-4ce^{-2ix}-8ae^{2x}$$
$$-8ibe^{2ix}+8ice^{-2ix}+8ae^{2x}+$$
$$+8be^{2ix}+8ce^{-2ix}=$$
$$=e^{2x}+\frac{e^{2ix}}{2i}-\frac{e^{-2ix}}{2i}$$

$a=\frac{1}{4}$ $b=-\frac{i-2}{40}$ $c=\frac{i-2}{40}$

$$y=e^{2x}\left(C_1e^{2ix}+C_2e^{-2ix}\right)+\frac{1}{4}e^{2x}$$
$$-\frac{i-2}{40}e^{2ix}+\frac{i-2}{40}e^{-2ix}$$

247(544)-misol.

$y''-9y=e^{3x}\cos x$

$\lambda^2-9=0$ $\lambda_1=3$ $\lambda_2=-3$

$$y=C_1e^{-3x}+C_2e^{3x}$$
$$y_1=e^{3x}(a\cos x+b\sin x)$$

$y'=3y_1+e^{3x}(-a\sin x+b\cos x)$

$$y''=3y'+3x+e^{3x}(-a\cos x-b\sin x)$$
$$9y_1+6x+e^{3x}(-a\cos x-b\sin x)$$
$$-9y_1=e^{3x}\cos x$$
$$e^{3x}((-6a-b)\sin x+$$
$$+(6b-a)\cos x)=e^{3x}\cos x$$

$\begin{cases}-6a-b=0\\6b-a=1\end{cases}$ $\begin{cases}a=-\frac{1}{37}\\b=\frac{6}{37}\end{cases}$

$$y=C_1e^{-3x}+C_2e^{3x}+e^{3x}(-\frac{\cos x}{37}+\frac{6}{37}\sin x)$$

248(545)-misol.

$y'' - 2y' + y = 6xe^x$

$$\lambda^2 - 2\lambda + 1 = 0 \quad \lambda = 1$$
$$y = e^x(C_1 + C_2x) \quad (1)$$
$$y_1 = x^2(ax + b)e^x =$$
$$= e^x(ax^3 + bx^2) \quad (2)$$
$$y' = y_1 + e^x(3ax^2 + 2bx)$$
$$y'' = y' + e^x(3ax^2 + 2bx) +$$
$$+e^x(6ax + 2b)$$
$$y' + e^x(3ax^2 + 2bx) +$$
$$+e^x(6ax + 2b) - 2y' + y_1 = 6xe^x$$
$$e^x(6ax + 2b) = 6xe^x \quad \begin{cases} a = 1 \\ b = 0 \end{cases}$$
$$(1) + (2)\ ) \quad <=>$$
$$y = e^x(C_1 + C_2x) + e^x x^3 = e^x(x^3$$
$$+C_1 + C_2x)$$

249(546)-misol.

$y'' + y = x \sin x$

$\lambda = \pm i$

$$C_1 \cos x + C_2 \sin x$$
$$y_2 = x\big((Ax + B)\sin x + (Cx + D)\cos x\big) =$$
$$Ax^2 \sin x + Bx \sin x +$$
$$+Cx^2 \cos x + Dx \cos x$$
$$y' = 2Ax \sin x + Ax^2 \cos x +$$
$$+B \sin x + Bx \cos x + 2Cx \cos x$$
$$-Cx^2 \sin x + D \cos x - Dx \sin x$$
$$y'' = (2D + 2A)\sin x + \cos x *$$
$$* (2B + 2C) + 4Ax \cos x -$$
$$-4Cx \sin x = x \sin x$$
$$A - D = 0 \quad B + C = 0$$
$$4A = 0 \quad -4C = 1$$

$D = 0 \quad A = 0 \quad B = \frac{1}{4} \quad C = -\frac{1}{4}$

$$y = C_1 \cos x + C_2 \sin x + \frac{x}{4}\sin x -$$
$$-\frac{x^2}{4}$$

250(547)-misol.

$y'' + 4y' + 4y = xe^{2x}$

$$\lambda^2 + 4\lambda + 4 = 0 \quad \lambda = -2$$
$$(C_1 + C_2x)e^{-2x}$$
$$xe^{2x}(ax + b)$$
$$y' = 2e^{2x}(ax + b) + ae^{2x}$$
$$y'' = 4e^{2x}(ax + b) + 4ae^{2x}$$
$$4e^{2x}(ax + b) + 4ae^{2x} +$$
$$8e^{2x}(ax + b) + 4ae^{2x} + 4$$
$$e^{2x}(ax + b) = xe^{2x}$$

$$16(ax+b)+8a=x$$

$a=\frac{1}{16}$ $16b+8a=0$ $b=-\frac{1}{32}$

$$y=e^{2x}(\frac{1}{16}x-\frac{1}{32})$$
$$y=(C_1+C_2x)e^{-2x}+$$
$$+e^{2x}(\frac{1}{16}x-\frac{1}{32})$$

251(548)-misol.

$y''-5y'=3x^2+\sin 5x$

$$\lambda^2-5\lambda=0 \quad \lambda=5$$
$$y=(C_1+C_2e^{5x}) \quad y=y_1+y_2$$

y_1 $y''-5y'=3x^2$

$$y_2 \quad y''-5y'=\sin 5x$$
$$y_1=x(ax^2+bx+c)=ax^3+$$
$$+bx^2+cx$$
$$y_1'=3ax^2+2bx+c$$
$$y_1''=6ax+2b$$
$$y''-5y'=3x^2$$
$$6ax+2b-15ax^2-10bx-5c=$$
$$=3x^2$$

$$\begin{cases}-15a=3\\6a-10b=0\\2b-5c=0\end{cases}$$

$$a=-0.2 \quad b=-0.12 \quad c=-0.048$$
$$y_1=-0.2x^3-0.12x^2-0.048x$$
$$y=a\cos 5x+b\sin 5x$$
$$y'=-5a\sin 5x+5b\cos 5x$$
$$y''=-25a\cos 5x-25b\sin 5x$$
$$y''-5y'=\sin 5x$$
$$(-25a-25b)\cos 5x+$$
$$+(25a-25b)\sin 5x=\sin 5x$$

$$\begin{cases}-25a-25b=0\\25a-25b=1\end{cases}$$

$$a=0.02 \quad b=-0.02$$
$$y_2=0.02(\cos 5x-\sin 5x)$$
$$y=C_1+C_2e^{5x}-0.2x^3-0.12x^2-$$
$$-0.048x+0.02(\cos 5x-\sin 5x)$$

252(549)-misol.

$y''-2y'+2y=e^x+x\cos x$

$$\lambda_1 = 1 + i \quad \lambda_2 = 1 - i$$

$$y'' - 2y' + 2y = e^x$$

$$y'' - 2y' + 2y = x\cos x$$

$$\gamma = 1 \quad \beta = 0 \;\; P_m(x) = 1 \;\; p = 0$$

$$\gamma + \beta i \neq \lambda_1 \;\; \gamma + \beta i \neq \lambda_2$$

$$y_1 = C_0 e^x \quad \gamma = 0 \;\; \beta = 1$$

$$P_m(x) = x \;\; Q_n(x) = 0 \;\; p = 1$$

$$\gamma + \beta i \neq \lambda_1 \;\; \gamma + \beta i \neq \lambda_2$$

$$y_2 = (a_0 x + a_1)\cos x + (b_0 x + b_1)\sin x)$$

$$y = C_0 e^x + (a_0 x + a_1)\cos x + (b_0 x + b_1)\sin x)$$

253(550)-misol.

$$y'' + 6y' + 10y =$$

$$= 3xe^{-3x} - 2e^{3x}\cos x$$

$$1)\; \lambda^2 + 6\lambda + 10 = 0 \;\; \lambda_{1,2} = -3 \pm i$$

$$y_0 = e^{3x}(C_1\cos x + C_2\sin x)$$

$$y'' + 6y' + 10y = 3xe^{-3x}$$

$$\alpha + 3i = -3 + 0i \;\; s = 0 \;\; m = 1$$

$$y_1 = e^{-3x}(ax + b)$$

$$y'' + 6y' + 10y = -2e^{3x}\cos x$$

$$\alpha = 3 \quad \beta = 1$$

$$\alpha + \beta i = 3 + i \quad s = 0 \;\; m = 0$$

$y_2 = e^{3x}(c\cos x + d\sin x)$

254(562)-misol. $y'' - 9y = e^{-3x}(x^2 + \sin 3x)$

Yechilishi: $y'' - 9y = e^{-3x}(x^2 + \sin 3x)$

$l^2 - 9 = 0$

$l = \pm 3$

$y = C_1 e^{-3x} + C_2 e^{-3x} + x e^{-3x}(ax^2 + bx + c) + e^{-3x}(a_1 x^2 + b_1 x + c_1) +$
$+ e^{-3x}(k_1 \sin 3x + k_2 \cos 3x)$

255(563)-misol. $y^{IV} + y'' = 7x - 3\cos x$

Yechilishi: $y^{IV} + y'' = 7x - 3\cos x$

$l^4 + l^2 = 0$

$l_1 = l_2 = 0; \quad l_{3,4} = \pm i$

1) $y^{IV} + y'' = 7x$ 2) $y^{IV} + y'' = -3\cos x$

$x(ax^2 + bx + c\cos x + d\sin x)$

256(564)-misol. $y'' + 4y = \cos x \cos 3x$

Yechilishi: $y'' + 4y = \cos x \cos 3x$

1) $l^2 + 4 = 0$

$l_{1,2} = \pm 2i$

$y = C_1 \cos 2x + C_2 \sin 2x$

2) $y'' + 4y = \cos x \cos 3x = \frac{e^{ix} + e^{-ix}}{2} \cdot \frac{e^{3ix} + e^{-3ix}}{2} =$

$= \frac{1}{4}\left(e^{4ix} + e^{-2ix} + e^{2ix} + e^{-4ix}\right) =$

$= \frac{1}{4}(\cos 4x + i\sin 4x + \cos(-2x) + i\sin(-2x) + \cos 2x + i\sin 2x +$
$\cos(-4x) + i\sin(-4x)) =$

$= \frac{1}{4}(2\cos 2x + 2\cos 4x) = \frac{1}{2}\cos 4x + \frac{1}{2}\cos 2x$

257(565)-misol. $y''' - 4y'' + 3y' = x^2 + xe^{2x}$

Yechilishi: $y''' - 4y'' + 3y' = x^2 + xe^{2x}$

$l^3 - 4l^2 + 3l = 0$

$l_1 = 0; \quad l_2 = 1; \quad l_3 = 3$

$y = C_1 + C_2 e^x + C_3 e^{3x} + x^2 + xe^{2x}$

258(566)-misol.. $y'' - 4y' + 5y = e^{2x}(\sin x)^2$

Yechilishi: $y'' - 4y' + 5y = e^{2x}(\sin x)^2$

$$y'' - 4y' + 5y = \frac{e^{2x}}{2} - \frac{e^{2x}\cos 2x}{2}$$

$$l^2 - 4l + 5 = 0$$

$$l = 2 \pm i$$

$$e^{2x}(\sin x)^2 = \frac{e^{2x}}{2} - \frac{e^{2x}\cos 2x}{2}$$

$$y = (a_0 + b_0 \cos 2x + b_1 \sin 2x)e^{2x}$$

259(567)-misol. $y'' + 3y' + 2y = e^{-x}(\cos x)^2$

Yechilishi: $y'' + 3y' + 2y = e^{-x}(\cos x)^2$

$$y'' + 3y' + 2y = e^{-x} \cdot \frac{1+\cos 2x}{2}$$

$$l^2 + 3l + 2 = 0$$

$$l_1 = -1; \quad l_2 = -2$$

$$y = C_1 e^{-x} + C_2 e^{-2x} - \frac{1}{4}\sin 2x\, e^{-x} + \frac{1}{4}\cos 2x\, e^{-x}$$

260(568)-misol.. $y'' - 2y' + 2y = (x + e^x)\sin x$

Yechilishi: $y'' - 2y' + 2y = x\sin x + e^x \sin x$

$$l^2 - 2l + 2 = 0$$

$$l = 1 \pm i$$

$y = e^x(C_1 \cos x + C_2 \sin x) + (ax + b)\sin x + (a_1 x + b_1)\cos x +$
$xe^x(a_2 \sin x + b_2 \cos x)$

270(569)-misol. $yIV + 5y'' + 4y = \sin x \cos 2x$

Yechilishi: $yIV + 5y'' + 4y = \sin x \cos 2x$

$$\sin x \cos 2x = \frac{1}{2}\sin 3x - \frac{1}{2}\sin x$$

$$l^4 + 5l^2 + 4 = 0$$

$$l_{1,2} = \pm 2i; \qquad l_{3,4} = \pm i$$

$$y_1 = a_0 \sin 3x + a_1 \cos 3x$$

$y_2 = x(b_0 \sin x + b_1 \cos x)$

$y = y_1 + y_2$

271(570)-misol. $y'' - 3y' + 2y = 2^x$

Yechilishi: $y'' - 3y' + 2y = 2^x$

$l^2 - 3l + 2 = 0$

$l_1 = 1; \qquad l_2 = 2$

$2^x = e^{xln2}$

$y = C_1 e^x + C_2 e^{2x} + Ae^{xln2}$

272(572)-misol. Masalalarda berilgan tenglamaning hususiy yechimi noma'lum koeffitsentlar usulida qanday ko'rinishda izohlashni yozing.

$y'' + 4y' + 3y = chx$

Yechilishi:

$y'' + 4y' + 3y = chx$

$y'' + 4y' + 3y = coshx$

$y'' + 4y' + 3y = \frac{1}{2} e^x + \frac{1}{2} e^{-x}$

$t^2 + 4t + 3 = 0$

$t_1 = -1 \ , t_2 = -3$

$y = C_1 e^{-x} + C_2 e^{-3x}$

$y = axe^{-x} + be^x$

273(573)-misol. Masalalarda berilgan tenglamaning hususiy yechimi noma'lum koeffitsentlar usulida qanday kurinishda izohlashni yozing.

$y'' + 4y = shx \cdot sin2x$

Yechilishi:

$y'' + 4y = shx \cdot \sin 2x$

$y'' + 4y = \frac{1}{2}e^{x}\sin 2x - \frac{1}{2}e^{-x}\sin 2x$

$t^2 + 4 = 0 \quad t_1 = -2, \; t_2 = 2$

$y'' + 4y = \frac{1}{2}e^{x}\sin 2x$

$y'' + 4y = -\frac{1}{2}e^{-x}\sin 2x$

$y = (a + c)\cos 2x + (b + d)\sin 2x$

274(575)-misol.. Tenglamalarni uzgarmasni variatsiyalash usulida yeching.

$y'' - 2y' + y = \frac{e^x}{x}$

Yechilishi:

$y'' - 2y' + y = \frac{e^x}{x}$

$t^2 - 2t + 1 = 0$

$t = 1(2)$

$y = (C_1 + C_2)e^x = C_1e^x + C_2xe^x$

$C_1x + C_2(e^x + xe^x) = \frac{e^x}{x}$

$C_2'^{e^x} = \frac{e^x}{x}$

$C_2 = lnx + C_3$

$C_1 = -x + C_4$

$y = -xe^x + C_4e^x + xlnxe^x + C_3xe^x = (C_3 - 1)xe^x + xlnxe^x + C_4e^x =$
$e^x(C_1x + C_2\text{+lnx})$

275(576)-misol. Tenglamani uzgarmas variatsiyalash usulida yeching.

$y'' + 3y' + 2y = \frac{1}{(e^x+1)}$

Yechilishi:

$y'' + 3y' + 2y = \frac{1}{(e^x+1)}$

$t^2 + 3t + 2 = 0$

$t_1 = -1 \ , t_2 = -2$

$y = C_1 e^{-x} + C_2 e^{-2x}$

$C_2 = -\int \quad \frac{e^{2x}}{e^x+1} dx = -\int (\frac{e^x}{1+e^x}) de^x = -\int \left(1 - \frac{1}{e^x+1}\right) de^x = -e^x + \ln(e^x +$

$1) + C_3$

$C_1 = \int (\frac{e^x}{e^x+1})\, dx = \int d\left(\frac{e^x}{e^x+1}\right) = \ln(e^x + 1) + C_4$

$y = \ln(e^x + 1)\, e^{-x} - e^{-x} + \ln(e^x + 1)\, e^{-2x} + C_4 e^{-x} + C_3 e^{-2x} = (e^{-x} +$

$e^{-2x}) \ln(e^x + 1) + (C_4 - 1)e^{-x} + C_3 e^{-2x} =$

$= (e^{-x} + e^{-2x}) \ln(e^x + 1) + C_1 e^{-x} + C_2 e^{-2x}$

$y = (e^{-x} + e^{-2x}) \ln(e^x + 1) + C_1 e^{-x} + C_2 e^{-2x}$

276(577)-misol. Tenglamani uzgarmaas variatsiyalash usulida yeching.

$$y'' + y = \frac{1}{\sin x}$$

Yechilishi:

$y'' + y = \frac{1}{\sin x}$

$y'' + y = 0$

$\varphi(t) = t^2 + 1$

$t_1 = i \ \ , t_2 = -i$

$y_1 = \cos x, \quad\ y_2 = -\sin x$

$y_0 = C_1 \cos x + C_2 \sin x$

$C_1(x) = -x + C_1$

$C_2(x) = \int ctg\ x\ dx = ln \sin x + C_2$

$y = (C_1 - x) \cos x + (C_2 + ln \sin x) \sin x = C_1 \cos x - x \cos x + C_2 \sin x +$

$[ln \sin x] \sin x = C_1 \cos x + C_2 \sin x + \sin x ln|\sin x| - x \cos x$

$y = C_1 \cos x + C_2 \sin x + \sin x\, ln|\sin x| - x \cos x$

277(578)-misol. Tenglamani uzgarmas variatsiyalash usulida yeching

$$y'' + 4y = 2\,\mathrm{tg}\,x$$

$y'' + 4y = 2tgx$

Yechilishi:

$t^2 + 4 = 0$

$t^2 = -4$

$t_1 = 2\ \ , t_2 = -2$

$y = C_1(x)\sin 2x + C_2\,(x)\cos 2x$

d $\ C_1(x) = -tgx\cos 2x\,dx$

d $\ C_1(x) = -\sin\frac{2x}{\cos x}((\cos x)^2 - (\sin x)^2)$dx

$d\,C_1(x) = -\sin x \cdot \cos x\,dx + \frac{(\sin x)^3}{\cos x}$dx

$C_1(x) = -\int \sin x\,d\sin x + \int \frac{(1-(cosx)^2}{\cos x}d\cos x = -(\sin\frac{x}{2})^2 + ln\cos x +$

$(\cos\frac{x}{2})^2 + C_1 = -ln|\cos x| + \cos x + C_1$

$C_2(x) = -2\int(\sin x)^2\,dx = -2\int\frac{(1-\cos 2x)}{2}dx = -x + \frac{\sin 2x}{2} + C_2$

$y = (ln|\cos x| + \frac{\cos 2x}{2} + C_1)\sin 2x + (x - \frac{\sin 2x}{2} + C_2\ )\cos 2x$

$y = \sin 2x\,ln|\cos x| + x\cos 2x + C_1\sin 2x + C_2\cos 2x$

278(579)-misol. Tenglamani uzliksiz variatsiyalash usuli bilan yeching.

$$y'' + 2y' + y = 3e^{-x}\sqrt{}(x+1)$$

Yechilishi

$y'' + 2y' + y = 3e^{-x}\sqrt{}(x+1)$

$t^2 + 2t + 1 = 0$

$t_{1,2} = -1$

$y = C_1(x)e^{-x} + C_2(x)xe^{-x}$

$C_1(x) = -\frac{6}{5}(x+1)^{\frac{5}{2}} + 2(x+1)^{\frac{3}{2}} + C_2$

$y = (-\frac{6}{5}(x+1)^{\frac{5}{2}} + 2\left(x+1)^{\frac{3}{2}} + C_1 + 2x(x+1)^{\frac{3}{2}} + xC_2\right)e^{-x} = (-\frac{6}{5}(x+1)^{\frac{5}{2}} +$

$2\left(x+1)^{\frac{3}{2}}(x+1) + C_1 + xC_2\right)e^{-x} =$

$= (-\frac{6}{5}(x+1)^{\frac{5}{2}} + 2\left(x+1)^{\frac{5}{2}} + C_1 + xC_2\right)e^{-x} = (\frac{4}{5}(x+1)^{\frac{5}{2}} + C_1 + xC_2)e^{-x}$

$y = (\frac{4}{5}(x+1)^{\frac{5}{2}} + C_1 + xC_2)e^{-x}$

279(580)-misol. Tenglamani uzliksiz variatsiyalash usuli bilan yeching.

$$y'' + y = 2\,(\sec x)^2$$

Yechilishi:

$y'' + y = 2(\sec x)^2$

$y'' + y = \frac{2}{(\cos x)^2}$

$t^2 + 1 = 0$

$t^2 = -1$

$t_1 = i\ \ , t_2 = -i$

$y = C_1(x)\sin x + C_2(x)\cos x$

$C_1(x) = 2\tan x + k_1$

$C_2(x) = \frac{1}{(\cos x)^2} + k_2$

$y = \left(2\tan x + k_1)\sin x + (-\frac{1}{(\cos x)^2} + k_2\right)\cos x =$

$= 2\tan x + k_1\sin x - \frac{1}{\cos x} + k_2\sin x =$

$= k_1\sin x + k_2\cos x + (\frac{2(\sin x)^2 - 1}{\cos x}) =$

$= k_1\sin x + k_2\cos x - \frac{\cos 2x}{\cos x}$

$y = k_1\sin x + k_2\cos x - \frac{\cos 2x}{\cos x}$

§-11.
Eyler tenglamalarini yeching

280(591)-misol. $x^3y''' + xy' - y = 0$. *Almashtirish kiritamiz*: $x = e^t$, $lnx = t$

$y' = \dot{y} \cdot \frac{1}{x}$, $y'' = (\ddot{y} - \dot{y}) \cdot \frac{1}{x^2}$, $y''' = (\dddot{y} - 3\ddot{y} + 2\dot{y}) \cdot \frac{1}{x^3}$, $\dddot{y} - 3\ddot{y} + 2\dot{y} + \dot{y} - y = 0$

Xarakteristik tenglamasini tuzamiz $k^3 - 3k^2 + 3k - 1 = 0 \Rightarrow (k-3)^2 = 0$ *dan,*

$$k = 1\,,3\ karrali \quad \Rightarrow \quad y(t) = C_1e^t + C_2te^t + C_3t^2e^t \quad \Rightarrow \quad Y(x) = C_1x + C_2lnx \cdot x + C_3ln^2x \cdot x$$

281(592) – *misol*

$x^2y''' = 2y' \Rightarrow x^2\,y''' - 2y' = 0$. *Almashtirish kiritamiz*: $x = e^t$

$$y_x{}' = \frac{y_t'}{e^t}\ ;\ y_{xx}{}'' = \frac{y_t''}{e^{2t}} - \frac{y_t'}{e^{2t}}\ ;\ y_{xxx} = \frac{y_{ttt}''' - 3y_{tt}'' + 2y_t'}{e^{3t}}$$

$$y_{ttt}''' - 3y_{tt}'' + 2y_t' - 2y_t' = 0 \Rightarrow y_{ttt}''' - 3y_{tt}'' = 0 \Rightarrow k^3 - 3k^2 = 0\ dan\ k = 0, k = 3$$

$$Y = (C_1 + C_2t) + C_3e^{3t}t \Rightarrow Y = C_1 + C_2lnx + C_3x^3$$

281(593) – *misol*. $x^2y'' - xy' - y + y = 8x^3$. *Almashtirish kiritamiz*: $x = e^t$. *Xarakteristik tenglamasini tuzamiz*

$$k(k-1) - k + 1 = 0 \Rightarrow k^2 - 2k + 1 = 0\ ;\ (k-1)^2 = 0\ dan\ ;\ k = 1\,,2\ karrali\ ;$$
$$y = (C_1 + C_2t)e^t\ buladi$$

$$y'' - 2y' + y = 8e^{3t}\ dan\ x = ae^{3t}\ almashtirish\ kiritamiz\ va\ ;$$
$$e^{3t}(9a - 6a + a) = 8e^{3t} \Rightarrow a = 2$$

$$y = (C_1 + C_2t)e^t + 2e^{3t} \Rightarrow Y = (C_1 + C_2lnx)x + 2x^3$$

282(594) – *misol*. $x^2y'' + xy' + 4y = 10x$ *Almashtirish kiritamiz*: $x = e^t$; $y'' - y' + y' + 4y = 10e^t$

$$y'' + 4y = 10e^t\ dan\ k_1 = 2i, k_2 = -2i \Rightarrow y = C_1cos2t + C_2sin2t + 2e^t\ dan$$

$$=> Y = C_1 cos2lnx + C_2 sin2lnx + 2x$$

283(595) – *misol*. $x^2y'' - 2xy = 6lnx$: xga $=> x^2y'' - 2y = 6\frac{lnx}{x} =$

$$> \{\frac{x = e^t}{y = y(t)} =>$$

$$y_x{}' = \frac{y_t'}{e^t} \; ; \; y_{xx}{}'' = \frac{y_t''}{e^{2t}} - \frac{y_t'}{e^{2t}}$$

$$y_{tt}'' - y_t' - 2y = 6\frac{t}{e^t} => k^2 - k - 2 = 0 \; dan \, , k_1 = -1, k_2 = -2 \quad =>$$

$$y = C_1e^{-t} + C_2e^{-2t}$$

$$Y = (at + b)t + e^{-t} = at^2e^{-t} + bte^{-2} => y' = 2ate^{-t} - at^2e^{-t} + be^{-t} - bte^{-t} =>$$

$$y'' = 2ae^{-t} - 2ae^{-t} - 2ae^{-t} - 2ae^{-t} + at^2e^{-t} - be^{-t} - be^{-t} + bte^{-t}$$

tengliklarni yuqoridagi tenglamaga olib borib quysak, quyidagi tenglikga ega bulamiz:

$$2ae^{-t} - 6ate^{-t} - 3be^{-t} = 6e^{-t}t \; => 2a - 6at - 3b = 6t \; dan \; a = -1, b = \frac{2}{3} \; ; y = C_1e^{-t} + C_2e^{2t} + (-t + \frac{2}{3})te^{-t}$$

$$y = C_1\frac{1}{x} + C_2x^2 + (-lnx + \frac{2}{3})lnx \cdot \frac{1}{x} \qquad => \quad Y = C_2x^2 + \frac{1}{x}\left(C_1 + \frac{2}{3}lnx - ln^2x\right)$$

284(596) – *misol*.

$x^2y'' - 3xy' + 5y = 3x^2$; $k(k-1) - 3k + 5 = 0 \; => k^2 - 4k + 5 = 0 \; => k_1 = 2 + i$,

$$k_2 = 2 - i$$

$$f(t) = 3e^{2t} => z_u = ae^{2t} => z_u' = 2ae^{2t} => z_u'' = 4ae^{2t} \; ; \; 7a - 8a + 5a = 3 \; dan$$

$$a = 3 \; va \; z_u = 3e^{2t}$$

$$z'' - 4z' + 5z = 0 \; dan; \quad \psi1(t) = e^{2t}cost \; ; \quad \psi2(t) = e^{2t}sint$$

$$z_0 = e^{2t}cost + Ce^{2t}sint \; => z = C_1e^{2t}cost + C_2e^{2t}sint + 3x^2$$

$$Y = C_1x^2coslnx + C_2x^2sinlnx + 3x^2$$

284(597)-misol. $x^2y'' - 6y = 5x^3 + 8x^2$. $k(k-1) - 6 = 0 \; => k^2 - k - 6 = 0 \; dan \; => k_1 = 3 \, , k_2 = -2$

$$Almashtirish \; kiritamiz: x = e^t \; ; \; y''t - y't - 6y = 5e^{2t} + 8e^{2t} \; dan$$

$$y_1 = Ae^{3t}t \; deb \; belgilasak; \;\; 6Ae^{3t} + 9Ae^{3t}t - Ae^{3t} - 3Ae^{3t} - 6Ae^{3t}t = 5e^{3t} \; dan \; =>$$

$$y_1 = e^{3t}t = x3lnx$$

$$y_2 = Ae^{2t} \; deb \; belgilasak \; ; 4Ae^{2t} - 2ae^{2t} - 6Ae^{2t} = 8e^{2t} \;\; dan \;\; A = -2 \; =>$$

$$y = -2e^{2t} \; = \; -2x^2$$

$$Y = C_1x^2 + C_2x^{-2} + C_3lnx - 2x^2$$

285(598)-misol. $x^2y'' - 2y = sinlnx. \;\; Almashtirish \; kiritamiz: x = e^t \; ; \; k(k-1) - 2 = 0 \;\; => \; k_1 = -1$

$$k_1 = -2 = 0 \; dan \; => k_1 = -1 \; , k_2 = 2$$

$$y_t'' - y_t' - 2\, y_t = sint \;\; ; \; y_t = Asint + Bcost \;\; ; \; y_t' = Acost - Bsint \; ; \;\; y_t'' = -Asint - Bcost$$

$$-Asint - Bcost - Acost + Bsint - 2sint - 2Bcost = sint \; tenglikdan \; => \;\; A = -0{,}3 \; ,$$

$$B = 0{,}1$$

$$y_t = C_1e^{-t} + C_2e^{2t} + 0{,}1cost - 0{,}3sint \;\; => \;\; Y = C_1x^{-1} + C_2x^2 + 0{,}1coslnx - 0{,}3sinlnx$$

286(599)-misol. $(x-2)^2\, y'' - 3(x-y)y' + 4y = x \;\;\; k(k-1) - 3k + 4 = 0 \;\; => \; k^2 - 4k + 4 = 0 \; dan \; =>$

$$k = 2$$

$$y = (C_1 + C_2t)e^t \;\; ; (\, k - 2) = e^t \; deb \; belgilasak \;\; => \; y'' - 4y' + 4y = e^t + 2 \; ; \;\; y_1' = C_1e^t \; , \qquad y_2 = e^t$$

$$C_1e^t - 4C_1e^t + 4C_1e^t = e^t \quad => \; C_1 - 4C_1 + 4C_1 = 1 \; ; \; y = \frac{1}{2}$$

$$y = \frac{1}{2} + e^t + (C_1 + C_2t)e^{2t} \;\; => \;\; Y = \frac{1}{2} + x - 2 + (C_1 + C_2(x-2))(x-2)^2$$

287(600)-misol. $(2x+3)3y''' + 3(2x+3)y' - 6y = 0. Almashtirish \; kiritamiz: (2x+3) = 2e^t \; ; \; x = e^t - \frac{3}{2}$

$$y_x{}' = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{y_t'}{x_t'} = \frac{y_t'}{e^t} \; ; \; y_x{}'' = \frac{y_x''}{e^{2x}} \cdot y_x{}' \; ; \; y_x{}''' = \frac{y_{ttt}''' - 3y_t'' + 2y_t'}{e^t} \quad hosil\ buladi \quad =>$$

$$8(y_t{}''' - 3y_t{}'' + 2y_t{}') + 6y_t{}' - 6y = 0 \quad => \; 8y_t{}''' - 24y_t{}'' + 22y_t{}' - 6y = 0 \quad => \; 8k^3 - 24k^2 + 22k - 6 = 0$$

$$(k - 1)(4k^2 - 8k + 3) = 0 \; dan \; , k_1 = 1 \, , k_2 = \frac{1}{2} \; , k_3 = \frac{3}{2} \quad =>$$

$$Y = C_1(x + \frac{3}{2}) + C_2(x + \frac{3}{2})0.5 + C_3(x + \frac{3}{2})1{,}5$$

Glossariy

1. ***Differentsial tenglama-*** hosila yoki differentsial qatnashgan tenglama.
2. ***Differentsial tenglama tartibi-***differntsial tenglamada qatnashgan hosilaning eng yuqori tartibiga aytiladi.
3. ***Echim-*** differentsial tenglamani qanoatlantiruvchi funktsiya.
4. ***Tenglamani integrallash-*** tenglamaning umumiy echimini topish.
5. ***Maxsus echim-*** umumiy echimlar orasida bo'lmagan echim.
6. ***Chiziqli tenglama-*** tenglama funktsiya va uning hosilasiga nisbatan chiziqli.
7. ***Bernulli, Rikkati tenglamasi-*** almashtirishlar bilan chiziqli tenglamaga keltiriladigan tenglamalar.
8. ***Lagranj, Klero tenglamasi-*** hosilaga nisbatan echilmaydigan tenglamalar.
9. ***Fundamental echimlar-*** yuqori tartibli, chiziqli tenglamaning o'zaro chiziqli bog'lamagan xususiy yechimlari majmuasi.

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