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# ANALITIK GEOMETRIYADAN MISOL VA MASALALAR

**o‘quv qo‘llanma**

**“Durdona” nashriyoti**
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Ushbu o‘quv qo‘llanma Oliy ta’lim muassasalarining ”Matematika” ta’lim yo‘nalishida tahsil olayotgan talabalar uchun mo‘ljallab yozilgan. Qo‘llanmada asosan Analitik geometriya faniga kirishning dastlabki tushunchalari, vektorlar va ular ustida chiziqli amallar, vektorning koordinatalari, moduli, yo‘naltiruvchi kosinuslari va koordinatalari bilan berilgan vektorlar ustida amallar, vektorlarning skalyar, vektor va aralash ko‘paytmalari, tekislikda to‘g‘ri chiziqlarning turli tenglamalari, fazoda tekislik va to‘g‘ri chiziq tenglamalari, tekislikda ikkinchi tartibli chiziqlar, tekislikda ikkinchi tartibli chiziqlarning qutb koordinatalar sistemasidagi tenglamalari, tekislikda ikkinchi tartibli chiziqlarning umumiy tenglamalari, ikkinchi tartibli chiziq va to‘g‘ri chiziqning o‘zaro vazoyati va ularning tenglamalarini soddalashtirish, ikkinchi tartibli sirtlar va ularning to‘g‘ri chiziqli yasovchilari, ikkinchi tartibli sirtlarlarning tenglamalarini kanonik ko‘rinishga keltirish, chiziqli va affin fazolar kabi tushunchalar batafsil yoritilgan. Barcha mavzularda nazariy ma’lumotlar, namunaviy masalalar yechimlari va talabalar mustaqil bajarishlari uchun mo‘ljallangan topshiriqlar keltirilgan. Bundan tashqari, qo‘llanmada keltirilgan mavzular bo‘yicha egallangan bilimlarni mustahkamlash uchun test topshiriqlari ham o‘z aksini topgan.

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## Mundarija

# SO‘Z BOSHI

”Analitik geometriya” fani Oliy matematikaning asosiy bo‘limlaridan biri hisoblanadi. Mazkur fanning vektorlar, tekislikda to‘g‘ri chiziq tenglamalari, fazoda to‘g‘ri chiziq va tekisliklar hamda ularning tenglamalari, o‘zaro joylashishlari, tekislikda ikkinchi tartibli chiziqlar, fazoda ikkinchi tartibli sirtlarning tenglamalari va xossalari, ikkinchi tartibli chiziqlar va sirtlar, tekislikda ikkinchi tartibli chiziqlar, tekislikda ikkinchi tartibli chiziqlarning qutb koordinatalar sistemasidagi tenglamalari va umumiy tenglamalari, ikkinchi tartibli chiziq va to‘g‘ri chiziqning o‘zaro vaziyati va ularning tenglamalarini soddalashtirish, ikkinchi tartibli sirtlarning to‘g‘ri chiziqli yasovchilari, ikkinchi tartibli sirtlarning tenglamalarini kanonik ko‘rinishga keltirish, chiziqli va affin fazolarni o‘rganish ko‘zda tutilgan.

Ushbu o‘quv qo‘llanma Analitik geometriya fani dasturida keltirilgan barcha mavzularga doir nazariy ma’lumotlar hamda misol va masalalarini qamrab olgan bir qo‘llanmadir. U Oliy ta’lim muassasalarining “Matematika” ta’lim yo‘nalishlarida tahsil olayotgan talabalar uchun mo‘ljallab yozilgan.

Mazkur qo‘llanmada yuqorida sanab o‘tilgan mavzularga oid qisqacha nazariy ma’lumotlar bayon qilingan. Ularga doir misol va masalalar dastlab sodda va muayyan tasavvur hosil qilinadigan, so‘ngra murakkabroq  masalalarni yechishga alohida e’tibor qaratilgan. Misol va masalalarni sharhlab, ularni yechib ko‘rsatishdan ko‘zlangan maqsad Analitik geometriya kursidan olingan nazariy bilimlardan misol va masalalarni yechishda foydalana olish ko‘nikmasini shakllantirishdir. Talabalar namuna sifatida yechib ko‘rsatilgan masalalarda qo‘llanilgan usullardan foydalanib mustaqil bajarishlari uchun ko‘plab misol va masalalar keltirilgan.

Qo‘llanmani o‘qish jarayonida talabalar o‘zlarining Analitik geometriya, Chiziqli algebra va analitik geometriya fanlaridan olgan bilimlarini to‘ldiradilar. Undan matematikaning ko‘plab sohalari bo‘yicha ilmiy-tadqiqot ishlari olib borayotgan magistrantlar, tayanch doktorantlar va mustaqil izlanuvchilar ham foydalanishlari mumkin.

# 1-MAVZU: ANALITIK GEOMETRIYA FANIGA KIRISH

**Reja:**

**1. Geometriyaning rivojlanish tarixidan.**

**2. Ikki nuqta orasidagi masofa.**

**3. Kesmani berilgan nisbatda bo‘lish.**

**Tayanch iboralar**: nuqta, kesma, kesmani berilgan nisbatda bo‘lish, ikki nuqta orasidagi masofa.

## 1.1. Geometriyaning rivojlanish tarixidan.

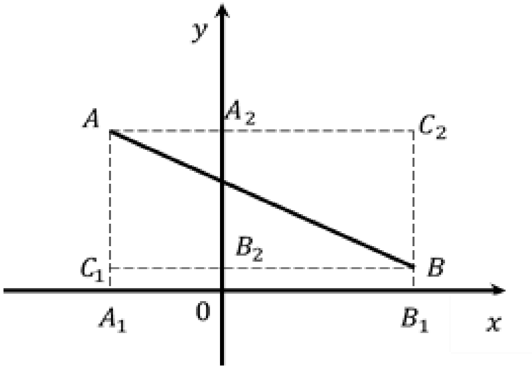
Geometriya fani qadimiy tarixga ega bo‘lib, unga oid boshlang‘ich tushunchalar bundan 4000 yil muqaddam Misr va Bobilda vujudga kelgan. Geometrik bilmlarning vujudga kelishi odamlarning amaliy faoliyati bilan bog‘liq. Bu ko‘pgina gemetrik figuralarning nomlarida o‘z aksini topgan. Masalan, trapesiya nomi yunoncha trapezion – so‘zidan olingan bo‘lib, “stolcha” ni bildiradi. “Chiziq” termini lotincha limem – “zig‘irip” so‘zidan hosil bo‘lgan. Qadimdayoq geometriya aksiomalar sistemasiga asosan tuzilgan qat’iy mantiqiy fanga aylangan. U uzluksiz rivojlanib yangi teoremalar, g‘oyalar va usullar bilan boyib borgan. Eramizdan avvalgi III asrda yunon olimi Yevklid “Negizlar” nomli asarini yozadi. Yevklid shu davrgacha bo‘lgan geometrik bilimlarni jamladi va bu fanning tugallangan aksiomatik bayonini berishga harakat qildi. Yevkliddan so‘ng yashagan olimlar uning “Negizlar”iga ba’zi mavzularni qo‘shdilar, aniqliklar kiritdilar. Geometriyaning hozirgi zamon fizikasi bilan bog‘lanishini kuzatish g‘oyat qiziqarli. Ko‘pincha matematikani boyitgan yangi tushunchalar fizika hamda kimyo va tabiatshunoslikning boshqa bo‘limlaridan keladi. Masalan, vektor mexanikadan olinganligi misol bo‘la oladi.

Geometriyaning kelgusi rivojlanishida esa matematikaning ichki talabi va o‘ziga xos mantiqiy rivojlanishi natijasida uning ichida vujudga kelgan, yangi geometrik tushunchalar yangi zamonaviy fizikani yaratishga yo‘l ochdi. Masalan, Lobachevskiy geometriyasi nisbiylik nazariyasini ochishga asos bo‘lib xizmat qildi.

Hozirgi zamon geometriyasi juda ko‘p yo‘nalishlarga ega. Ulardan biri geometriyani sonlar nazariyasi bilan, ikkinchisi kvant fizikasi bilan, uchinchisi esa matematik tahlil bilan yaqinlashtiradi. Hozirgi zamon matematikasi bo‘limlari shundayki unda geometriya ko‘proqmi, algebrami yoki tahlil(analiz) aytish qiyin. Geometriyaning rivojlanishida Markaziy Osiyodan chiqqan matematiklar Muhammad ibn Muso al-Xorazmiy, Abu Rayhon Beruniy, Abu Ali ibn Sino, Abdurahmon al-Xaziniy, Abul Vafo Buzmoniy, Umar Xayyom, Mirzo Ulug‘bek, G‘iyosiddin al-Koshiy va boshqalarning xizmati kattadir. XVII asrda fransuz matematigi va filosofi Rene Dekart ishlari tufayli, butun matematikani, xususan geometriyani inqilobiy qayta qurgan koordinatlar usuli(metodi) vujudga keldi. Algebraik tenglik(tengsizlik) larni geometrik obraz(grafik)lar orqali talqin qilish va aksincha geometrik masalalarni yechishni analitik, formulalar, tenglamalar sistemalari yordamida izlash imkoniyatini paydo qildi. Matematika fanining yangi tarmog‘i analitik geometriya vujudga keldi. Analitik geometriyaning mohiyati, geometrik ob’yektlarga uning algebraik(analitik) ifodasini mos qo‘yib, ularning xususiyatlarini o‘rganishni, unga mos algebraik ifodalarni tekshirish orqali amalga oshiriladi.

## 1.2. Ikki nuqta orasidagi masofa.

$A(x_1, y_1)$ va $B(x_2, y_2)$ nuqtalardan o‘tuvchi hamda $Ox$ va $Oy$ o‘qlariga paralel bo‘lmagan to‘g‘ri chiziq berilgan bo‘lsin: $A$ va $B$ nuqtalardan $Ox$ va $Oy$ o‘qlariga paralel hamda ular bilan $A_1$ , $A_2$, $B_1$, $B_2$ nuqtalarda kesishguncha to‘g‘ri chiziqlar o‘tkazamiz.




**1.2.1-chizma**

Tekislikda hosil bo‘lgan $A_1C_1BC_2$ to‘g‘ri to‘rtburchakni qaraylik. Undagi $A_1C_1B$ to‘g‘ri burchakli uchburchakdan Pifagor teoremasiga asosan,

$$|AB|^2 = |AC_1|^2 + |C_1B|^2 \qquad (1.1)$$

bundan, $|C_1B| = |A_1B_1| = |x_2 - x_1|$,

$$|AC_1| = |A_2B_2| = |y_2 - y_1|. \qquad (1.2)$$

$|AB| = d$ belgilash kiritamiz. U holda (1.1) va (1.2) lardan:

$$d^2 = |AB|^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

yoki

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad (1.3)$$

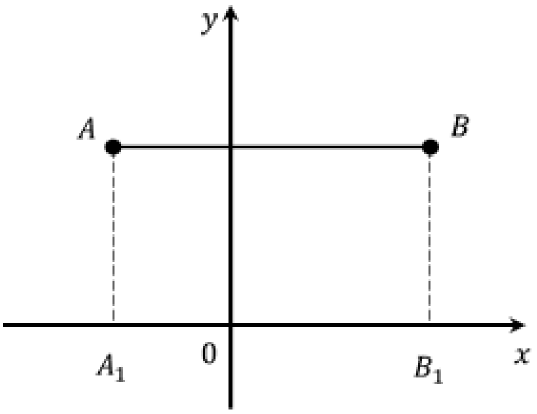
(1.3) formula ikki nuqta orasidagi masofani(kesma uzunligini) topish formulasidir. Bu formula umumiy formula bo‘lib, $A$ va $B$ nuqtaning tekislikdagi har qanday holatida ham quyidagicha bo‘ladi:

$$|AC_2| = |C_1B| = |x_2 - x_1| \text{ va}$$
$$|AC_1| = |C_2B| = |y_2 - y_1|. \qquad (1.4)$$

Agar ikki nuqtadan o‘tuvchi to‘g‘ri chiziq absissa yoki ordinata o‘qlaridan biriga paralel bo‘lsa, masalan $Ox$ o‘qqa paralel bo‘lsa,

$$|AB| = |A_1B_1| = |x_2 - x_1| \qquad (1.5)$$

dan iborat bo‘ladi. Bunda $y_2 - y_1 = 0$, chunki $y_2 = y_1$ .




**1.2.2-chizma**

Agar $A(x_1, y_1)$ nuqta $O(0;0)$ nuqta (koordinatalar boshi) bilan ustma-ust tushsa, (1.1) formulani quyidagicha yozish mumkin:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_2 - 0)^2 + (y_2 - 0)^2} =$$
$$= \sqrt{{x_2}^2 + {y_2}^2},$$

bundan

$$d = \sqrt{{x_2}^2 + {y_2}^2} \tag{1.6}$$

**1-misol.** $M(4;-1)$ va $N(-2;5)$ nuqtalar berilgan bo‘lsa $MN$ kesmaninig uzunligini toping.

**Yechish.** Berilganlarga ko‘ra: $x_1 = 4$, $y_1 = -1$, $x_2 = -2$, $y_2 = 5$. Bu qiymatlarni (1.3) formulaga qo‘ysak:

$$d = |MN| = \sqrt{(-2-4)^2 + \left(5-(-1)\right)^2} = \sqrt{36+36} = \sqrt{72} =$$
$$= 6\sqrt{2}$$

Demak, $MN$ kesmaning uzunligi $6\sqrt{2}$ o‘lchov birligiga teng ekan.

**2-misol.** $M(5;3)$ va $N(2;-1)$ nuqtalar orasidagi masofani toping.

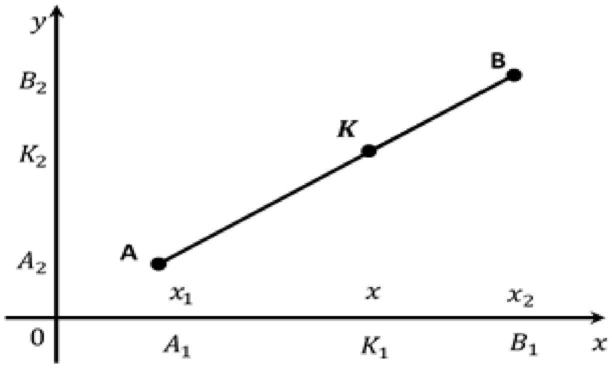
**Yechish.** Shartga ko‘ra: $x_1 = 5$, $y_1 = 3$, $x_2 = 2$, $y_2 = -1$. Bu qiymatlarni (1.3) formulaga qo‘ysak:

$$MN = \sqrt{(2-5)^2 + (-1-3)^2} = \sqrt{9+16} = \sqrt{25} = 5 \text{ bo‘ladi.}$$

### 1.3. Kesmani berilgan nisbatda bo‘lish

Uchlari $A(x_1, y_1)$ va $B(x_2, y_2)$ nuqtalardan iborat $AB$ kesma berilgan bo‘lsin. Shu kesmada yotgan hamda uni ixtiyoriy nisbatda bo‘luvchi biror $K(x, y)$ nuqtaning koordinatalarini topish talab qilinsin.

Koordinatalari izlangan nuqtani $AB$ kesmaning ixtiyoriy nuqtasiga joylashtiramiz. Natijada, $\frac{|AK|}{|KB|}$ nisbat hosil bo‘ladi. Bu nisbatni $\lambda$ bilan belgilasak, $\frac{|AK|}{|KB|} = \lambda$ bo‘ladi. Bunda $\lambda > 0$. Agar $K$ nuqta $AB$ kesmadan tashqarida yotsa $\lambda < 0$ bo‘lar edi.




**1.3.1-chizma**

$AB$ kesma absissa yoki ordinata o‘qlaridan hech biriga parallel bo‘lmagan holni qaraymiz. $A, K, B$ nuqtalardan $Ox$ va $Oy$ o‘qlarga ular bilan kesishguncha perpendikulyar to‘g‘ri chiziqlar o‘tkazamiz. Kesishish nuqtalarini mos ravishda $A_1, K_1, B_1, A_2, K_2$ va $B_2$ harflar bilan belgilaymiz. U holda Fales teoremasiga asosan:

$$\frac{|AK|}{|KB|} = \frac{|A_1K_1|}{|K_1B_1|} = |\lambda|. \tag{1.7}$$

Bundan $|A_1K_1| = x - x_1$ va $|K_1B_1| = x_2 - x$. Bularni (1.7) ga qo‘ysak quyidagi tenglama hosil bo‘ladi: $\frac{x-x_1}{x_2-x} = \lambda$ yoki

$$x = \frac{x_1+\lambda x_2}{1+\lambda} \tag{1.8}$$

$(1.8) -$ izlanayotgan $K$ nuqtaning absissasini topish formulasidir. Shuningdek, $K$ ning ordinatasi quyidagi formula yordamida topiladi:

$$y = \frac{y_1+\lambda y_2}{1+\lambda} \tag{1.9}$$

Demak, kesmani berilgan nuqtada bo‘luvchi ixtiyoriy nuqtaning koordinatalari

$$K\left(\frac{x_1+\lambda x_2}{1+\lambda}; \frac{y_1+\lambda y_2}{1+\lambda}\right) \tag{1.10}$$

formula orqali topiladi. Bunda $\lambda \neq -1$.

Agar $\lambda = 1$ bo‘lsa (1.10) dagi $K$ nuqtaning koordinatalari quyidagi ko‘rinishda bo‘ladi:

$$K\left(\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2}\right) \tag{1.11}$$

bunda $K$ nuqta $AB$ kesmaning o‘rtasida yotadi.

**3-misol.** Tekislikda A$(5;3)$ va $B(2;1)$ nuqtalar berilgan. $AB$ kesmani $\frac{AC}{CB} = \lambda = 0{,}2$ nisbatda bo‘luvchi $C(x,y)$ nuqtaning koordinatlarini toping.

**Yechish.** Shartga ko‘ra $x_1 = 5$, $y_1 = 3$, $x_2 = 2$, $y_2 = 1$, $\lambda = 0{,}2$ (1.4) formulaga asosan:

$$x = \frac{x_1+\lambda x_2}{1+\lambda} = \frac{5+0{,}2\cdot 2}{1+0{,}2} = \frac{5{,}4}{1{,}2} = \frac{54}{12} = \frac{27}{6} = \frac{9}{2},$$

$$y = \frac{y_1+\lambda y_2}{1+\lambda} = \frac{3+0{,}2\cdot 1}{1+0{,}2} = \frac{3{,}2}{1{,}2} = \frac{32}{12} = \frac{8}{3}$$

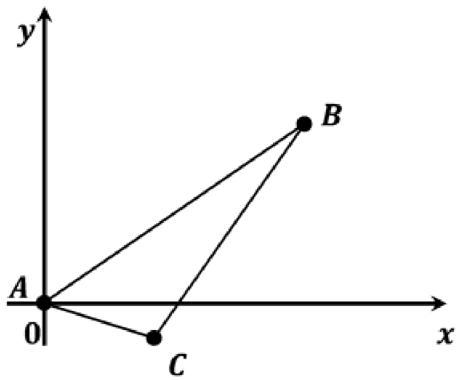
Shunday qilib, $C(4,5;\frac{8}{3})$ bo‘ladi.

**4-misol.** Uchlari $A(0;0)$, $B(12;5)$ va $C(4;-3)$ nuqtalarda yotgan uchburchak berilgan. $A$ burchagidan chiqqan bissektrisa va shu burchak qarshisidagi tomonnning kesishish nuqtasi $D(x,y)$ ning koordinatalarini toping.

**Yechish**. Berilgan nuqtalarning koordinatalari yordamida $ABC$ uchburchakni yasaymiz.

Ma’lumki, $D(x,y)$ nuqta $BC$ tomonni $\lambda > 0$, $\lambda = \frac{|BD|}{|CD|}$ nisbatda bo‘ladi. Buni quyidagicha ham yozish mumkin:

$$\frac{|BD|}{|CD|} = \frac{|AB|}{|AC|} = \lambda.$$




**1.3.2-chizma**

Ma’lumki, $D(x,y)$ nuqta $BC$ tomonni $\lambda > 0$, $\lambda = \frac{|BD|}{|CD|}$ nisbatda bo‘ladi. Buni quyidagicha ham yozish mumkin:

$$\frac{|BD|}{|CD|} = \frac{|AB|}{|AC|} = \lambda.$$

$AB$ va $AC$ kesmalarning uzunliklarini topamiz.

$$|AB| = \sqrt{12^2 + 5^2} = \sqrt{169} = 13 \text{ va } |AC| = \sqrt{4^2 + 3^2} = \sqrt{25} = 5.$$

Bulardan $\lambda = \frac{13}{5}$,

$$x = \frac{12 + \frac{13}{5} \cdot 4}{1 + \frac{13}{5}} = 6\frac{2}{9} \quad \text{va} \quad y = \frac{5 + \frac{13}{5} \cdot (-3)}{1 + \frac{13}{5}} = -\frac{7}{9}$$

demak, izlanayotgan nuqtaning koordinatalari $D(6\frac{2}{9};-\frac{7}{9})$ dan iborat bo‘ladi.

## Mustahkamlash uchun topshiriqlar

### 1.2. Ikki nuqta orasidagi masofaga doir misollar

**1.2.1.** Quyidagi hollarning har birida $A, B$ nuqtalar orasidagi $d$ masofa topilsin:

1) $A(4;3),\ B(7;7)$ 3) $A(12;-1),\ B(0;4)$
2) $A(3;1),\ B(-2;4)$ 4) $A(3;5),\ B(4;6)$

**1.2.2**. Koordinatalar boshidan quyidagi nuqtalargacha bo‘lgan masofalar topilsin:

1) $A(11;4)$ 3) $A(-11;0)$
2) $A(-3;-4)$ 4) $A(5;12)$

**1.2.3**. Koordinata tekisligida $A(1;1)$ va $B(3;7)$ nuqtalardan teng uzoqlikda yotgan $C(2;y)$ joylashgan nuqtalar topilsin.

**1.2.4**. Koordinata tekisligida $A(-2;2)$ va $B(4;8)$ nuqtalardan teng uzoqlikda yotgan $C(3;y)$ joylashgan nuqtalar topilsin.

**1.2.5.** Koordinata tekisligida $A(-3;2)$ va $B(9;3)$ nuqtalardan teng uzoqlikda yotgan $C(x;6)$ joylashgan nuqtalar topilsin.

**1.2.6**. Ordinata o‘qidan shunday nuqtani topingki koordinata boshidan va $A(-8;-4)$ nuqtadan teng uzoqlikda bo‘lsin.

**1.2.7.** Ordinata o‘qidan shunday nuqtani topingki koordinata boshidan va $B(6;4)$ nuqtadan teng uzoqlikda bo‘lsin.

**1.2.8.** Absissa o‘qidan shunday nuqtani topingki koordinata boshidan va $C(-3;1)$ nuqtadan teng uzoqlikda bo‘lsin.

**1.2.9**. Absissa o‘qidan shunday nuqtani topingki koordinata boshidan va $D(5;8)$ nuqtadan teng uzoqlikda bo‘lsin.

**1.2.10.** $A(4;5)$ va $B(3;2)$ nuqtalardan teng uzoqlikda yotgan $C(5;y)$ nuqtani toping.

**1.2.11**. $ABC$ uchburchak uchlarining koordinatalari berilgan: $A(3;1)$, $\mathrm{B}(7;5)$, $\mathrm{C}(5;-1)$. U o‘tkir burchaklimi, to‘g‘ri burchaklimi yoki o‘tmas burchaklimi?

**1.2.12**. Koordinata o‘qlarida $A(-5;9)$ nuqtadan 15 birlik uzoqlikda joylashgan nuqtalar topilsin.
**1.2.13**. Koordinata o‘qlarida $B(-2;11)$ nuqtadan 10 birlik uzoqlikda joylashgan nuqtalar topilsin.
**1.2.14**. Markazi $C(6;7)$ nuqtada va radiusi $r=5$ bo‘lgan aylana berilgan. $A(7;14)$ nuqtadan bu aylanaga urinmalar o‘tkazilgan. A nuqtadan urinish nuqtalargacha bo‘lgan masofalar topilsin.
**1.2.15**. Radiusi $r=10$ bo‘lgan aylana markazi $C(-4;-6)$ nuqtada. Koordinata burchaklar bissektrisalari bilan aylananing kesishish nuqtalari topilsin.
**1.2.16**. $ABC$ uchburchak uchlari berilgan: $A(2;-3)$, $\mathrm{B}(1;3)$, $\mathrm{C}(-6;-4)$. $A(2;-3)$ nuqtaga $BC$ tomonga nisbatan simmetrik bo‘lgan $M$ nuqta topilsin.
**1.2.17**. Uchlari $A(2;2)$, $\mathrm{B}(-5;1)$, $\mathrm{C}(3;-5)$ nuqtalarda bo‘lgan $ABC$ uchburchakka tashqi chizilgan aylana markazi va radiusi topilsin.
**1.2.18**. Rombning ikkita qarama-qarshi uchi $A(8;-3)$, $\mathrm{C}(10;11)$ berilgan. $AB$ tomon 10 ga teng. Qolgan uchlarining koordinatalari topilsin.
**1.2.19**. $A(-4;2)$ nuqtadan o‘tib $Ox$ o‘qiga $B(2;0)$ nuqtada urinadigan aylana markazi topilsin.
**1.2.20**. $A(2;-1)$ nuqtadan o‘tgan va ikkala koordinata o‘qlariga urinadigan aylana tenglamasi tuzilsin.
**1.2.21**. $B(3;1)$ nuqtadan o‘tgan va ikkala koordinata o‘qlariga urinadigan aylana tenglamasi tuzilsin.
**1.2.22.** Koordinatalar boshidan $A(-3;4)$ nuqtagacha bo‘lgan masofani toping.
**1.2.23.** Koordinatalar boshidan $A(2;-5)$ nuqtagacha bo‘lgan masofani toping.
**1.2.24.** Uchlari $A(4;3)$, $B(0;0)$ va $C(10;5)$ nuqtalarda bo‘lgan uchburchakning perimetrini toping.
**1.2.25.** $A(5;4)$ nuqta va $AB$ kesmaning o‘rtasi $C(0;3)$ berilgan. Kesmaning ikkinchi $B(x;y)$ uchini toping.

**1.2.26.** Uchlari $A(3;4)$, $B(7;7)$ va $\mathrm{C}(4;3)$ nuqtalarda bo‘lgan uchburchakning teng yonli ekanligini ko‘rsating.
**1.2.27.** $A(2;8)$ va $\mathrm{B}(6;-4)$ nuqtalar bilan chegaralangan $AB$ kesma $C,D,E$ nuqtalar bilan 4 ta teng bo‘laklarga bo‘lingan. $C,D$ va $E$ nuqtalarni toping.
**1.2.28.** $AB$ kesma $C(-1;-2)$ va $D(2;0)$ nuqtalar orqali teng uch bo‘laklarga bo‘lingan. $A$ va $B$ nuqtalarni toping.
**1.2.29.** Uchlari $A(2;5)$, $B(6;3)$ va $C(4;0)$ nuqtalarda bo‘lgan uchburchakning yuzi hisoblansin.
**1.2.30.** Uchlari $A(3;1)$, $B(4;6)$, $C(6;3)$ va $D(5;-2)$ nuqtalarda bo‘lgan to‘rtburchakning yuzi hisoblansin.

## 1.3. Kesmani berilgan nisbatda bo‘lishga doir misollar

**1.3.1**. $A(-3;8)$, $B(4;-6)$ nuqtalar bilan chegaralangan $AB$ kesmani $\lambda=\frac{3}{4}$ nisbatga bo‘luvchi $C$ nuqtaning koordinatalari topilsin.
**1.3.2**. $M(-1;3)$, $N(4;-7)$ nuqtalar bilan chegaralangan $AB$ kesmani $\lambda=\frac{2}{3}$ nisbatga bo‘luvchi $P$ nuqtaning koordinatalari topilsin.
**1.3.3**. $A(4;-5)$, $B(-2;7)$ nuqtalar bilan chegaralangan $AB$ kesmani $\lambda=\frac{1}{5}$ nisbatga bo‘luvchi $C$ nuqtaning koordinatalari topilsin.
**1.3.4**. $M(1;-4)$, $N(-7;12)$ nuqtalar bilan chegaralangan $AB$ kesmani $\lambda=\frac{3}{5}$ nisbatga bo‘luvchi $P$ nuqtaning koordinatalari topilsin.
**1.3.5**. $A(-2;-3)$, $B(3;7)$ nuqtalar bilan chegaralangan $AB$ kesmani $\lambda=\frac{3}{2}$ nisbatga bo‘luvchi $C$ nuqtaning koordinatalari topilsin.
**1.3.6**. Quyidagi hollarning har birida $AB$ kesma o‘rtasining koordinatalari topilsin:
1) $A(2;3)$, $B(-4;7)$;
2) $A(-2;4)$, $B(2;-4)$;
3) $A(0;0)$, $B(1;1)$.
**1.3.7**. $A(3;4)$ va $B(2;-1)$ nuqtalar berilgan. $AB$ to‘g‘ri chiziqning koordinata o‘qlari bilan kesishish nuqtalari topilsin.

**1.3.8**. Uchlari $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ nuqtalarda joylashgan uchburchakning og‘irlik markazi topilsin.
**1.3.9**. Uchburchak tomonlarining o‘rtalari $M_1(2;4)$, $M_2(-3;0)$, $M_3(2;1)$ berilgan. Uning uchlari topilsin.
**1.3.10**. Uchburchakning uchlari $A(1;1)$, $B(7;1)$, $C(1;7)$ berilgan. Uchburchak tomonlarining o‘rtalari topilsin.
**1.3.11**. $AB$ kesmaning bir uchi $A(2;3)$ nuqtada joylashgan. $M(1;-2)$ nuqta uning o‘rtasi. Kesmaning ikkinchi uchi topilsin.
**1.3.12**. $MN$ kesmaning bir uchi $K(-2;1)$ nuqtada joylashgan. $M(2;5)$ nuqta uning o‘rtasi. Kesmaning ikkinchi uchi topilsin.
**1.3.13**. $AB$ kesmaning bir uchi $A(-4;2)$ nuqtada joylashgan. $M(4;-1)$ nuqta uning o‘rtasi. Kesmaning ikkinchi uchi topilsin.
**1.3.14**. Parallelogrammning qo‘shni uchlari $A(-4;-7)$, $B(2;6)$ va diagonallari kesishgan $M(3;1)$ nuqta berilgan. Uning qolgan ikki uchining koordinatalari topilsin.
**1.3.15**. $Ox$, $Oy$ o‘qlariga mos ravishda $OA = 8$, $OB = 4$ kesmalar joylashgan. Koordinatalar boshidan $AB$ to‘g‘ri chiziqqa perpendikulyar tushirilgan. Perpendikulyar asosi $AB$ kesmani qanday nisbatda bo‘ladi?(Dekart koordinatalar sistemasi).
**1.3.16**. $A(-3;1)$, $B(2;-3)$ nuqtalar orqali o‘tgan to‘g‘ri chiziqqa shunday $M$ nuqta topilsaki, $\overline{AM} = 3\overline{AB}$ tenglik bajarilsin.
**1.3.17**. Trapetsiyaning uchta ketma-ket joylashgan $A(-2;-3)$, $B(1;4)$, $C(3;1)$ uchlari berilgan. Agar $AD$ asosi $BC$ asosidan 5 marta katta bo‘lsa, trapetsiyaning to‘rtinchi $D$ uchi topilsin.
**1.3.18**. $A(-4;2)$ va $B(8;-7)$ nuqtalar berilgan. $AB$ kesmani uchta teng bo‘lakka bo‘luvchi $C, D$ nuqtalar topilsin.
**1.3.19**. $A(3;4)$ va $B(-6;11)$ nuqtalar berilgan. $AB$ kesmani uchta teng bo‘lakka bo‘luvchi $C, D$ nuqtalar topilsin.
**1.3.20**. $C(2;2)$, $D(1;5)$ nuqtalar $AB$ kesmani uchta teng bo‘lakka bo‘lsa, uning $A, B$ uchlari topilsin.
**1.3.21**. $C(-2;4)$, $D(1;8)$ nuqtalar $AB$ kesmani uchta teng bo‘lakka bo‘lsa, uning $A, B$ uchlari topilsin.

**1.3.22**. $A(2;4)$ nuqta berilgan. $AB$ to‘g‘ri chiziq ordinata o‘qini $C$ nuqtada, abssissa o‘qini $D$ nuqtada kesib o‘tadi. $C$ nuqta $AB$ kesmani $\frac{2}{3}$ nisbatda va $D$ nuqta $-\frac{3}{4}$ nisbatda bo‘lishini bilgan holda $B$ nuqtaning koordinatalari topilsin.

**1.3.23**. Kesmaning uchlari $M(3;-2)$ va $N(10;-9)$ nuqtalarda yotadi. $C$ nuqta kesmani $\lambda=\frac{2}{5}$ nisbatda bo‘lsa, shu nuqtaning koordinatalarini toping.

**1.3.24**. $B(-3;4)$ nuqta $AC$ kesmani $\lambda=\frac{2}{3}$ nisbatda bo‘lsa, $A(1;2)$ ni bilgan holda $C(x;y)$ni koordinatalarini toping.

**1.3.25**. $C(-5;4)$ nuqta $AB$ kesmani $\frac{3}{4}$ nisbatda, $D(6;-5)$ nuqta esa $\frac{2}{3}$ bo‘lsa, $A, B$ nuqtalarning koordinatalari topilsin.

**1.3.26**. Uchlari $A(5;-4)$, $B(-1;2)$, $C(5;1)$ nuqtalarda bo‘lgan uchburchakning $AD$ medianasining uzunligini topilsin.

**1.3.27**. $(4;2)$, $(0;-1)$ nuqtalardan o‘tadigan to‘g‘ri chiziqda $(-4;-4)$ nuqtadan 5 birlik masofada joylashgan nuqtalar topilsin.

**1.3.28**. $(4;8)$, $(-1;-4)$ nuqtalardan o‘tadigan to‘g‘ri chiziqda $(-1;-4)$ nuqtadan 4 birlik masofada joylashgan nuqtalar topilsin.

**1.3.29**. $ABC$: $A(4;1)$, $B(7;5)$, $C(-4;7)$ uchburchakning $AD$ bissektrisasining uzunligi hisoblansin.

**1.3.30**. Trapetsiyaning uchta ketma-ket $A(-1;-2), B(1;3), C(9;9)$ uchlari berilgan. Trapetsiyaning asosi $AD = 15$ bo‘lsa, uning to‘rtinchi $D$ uchi topilsin.

# 2- MAVZU: VEKTORLAR VA ULAR USTIDA CHIZIQLI AMALLAR

**Reja:**

**1. Ta’rif va tushunchalar.**

**2. Vektorlar ustida amallar. Vektorni qo‘shish, ayirish va songa ko‘paytirish.**

**3. Berilgan vektorni bazis vektorlar bo‘yicha yoyish.**

**Tayanch iboralar:** nol vektor, kollinear, birlik vektor, teng vektor, vektorning yig‘indisi, vektorlarning ayirmasi, chiziqli bog‘liq, komplanar, yoyilgan, bazis.

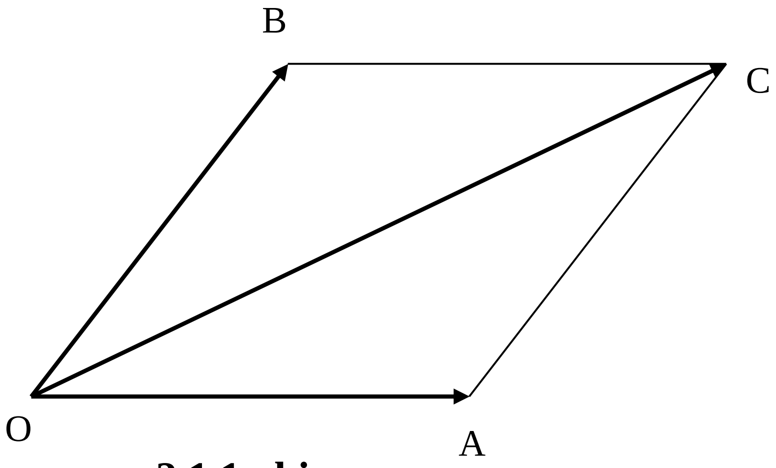
## 2.1. Ta’rif va tushunchalar.

Yo‘naltirilgan kesmaga vektor deyiladi. Ma’lumki kesma ikki nuqtani tutashtirishdan hosil bo‘ladi. Birinchi nuqta vektorning – ***boshi***, ikkinchisi – ***oxiri*** deyiladi. Boshi bilan oxiri ustma-ust tushgan vektor ***nol*** vektor deyiladi. Nol vektordan farqli har qanday vektor yo‘nalgan kesma bilan tasvirlanadi.

Vektorlar **AB, CD,** $\overline{AB}, \overline{CD}$ yoki $\overrightarrow{AB}, \overrightarrow{CD}$ kabi belgilanadi.

$AB, CD$ to‘g‘ri chiziqlar parallel yoki ustma-ust tushsa, ikkita nol bo‘lmagan $\overrightarrow{AB}, \overrightarrow{CD}$ vektorlar ***kollinear*** deyiladi. Nol vektor har qanday vektorga kollinear hisoblanadi.

Noldan farqli $\overrightarrow{AB},$ vektorning uzunligi deb, $\boldsymbol{AB}$ kesmaning uzunligiga aytiladi va $|\overrightarrow{AB}|$ kabi belgilanadi. Ta’rif bo‘yicha nol vektorning uzunligi (moduli) nolga teng. Uzunligi birga teng bo‘lgan vektor ***birlik vektor*** deyiladi.




**2.1.1-chizma**

Nol bo‘lmagan ikkita $\overrightarrow{AB},\ \overrightarrow{CD}$ vektorlarning uzunliklari teng va bir xil yo‘nalgan bo‘lsa, bunday vektorlar ***teng vektorlar*** deyiladi.

Noldan farqli $\vec{a}$ vektor uchun $\frac{\vec{a}}{|\vec{a}|} = \vec{a}^0$ dan iborat vektor $\vec{a}$ bilan bir xil yo‘nalgan birlik vektor bo‘ladi, bu yerda: $\vec{a} = |\vec{a}| \cdot \vec{a}^0$.

**1-misol.** $A(3;-1;2)$ nuqta $\overrightarrow{AB}$ vektorning boshi $B(-1;2;1)$ nuqta esa oxiri bo‘lsa, $\overrightarrow{AB}$ vektorning koordinatalarini toping.

**Yechish:** $\overrightarrow{AB}$ vektorning koordinatalarini topish uchun mos ravishda $B$ nuqtaning koordinatalaridan $A$ nuqtaning koordinatalarini ayiramiz.

$$\overrightarrow{AB}(-1-3; 2-(-1); 1-2)$$
$$\overrightarrow{AB}(-4;3;-1).$$

**2-Misol.** $\vec{a}(3;-6;-2)$ vektorga yo‘nalishdosh bo‘lgan birlik vektorni toping.

**Yechish:** Birlik vektorni quyidagicha yozish mumkin.

$$\vec{a}^0 = \vec{\imath}\,\vec{a}_x^0 + \vec{\jmath}\ \vec{a}_y^0 + \vec{k}\ \vec{a}_z^0$$

endi $\vec{a}_x^0,\ \vec{a}_y^0,\ \vec{a}_z^0$ larni topamiz.

$$\vec{a}_x^0 = \frac{\vec{a}_x}{|a|};\ \ \vec{a}_y^0 = \frac{\vec{a}_y}{|a|};\ \vec{a}_z^0 = \frac{\vec{a}_z}{|a|}$$

$|a| = \sqrt{6^2 + (-2)^2 + (-3)^2} = \sqrt{49} = 7$ bo‘lgani uchun. Bundan $\vec{a}_x^0 = \frac{3}{7};\quad \vec{a}_y^0 = -\frac{6}{7};\ \ \vec{a}_z^0 = -\frac{2}{7}$ ga ega bo‘lamiz. Demak, berilgan vektorga yo‘nalishdosh birlik vektor quyidagicha $\vec{a}^0 = \left(\frac{3}{7}, -\frac{6}{7}, -\frac{2}{7}\right)$ bo‘ladi.

**2.2. Vektorlar ustida amallar. Vektorni qo‘shish, ayirish va songa ko‘paytirish.**

Ikki vektorni qo‘shish deb, birinchi vektorning oxiriga ikkinchi vektorning boshi keltirib qo‘yilganda birinchi vektorning boshidan chiqib ikkinchi vektorning oxiriga tomon yo‘nalgan vektorga aytiladi. $\vec{a}, \vec{b}, \vec{c}, \dots, \vec{k}$ vektorlarning yig‘indisi deb, quyidagicha yasaladigan $\vec{a} + \vec{b} + \vec{c} \dots + \vec{k}$ vektorga aytiladi.

Ixtiyoriy $O$ nuqtaga $\vec{a}$ vektor qo‘yiladi, uning oxiriga $\vec{b}$ vektorning boshi qo‘yiladi va hokazo. Olingan $O$ nuqta $\vec{a} + \vec{b} + \vec{c} \dots + +\vec{k}$ vektorning boshi, eng so‘ngi vektorning oxiri esa, yig‘indining oxiri deyiladi.

Vektorlarning yig‘indisi $O$ nuqtani tanlab olishga bog‘liq emas.

Kollinear bo‘lmagan ikkita $\vec{a}, \vec{b}$ vektorlarning yig‘indisi quyidagicha ham yasalishi mumkin (parallelogramm qoidasi): ikkala $\vec{a}, \vec{b}$ vektorni bitta $O$ nuqtadan boshlab $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ vektorlar qo‘yiladi; tomonlari $OA$, $OB$ bo‘lgan $OBCA$ parallelogramm yasaladi, u holda $\overrightarrow{OC} = \overrightarrow{AB} + \overrightarrow{OB} = \vec{a} + \vec{b}$ hosil bo‘ladi.

$$\vec{x} + \vec{b} = \vec{a} \tag{2.1}$$

shartni qanoatlantiruvchi $\vec{x}$ vektorga $\vec{a}, \vec{b}$ vektorlarning ayirmasi deyiladi.

$\vec{a}, \vec{b}$ vektorlarning $\vec{a} - \vec{b}$ ayirmasini yasash uchun quyidagicha ish ko‘riladi: $\vec{a}, \vec{b}$ vektorlar bitta nuqtadan qo‘yiladi $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$. U hol $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ da $\overrightarrow{BA} = \overrightarrow{OA} + \overrightarrow{OB} = \vec{a} + \vec{b}$.

$\vec{a} \neq 0$ vektorga qarama - qarshi vektor deb $\vec{a}$ vektorga kollinear, moduli shu vektor moduliga teng, yo‘nalishi esa $\vec{a}$ vektor yo‘nalishiga qarama - qarshi bo‘lgan vektorga aytiladi. Ravshanki, qo‘shish amalining xossalari quyidagicha bo‘ladi:

$$\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c} \quad \text{(assotsiativlik)}$$

$$\vec{a} + 0 = \vec{a}$$

$$\vec{a} + (-\vec{a}) = 0$$

$$\vec{a} + \vec{b} = \vec{b} + \vec{a} \qquad \text{(kommutativlik)} \qquad (2.2)$$

$\lambda$ son bilan $\vec{a} \neq 0$ vektorning ko‘paytmasi deb, $\vec{a}$ vektorga kollinear, moduli $|\lambda|\,|\vec{a}|$ bo‘lgan $\vec{a}$ vektor bilan bir xil, $\lambda < 0$ holda unga qarama - qarshi yo‘nalgan $\lambda\vec{a}$ vektorga aytiladi. Agar $\lambda = 0$ yoki $\vec{a} = 0$ bo‘lsa $\lambda\vec{a} = 0$ bo‘ladi. Vektorni songa ko‘paytirish amalining xossalari:

$$1 \cdot \vec{a} = \vec{a}$$

$$\lambda(\mu\vec{a}) = (\lambda\mu) \cdot \vec{a}$$

$$\lambda(\vec{a} + \vec{b}) = \lambda \cdot \vec{a} + \lambda \cdot \vec{b}$$

$$(\lambda + \mu) \cdot \vec{a} = \lambda \cdot \vec{a} + \mu \cdot \vec{a} \qquad (2.3)$$

Agar $\vec{a}$, $\vec{b}$ vektorlar kollinear va $\vec{b} \neq 0$ bo‘lsa, $\frac{\vec{a}}{\vec{b}}$ nisbat deb shunday $\lambda$ songa aytiladiki, unda $\vec{a} = \lambda \cdot \vec{b}$ bo‘ladi.

**3-Misol.** $\vec{a}(-2; 3; 1)$ va $\vec{b}(8; -4; -6)$ vektorlar berilgan. Quyidagi $3\vec{a} - \frac{1}{2}\vec{b}$ vektorning koordinata o‘qlaridagi proyeksiyalarini aniqlang.

**Yechish:** Endi $3\vec{a}$ va $\frac{1}{2}\vec{b}$ vektorlarni aniqlaymiz. $3\vec{a} = \{-6; 9; 3\}$, $\frac{1}{2}\vec{b} = \{4; -2; -3\}$. Demak,

$$3\vec{a} - \frac{1}{2}\vec{b} = \{-6 - 4; 9 - (-2); 3 - (-3)\}$$

$$3\vec{a} - \frac{1}{2}\vec{b} = \{-10; 11; 6\}.$$

**4-Misol.** $\vec{a} = \vec{\imath} + 3\vec{\jmath} - 2\vec{k}$ va $\vec{b} = 2\vec{\imath} + \vec{\jmath} + 4\vec{k}$ vektorlar berilgan $2\vec{a} + 3\vec{b}$ vektorlar yig‘indisini toping.

**Yechish:** $\vec{a}$ vektorni koordinatalari, $\vec{a}(1; 3; -2)$ xuddi shuningdek $\vec{b}(2; 1; 4)$. Endi $2\vec{a}$ va $3\vec{b}$ vektorlarni aniqlaymiz. $2\vec{a} = \{2; 6; -4\}$, $3\vec{b} = \{6; 3; 12\}$. Demak,

$$2\vec{a} + 3\vec{b} = \{2 + 6; 6 + 3; -4 + 12\}$$

$$2\vec{a} + 3\vec{b} = \{8; 9; 8\}.$$

### 2.3. Berilgan vektorni bazis vektorlar bo‘yicha yoyish.

Vektorlar uchun bir vaqtning o‘zida nolga teng bo‘lmagan $\alpha, \beta, \gamma, \dots, x$ sonlar mavjud bo‘lib, $\alpha \cdot \vec{a} + \beta \cdot \vec{b} + \gamma \cdot \vec{k} = 0$ tenglik bajarilsa, $\vec{a}, \vec{b}, \vec{c}, \dots, \vec{k}$ vektorlar ***chiziqli bog‘liq*** deb ataladi.

Ikki vektorning kollinear bo‘lishi uchun ular chiziqli bog‘liq bo‘lishi zarur va yetarlidir.

Agar uchta $\vec{a}, \vec{b}, \vec{c}$ vektorni bitta nuqtaga keltirgandan keyin ular bitta tekislikda yotsa, $\vec{a}, \vec{b}, \vec{c}$ vektorlar ***komplanar*** deyiladi. Uchta vektorning komplanar bo‘lishi uchun ular chiziqli bog‘liqli bo‘lishi zarur va yetarli.

Agar $\vec{a}, \vec{b}$ vektorlar kollinear bo‘lmasa va $\vec{a}, \vec{b}, \vec{c}$ vektorlar esa komplanar bo‘lmasa, u holda $\vec{c}$ vektor yagona usulda $\vec{a}, \vec{b}$ vektorlarning chiziqli kombinatsiyasi ko‘rinishida ifodalanishi mumkin:

$$\vec{c} = \alpha \cdot \vec{a} + \beta \cdot \vec{b} \qquad (2.4)$$

Bu holda $\vec{c}$ vektor $\vec{a}, \vec{b}$ vektorlar orqali ***yoyilgan*** deyiladi.

Agar $\vec{a}, \vec{b}, \vec{c}$ vektorlar komplanar bo‘lmasa, u holda ixtiyoriy $\vec{d}$ vektorni $\vec{a}, \vec{b}, \vec{c}$ vektorlarning kombinatsiyasi ko‘rinishida yagona ravishda ifodalash mumkin:

$$\vec{d} = \alpha \cdot \vec{a} + \beta \cdot \vec{b} + \gamma \cdot \vec{c} \qquad (2.5)$$

Bu holda ham $\vec{d}$ vektor $\vec{a}, \vec{b}, \vec{c}$ vektorlar yo‘nalishilari bo‘yicha ***yoyilgan*** deyiladi. $M$ nuqtaning radius vektori $\vec{r}$ deb $\overrightarrow{OM}$ vektorga aytiladi, bu yerda $O -$ tayin bir nuqta.

Agar $M$ nuqta $M_1, M_2$ kesmani $\lambda$ nisbatda bo‘lsa, u holda $M$ nuqtaning $\vec{r}$ radius-vektori, $M_1, M_2$ nuqtalarning $\vec{r}_1, \vec{r}_2$ radius-vektorlari orqali quyidagicha ifodalanadi:

$$\vec{r} = \frac{\vec{r}_1 + \lambda \vec{r}_2}{1 + \lambda}$$

Xususan, $M$ nuqta $M_1, M_2$ kesmaning o‘rtasi bo‘lsa, $\vec{r} = \frac{\vec{r}_1 + \vec{r}_2}{2}$ tartiblangan nokomplanar uchta $\vec{e}_1, \vec{e}_2, \vec{e}_3$ vektor fazo ***bazisi*** deb ataladi, agar $\vec{e}_1, \vec{e}_2, \vec{e}_3$ vektorlar birlik va jufti-jufti bilan ortogonal bo‘lsa, u holda bazis ***ortonormallangan*** deyiladi. Ortonormallangan bazis vektorlari ko'pincha $\vec{i}, \vec{j}, \vec{k}$ bilan belgilanadi.

$\vec{a}$ vektorning $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisdagi koordinatalari deb shunday $x, y, z$ sonlarga aytiladiki, bunda

$$\vec{a} = x \cdot \vec{e}_1 + y \cdot \vec{e}_2 + z \cdot \vec{e}_3 \quad (2.6)$$

koordinatalari $x, y, z$ dan iborat $\vec{a}$ vektor $\vec{a} = \{x, y, z\}$ ko'rinishda yoziladi. Mos koordinatalarigina teng bo'lgan ikki vektor o'zaro teng bo'ladi:

$$x = x'; \; y = y'; \; z = z' \Rightarrow \vec{a}(x, y, z) = \vec{b}(x', y', z').$$

Ikki $\vec{a}(x, y, z) \neq 0, \vec{b}(x', y', z') \neq 0$ vektorning kollinear bo'lishligining zaruriy va yetarli sharti ularning mos koordinatalarining proportsionalligidan iborat:

$$x' = \lambda \cdot x; \; y' = \lambda \cdot y; \; z' = \lambda \cdot z \quad (2.7)$$

bunda $\lambda$ son $\vec{b}$ vektorning $\vec{a}$ vektorga nisbatini bildiradi.

$\vec{a} = \{x, y, z\}, \vec{b} = \{x', y', z'\}$ vektorlar uchun quyidagi munosabatlar o'rinli: $\vec{a} + \vec{b} = \{x + x', y + y', z + z'\}$

$$\vec{a} - \vec{b} = \{x - x', y - y', z - z'\}$$
$$\lambda \cdot \vec{a} = \{\lambda \cdot x, \lambda \cdot y, \lambda \cdot z\}$$

Uchta $\vec{a} = \{x, y, z\}, \vec{b} = \{x', y', z'\}, \vec{c} = \{x'', y'', z''\}$ vektorlar komplanar bo'lishining zarur va yetarli sharti ushbu

$$\begin{vmatrix} x & y & z \\ x' & y' & z' \\ x'' & y'' & z'' \end{vmatrix} = 0$$

tenglikning bajarilishidan iborat.

**5-Misol.** Tekislikda ikkita vektorlar $\vec{b}(-1; 4)$, $\vec{c}(3; -2)$ berilgan. $\vec{a}(-11; 14)$ vektorning $\vec{b}$, $\vec{c}$ bazisi bo'yicha yoyilmasini toping.

**Yechish.** $\vec{a}$ vektorni $\vec{b}$ va $\vec{c}$ vektorlar bo'yicha yoyish, $\vec{a}$ vektorni chiziqli kombinatsiya ko'rinishida ifodalash demakdir. $\vec{a} = \lambda\vec{b} + \mu\vec{c}$, bu yerda $\lambda$ va $\mu$ topilishi kerak bo'lgan sonlar.

Koordinata ko'rinishida bu quyidagicha bo'ladi.

$$-11\vec{i} + 14\vec{j} = (-\lambda + 3\mu)\vec{i} + (4\lambda - 2\mu)\vec{j}$$

Natijada quyidagi tenglamalar sistemasini hosil qilamiz.

$$\begin{cases} -\lambda + 3\mu = -11 \\ 4\lambda - 2\mu = 14 \end{cases}$$

Bu sistemani yechib, $\lambda = 2;\ \mu = -3$ ekanligini topamiz.

Demak, $\vec{a} = 2\vec{b} - 3\vec{c}$.

**6-Misol.** $\vec{a}(4;2;0)$ vektorni $\vec{p}(1;-1;2)$, $\vec{q}(2;2;-1)$ va $\vec{r}(3;7;-7)$ vektorlar bo‘yicha yoying.

**Yechish.** $\vec{a}$ vektorni $\vec{p}$, $\vec{q}$ va $\vec{r}$ vektorlar bo‘yicha yoyish, $\vec{a}$ vektorni chiziqli kombinatsiya ko‘rinishida ifodalash demakdir.

$\vec{a} = c_1\vec{p} + c_2\vec{q} + c_3\vec{r}$, bu yerda $c_1, c_2$ va $c_3$ - topilishi kerak bo‘lgan sonlar.

Koordinata ko‘rinishida bu quyidagicha bo‘ladi.

$4\vec{i} + 2\vec{j} + 0 \cdot \vec{k} = (c_1 + 2c_2 + 3c_3)\vec{i} + (-c_1 + 2c_2 + 7c_3)\vec{j} +$
$+(2c_1 - c_2 - 7c_3)\vec{k}$

Natijada quyidagi tenglamalar sistemasini hosil qilamiz.

$$\begin{cases} c_1 + 2c_2 + 3c_3 = 4 \\ -c_1 + 2c_2 + 7c_3 = 2 \\ 2c_1 - c_2 - 7c_3 = 0 \end{cases}$$

Bu sistemani yechib, $c_1 = 3;\ c_2 = -1;\ c_3 = 1$ ekanligini topamiz. Demak,

$$\vec{a} = 3\vec{p} - \vec{q} + \vec{r}.$$

## Mustaqil yechish uchun topshiriqlar

### 2.1. Ta’rif va tushunchalarga doir misollar

**2.1.1.** $A(5;7)$ nuqta $\overrightarrow{AB}$ vektorning boshi $B(-2;4)$ nuqta esa oxiri bo‘lsa, $\overrightarrow{AB}$ vektorning koordinatalarini toping.

**2.1.2.** $C(-3;4;-2)$ nuqta $\overrightarrow{CD}$ vektorning boshi $D(5;-6;3)$ nuqta esa oxiri bo‘lsa, $\overrightarrow{CD}$ va $\overrightarrow{DC}$ vektorlar koordinatalarini toping.

**2.1.3.** $A(8;-7;-3)$ nuqta $\overrightarrow{AB}$ vektorning boshi $B(-4;-9;4)$ nuqta esa oxiri bo‘lsa, $\overrightarrow{AB}$ va $\overrightarrow{BA}$ vektorlar koordinatalarini toping.

**2.1.4.** $\overrightarrow{AB}(6;-8)$ vektorning boshi $A(8;-7)$ ekanligi ma’lum bo‘lsa, vektorning oxiri $B(x;y)$ nuqtani toping.

**2.1.5.** $\overrightarrow{AB}(-2;3;-7)$ vektorning boshi $A(-3;5;6)$ ekanligi ma'lum bo'lsa, vektorning oxiri $B(x;y;z)$ nuqtani toping.

**2.1.6.** $\overrightarrow{AB}(5;4;-2)$ vektorning boshi $A(2;-5;6)$ ekanligi ma'lum bo'lsa, vektorning oxiri $B(x;y;z)$ nuqtani toping.

**2.1.7.** $\overrightarrow{AB}(-5;7)$ vektorning oxiri $B(4;-1)$ ekanligi ma'lum bo'lsa, vektorning boshi $A(x;y)$ nuqtani toping.

**2.1.8.** $\overrightarrow{AB}(-2;-1;4)$ vektorning oxiri $B(-6;7;-3)$ ekanligi ma'lum bo'lsa, vektorning boshi $A(x;y;z)$ nuqtani toping.

**2.1.9.** $\overrightarrow{AB}(1;7;-9)$ vektorning oxiri $B(1;-3;-2)$ ekanligi ma'lum bo'lsa, vektorning boshi $A(x;y;z)$ nuqtani toping.

**2.1.10.** $\vec{a}(-4;3)$ vektorga yo'nalishdosh bo'lgan birlik vektorni toping.

**2.1.11.** $\vec{b}(-8;-6)$ vektorga yo'nalishdosh bo'lgan birlik vektorni toping.

**2.1.12.** $\vec{c}(9;-12)$ vektorga qarama-qarshi yo'nalgan birlik vektorni toping.

**2.1.13.** $\vec{d}(6;-2;-3)$ vektorga yo'nalishdosh bo'lgan birlik vektorni toping.

**2.1.14.** $\vec{a}(-4;3;12)$ vektorga yo'nalishdosh bo'lgan birlik vektorni toping.

**2.1.15.** $\vec{b}(2;-6;-9)$ vektorga yo'nalishdosh bo'lgan birlik vektorni toping.

**2.1.16.** $\vec{c}(3;4;-12)$ vektorga qarama-qarshi yo'nalgan birlik vektorni toping.

**2.1.17.** $\vec{d}(-1;12;-12)$ vektorga qarama-qarshi yo'nalgan birlik vektorni toping.

**2.1.18.** $\vec{b}(2;-10;11)$ vektorga qarama-qarshi yo'nalgan birlik vektorni toping.

**2.1.19.** $\vec{a}(6;-8)$ vektor berilgan. $\vec{a}$ ga kollinear va; 1) $\vec{a}$ bilan bir xil yo'nalgan;
2) $\vec{a}$ bilan qarama-qarshi yo'nalgan birlik vektor topilsin.

**2.1.20.** $\vec{b}(-0{,}9; 0{,}1)$ vektor berilgan. $\vec{b}$ ga kollinear va; 1) $\vec{b}$ bilan bir xil yo‘nalgan; 2) $\vec{b}$ bilan qarama-qarshi yo‘nalgan birlik vektor topilsin.
**2.1.21.** $\vec{c}(11; -7; 3)$ vektor berilgan. $\vec{c}$ ga kollinear va; 1) $\vec{c}$ bilan bir xil yo‘nalgan; 2) $\vec{c}$ bilan qarama-qarshi yo‘nalgan birlik vektor topilsin.
**2.1.22.** $\vec{d}(-8; 4; 1)$ vektor berilgan. $\vec{d}$ vektor bilan bir xil yo‘nalgan birlik vektor topilsin.
**2.1.23.** $\vec{a}(9; -2; 6)$ vektor berilgan. $\vec{a}$ vektor bilan bir xil yo‘nalgan birlik vektor topilsin.
**2.1.24.** $\vec{b}(10; 2; -11)$ vektor berilgan. $\vec{b}$ vektor bilan qarama-qarshi yo‘nalgan birlik vektor topilsin.
**2.1.25.** $\vec{a}(3; -5)$ vektorga yo‘nalishdosh, uzunligi 3 ga teng bo‘lgan vektorni toping.
**2.1.26.** $\vec{b}(-2; 4; -3)$ vektorga yo‘nalishdosh, uzunligi 5 ga teng bo‘lgan vektorni toping.
**2.1.27.** $\vec{c}(-6; 1)$ vektorga qarama-qarshi, uzunligi 4 ga teng bo‘lgan vektorni toping.
**2.1.28.** $\vec{d}(-6; 1; -3)$ vektorga qarama-qarshi, uzunligi 6 ga teng bo‘lgan vektorni toping.
**2.1.29*.** Bitta nuqtadan $\vec{a}(-12; 16)$, $\vec{b}(12; 5)$ vektorlar o‘tkazilgan. $\vec{a}$ bilan $\vec{b}$ vektorlar orasidagi burchakni teng ikkiga bo‘ladigan va shu nuqtadan chiqqan birlik vektorning koordinatalari topilsin.
**2.1.30*.** Bitta nuqtadan $\vec{a}(-3; 0; 4)$, $\vec{b}(5; -2; -14)$ vektorlar o‘tkazilgan. $\vec{a}, \vec{b}$ vektorlar orasidagi burchakni teng ikkiga bo‘ladigan birlik vektor topilsin.

### 2.2. Vektorlar ustida amallar. Vektorni qo‘shish, ayirish va songa ko‘paytirishga doir misollar

**2.2.1.** $\vec{a}$ va $\vec{b}$ berilgan vektorlardan foydalanib, quyidagi vektorlarni har birini bo‘yicha har bir vektorlarni yasang:
1) $\vec{a} + \vec{b}$; 2) $\vec{a} - \vec{b}$; 3) $\vec{b} - \vec{a}$; 4) $-\vec{a} - \vec{b}$.

**2.2.2.** Berilgan $\vec{a}$ va $\vec{b}$ vektorlar bo‘yicha quyidagi har bir vektorlarni yasang:

1) $3\vec{a}$; 2) $-\frac{1}{2}\vec{b}$; 3) $2\vec{a}+\frac{1}{3}\vec{b}$; 4) $\frac{1}{2}\vec{a}-3\vec{b}$.

**2.2.3.** $\vec{a}(12;-3)$ va $\vec{b}(-3;6)$ vektorlar berilgan. Quyidagi vektorlarning koordinata o‘qlaridagi proyeksiyalarini aniqlang:

1) $2\vec{a}+\vec{b}$; 2) $\vec{a}-2\vec{b}$; 3) $-3\vec{a}$; 4) $-\frac{1}{3}\vec{b}$; 5) $4\vec{a}+3\vec{b}$;

6) $\frac{1}{3}\vec{a}-2\vec{b}$.

**2.2.4.** $\vec{a}(-4;1)$ va $\vec{b}(6;-8)$ vektorlar berilgan. Quyidagi vektorlarning koordinata o‘qlaridagi proyeksiyalarini aniqlang:

1) $\vec{a}+\vec{b}$; 2) $\vec{a}-\vec{b}$; 3) $2\vec{a}$; 4) $-\frac{1}{2}\vec{b}$; 5) $2\vec{a}+3\vec{b}$; 6) $\frac{1}{4}\vec{a}-\vec{b}$.

**2.2.5.** $\vec{a}(8;-4)$ va $\vec{b}(-9;-3)$ vektorlar berilgan. Quyidagi vektorlarning koordinata o‘qlaridagi proyeksiyalarini aniqlang:

1) $3\vec{a}-2\vec{b}$; 2) $\vec{a}+2\vec{b}$; 3) $-2\vec{a}$; 4) $\frac{1}{3}\vec{b}$; 5) $-3\vec{a}+2\vec{b}$

6) $\frac{1}{4}\vec{a}-2\vec{b}$.

**2.2.6.** $\vec{a}(-2;3;-4)$ va $\vec{b}(0;-2;6)$ vektorlar berilgan. Quyidagi vektorlarning koordinata o‘qlaridagi proyeksiyalarini aniqlang:

1) $-\vec{a}+2\vec{b}$; 2) $\vec{a}-3\vec{b}$; 3) $-4\vec{a}$; 4) $-\frac{2}{3}\vec{b}$; 5) $4\vec{a}+\vec{b}$;

6) $\frac{1}{2}\vec{a}-3\vec{b}$.

**2.2.7.** $\vec{a}(-1;5;-6)$ va $\vec{b}(4;-9;-3)$ vektorlar berilgan. Quyidagi vektorlarning koordinata o‘qlaridagi proyeksiyalarini aniqlang:

1) $3\vec{a}-2\vec{b}$; 2) $\vec{a}+2\vec{b}$; 3) $-2\vec{a}$; 4) $\frac{1}{3}\vec{b}$; 5) $-3\vec{a}+2\vec{b}$

6) $\frac{1}{4}\vec{a}-2\vec{b}$.

**2.2.8.** $\vec{a}(3;-1;5)$ va $\vec{b}(-2;4;-6)$ vektorlar berilgan. Quyidagi vektorlarning koordinata o‘qlaridagi proyeksiyalarini aniqlang:

1) $3\vec{a}+\vec{b}$; 2) $\vec{a}-2\vec{b}$; 3) $-4\vec{a}$; 4) $\frac{1}{2}\vec{b}$; 5) $3\vec{a}+4\vec{b}$;

6) $\frac{1}{3}\vec{a}+\vec{b}$.

**2.2.9.** $\vec{a}(3;-2;6)$ va $\vec{b}(-2;1;0)$ vektorlar berilgan. Quyidagi vektorlarning koordinata o‘qlaridagi proyeksiyalarini aniqlang:

1) $\vec{a}+\vec{b}$; 2) $\vec{a}-\vec{b}$; 3) $2\vec{a}$; 4) $-\frac{1}{2}\vec{b}$; 5) $2\vec{a}+3\vec{b}$; 6) $\frac{1}{3}\vec{a}-\vec{b}$.

**2.2.10.** Uchta $\vec{a}=\{2;4\}$, $\vec{b}=\{-3;1\}$, $\vec{c}=\{5;-2\}$ vektor berilgan.

1) $2\vec{a}+3\vec{b}-5\vec{c}$; 2) $\vec{a}-14\vec{b}+14\vec{c}$ vektorlar topilsin.

**2.2.11.** Uchta $\vec{a}=\{7;-1\}$, $\vec{b}=\{2;-3\}$, $\vec{c}=\{4;-1\}$ vektor berilgan.

1) $3\vec{a}-2\vec{b}+\vec{c}$; 2) $5\vec{a}+\vec{b}+2\vec{c}$ vektorlar topilsin.

**2.2.12.** Uchta $\vec{a}=\{-3;1\}$, $\vec{b}=\{6;-2\}$, $\vec{c}=\{-4;-7\}$ vektor berilgan.

1) $2\vec{a}-4\vec{b}+7\vec{c}$; 2) $5\vec{a}+3\vec{b}-4\vec{c}$ vektorlar topilsin.

**2.2.13.** Uchta $\vec{a}=\{5;7;2\}$, $\vec{b}=\{3;0;4\}$, $\vec{c}=\{-6;1;-1\}$ vektor berilgan.

1) $3\vec{a}-2\vec{b}+\vec{c}$; 2) $5\vec{a}+6\vec{b}+4\vec{c}$ vektorlar topilsin.

**2.2.14.** Uchta $\vec{a}=\{-4;2;-1\}$, $\vec{b}=\{2;-1;0\}$, $\vec{c}=\{3;-2;5\}$ vektor berilgan.

1) $2\vec{a}+4\vec{b}-3\vec{c}$; 2) $5\vec{a}-3\vec{b}+4\vec{c}$ vektorlar topilsin.

**2.2.15.** Uchta $\vec{a}=\{6;-3;-2\}$, $\vec{b}=\{-1;4;-3\}$, $\vec{c}=\{-4;3;-6\}$ vektor berilgan.

1) $2\vec{a}-3\vec{b}+5\vec{c}$; 2) $\vec{a}+9\vec{b}-7\vec{c}$ vektorlar topilsin.

**2.2.16.** $\vec{a}(2;4)$ va $\vec{b}(2;-1)$ vektorlar yig‘indisi va ayirmasi toping.

**2.2.17.** $\vec{a}(-2;3)$ va $\vec{b}(-4;5)$ vektorlar yig‘indisi va ayirmasi toping.

**2.2.18.** $\vec{a}(8;6)$ va $\vec{b}(4;3)$ vektorlar yig‘indisi va ayirmasi toping.

**2.2.19.** $\vec{a}(3;-5;8)$ va $\vec{b}(-1;1;-4)$ vektorlar yig‘indisi va ayirmasi toping.

**2.2.20.** $\vec{a}(-3;-1;4)$ va $\vec{b}(1;7;5)$ vektorlar yig‘indisi va ayirmasi toping.

**2.2.21.** $\vec{a}=2\vec{i}+3\vec{j}-3\vec{k}$ va $\vec{b}=\vec{i}+3\vec{j}+2\vec{k}$ vektorlar berilgan. $2\vec{a}-$ $-5\vec{b}$ vektorlar ayirmasini toping.

**2.2.22.** $\overrightarrow{AB}=\{2;6;-4\}$ va $\overrightarrow{AC}=\{4;2;-2\}$ vektorlar $ABC$ uchburchakning yon tomonlariga mos keladi. Uchburchakning medianalariga to‘g‘ri keluvchi $\overrightarrow{AM}$, $\overrightarrow{BN}$, $\overrightarrow{CP}$ vektorlarning koordinatalarini aniqlang.

**2.2.23.** $ABC$ uchburchakda $AD$ mediana o‘tkazilgan. $\overrightarrow{AD}$ vektorni $\overrightarrow{AB}$**,** $\overrightarrow{AC}$ vektorlar orqali ifodalang.

**2.2.24.** $ABC$ uchburchakda $AD,\ BE,\ CF$ medianalar o‘tkazilgan. $\overrightarrow{AD}+\overrightarrow{BC}+\overrightarrow{CF}$ vektorlar yig‘indisi topilsin.

**2.2.25.** $\overrightarrow{AB}=\vec{p}$, $\overrightarrow{AF}=\vec{q}$ vektorlar muntazam $ABCDEF$ oltiburchakning ikkita qo‘shni tomonlari. Bu oltiburchakning tomonlari bo‘ylab qo‘yilgan $\overrightarrow{BC},\ \overrightarrow{CD},\ \overrightarrow{DE},\ \overrightarrow{EF}$ vektorlarni $\vec{p},\ \vec{q}$ vektorlar orqali ifodalang.

**2.2.26.** Uchburchak tekisligida shunday nuqta topilsinki, shu nuqtadan uchburchak uchlariga yo‘nalgan vektorlar yig‘indisi nolga teng bo‘lsin.

**2.2.27.** $\overrightarrow{AB}=\vec{m}$ va $\overrightarrow{AC}=\vec{n}$ vektorlardan tashkil topgan $ABC$ uchburchakda quyidagi vektorlarni yasang:

1) $\dfrac{\vec{m}+\vec{n}}{2}$, 2) $\dfrac{\vec{m}-\vec{n}}{2}$, 3) $\dfrac{\vec{n}-\vec{m}}{2}$, 4) $-\dfrac{\vec{m}+\vec{n}}{2}$

**2.2.28.** $O$ nuqta $ABC$ uchburchakning og‘irlik markazi hisoblanadi. $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}=0$ ekanligini isbotlang.

**2.2.29.** $ABCDE$ to‘g‘ri burchakli beshburchakda uning tomonlariga to‘g‘ri keladigan vektorlar berilgan: $\overrightarrow{AB}=\vec{m},\ \overrightarrow{BC}=\vec{n},\ \overrightarrow{CD}=\vec{p}$, $\overrightarrow{EA}=\vec{r}$. Quyidagi vektorlarni yasang:

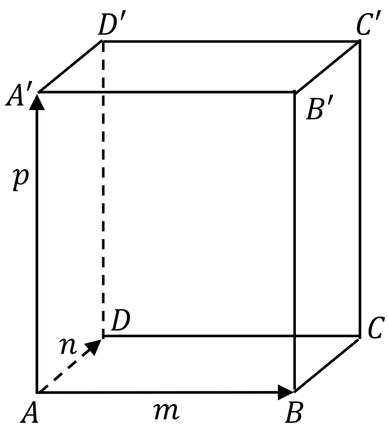
1) $\vec{m}-\vec{n}+\vec{p}-\vec{q}+\vec{r}$; 2) $\vec{m}+2\vec{p}+\frac{1}{2}\vec{q}+\vec{r}$;

3) $2\vec{m}+\frac{1}{2}\vec{n}-3\vec{m}-\vec{q}+2\vec{r}$.

**2.2.30.** $ABCDA'B'C'D'$ parallelopipedning qirralariga mos vektorlar berilgan: $\overrightarrow{AB}=\vec{m}$, $\overrightarrow{AD}=\vec{n},$ va $\overrightarrow{AA}=\vec{p}$ (2.2.1-chizma). Quyidagi har bir vektorni yasang:

1) $\vec{m}+\vec{n}+\vec{p}$; 2) $\vec{m}+\vec{n}+\frac{1}{2}\vec{p}$;

3) $\frac{1}{2}\vec{m}-\frac{1}{2}\vec{n}+\vec{p}$; 4) $\vec{m}+\vec{n}-\vec{p}$




**2.2.1-chizma**

5) $-\vec{m}-\vec{n}+\frac{1}{2}\vec{p}$.

### 2.3. Berilgan vektorni bazis vektorlar bo‘yicha yoyishga doir misollar

**2.3.1.** $\vec{i},\vec{j},\vec{k}$ bazis bo‘yicha vektorlar yoyilmasi berilgan: $\vec{c}=16\vec{i}-$ $-15\vec{j}+12\vec{k}$. Shu bazis bo‘yicha $\vec{c}$ vektorga parallel va qarama-qarshi $\vec{d}$ vektorning yoyilmasini aniqlang, bunda $|\vec{d}|=75$ ga teng.

**2.3.2.** Tekislikda $\vec{p}(2;-3)$, $\vec{q}(1;2)$ vektorlar berilgan bo‘lsin. $\vec{a}(9;4)$ vektorni $\vec{p}\,,\vec{q}$ bazis bo‘yicha yoyilmasini toping.

**2.3.3.** Tekislikda $\vec{p}(-4;1)$, $\vec{q}(3;-5)$ vektorlar berilgan bo‘lsin. $\vec{a}(11;-7)$ vektorni $\vec{p},\vec{q}$ bazis bo‘yicha yoyilmasini toping.

**2.3.4.** Tekislikda $\vec{p}(3;-2)$, $\vec{q}(-4;1)$ vektorlar berilgan bo‘lsin. $\vec{a}(17;-8)$ vektorni $\vec{p},\vec{q}$ bazis bo‘yicha yoyilmasini toping.

**2.3.5.** Tekislikda $\vec{a}(3;-2)$, $\vec{b}(-2;1)$ va $\vec{c}(7;-4)$ vektorlar berilgan. Har bir vektorni, qolgan ikki vektorni bazis sifatida qabul qilib, yoyilmasini aniqlang.

**2.3.6.** $\vec{p}(3;-2;1)$, $\vec{q}(-1;1;-2)$, $\vec{r}(2;1;-3)$ va $\vec{c}(11;-6;5)$ vektorlar berilgan. $\vec{p},\vec{q},\vec{r}$ bazis bo‘yicha $\vec{c}=\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r}$ vektorning yoyilmasini toping.

**2.3.7.** $\vec{p}(3;-2;1)$, $\vec{q}(-1;1;-2)$, $\vec{r}(2;1;-3)$ va $\vec{c}(11;-6;5)$ vektorlar berilgan. $\vec{c},\vec{q},\vec{r}$ bazis bo‘yicha $\vec{p}=\alpha\vec{c}+\beta\vec{q}+\gamma\vec{r}$ vektorning yoyilmasini toping.

**2.3.8.** $\vec{p}(3;-2;1)$, $\vec{q}(-1;1;-2)$, $\vec{r}(2;1;-3)$ va $\vec{c}(11;-6;5)$ vektorlar berilgan. $\vec{p},\vec{c},\vec{r}$ bazis bo‘yicha $\vec{q}=\alpha\vec{p}+\beta\vec{c}+\gamma\vec{r}$ vektorning yoyilmasini toping.

**2.3.9.** $\vec{p}(3;-2;1)$, $\vec{q}(-1;1;-2)$, $\vec{r}(2;1;-3)$ va $\vec{c}(11;-6;5)$ vektorlar berilgan. $\vec{p},\vec{q},\vec{c}$ bazis bo‘yicha $\vec{r}=\alpha\vec{p}+\beta\vec{q}+\gamma\vec{c}$ vektorning yoyilmasini toping.

**2.3.10.** $\vec{p}(1;-2;1)$, $\vec{q}(-1;5;3)$, $\vec{r}(7;1;-1)$ vektorlar berilgan. $\vec{p},\vec{q},\vec{r}$ bazis bo‘yicha $\vec{c}(12;-9;6)$ vektorning yoyilmasini toping.

**2.3.11.** $\vec{a}(3;-1)$, $\vec{b}(1;-2)$, $\vec{c}(-1;7)$ vektorlar berilgan. $\vec{a},\vec{b}$ bazis bo‘yicha $\vec{p}=\vec{a}+\vec{b}+\vec{c}$ vektorning yoyilmasini aniqlang.

**2.3.12.** $\vec{a}(2;1;0)$, $\vec{b}(1;-1;2)$, $\vec{c}(2;2;-1)$ va $\vec{d}(3;7;-7)$ vektorlar berilgan bo‘lsin. Har bir vektorning yoyilmasini qolgan uchta vektorni bazis sifatida qabul qilib aniqlang.

**2.3.13.** $\vec{a}(2;-1;3)$ va $\vec{b}(-6;3;-9)$ vektorlar kollinearligini tekshiring. Ularning qaysi biri necha marta uzunligini, qanday yo‘nalganligini, bir tomonga yoki qarama-qarshi ekanligini ko‘rsating.

**2.3.14.** $\alpha,\beta$ ning qanday qiymatida $\vec{a}=-2\vec{\imath}+3\vec{\jmath}+\beta\vec{k}$ va $\vec{b}=\alpha\vec{\imath}-$ $-6\vec{\jmath}+2\vec{k}$ vektorlar kollinear bo‘ladi?

**2.3.15.** $\vec{a}=-\vec{\imath}+2\vec{\jmath}+\beta\vec{k}$ va $\vec{b}=\alpha\vec{\imath}+6\vec{\jmath}-2\vec{k}$ vektorlar kollinear bo‘lsa, $\alpha$ va $\beta$ ni toping.

**2.3.16.** $\vec{a}(2;-1;3)$, $\vec{b}(-6;3;-9)$ , $\vec{c}(1;2;3)$, $\vec{d}(-6;12;18)$ vektorlar berilgan. Ulardan qaysilari o‘zaro kollinear?

**2.3.17.** $\vec{a}(\lambda \mathrm{n};\mathrm{n}-2;\mathrm{n}+1)$ va $\vec{b}(\mathrm{n}-3;\mu \mathrm{n};\mathrm{n}-1)$ vektorlar $\lambda$ va $\mu$ parametrlarning qanday qiymatlarida kollinear bo‘lishini aniqlang.

**2.3.18.** Berilgan $\vec{a}(\mathrm{n};2\mathrm{n}+1;1-\mathrm{n})$, $\vec{b}(\mathrm{n}+1;\mathrm{n}-1;\lambda)$ va $\vec{c}(\mathrm{n}-$ $-1;3\mathrm{n};1)$ vektorlar $\lambda$ - parametrning qanday qiymatida komplanar bo‘ladi?

**2.3.19.** Quyidagi hollarning har birida $\vec{c}$ vektorni $\vec{a}$, $\vec{b}$ vektorlarning chiziqli kombinatsiyasi shaklida ifodalang:

a) $\vec{a}=\{4;-2\}$, $\vec{b}=\{3;5\}$, $\vec{c}=\{1;-7\}$;

b) $\vec{a}=\{5;4\}$, $\vec{b}=\{-3;0\}$, $\vec{c}=\{19;8\}$;

c) $\vec{a}=\{-6;2\}$, $\vec{b}=\{4;7\}$, $\vec{c}=\{9;-3\}$.

**2.3.20.** Quyidagi hollarning har birida $\vec{a}$ vektorni $\vec{b}$, $\vec{c}$ vektorlarning chiziqli kombinatsiyasi shaklida ifodalang:

a) $\vec{a}=\{-8;7\}$, $\vec{b}=\{-2;3\}$, $\vec{c}=\{-4;1\}$;

b) $\vec{a}=\{14;-16\}$, $\vec{b}=\{2;-1\}$, $\vec{c}=\{-4;5\}$;

c) $\vec{a}=\{-1;2\}$, $\vec{b}=\{-2;4\}$, $\vec{c}=\{-1;3\}$.

**2.3.21.** Quyidagi hollarning har birida $\vec{b}$ vektorni $\vec{a},\ \vec{c}$ vektorlarning chiziqli kombinatsiyasi shaklida ifodalang:

a) $\vec{a} = \{-1; 2\},\ \vec{b} = \{3; -4\},\ \vec{c} = \{2; -3\};$

b) $\vec{a} = \{3; -4\},\ \vec{b} = \{-13; 13\},\ \vec{c} = \{-4; 1\};$

c) $\vec{a} = \{6; -2\},\ \vec{b} = \{-3; 1\},\ \vec{c} = \{9; -3\}.$

**2.3.22.** Quyidagi hollarning har birida $\vec{d}$ vektorni $\vec{a},\ \vec{b},\ \vec{c}$ vektorlarning chiziqli kombinatsiyasi shaklida ifodalang:

a) $\vec{a} = \{2; 3; 1\},\ \vec{b} = \{5; 7; 0\},\ \vec{c} = \{3; -2; 4\},\ \vec{d} = \{4; 12; -3\};$

b) $\vec{a} = \{5; -2; 0\},\ \vec{b} = \{0; -3; 4\},\ \vec{c} = \{-6; 0; 1\},$
$\vec{d} = \{25; -22; 16\};$

c) $\vec{a} = \{3; 5; 6\},\ \vec{b} = \{2; -7; 1\},\ \vec{c} = \{12; 0; 6\},\ \vec{d} = \{0; 20; 18\}.$

**2.3.23.** Quyidagi hollarning qaysi birida uchta $\vec{a}, \vec{b}, \vec{c}$ vektor chiziqli bog‘liq bo‘lishini va basharti ular chiziqli bog‘liq bo‘lgan holda $\vec{c}$ vektorni $\vec{a}, \vec{b}$ vektorlarning chiziqli kombinatsiyasi sifatida ifodalang:

a) $\vec{a} = \{5; 2; 1\},\ \vec{b} = \{-1; 4; 2\},\ \vec{c} = \{-1; -1; 6\};$

b) $\vec{a} = \{6; 4; 2\},\ \vec{b} = \{-9; 6; 3\},\ \vec{c} = \{-3; 6; 3\};$

c) $\vec{a} = \{6; -18; 12\},\ \vec{b} = \{-8; 24; -16\},\ \vec{c} = \{8; 7; 3\}.$

**2.3.24.** Uchta $\vec{a},\ \vec{b},\ \vec{c}$ vektor va uchta $\lambda,\ \mu,\ \nu$ son qanday bo‘lmasin, siz $\lambda\vec{a} - \mu\vec{b},\ \ \nu\vec{b} - \lambda\vec{c},\ \mu\vec{c} - \nu\vec{a}$ vektorlarning komplanar ekanligini isbotlang.

**2.3.25.** $\vec{a}(m; -12; -2),\ \ \vec{b}(0; m; 1)$ va $\vec{c}(1; 2; 3)$ vektorlar $m$ parametrning qanday qiymatlarida komplanar bo‘lishini toping.

**2.3.26.** Berilgan $\vec{a}(n; 2n+1; 1-n),\ \ \vec{b}(n+1; n-1; \lambda)$ va $\vec{c}(n - -1; 3n; 1)$ vektorlar $\lambda$ parametrning qanday qiymatida komplanar bo‘ladi?

**2.3.27.** $\lambda$ parametrning qanday qiymatida $\vec{a}(\lambda n;\ n-2;\ n+1)$ va $\vec{b}(n-3;\ \lambda n;\ n-1)$ vektorlar ortogonal bo‘lishini aniqlang.

**2.3.28.** $\vec{x}(n; n+4; n-1)$ vektorni $\vec{e}_1(1; 1; 0),\ \ \vec{e}_2(1; 0; 1)$ va $\vec{e}_3(0; 1; 1)$ bazisdagi yoyilmasini toping .

**2.3.29.** $\vec{\boldsymbol{a}}(\boldsymbol{2n}; \boldsymbol{n}+\boldsymbol{3}; \boldsymbol{n}-\boldsymbol{1})$, $\vec{\boldsymbol{b}}(\boldsymbol{n}; \boldsymbol{2n}-\boldsymbol{13}; \boldsymbol{4n})$ va $\vec{\boldsymbol{c}}(\boldsymbol{2n}; \boldsymbol{13}-$ $-\boldsymbol{5n}; -\boldsymbol{13n}-\boldsymbol{3})$ vektorlar chiziqli bog‘liq ekanligini ko‘rsating va bu bog‘lanishni toping.

**2.3.30.** $\vec{e}_1(n; n-1; 2n)$, $\vec{e}_2(n+1; 0; n+2)$ va $\vec{e}_3(1; n; n-3)$ vektorlar bazis tashkil etishini ko‘rsating.

# 3-MAVZU. KOORDINATALAR SISTEMASI

**Reja:**

**1. Vektorning koordinatalari. Vektorning moduli va yo‘naltiruvchi kosinuslari.**

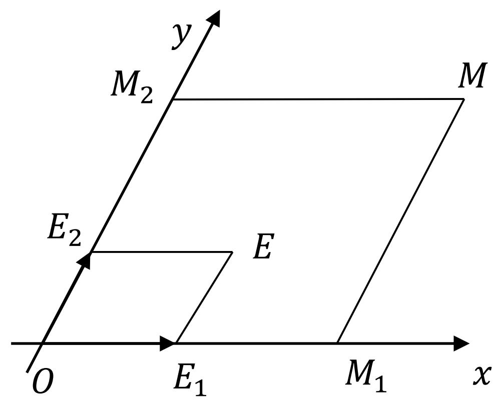
**2. Koordinatalari bilan berilgan vektorlar ustida amallar.**

**Tayanch iboralar:** masshtab vektor, birlik nuqta, vektorning koordinatalari, vektorning moduli, yo‘naltiruvchi kosinuslar, vektorlarni qo‘shish va ayirish.

## 3.1. Vektorning koordinatalari. Vektorning moduli va yo‘naltiruvchi kosinuslari.

Ma’lum tartibda olingan va kesishadigan ikkita $Ox, Oy$ o‘qlar jufti tekislikda umumiy dekart yoki affin koordinatalar sistemasi deb ataladi. Koordinatalar sistemasining boshi sifatida $Ox, Oy$ o‘qlarning umumiy nuqtasi olinadi. $Ox$ o‘qi- absissalar o‘qi, $Oy$ -o‘qi ordinatalar o‘qi deb ataladi(3.1.1-chizma). $OE_1 = \vec{l}_1, OE_2 = \vec{l}_2$ vektorlar $Ox, Oy$ o‘qlarning ***masshtab vektorlari*** deyiladi.

$E_1 E_2$ nuqtalar $Ox, Oy$ o‘qlarning birlik nuqtalari deb ataladi.




**3.1.1-chizma**

Ixtiyoriy $M$ nuqtadan $Ox$, $Oy$ o‘qlariga parallel to‘g‘ri chiziqlar o‘tkazamiz. $M_1$ va $M_2$ nuqtalar bu to‘g‘ri chiziqlarning mos ravishda $Ox$ va $Oy$ o‘qlari bilan kesishish nuqtalari bo‘lsin. $M$ nuqtaning $Ox$ o‘qidagi koordinatasini $x$ va $M_2$ nuqtaning $Oy$ o‘qdagi koordinatasini $y$ bilan belgilaymiz. $x$ va $y$ sonlari mos ravishda $M$ nuqtaning ***absissasi*** va ***ordinatasi*** deyiladi va $M(x, y)$ ko‘rinishda yoziladi. $E(1,1)$ nuqta ***birlik nuqta*** deb ataladi .

$Ox, Oy$ nuqtalar orasidagi burchak $\frac{\pi}{2}$ ga teng bo‘lib, masshtab vektorlari bir xil uzunlikka ega bo‘lsa, umumiy dekart sistemasi to‘g‘ri burchakli deyiladi. O‘qlardagi masshtab vektorlarining uzunliklari teng bo‘lib, o‘qlar orasidagi burchak $\frac{\pi}{2}$ dan farqli bo‘lsa, sistema ***qiyshiq burchakli*** deyiladi.

Ushbu mavzudagi masalalarni yechish uchun zarur bo‘lgan formulalarni keltiramiz.

$A(x_1, y_1),\ B(x_2, y_2),\ C(x_3, y_3)$ nuqtalarning bir to‘g‘ri chiziqda yotishi uchun

$$\begin{vmatrix} x_1 - x_3 & y_1 - y_3 \\ x_2 - x_3 & y_2 - y_3 \end{vmatrix} = 0 \tag{3.1}$$

yoki

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0 \tag{3.2}$$

shartning bajarilishi zarur va yetarlidir.

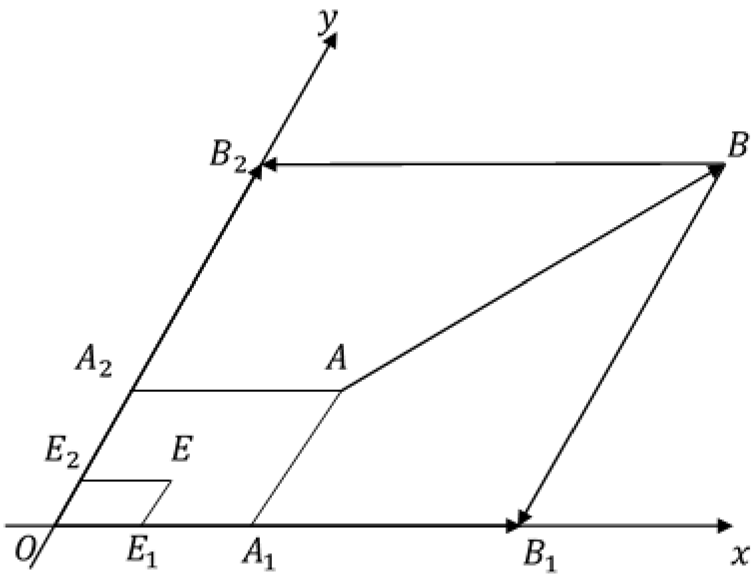
Uchlari $A(x_1, y_1),\ B(x_2, y_2)$ nuqtalarda bo‘lgan $AB$ kesmani $\lambda \neq -1$ nisbatda bo‘luvchi $C(x, y)$ nuqtaning koordinatalari quyidagi munosabatlardan aniqlanadi:

$$x = \frac{x_1 + \lambda x_2}{1 + \lambda},\ y = \frac{y_1 + \lambda y_2}{1 + \lambda}. \tag{3.3}$$

$AB$ kesmani teng ikkiga bo‘luvchi nuqta koordinatalari kesma uchlariga tegishli koordinatalar yig‘indisining yarmiga teng.

$$x = \frac{x_1 + x_2}{2},\ y = \frac{y_1 + y_2}{2}. \tag{3.4}$$

(3.1) va (3.4) formulalar affin koordinatalar sistemasida ham o‘rinlidir. $\overrightarrow{AB}$ vektorlarning koordinatalari quyidagicha aniqlanadi: $A, B$ nuqtalardan $Ox, Oy$ o‘qlarga parallel to‘g‘ri chiziqlar o‘tkazamiz. Bu to‘g‘ri chiziqlarning $Ox$ o‘qi bilan kesishish nuqtalarini $A_1, B_1$ bilan, $Oy$ o‘qi bilan kesishish nuqtalarini $A_2, B_2$ orqali belgilaymiz. $\overrightarrow{A_1B_1}$ vektorning $Ox$ o‘qdagi koordinatasi $x$ va $\overrightarrow{A_2B_2}$ vektorning $Oy$ o‘qdagi koordinatasi $y$ bilan birgalikda $\overrightarrow{AB}$ vektorning umumiy dekart $Oxy$ sistemasidagi koordinatalari deb ataladi(3.1.2-chizma).




**3.1.2-chizma**

Agar $(x_1, y_1) - A$ nuqtaning va $(x_2, y_2) - B$ nuqtaning koordinatalari bo‘lsa, u holda $\overrightarrow{AB}$ vektorning koordinatalari quyidagicha bo‘ladi:

$$x = x_2 - x_1,\ y = y_2 - y_1 \tag{3.5}$$

Agar $x, y - \overrightarrow{AB}$ vektorning koordinatalari bo‘lsa,

$$\overrightarrow{AB} = \{x, y\}$$

shaklida yoziladi.

$A(x_1, y_1)$ va $B(x_2, y_2)$ nuqtalar orasidagi masofa quyidagi formula bo‘yicha hisoblanadi:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad (3.6)$$

yoki

$$d = \sqrt{X^2 + Y^2} \qquad (3.7)$$

bu yerda $X,\ Y$ sonlar $\overrightarrow{AB}$ vektorning koordinatalari.

Uchlari $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ nuqtalardagi uchburchakning yuzi to‘g‘ri burchakli koordinatalar sistemasida quyidagicha topiladi:

$$S = \frac{1}{2} mod \begin{vmatrix} x_1 - x_3 & y_1 - y_3 \\ x_2 - x_3 & y_2 - y_3 \end{vmatrix} \qquad (3.8)$$

yoki

$$S = \frac{1}{2} mod \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \qquad (3.9)$$

O‘zlarining son qiymati va yo‘nalishi bilan aniqlanadigan miqdorlar ***vektorlar*** deb ataladi.$M_1(x_1, y_1, z_1)$ va $M_2(x_2, y_2, z_2)$ nuqtalar mos ravishda $\vec{a}$ vektorning boshi va oxiri bo‘lsin. U holda $\vec{a}$ vektorning koordinatalari quyidagicha aniqlanadi.

$$\vec{a} = \overrightarrow{M_1M_2} = \{x_2 - x_1;\ y_2 - y_1;\ z_2 - z_1\}$$

$\vec{a}$ vektorning uzunligiga teng bo‘lgan son uning moduli deyiladi va quyidagicha aniqlanadi.

$$|\vec{a}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Agar $\vec{a}$ vektor koordinata o‘qlari bilan mos ravishda $\alpha,\ \beta$ va $\gamma$ burchaklar hosil qilsa, u holda $\cos\alpha, \cos\beta$ va $\cos\gamma$, $\vec{a}$ vektorning yo‘naltiruvchi kosinuslari deyiladi va quyidagicha aniqlanadi:

$$\cos\alpha = \frac{X}{|\vec{a}|}; \quad \cos\beta = \frac{Y}{|\vec{a}|}; \quad \cos\gamma = \frac{Z}{|\vec{a}|} \qquad (3.10)$$

bu yerda: $X = x_2 - x_1, \quad Y = y_2 - y_1, \quad Z = z_2 - z_1$

Vektorning o‘qqa proyeksiyasi. $\vec{a}$ vektorning $Y$ o‘qqa proyeksiyasi, uning moduli va $Y$ o‘q bilan tashkil qilgan burchagi $\varphi$ orqali quyidagicha aniqlanadi.

$$pr_Y\vec{a} = |\vec{a}|\cos\varphi$$

Ixtiyoriy $\vec{a}$ vektorning berilgan koordinatalar sistemasiga proyeksiyasini $X, Y, Z$ orqali belgilaylik. U holda $\vec{a} = \{X, Y, Z\}$ va $|\vec{a}| = \sqrt{X^2 + Y^2 + Z^2}$ bo‘ladi.

**1-Misol.** $\vec{a}(-3; 6; -2)$ vektorning modulini toping.

**Yechish:** Modulni topish formulasiga asosan

$$\vec{a} = \sqrt{(-3)^2 + 6^2 + (-2)^2} = \sqrt{9 + 36 + 4} = \sqrt{49} = 7$$

**2-Misol.** $\vec{a}(15, -12, -16)$vektorning yo‘naltiruvchi kosinuslarini aniqlang.

**Yechish:**$|\vec{a}| = \sqrt{15^2 + (-12)^2 + (-16)^2} =.$

$$= \sqrt{225 + 144 + 256} = 25$$

Endi $x = 15;\ y = -12;\ z = -16$ ekanligini e’tiborga olib yo‘naltiruvchi kosinuslarni aniqlaymiz.

$$\cos\alpha = \frac{15}{25};\ \cos\beta = -\frac{12}{25};\ \cos\gamma = -\frac{16}{25}.$$

### 3.2. Koordinatalari bilan berilgan vektorlar ustida amallar.

Vektorlarni qo‘shish va ayirish: Agar $\vec{a}$ va $\vec{b}$ vektorlar koordinatalari berilgan bo‘lsa, ya’ni

$$\vec{a}(x_1, y_1, z_1) \text{ va } \vec{b}(x_2, y_2, z_2)$$

u holda

$$\vec{a} + \vec{b} = \{x_1 + x_2;\ \ y_1 + y_2;\ \ z_1 + z_2\},$$

$\vec{a} - \vec{b} = \{x_1 - x_2;\ \ y_1 - y_2;\ \ z_1 - z_2\}$ ko‘rinishida bo‘ladi.

Agar $\vec{a}(x_1, y_1, z_1)$ bo‘lsa, u holda har qanday $\alpha$ son uchun quyidagi formula o‘rinli

$$\alpha\vec{a} = \{\alpha x_1, \alpha y_1, \alpha z_1\}. \qquad (3.11)$$

Bir to‘g‘ri chiziqda yoki parallel to‘g‘ri chiziqlarda yotuvchi vektorlar kollinear vektorlar deb ataladi. $\vec{a}(x_1, y_1, z_1)$ va $\vec{b}(x_2, y_2, z_2)$ vektorlarning kollinearlik sharti quyidagicha bo‘ladi:

$$\frac{x_2}{x_1} = \frac{y_2}{y_1} = \frac{z_2}{z_1}$$

$\vec{\imath}$, $\vec{\jmath}$, $\vec{k}$ uchlik vektorlar bazis koordinatalari deyiladi, agar quyidagi uchta shart bajarilsa,

1) $\vec{\imath}$ vektor $OX$ o'qida, $\vec{\jmath}$ vektor $OY$ o'qida, $\vec{k}$ vektor $OZ$ o'qida yotadi.

2) har bir $\vec{\imath}$, $\vec{\jmath}$, $\vec{k}$ vektorlar o'z o'qlarida musbat tomonga yo'nalgan bo'ladi.

3) $\vec{\imath}$, $\vec{\jmath}$, $\vec{k}$ vektorlar, birlik vektorlar, ya'ni $|\vec{\imath}| = 1$, $|\vec{\jmath}| = 1$, $|\vec{k}| = 1$, $\vec{a}$ vektor qanday bo'lishidan qat'iy nazar uni har doim $\vec{\imath}$, $\vec{\jmath}$, $\vec{k}$ bazislar bo'yicha yoyish mumkin, ya'ni $\vec{a} = x_1\vec{\imath} + y_1\vec{\jmath} + z_1\vec{k}$. Bu yerda $x_1, y_1, z_1 - \vec{a}$ vektorning koordinatalari.

**3-Misol.** $\vec{a}(3;2;-1)$, $\vec{b}(-2;-3;4)$ va $\vec{c}(5;4;-6)$ vektorlar berilgan bo'lsa $\vec{d} = 4\vec{a} + 3\vec{b} - 2\vec{c}$ vektorni $\vec{\imath}$, $\vec{\jmath}$, $\vec{k}$ vektor bo'yicha yoying.

**Yechish:** Endi $4\vec{a}$, $3\vec{b}$ va $2\vec{c}$ vektorlarni aniqlaymiz.
$4\vec{a} = \{12;8;-4\}$, $3\vec{b} = \{-6;-9;12\}$ va $2\vec{c} = \{10;8;-12\}$.
Demak,

$$\vec{d} = 4\vec{a} + 3\vec{b} - 2\vec{c} =$$
$$= \{12 + (-6) - 10;\ 8 + (-9) - 8; -4 + 12 - (-12)\}$$
$$\vec{d} = 4\vec{a} + 3\vec{b} - 2\vec{c} = \{-4;-9;20\}.$$
$$\vec{d} = -4\vec{\imath} - 9\vec{\jmath} + 20\vec{k}$$

**4-Misol.** Uchlari $A(2;-3;1)$ va $B(16;11;15)$ nuqtalarda joylashgan $AB$ kesmani $\lambda = 2:5$ nisbatda bo'luvchi nuqtaning koordinatasini toping.

**Yechish:** $AB$ kesmani $\lambda = 2:5$ nisbatda bo'luvchi nuqtaning koordinatasini, yuqorida berilgan $(3.3)$ formula asosan topimiz:

$$x_0 = \frac{2 + \frac{2}{5}\cdot 16}{1 + \frac{2}{5}} = 6, \quad y_0 = \frac{-3 + \frac{2}{5}\cdot 11}{1 + \frac{2}{5}} = 1, \quad z_0 = \frac{1 + \frac{2}{5}\cdot 15}{1 + \frac{2}{5}} = 7$$

Natijada, $AB$ kesmani $\lambda = 2:5$ nisbatda bo'luvchi $C(6;1;7)$ nuqtaning koordinatasini topdik.

## Mustaqil yechish uchun topshiriqlar

**3.1. Vektorning koordinatalari. Vektorning moduli va yo‘naltiruvchi kosinuslariga doir misollar.**

**3.1.1.** $\vec{b}(8;-6)$ vektorning modulini toping.

**3.1.2.** $\vec{d}(-2;3;-6)$ vektorning modulini toping.

**3.1.3.** $\vec{a}(9;-2;6)$ vektorning modulini toping.

**3.1.4.** $\vec{c}(-4;12;-3)$ vektorning modulini toping.

**3.1.5.** $\vec{d}(12;-1;12)$ vektorning modulini toping.

**3.1.6.** $\vec{c}(12;-9)$ vektorning yo‘naltiruvchi kosinuslarini aniqlang.

**3.1.7.** $\vec{b}(-10;2;11)$ vektorning yo‘naltiruvchi kosinuslarini aniqlang.

**3.1.8.** $\vec{a}(12;-15;16)$ vektorning yo‘naltiruvchi kosinuslarini aniqlang.

**3.1.9.** $\vec{c}(1;-12;12)$ vektorning yo‘naltiruvchi kosinuslarini aniqlang.

**3.1.10.** $\overrightarrow{OP}(3;-6;2)$ vektorning yo‘naltiruvchi kosinuslarini toping.

**3.1.11.** $\vec{a}(12;-15;-16)$ vektorning yo‘naltiruvchi kosinuslarini toping.

**3.1.12.** Boshi $A(-3;5)$ oxiri $B(5;-1)$ nuqtalarda bo‘lgan $\overrightarrow{AB}$ vektorning yo‘naltiruvchi kosinuslari va uzunligi topilsin.

**3.1.13.** Boshi $C(4;-9;6)$ oxiri $D(-8;3;5)$ nuqtalarda bo‘lgan $\overrightarrow{CD}$ vektorning yo‘naltiruvchi kosinuslari va moduli topilsin.

**3.1.14.** Boshi $M(-2;1;-3)$ oxiri $N(0;-1;2)$ nuqtalarda bo‘lgan $\overrightarrow{MN}$ vektorning yo‘naltiruvchi kosinuslari va moduli topilsin.

**3.1.15.** $\overrightarrow{AK}$ vektorning yo‘naltiruvchi kosinuslari $\frac{2}{7},\frac{3}{7},\frac{6}{7}$ ekanini bilgan holda, $A(0;0;12)$ nuqtadan 7 birlik masofada joylashgan $K$ nuqtaning koordinatalari topilsin.

**3.1.16.** $\overrightarrow{CD}$ vektorning yo‘naltiruvchi kosinuslari $-\frac{12}{13},\frac{4}{13},-\frac{3}{13}$ ekanini bilgan holda, $C(2;-1;4)$ nuqtadan 13 birlik masofada joylashgan $D$ nuqtaning koordinatalari topilsin.

**3.1.17.** *A*gar vektor koordinata o‘qlari bilan bir xil burchaklar hosil qilsa va uning moduli 3 ga teng bo‘lsa, shu vektorning koordinatalarini toping.

**3.1.18.** *A*gar vektor koordinata o‘qlari bilan bir xil burchaklar hosil qilsa va uning moduli 17 ga teng bo‘lsa, shu vektorning koordinatalarini toping.

**3.1.19.** Bitta nuqtadan $\vec{a}(-12;16)$, $\vec{b}(12;5)$ vektorlar o‘tkazilgan. $\vec{a}$ bilan $\vec{b}$ vektorlar orasidagi burchakni teng ikkiga bo‘ladigan va shu nuqtadan chiqqan birlik vektorning koordinatalari topilsin.

**3.1.20.** Vektor $Ox$ va $Oz$ o‘qlari bilan $\alpha = 120^0$, $\gamma = 45^0$ burchaklar tashkil qiladi. Shu vektor $Oy$ o‘qi bilan qanday burchak hosil qiladi?

**3.1.21.** Vektor $Oy$ va $Oz$ o‘qlari bilan $\beta = 45^0$, $\gamma = 60^0$ burchaklar tashkil qiladi. Shu vektor $Ox$ o‘qi bilan qanday burchak hosil qiladi?

**3.1.22.** Vektorning 2 ta koordinatasi $x = 4$, $y = -12$ berilgan. $|\vec{a}| = 13$ bo‘lgan holda vektorning uchinchi $z$ o‘qining koordinatasini aniqlang.

**3.1.23.** Vektorning 2 ta koordinatasi $x = -16$, $z = 15$ berilgan. $|\vec{a}| = 25$ bo‘lgan holda vektorning uchinchi $y$ o‘qining koordinatasini aniqlang.

**3.1.24.** Birinchi koordinatalari mos ravishda $x = 7$, $y = 6$ ga teng bo‘lib, uzunligi 11 ga teng vektorning boshi $A(2;-1;\ 5)$ nuqtada joylashgan bo‘lsa, bu vektor oxirining koordinatalari topilsin.

**3.1.25.** Birinchi koordinatalari mos ravishda $y = -3$, $z = 4$ ga teng bo‘lib, uzunligi 13 ga teng vektorning oxiri $B(-5;3;-2)$ nuqtada joylashgan bo‘lsa, bu vektor boshining koordinatalari topilsin.

**3.1.26.** Birinchi koordinatalari mos ravishda $x = 4$, $z = 12$ ga teng bo‘lib, uzunligi 13 ga teng vektorning boshi $A(4;-2;-3)$ nuqtada joylashgan bo‘lsa, bu vektor oxirining koordinatalari topilsin.

**3.1.27.** Vektor 2 ta koordinata o‘qlari bilan quyidagi burchaklarni hosil qilib biladimi:

1) $\alpha = 30^0$, $\beta = 45^0$;

2) $\beta = 60^0$, $\gamma = 60^0$;

3) $\alpha = 150^0,\ \gamma = 30^0$

**3.1.28.** Vektor koordinata o‘qlari bilan quyidagi burchaklarni hosil qilib biladimi:

1) $\alpha = 45^0,\ \beta = 60^0,\ \gamma = 120^0$;

2) $\alpha = 45^0,\ \beta = 135^0,\ \gamma = 60^0$;

3) $\alpha = 90^0,\ \beta = 150^0,\ \gamma = 60^0$

**3.1.29.** Boshi $A(n; 2n+3; 5-2n)$, oxiri esa $B(2n+3; 2n-1; n)$ nuqtada joylashgan $\overrightarrow{AB}$ vektorning koordinatalarini toping.

**3.1.30.** Boshi $C(n-2; n+3; n)$ va oxiri $D(n+1; n-3; n-1)$ nuqtada joylashgan $\overrightarrow{CD}$ vektorning koordinatalarini toping.

### 3.2. Koordinatalari bilan berilgan vektorlar ustida amallarga doir misollar.

**3.2.1.** $\vec{a}(4; -1; 6)$ va $\vec{b}(-1; 4; -5)$ vektorlar berilgan bo‘lsa, $\vec{c} = 5\vec{a} - 2\vec{b}$ vektorni $\vec{i}, \vec{j}, \vec{k}$ vektorlar bo‘yicha yoying.

**3.2.2.** $\vec{a}(-9; 7; -5)$ va $\vec{b}(2; -1; 3)$ vektorlar berilgan bo‘lsa, $\vec{c} = 2\vec{a} + 6\vec{b}$ vektorni $\vec{i}, \vec{j}, \vec{k}$ vektorlar bo‘yicha yoying.

**3.2.3.** $\vec{a}(3; -4; 2)$, $\vec{b}(-4; 6; -3)$ va $\vec{c}(-5; 4; 7)$ vektorlar berilgan bo‘lsa, $\vec{d} = 5\vec{a} - 2\vec{b} + \vec{c}$ vektorni $\vec{i}, \vec{j}, \vec{k}$ vektorlar bo‘yicha yoying.

**3.2.4.** $\vec{a}(-5; 2; -3)$, $\vec{b}(1; -6; 4)$ va $\vec{c}(4; -1; 7)$ vektorlar berilgan bo‘lsa, $\vec{d} = 2\vec{a} + 3\vec{b} - 6\vec{c}$ vektorni $\vec{i}, \vec{j}, \vec{k}$ vektorlar bo‘yicha yoying.

**3.2.5.** Agar $\vec{b}(3; -1; 4)$ vektorning boshi $M(1; 2; -3)$ nuqta bilan boshlansa, vektorning oxiri $N$ nuqtani toping.

**3.2.6.** Agar $\vec{a}(2; -3; -1)$ vektorning oxiri $N(1; -1; 2)$ nuqta bilan tugasa, vektorning boshini toping.

**3.2.7.** Vektor $Ox$ va $Oz$ o‘qlari bilan $\alpha = 120^0$ va $\gamma = 45^0$ burchaklar tashkil qiladi. Shu vektor $Oy$ o‘qi bilan qanday burchak hosil qiladi?

**3.2.8.** $|\vec{a}| = 2$ vektorning moduli va $\alpha = 45^0,\ \beta = 60^0,\ \gamma = 120^0$ burchaklar berilgan. $\vec{a}$ vektorning koordinata o‘qiga proyeksiyasini toping.

**3.2.9.** $Ox$ va $Oy$ koordinata o‘qlari bilan $\vec{a}$ vektor $\alpha = 60^0, \beta = 120^0$ burchaklar hosil qiladi. $|\vec{a}| = 2$ bo‘lganda uning koordinatalarini hisoblang.
**3.2.10.** Koordinatalarning to‘g‘ri burchakli sistemasida quyidagi nuqtalar yasalsin:
$A(2;3), B(0;4), C(-2;1), D(-3;-5), F(6;-2), G(5;0), K(0;-1),$
$S(-3;0), T(0;7)$.
**3.2.11.** Parallelogrammning ketma-ket keluvchi uchta $A(-2;1)$, $B(1;3), C(4;0)$ uchlari berilgan, uning to‘rtinchi uchini toping.
**3.2.12.** Parallelogrammning ketma-ket keluvchi uchta $A(2;2)$, $B(-1;3)$, $C(-2;0)$ uchlari berilgan, uning to‘rtinchi uchini toping.
**3.2.13.** Parallelogrammning ketma-ket keluvchi uchta $A(-3;0)$, $C(1;-1)$, $D(-1;-3)$ uchlari berilgan, uning to‘rtinchi uchini toping.
**3.2.14.** Parallelogrammning uchta $A$, $B$, $C$ uchlari berilgan, uning to‘rtinchi uchini toping (Masala nechta yechimga ega).
**3.2.15.** Berilgan $\vec{a}(n-2;\ n+3;\ n-1)$ va $\vec{b}(n;\ n-4;\ n+2)$ vektorlar bo‘yicha $n\vec{a}$, $\vec{a}+\vec{b}$, $\vec{a}-\vec{b}$ va $3\vec{a}+n\vec{b}$ vektorlarni toping.
**3.2.16.** Uchlari $A(2;-3;1)$ va $B(16;11;15)$ nuqtalarda joylashgan $AB$ kesmaning o‘rta nuqtasining koordinatasini toping.
**3.2.17.** Uchlari $A(-5;8;-2)$ va $B(13;-4;12)$ nuqtalarda joylashgan $AB$ kesmaning o‘rta nuqtasining koordinatasini toping.
**3.2.18.** Uchlari $A(-3;4;-1)$ va $B(7;-8;5)$ nuqtalarda joylashgan $AB$ kesmaning o‘rta nuqtasining koordinatasini toping.
**3.2.19.** Uchlari $A(4;-5;1)$ va $B(-8;7;9)$ nuqtalarda joylashgan $AB$ kesmani $\lambda = 1:3$ nisbatda bo‘luvchi nuqtaning koordinatasini toping.
**3.2.20.** Uchlari $A(-1;9;-13)$ va $B(-5;1;-5)$ nuqtalarda joylashgan $AB$ kesmani $\lambda = 3:5$ nisbatda bo‘luvchi nuqtaning koordinatasini toping.
**3.2.21.** $\vec{a}(m;3;2)$ va $\vec{b}(4;6;n)$ vektorlar $m$ va $n$ parametrlarning qanday qiymatlarida kollinear bo‘lishini aniqlang.
**3.2.22.** $\vec{c}(6;l;2)$ va $\vec{d}(k;-8;4)$ vektorlar $l$ va $k$ parametrlarning qanday qiymatlarida kollinear bo‘lishini aniqlang.

**3.2.23.** $\vec{a}(-2;3), \vec{b}(4;-5)$ va $\vec{c}(3;-6)$ vektorlar berilgan bo‘lsa, $5\vec{a}-$ $-3\vec{b}$ va $2\vec{a}+3\vec{c}$ vektorlarning uzunligini taqqoslang.

**3.2.24.** $\vec{a}=3\vec{i}-5\vec{j},\ \vec{b}=-6\vec{i}+5\vec{j}$ va $\vec{c}=-\vec{i}+2\vec{j}$ vektorlar berilgan bo‘lsa, $2\vec{a}+3\vec{c}$ va $4\vec{b}-\vec{c}$ vektorlarning uzunligini taqqoslang.

**3.2.25.** $\vec{a}(4;-3;5),\ \vec{b}(-2;1;-1)$ va $\vec{c}(1;-6;4)$ vektorlar berilgan bo‘lsa, $2\vec{a}+5\vec{b}$ va $6\vec{b}-3\vec{c}$ vektorlarning uzunligini taqqoslang.

**3.2.26.** $\vec{a}(-3;2;-4), \vec{b}(4;-5;-1)$ va $\vec{c}(-3;6;-4)$ vektorlar berilgan bo‘lsa, $3\vec{a}+2\vec{b}-4\vec{c}$ va $2\vec{b}-3\vec{c}-\vec{a}$ vektorlarning uzunligini taqqoslang.

**3.2.27.** $\vec{a}=-\vec{i}+3\vec{j}-2\vec{k},\ \vec{b}=4\vec{i}-\overrightarrow{5j}+3\vec{k}$ va $\vec{c}=-2\vec{i}+6\vec{j}+\vec{k}$ vektorlar berilgan bo‘lsa, $3\vec{a}+5\vec{b}-4\vec{c}$ va $2\vec{b}-3\vec{c}-\vec{a}$ vektorlarning uzunligini taqqoslang.

**3.2.28.** Uchlari $A(n-2;\ n+3;\ n)$ va $B(n+1;\ n-3;\ n-1)$ nuqtalarda joylashgan $AB$ kesmani $\lambda=(n-1):(n+2)$ nisbatda bo‘luvchi $C(x,y,z)$ nuqta koordinatalarini aniqlang.

**3.2.29.** Fazoda uchlari $M_1(-3;2;\ 4),\ M_2(6;0;1)$ nuqtalarda bo‘lgan $M_1M_2$ kesmani $\lambda=2$ nisbatda bo‘luvchi nuqta topilsin.

**3.2.30.** Fazoda uchlari $N_1(-1;4;\ -7),\ N_2(5;4;7)$ nuqtalarda bo‘lgan $N_1N_2$ kesmani $\lambda=\frac{3}{4}$ nisbatda bo‘luvchi nuqta topilsin.

# 4-MAVZU: VEKTORLARNI SKALYAR, VEKTOR VA ARALASH KO‘PAYTMALARI

**Reja:**

**1. Vektorlarning skalyar ko‘paytmasi.**

**2. Vektorning vektor va aralash ko‘paytmasi.**

**Tayanch iboralar:** skalyar ko‘paytma, vektor va aralash ko‘paytma.

### 4.1. Vektorlarning skalyar ko‘paytmasi.

Berilgan ikki vektorning uzunligi va ular orasidagi burchak kosinusining ko‘paytmasiga shu ikki vektorning ***skalyar ko‘paytmasi*** deyiladi.

Demak, $\vec{a}$ va $\vec{b}$ vektorlarning ***skalyar ko‘paytmasi***

$$\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cdot cos\varphi \qquad (4.1)$$

ko‘rinishida bo‘ladi. Bu yerda $\varphi$ burchak $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchakdir.

$\vec{a} = 0$ yoki $\vec{b} = 0$ holda ta’rifga ko‘ra $\vec{a}\cdot\vec{b} = 0$. $\vec{a}\perp\vec{b}$ yoki $\vec{a} = 0$, yoki $\vec{b} = 0$ holdagina $\vec{a}\cdot\vec{b}$ ***skalyar ko‘paytma*** nolga teng.

Skalyar ko‘paytirish amalining xossalari:

$$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$$

$$\lambda(\vec{a}\cdot\vec{b}) = (\lambda\cdot\vec{a})\vec{b} \qquad \text{(kommutativlik)} \qquad (4.2)$$

$$(\vec{a}+\vec{b})\cdot\vec{c} = \vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{c}$$

$$\vec{a}\cdot\vec{a} = \vec{a}^2 \geq 0 \qquad \text{(distributivlik)} \qquad (4.3)$$

ammo faqat $\vec{a} = 0$ holdagina $\vec{a}\cdot\vec{a} = 0$ bo‘ladi.

Fazodagi Dekart koordinatalar sistemasida

$$\vec{a}(x;y;z), \quad \vec{b}(x';y';z')$$

berilgan bo‘lsa, skalyar ko‘paytmaning xossalaridan foydalanamiz:

$$\vec{a}\cdot\vec{b} = (x\vec{\imath}+y\vec{\jmath}+z\vec{k})(x'\vec{\imath}+y'\vec{\jmath}+z'\vec{k}) = xx'\vec{\imath}^2+xy'\vec{\imath}\vec{\jmath}+xz'\vec{\imath}\vec{k}+$$

$$+x'y\vec{\imath}\vec{\jmath}+yy'j^2+yz'\vec{\jmath}\vec{k}+x'z\vec{\imath}\vec{k}+y'z\vec{\jmath}\vec{k}+zz'\vec{k}^2 =$$

$$= xx'\cdot 1+xy'\cdot 0+xz'\cdot 0+x'y\cdot 0+yy'\cdot 1+yz'\cdot 0+x'z\cdot 0+$$

$$+y'z\cdot 0+zz'\cdot 1 = xx'+yy'+zz'$$

Skalyar ko‘paytmaning xossalaridan, ortlar uchun ushbu tengliklar o‘rinli ekanligini ko‘ramiz:

$$\vec{\imath}^2 = \vec{\imath}\cdot\vec{\imath} = |\vec{\imath}|\cdot|\vec{\imath}|\cdot cos\varphi = 1\cdot 1\cdot cos0^0 = 1,\ \vec{\jmath}^2 = 1,\ \vec{k}^2 = 1.$$

$$\vec{\imath}\cdot\vec{\jmath} = |\vec{\imath}|\cdot|\vec{\jmath}|\cdot cos\varphi = 1\cdot 1\cdot cos90^0 = 0,\ \ \vec{\imath}\cdot\vec{k} = 0,\ \ \vec{\jmath}\cdot\vec{k} = 0.$$

Demak,

$$\vec{a}\cdot\vec{b} = xx'+yy'+zz', \qquad (4.4)$$

$$|\vec{a}| = \sqrt{x^2+y^2+z^2}$$

vektorlarning skalyar ko‘paytmasi ularning mos koordinatalari ko‘paytmalarining yig‘indisiga teng bo‘ladi.

$|\vec{a}| = 0,\ |\vec{b}| = 0$ shartlarida $\vec{a},\ \vec{b}$ vektorlar orasidagi burchak kosinusi quyidagi formula bo‘yicha topiladi.

$(4.1)$ va $(4.4)$ formulalardan,

$$cos\varphi = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$$

$$cos\varphi = \frac{xx' + yy' + zz'}{\sqrt{x^2 + y^2 + z^2} \cdot \sqrt{x'^2 + y'^2 + z'^2}} \qquad (4.5)$$

kelib chiqadi. $\vec{a}(x; y; z)$, $\vec{b}(x'; y'; z')$ vektorlar ortogonal (perpendikulyar) bo‘lishligining zarur va yetarli sharti ushbu tenglikdan iborat (fazoda):

$$xx' + yy' + zz' = 0. \qquad (4.6)$$

**1-Misol.** $\vec{a}(3; 6)$ va $\vec{b}(5; -2)$ vektorlarning skalyar ko‘paytmasini toping.

**Yechish:** $\vec{a}$ va $\vec{b}$ vektorlarning skalyar ko‘paytmasi ularning mos koordinatalari ko‘paytmalarining yig‘indisiga tengligidan,

$$\vec{a} \cdot \vec{b} = xx' + yy' = 3 \cdot 5 + 6 \cdot (-2) = 3$$

kelib chiqadi. Demak, berilgan $\vec{a}$ va $\vec{b}$ vektorlarning skalyar ko‘paytmasi 3 ga teng bo‘ladi.

**2-Misol.** Koordinatalari bilan berilgan quyidagi $\vec{a}(4; 3)$ va $\vec{b}(1; 7)$ vektorlar orasidagi $\varphi$ burchakni toping.

**Yechish:** $\vec{a}$ va $\vec{b}$ vektorlarning orasidagi $\varphi$ burchakni topish formulasidan,

$$cos\varphi = \frac{xx' + yy'}{\sqrt{x^2 + y^2} \cdot \sqrt{x'^2 + y'^2}} = \frac{4 \cdot 1 + 3 \cdot 7}{\sqrt{16 + 9} \cdot \sqrt{1 + 49}} = \frac{25}{5 \cdot 5\sqrt{2}} =$$

$$= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$cos\varphi = \frac{\sqrt{2}}{2}$ kelib chiqadi, hamda $\vec{a}$ va $\vec{b}$ vektorlarning orasidagi burchak $\varphi = \frac{\pi}{4}$ ni tashkil qiladi.

## 4.2. Vektorning vektor va aralash ko‘paytmasi.

Ikki vektor $\vec{a}$ va $\vec{b}$ ning ***vektor ko‘paytmasi*** deb quyidagi xossalarga ega bo‘lgan $\vec{c}$ vektorga aytiladi:

1. $\vec{c}$ vektorning uzunligi $\vec{a}$ va $\vec{b}$ vektorlardan yasalgan parallelogrammning yuziga teng, ya’ni

$$|\vec{c}| = |\vec{a}| \cdot |\vec{b}| \sin\varphi \qquad (\varphi = \vec{a}^\wedge\vec{b}) \qquad (4.7)$$

2. $\vec{c}$ vektor shu parallelogramm tekisligiga perpendikulyar, ya’ni u ham $\vec{a}$ vektorga, ham $\vec{b}$ vektorga perpendikulyardir:

$$\vec{a} \cdot \vec{c} = 0 \ \text{ va } \ \vec{b} \cdot \vec{c} = 0 \qquad (4.8)$$

3. $\vec{a}, \vec{b}, \vec{c}$ vektorlar ko‘rsatilgan tartibda olinganda vektorlarning o‘ng uchligini tashkil etadi.

Fazoda berilgan

$$\vec{a}(x; y; z), \quad \vec{b}(x'; y'; z')$$

vektorlar uchun

$$[\vec{a}, \vec{b}] = \left\{ \begin{vmatrix} y & z \\ y' & z' \end{vmatrix}; \begin{vmatrix} z & x \\ z' & x' \end{vmatrix}; \begin{vmatrix} x & y \\ x' & y' \end{vmatrix} \right\} \qquad (4.9)$$

yoki

$$[\vec{a}, \vec{b}] = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ x' & y' & z' \end{vmatrix} \qquad (4.10)$$

tenglik o‘rinli.

Fazoda uch vektorning ***aralash ko‘paytmasi*** deb, birinchi ikki vektorning vektor ko‘paytmasiga uchinchi vektorni skalyar ko‘paytirishga aytiladi.

$$\vec{a}\vec{b}\vec{c} = [\vec{a}, \vec{b}]\vec{c} \qquad (4.11)$$

Fazoda $\vec{a}(x; y; z)$, $\vec{b}(x'; y'; z')$ va $\vec{c}(x''; y''; z'')$ vektorlar koordinatalar bilan berilgan bo‘lsin. Ularning aralash ko‘paytmasi formulasini keltirib chiqaraylik. Buning uchun $\vec{a}$ va $\vec{b}$ vektorlarni vektor ko‘paytiramiz.

$$[\vec{a},\vec{b}] = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ x' & y' & z' \end{vmatrix}$$

bu vektor ko‘paytmani esa $\vec{c}(x''; y''; z'')$ vektorga skalyar ko‘paytiramiz va

$$[\vec{a},\vec{b}]\cdot\vec{c} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ x' & y' & z' \end{vmatrix} \cdot \left(x''\vec{i} + y''\vec{j} + z''\vec{k}\right) =.$$

$$= \left(\begin{vmatrix} y & z \\ y' & z' \end{vmatrix}\vec{i} - \begin{vmatrix} x & z \\ x' & z' \end{vmatrix}\vec{j} + \begin{vmatrix} x & y \\ x' & y' \end{vmatrix}\vec{k}\right)\left(x''\vec{i} + y''\vec{j} + z''\vec{k}\right) =$$

$$= \left(\begin{vmatrix} y & z \\ y' & z' \end{vmatrix}x'' - \begin{vmatrix} x & z \\ x' & z' \end{vmatrix}y'' + \begin{vmatrix} x & y \\ x' & y' \end{vmatrix}z''\right) = \begin{vmatrix} x'' & y'' & z'' \\ x & y & z \\ x' & y' & z' \end{vmatrix} =$$

$$= -\begin{vmatrix} x & y & z \\ x'' & y'' & z'' \\ x' & y' & z' \end{vmatrix} = \begin{vmatrix} x & y & z \\ x' & y' & z' \\ x'' & y'' & z'' \end{vmatrix} \qquad (4.12)$$

hosil bo‘ladi.

**3-Misol.** $\vec{a}(1;-3;4)$ va $\vec{b}(3;-4;2)$ vektorlarning vektor ko‘paytmasini toping.

**Yechish:** Yuqorida berilgan $(4.10)$ formuladan foydalanib,

$$[\vec{a},\vec{b}] = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -3 & 4 \\ 3 & -4 & 2 \end{vmatrix} = \left(\begin{vmatrix} -3 & 4 \\ -4 & 2 \end{vmatrix}\vec{i} - \begin{vmatrix} 1 & 4 \\ 3 & 2 \end{vmatrix}\vec{j} + \begin{vmatrix} 1 & -3 \\ 3 & -4 \end{vmatrix}\vec{k}\right) =$$

$$= 10\vec{i} + 10\vec{j} + 5\vec{k}$$

$\vec{c}(10; 10; 5)$ vektorning koordinatasini topdik.

**4-Misol.** $\vec{a}(3;-4;2)$, $\vec{b}(-1;2;5)$ va $\vec{c}(2;3;-4)$ vektorlarning aralash ko‘paytmasini toping.

**Yechish:** Yuqorida berilgan $(4.12)$ formuladan foydalanib,

$$[\vec{a},\vec{b}]\cdot\vec{c} = \begin{vmatrix} x & y & z \\ x' & y' & z' \\ x'' & y'' & z'' \end{vmatrix} = \begin{vmatrix} 3 & -4 & 2 \\ -1 & 2 & 5 \\ 2 & 3 & -4 \end{vmatrix} =$$

$$= -24 - 6 - 40 - 8 - 45 + 16 = -107$$

ya’ni, $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlarning aralash ko‘paytmasi $-107$ ga teng ekan.

## Mustaqil yechish uchun topshiriqlar.

### 4.1. Vektorlarning skalyar ko'paytmasiga doir misollar.

**4.1.1.** $\vec{a}(6;-8)$, $\vec{b}(12;9)$ va $\vec{c}(-4;3)$ vektorlar uchun

1) $\vec{a}\vec{b}$; 2) $\vec{a}\vec{c}$; 3) $\vec{b}\vec{c}$ ni hisoblang.

**4.1.2.** $\vec{a}(3;5;7)$, $\vec{b}(-2;6;1)$ va $\vec{c}(2;-4;0)$ vektorlar uchun

1) $\vec{a}\vec{b}$; 2) $\vec{a}\vec{c}$; 3) $\vec{b}\vec{c}$; 4) $(2\vec{a}-\vec{b})(3\vec{b}+\vec{c})$;

5) $(3\vec{a}+2\vec{c})(2\vec{b}-\vec{c})$ skalyar ko'paytmasini hisoblang.

**4.1.3.** Koordinatalari bilan berilgan $\vec{a}(6;-8)$, $\vec{b}(12;9)$, $\vec{c}(2;-5)$, $\vec{d}(3;7)$, $\vec{m}(-2;6)$ va $\vec{n}(3;-9)$ vektorlar orasidagi

1) $\vec{a}\,^\wedge\vec{b}$; 2) $\vec{c}\,^\wedge\vec{d}$; 3) $\vec{m}\,^\wedge\vec{n}$ ni toping.

**4.1.4.** Koordinatalari bilan berilgan $\vec{a}(8;4;1)$, $\vec{b}(2;-2;1)$, $\vec{c}(2;5;4)$ va $\vec{d}(6;0;-3)$ vektorlar orasidagi

1) $\vec{a}\,^\wedge\vec{b}$; 2) $\vec{c}\,^\wedge\vec{d}$ ni toping.

**4.1.5.** $|\vec{a}|=8$, $|\vec{b}|=5$, $(\vec{a}\,^\wedge\vec{b})=60^0$ berilgan bo'lsa, $\vec{a}$ va $\vec{b}$ vektorlarning skalyar ko'paytmasini toping.

**4.1.6.** $\vec{c}$ va $\vec{d}$ birlik vektor va $(\vec{c}\,^\wedge\vec{d})=135^0$ berilgan bo'lsa, $\vec{c}$ va $\vec{d}$ vektorlarning skalyar ko'paytmasini toping.

**4.1.7.** $|\vec{a}|=3$, $|\vec{b}|=6$, $\vec{a}\downarrow\downarrow\vec{b}$ berilgan bo'lsa, $\vec{a}$ va $\vec{b}$ vektorlarning skalyar ko'paytmasini toping.

**4.1.8.** $|\vec{c}|=3$, $|\vec{d}|=7$, $\vec{c}\downarrow\uparrow\vec{d}$ berilgan bo'lsa, $\vec{c}$ va $\vec{d}$ vektorlarning skalyar ko'paytmasini toping.

**4.1.9.** $\vec{a}$ va $\vec{b}$ vektorlar o'zaro $\varphi=\frac{2\pi}{3}$ burchak tashkil qiladi. $|\vec{a}|=3$ va $|\vec{b}|=4$ bo'lsa, quyidagilarni hisoblang:

1) $\vec{a}\vec{b}$; 2) $\vec{a}^2$; 3) $\vec{b}^2$; 4) $(\vec{a}+\vec{b})^2$; 5) $(\vec{a}-\vec{b})^2$;

6) $(3\vec{a}+2\vec{b})^2$; 7) $(2\vec{a}-3\vec{b})^2$; 8) $(3\vec{a}-2\vec{b})(\vec{a}+2\vec{b})$.

**4.1.10.** $\vec{a}$ va $\vec{b}$ vektorlar o'zaro perpendikulyar, $\vec{c}$ vektor ularning har biri bilan $\varphi=\frac{\pi}{3}$ burchak hosil qilib, $|\vec{a}|=3$, $|\vec{b}|=5$, $|\vec{c}|=8$ ga teng bo'lsa, quyidagilarni hisoblang:

1) $(3\vec{a}-2\vec{b})(\vec{b}+3\vec{c})$;  2) $(\vec{a}+\vec{b}+\vec{c})^2$;  3) $(\vec{a}+2\vec{b}-3\vec{c})^2$;
4) $(\vec{a}+\vec{b}-\vec{c})(\vec{a}+\vec{b}+\vec{c})$;  5) $(2\vec{a}-\vec{b}+3\vec{c})(2\vec{a}+\vec{b}-3\vec{c})$

**4.1.11.** $\vec{a}(5;-6;1)$, $\vec{b}(-4;3;0)$, $\vec{c}(5;-8;10)$ vektorlar berilgan bo‘lsa,

1) $3\vec{a}^2-4\vec{a}\vec{b}+2\vec{c}^2$;

2) $3\vec{a}\vec{b}-4\vec{b}\vec{c}-5\vec{a}\vec{c}$;

3) $2\vec{a}^2+4\vec{b}^2-5\vec{c}^2$ ifodalarni hisoblang.

**4.1.12.** $\vec{a}(3;1;2)$, $\vec{b}(2;7;4)$, $\vec{c}(1;2;1)$ vektorlar berilgan bo‘lsa,

1) $(\vec{a}\,\vec{b})\,\vec{c}$;

2) $\vec{a}^2(\vec{b}\vec{c})$;

3)$\vec{a}^2\vec{b}+\vec{b}^2\vec{c}+\vec{c}^2\vec{a}$ ifodalarni hisoblang.

**4.1.13.** $A(-1;3;-7)$, $B(2;-1;5)$ va $C(0;1;-5)$ nuqtalar berilgan bo‘lsa,

1)$\sqrt{\overrightarrow{AB}^2}$;  2) $\sqrt{\overrightarrow{AC}^2}$;  3) $\sqrt{\overrightarrow{BC}^2}$;  4) $(2\overrightarrow{AB}-\overrightarrow{CB})(2\overrightarrow{BC}+\overrightarrow{BA})$;
5) $(3\overrightarrow{AB}-2\overrightarrow{CB})(3\overrightarrow{BC}+2\overrightarrow{AC})$ ifodalarni hisoblang.

**4.1.14.** $\vec{a}(2;-4;4)$ va $\vec{b}(-3;2;6)$ vektorlar hosil qilgan burchak kosinusini toping.

**4.1.15.** $\vec{a}(5;2)$, $\vec{b}(7;-3)$ vektorlar berilgan. Bir vaqtning o‘zida ikkita $\vec{a}\vec{x}=38$, $\vec{b}\vec{x}=30$ tenglamani qanoatlantiradigan $\vec{x}$ vektor topilsin.

**4.1.16.** $\vec{a}(3;4)$, $\vec{b}(6;-7)$ vektorlar berilgan. Bir vaqtning o‘zida ikkita $\vec{a}\vec{x}=2$, $\vec{b}\vec{x}=19$ tenglamani qanoatlantiradigan $\vec{x}$ vektor topilsin.

**4.1.17.** $\vec{a}(3;-2;4)$, $\vec{b}(5;1;6)$, $\vec{c}(-3;0;2)$ vektorlar berilgan. Bir vaqtning o‘zida $\vec{a}\cdot\vec{x}=4$, $\vec{b}\cdot\vec{x}=35$, $\vec{c}\cdot\vec{x}=0$ tenglamalarni qanoatlantiradigan $\vec{x}$ vektor topilsin.

**4.1.18.** $\vec{a}(2;1;-1)$ vektorga kollinear va $\vec{x}\vec{a}=3$ shartni qanoatlantiruvchi $\vec{x}$ vektorni toping.

**4.1.19.** $\vec{x}$ vektor $\vec{a}=3\vec{i}+2\vec{j}+2\vec{k}$ va $\vec{b}=18\vec{i}-22\vec{j}-5\vec{k}$ vektorlarga perpendikulyar, $Oy$ o‘qi bilan o‘tmas burchak hosil qiladi. $|\vec{x}|=14$ bo‘lsa, uning koordinatalarini toping.

**4.1.20.** Uchta $\vec{a} = 2\vec{\imath} - \vec{\jmath} + 3\vec{k}$, $\vec{b} = \vec{\imath} - 3\vec{\jmath} + 2\vec{k}$ va $\vec{c} = 3\vec{\imath} + 2\vec{\jmath} -$ $-4\vec{k}$ vektorlar berilgan. $\vec{x}\vec{a} = -5$, $\vec{x}\vec{b} = -11$ va $\vec{x}\vec{c} = 20$ shartlarni qanoatlantiruvchi $\vec{x}$ vektorni toping.

**4.1.21.** Tomonlari birga teng bo‘lgan teng tomonli $ABC$ uchburchak berilgan. $\overrightarrow{BC} = \vec{a}$, $\overrightarrow{CA} = \vec{b}$, $\overrightarrow{AB} = \vec{c}$ deb $\vec{a}\,\vec{b} + \vec{b}\,\vec{c} + \vec{a}\,\vec{c}$ ifoda hisoblansin.

**4.1.22.** $\vec{a} = \alpha\vec{\imath} - 3\vec{\jmath} + 2\vec{k}$ va $\vec{b} = \vec{\imath} + 2\vec{\jmath} - \alpha\vec{k}$ vektorlar $\alpha$ ning qanday qiymatida o‘zaro perpendikulyar bo‘ladi?

**4.1.23.** $ABC$ uchburchak tomonlarining uzunliklari berilgan:$|BC| = 5$, $|CA| = 6$, $|AB| = 7$ bo‘lsa,

1) $\overrightarrow{BA}$ va $\overrightarrow{BC}$; 2) $\overrightarrow{AB}$ va $\overrightarrow{BC}$; 3) $\overrightarrow{AB}$ va $\overrightarrow{AC}$;

4) $\overrightarrow{BA}$ va $\overrightarrow{CA}$; 5) $\overrightarrow{CA}$ va $\overrightarrow{BC}$ vektorlarning skalyar ko‘paytmasi topilsin.

**4.1.24.** $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlar, $\vec{a} + \vec{b} + \vec{c} = 0$ shart bilan quyidagilar $|\vec{a}| = 3$, $|\vec{b}| = 1$, $|\vec{c}| = 4$ berilgan bo‘lsa, $\vec{a}\,\vec{b} + \vec{b}\,\vec{c} + \vec{c}\,\vec{a}$ ni hisoblang.

**4.1.25.** $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlar bir-birlari bilan $60^0$ ga teng bo‘lgan burchak tashkil qilsa, hamda $|\vec{a}| = 4$, $|\vec{b}| = 2$ va $|\vec{c}| = 6$ berilgan bo‘lsa, $\vec{p} = \vec{a} + \vec{b} + \vec{c}$ vektorning modulini aniqlang.

**4.1.26.** $|\vec{a}| = 3$, $|\vec{b}| = 5$ berilgan. $\alpha$ ning qanday qiymatida $\vec{a} + \alpha\vec{b}$ va $\vec{a} - \alpha\vec{b}$ vektorlar perpendikulyar bo‘ladi?

**4.1.27.** $\vec{a} + \vec{b}$ vektor $\vec{a} - \vec{b}$ vektorga perpendikulyar bo‘lishi uchun $\vec{a}$ va $\vec{b}$ vektorlar qanday shartni qanoatlantirishi kerak?

**4.1.28.** $\vec{a}$ va $\vec{b}$ vektorlar $\varphi = \frac{\pi}{6}$ burchak hosil qiladi. $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 1$ bo‘lsa, $\vec{p} = \vec{a} + \vec{b}$ va $\vec{q} = \vec{a} - \vec{b}$ vektorlar orasidagi $\alpha$ burchakni toping.

**4.1.29.** $A(-1; -2; 4)$, $B(-4; -2; 0)$ va $C(3; -2; 1)$ uchburchakning uchlari berilgan. Uning $B$ uchidagi ichki burchakni toping.

**4.1.30.** Uchburchakning $A(3; 2; -3)$, $B(5; 1; -1)$ va $C(1; -2; 1)$ uchlari berilgan. Uning $A$ uchidagi ichki burchakni aniqlang.

## 4.2. Vektorning vektor va aralash ko‘paytmasiga doir misollar.

**4.2.1.** Determinantlarni hisoblang.

1) $\begin{vmatrix} 5 & -2 \\ 3 & 4 \end{vmatrix}$; 2) $\begin{vmatrix} 3 & -3 \\ 2 & -2 \end{vmatrix}$; 3) $\begin{vmatrix} \sqrt[4]{5} & \sqrt{2} \\ \sqrt{2} & \sqrt[4]{125} \end{vmatrix}$;

4) $\begin{vmatrix} 3 & x+1 \\ -4 & -21 \end{vmatrix} = 1$; 5) $\begin{vmatrix} cosx & -sinx \\ sinx & cosx \end{vmatrix}$; 6) $\begin{vmatrix} x-1 & 3 \\ 2 & 3 \end{vmatrix} = 0$;

**4.2.2.** Determinantlarni hisoblang.

1) $\begin{vmatrix} 3 & 2 & 1 \\ 2 & 5 & 3 \\ 3 & 4 & 2 \end{vmatrix}$; 2) $\begin{vmatrix} 4 & -3 & 5 \\ 3 & -2 & 8 \\ 1 & -7 & -5 \end{vmatrix}$; 3) $\begin{vmatrix} 2 & -1 & 2 \\ 3 & 5 & 3 \\ 1 & 6 & x+5 \end{vmatrix} = 0$;

4) $\begin{vmatrix} x^2 & 3 & 2 \\ x & -1 & 1 \\ 0 & 1 & 4 \end{vmatrix} = 0$; 5) $\begin{vmatrix} x^2 & 4 & 9 \\ x & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 0$.

**4.2.3.** Quyidagi chiziqli tenglamalar sistemasini Kramer usulida yeching.

1) $\begin{cases} 3y - x = -17 \\ 5x + 3y = -5 \end{cases}$; 2) $\begin{cases} 2x - 3y = 5 \\ 4x - 5y = 7 \end{cases}$; 3) $\begin{cases} xcos\alpha - ysin\alpha = cos2\alpha \\ xsin\alpha + ycos\alpha = sin2\alpha \end{cases}$

**4.2.4.** Quyidagi chiziqli tenglamalar sistemasini Kramer usulida yeching.

1) $\begin{cases} x + y + 4z = 1 \\ 2x + y + 6z = 2 \\ 3x + 3y + 13z = 2 \end{cases}$; 2) $\begin{cases} x + 2y - z = -6 \\ 2x - y + z = 7 \\ 3x + 5y + 2z = -1 \end{cases}$;

3) $\begin{cases} x + 2y + 3z - 13 = 0 \\ 3x + 2y + 2z - 16 = 0 \\ 4x - 2y + 5z - 5 = 0 \end{cases}$; 4) $\begin{cases} 2x - 3y + z = 2 \\ 2x + y - 4z = 9 \\ 6x - 5y + 2z = 17 \end{cases}$.

**4.2.5.** $\vec{a}$ va $\vec{b}$ vektorlar o‘zaro $\varphi = \frac{\pi}{6}$ burchak hosil qiladi. Agar $|\vec{a}| = 6$, $|\vec{b}| = 5$ bo‘lsa, $|[\vec{a}\vec{b}]|$ ni hisoblang.

**4.2.6.** $|\vec{a}| = 10$, $|\vec{b}| = 2$ va $\vec{a}\,\vec{b} = 12$ berilgan bo‘lsa, $|[\vec{a}\vec{b}]|$ ni hisoblang.

**4.2.7.** $|\vec{a}| = 3$, $|\vec{b}| = 26$ va $|[\vec{a}\vec{b}]| = 72$ bo‘lsa, $\vec{a}\,\vec{b}$ ni toping.

**4.2.8.** $\vec{a}$ va $\vec{b}$ vektorlar o‘zaro perpendikulyar. $|\vec{a}| = 3$ va $|\vec{b}| = 4$ ni bilgan holda, quyidagilarni hisoblang:

1) $|[(\vec{a}+\vec{b})(\vec{a}-\vec{b})]|$;

2) $|[(3\vec{a}-\vec{b})(\vec{a}-2\vec{b})]|$.

**4.2.9.** $\vec{a}$ va $\vec{b}$ vektorlar o‘zaro $\varphi = \frac{2\pi}{3}$ burchak hosil qiladi. $|\vec{a}| = 1$, $|\vec{b}| = 2$ ni bilgan holda, quyidagilarni hisoblang:

1) $[\vec{a},\vec{b}]^2$; 2) $[(2\vec{a}+\vec{b})(\vec{a}+2\vec{b})]^2$; 3) $[(\vec{a}+3\vec{b})(3\vec{a}-\vec{b})]^2$.

**4.2.10.** Ixtiyoriy $\vec{p}$, $\vec{q}$, $\vec{r}$, $\vec{n}$ vektorlar berilgan. $\vec{a} = [\vec{p}\,\vec{n}]$, $\vec{b} = [\vec{q}\,\vec{n}]$ va $\vec{c} = [\vec{r}\,\vec{n}]$ vektorlarni komplanar ekanligini isbotlang.

**4.2.11.** $\vec{a}(3;-1;-2)$ va $\vec{b}(1;2;-1)$ vektorlar berilgan. Vektor ko‘paytmalar koordinatalarini toping:

1) $[\vec{a}\,\vec{b}]$; 2) $[(2\vec{a}+\vec{b})\vec{b}]$; 3) $[(2\vec{a}-\vec{b})(2\vec{a}+\vec{b})]$.

**4.2.12.** $A(2;-1;2)$, $B(1;2;-1)$ va $C(3;2;1)$ nuqtalar berilgan. Vektor ko‘paytmalar koordinatalarini toping:

1) $[\overrightarrow{\mathrm{AB}}\,\overrightarrow{\mathrm{BC}}]$; 2) $[(\overrightarrow{\mathrm{BC}}-2\overrightarrow{\mathrm{CA}})\overrightarrow{CB}]$.

**4.2.13.** $A(1;2;0)$, $B(3;0;-3)$ va $C(5;2;6)$ nuqtalar berilgan. $ABC$ uchburchak yuzasini hisoblang.

**4.2.14.** Uchburchakning $A(1;-1;2)$, $B(5;-6;2)$ va $C(1;3;-1)$ uchlari berilgan. $B$ uchidan $AC$ yon tomonga tushirilgan balandlik uzunligini hisoblang.

**4.2.15.** $\vec{a}(2;-2;1)$ va $\vec{b}(2;3;6)$ vektorlar orasidagi burchak sinusini hisoblang.

**4.2.16.** $A(3;-2;5)$, $B(1;4;-3)$ va $C(-6;2;4)$ nuqtalar berilgan bo‘lsa,

1) $[\overrightarrow{\mathrm{AB}}\,\overrightarrow{\mathrm{BC}}]\overrightarrow{\mathrm{AC}}$; 2) $[\overrightarrow{\mathrm{AB}}\,\overrightarrow{\mathrm{AC}}]\overrightarrow{\mathrm{BC}}$; 3) $[\overrightarrow{\mathrm{BC}}\,\overrightarrow{\mathrm{AC}}]\overrightarrow{\mathrm{AB}}$

aralash ko‘paytmasini toping.

**4.2.17.** $C(-2;4;3)$, $D(1;-5;6)$ va $E(3;7;-4)$ nuqtalar berilgan bo‘lsa,

1) $[\overrightarrow{\mathrm{CD}}\,\overrightarrow{\mathrm{DE}}]\overrightarrow{\mathrm{CE}}$; 2) $(2\overrightarrow{\mathrm{CD}}-3\overrightarrow{\mathrm{DE}})(\overrightarrow{\mathrm{DC}}+3\overrightarrow{\mathrm{CE}})(2\overrightarrow{\mathrm{CD}}-\overrightarrow{\mathrm{ED}})$;

3) $\left(3\overrightarrow{DE}+\overrightarrow{ED}\right)\left(2\overrightarrow{CD}-\overrightarrow{EC}\right)\left(\overrightarrow{DC}+2\overrightarrow{CE}\right)$ aralash ko‘paytmasini toping.

**4.2.18.** $\vec{a}=\vec{\imath}+6\vec{\jmath}-4\vec{k}$, $\vec{b}=-3\vec{\imath}+2\vec{\jmath}+7\vec{k}$ va $\vec{c}=-5\vec{\imath}-6\vec{\jmath}+2\vec{k}$ vektorlar berilgan bo‘lsa,

1) $[\vec{a}\,\vec{b}]\vec{c}$; 2) $[\vec{a}\,\vec{c}]\vec{b}$; 3) $[\vec{b}\,\vec{c}]\vec{a}$;

4) $\left(\vec{b}+2\vec{\vec{a}}\right)\left(\vec{c}+3\vec{b}\right)(2\vec{a}-\vec{c})$; 5) $(3\vec{a}-\vec{c})\left(2\vec{b}+\vec{a}\right)\left(4\vec{c}+3\vec{b}\right)$

aralash ko‘paytmasini toping.

**4.2.19.** $\vec{a}(2;-3;1)$, $\vec{b}(-3;1;2)$ va $\vec{c}(1;2;3)$ vektorlar berilgan bo‘lsa, $\left[[\vec{a},\vec{b}]\vec{c}\right]$ va $\left[\vec{a}[\vec{b}\,\vec{c}]\right]$ ni hisoblang.

**4.2.20.** $\vec{a}(6;-4;8)$ va $\vec{b}(-2;4;0)$ vektorlar berilgan bo‘lsa:

1) $\left[(\vec{a}+\vec{b})(\vec{a}-\vec{b})\right]$;

2) $\left[\vec{a}(\vec{a}+\vec{\vec{b}})\right]$;

3) $\left[\frac{\vec{a}+\vec{b}}{2}(\vec{b}-\frac{\vec{a}}{2})\right]$ topilsin.

**4.2.21.** Quyidagi hollarning har birida $\left[\vec{a}\,\vec{b}\right]$ vektor ko‘paytma topilsin:

1) $\vec{a}(2;3;1)$, $\vec{b}(5;6;4)$;

2) $\vec{a}(5;-2;1)$, $\vec{b}(4;0;6)$;

3) $\vec{a}(-2;6;-4)$, $\vec{b}(3;-9;6)$.

**4.2.22.** $\vec{a}(8;4;1)$ va $\vec{b}(2;-2;1)$ vektorlardan yasalgan parallelogramm yuzi hisoblansin.

**4.2.23.** $\vec{a}(3;1;2)$, $\vec{b}(2;7;4)$ va $\vec{c}(1;2;1)$ vektorlar berilgan:

1) $\vec{a}\vec{b}\vec{c}$; 2) $\left[[\vec{a}\vec{b}]\vec{c}\right]$; 3) $\left[\vec{a}[\vec{b}\vec{c}]\right]$ topilsin.

**4.2.24.** Berilganlarga ko‘ra $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlarning aralash ko‘paytmasini toping.

1) $\vec{a}=\vec{k}$, $\vec{b}=\vec{\imath}$, $\vec{c}=\vec{\jmath}$; 2) $\vec{a}=\vec{\imath}$, $\vec{b}=\vec{k}$, $\vec{c}=\vec{\jmath}$;

3) $\vec{a}=\vec{\jmath}$, $\vec{b}=\vec{\imath}$, $\vec{c}=\vec{k}$; 4) $\vec{a}=\vec{\imath}+\vec{\jmath}$, $\vec{b}=\vec{\jmath}$, $\vec{c}=\vec{k}$;

5) $\vec{a}=\vec{\imath}+\vec{\jmath}$, $\vec{b}=\vec{\imath}-\vec{\jmath}$, $\vec{c}=\vec{\jmath}$; 6) $\vec{a}=\vec{\imath}+\vec{\jmath}$, $\vec{b}=\vec{\imath}-\vec{\jmath}$, $\vec{c}=\vec{k}$.

**4.2.25.** $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlar o‘zaro perpendikulyar hamda $|\vec{a}| = 4$, $|\vec{b}| = 2$ va $|\vec{c}| = 3$ berilgan bo‘lsa, $\vec{a}\vec{b}\vec{c}$ ni toping.

**4.2.26.** $\vec{a}$ va $\vec{b}$ vektorlar o‘zaro $\varphi = \frac{\pi}{6}$ burchak tashkil qiladi va $\vec{c}$ vektor bilan perpendikulyar. $|\vec{a}| = 6$, $|\vec{b}| = 3$ va $|\vec{c}| = 4$ berilgan bo‘lsa, $\vec{a}\vec{b}\vec{c}$ ni toping.

**4.2.27.** $\vec{a}(1;-1;3)$, $\vec{b}(-2;2;1)$ va $\vec{c}(3;-2;5)$ vektorlar berilgan bo‘lsa, $\vec{a}\vec{b}\vec{c}$ ni toping.

**4.2.28.** $\vec{a} = 2\vec{i} + 3\vec{j} + 4\vec{k}$, $\vec{b} = 3\vec{i} + 2\vec{j} + \vec{k}$ va $\vec{c} = \vec{j} - \vec{k}$ vektorlar berilgan bo‘lsa, quyidagilarni toping.

1) $[\vec{a}\vec{b}]$; 2) $[\vec{b}\vec{c}]$; 3) $[\vec{a}\vec{c}]$; 4) $\left[[\vec{a}\vec{c}]\vec{b}\right]$; 5) $\left[[\vec{a}\vec{b}]\vec{c}\right]$;

6) $\left[\vec{a}[\vec{b}\vec{c}]\right]$; 7) $\left([\vec{a}\vec{b}]\vec{c}\right)$; 8) $\left([\vec{a}\vec{c}]\vec{b}\right)$; 9) $\left(\vec{a}[\vec{b}\vec{c}]\right)$;

10) $\left[(2\vec{a} - 3\vec{b})(4\vec{a} - 5\vec{c})\right]$; 11) $\left[(\vec{a} - 2\vec{c})(3\vec{b} - 2\vec{a})\right]$;

12) $\left(\left[(2\vec{b} + \vec{c})(3\vec{a} - \vec{c})\right](\vec{b} - 2\vec{a})\right)$;

13) $\left(\left[(2\vec{a} - 5\vec{c})(2\vec{b} + 3\vec{c})\right](3\vec{b} - \vec{c})\right)$; 14) $\left((\vec{a}\vec{b})\vec{c}\right)$; 15) $\left((\vec{b}\vec{c})\vec{a}\right)$.

**4.2.29.** $\vec{a} = -\vec{i} + 3\vec{j} + \vec{k}$, $\vec{b} = \vec{j} + 2\vec{k}$ va $\vec{c} = 2\vec{a} - 3\vec{b}$ vektorlar berilgan bo‘lsa, quyidagilarni toping:

1) $\left(\vec{a}[\vec{c}\vec{b}]\right)$; 2) $\left(\vec{c}[\vec{a}\vec{b}]\right)$; 3) $\left[\vec{a}[\vec{b}\vec{c}]\right]$; 4) $\left[\vec{b}[\vec{a}\vec{c}]\right]$;

5) $\left(\left[\vec{a}[\vec{b}\vec{c}]\right]\vec{c}\right)$; 6) $\left(\left[\vec{a}[\vec{a}\vec{c}]\right]\left[\vec{b}[\vec{a}\vec{c}]\right]\right)$.

**4.2.30.** Ayniyatni isbotlang:

1) $\left[\vec{a}[\vec{b}\vec{c}]\right] + \left[\vec{b}[\vec{a}\vec{c}]\right] + \left[\vec{c}[\vec{a}\vec{b}]\right] = 0$;

2) $[\vec{a}\vec{b}][\vec{c}\vec{d}] = (\vec{a}\vec{c})(\vec{b}\vec{d}) - (\vec{a}\vec{d})(\vec{b}\vec{c})$;

3) $[\vec{a}\vec{b}][\vec{c}\vec{d}] + [\vec{a}\vec{c}]\,[\vec{d}\vec{b}] + [\vec{a}\vec{d}]\,[\vec{b}\vec{c}] = 0$;

4) $\left[[\vec{a}\vec{b}][\vec{c}\vec{d}]\right] = \vec{c}(\vec{a}\vec{b}\vec{d}) - \vec{d}(\vec{a}\vec{b}\vec{c})$;

5) $[\vec{a}\vec{b}][\vec{b}\vec{c}][\vec{c}\vec{a}] = (\vec{a}\vec{b}\vec{c})^2$;

6) $\left[\vec{a}\left[\vec{a}\left[\vec{a}[\vec{a}\vec{b}]\right]\right]\right] = \vec{a}^4\vec{b}$ bu yerda, $\vec{a}$ va $\vec{b}$ vektorlar o‘zaro perpendikulyar;

7) $[\vec{a}(\vec{b}[\vec{c}\vec{d}])] = [\vec{a}\vec{c}](\vec{b}\vec{d}) - [\vec{a}\vec{d}](\vec{b}\vec{c})$;

8) $\left[\vec{a}\left[\vec{b}[\vec{c}\vec{d}]\right]\right] = (\vec{a}\vec{c}\vec{d})\vec{b} - (\vec{a}\vec{b})[\vec{c}\vec{d}]$;

9) $[\vec{a}\vec{b}]^2[\vec{a}\vec{c}]^2 - ([\vec{a}\vec{b}][\vec{a}\vec{c}]) = \vec{a}^2(\vec{a}\vec{b}\vec{c})^2$;

10) $\left[[\vec{a}\vec{b}][bc]\right]\left[[\vec{b}\vec{c}][\vec{c}\vec{a}]\right]\left[[\vec{c}\vec{a}][\vec{a}\vec{b}]\right] = (\vec{a}\vec{b}c)^4$;

11) $(\vec{a}\vec{b})[\vec{c}\vec{d}] + (\vec{a}\vec{c})[\vec{d}\vec{b}] + (\vec{a}\vec{d})[\vec{b}\vec{c}] = \vec{a}(\vec{b}\vec{c}\vec{d})$;

12) $(\vec{a}\vec{b}\vec{c})(\vec{a}\vec{d}\vec{e}) = \begin{vmatrix} \vec{a}\vec{b}\vec{d} & \vec{a}\vec{b}\vec{e} \\ \vec{a}\vec{c}\vec{d} & \vec{a}\vec{c}\vec{e} \end{vmatrix}$.

## 5-MAVZU: TEKISLIKDA TO‘G‘RI CHIZIQLARNING TURLI TENGLAMALARI

**Reja:**

**1. To‘g‘ri chiziqning burchak koeffitsiyentli tenglamasi. To‘g‘ri chiziq orasidagi burchak.**

**2. To‘g‘ri chiziqning umumiy tenglamasi. Berilgan ikki nuqtadan o‘tuvchi tenglamasi. To‘g‘ri chiziqning koordinata o‘qlaridan ajratgan kesmalar bo‘yicha tenglamasi.**

**3. To‘g‘ri chiziqning normal tenglamasi. To‘g‘ri chiziq tenglamasini normal holda keltirish. Berilgan nuqtadan to‘g‘ri chiziqqacha bo‘lgan masofa.**

**4. Tekislikda to‘g‘ri chiziqlarga doir aralash masalalar.**

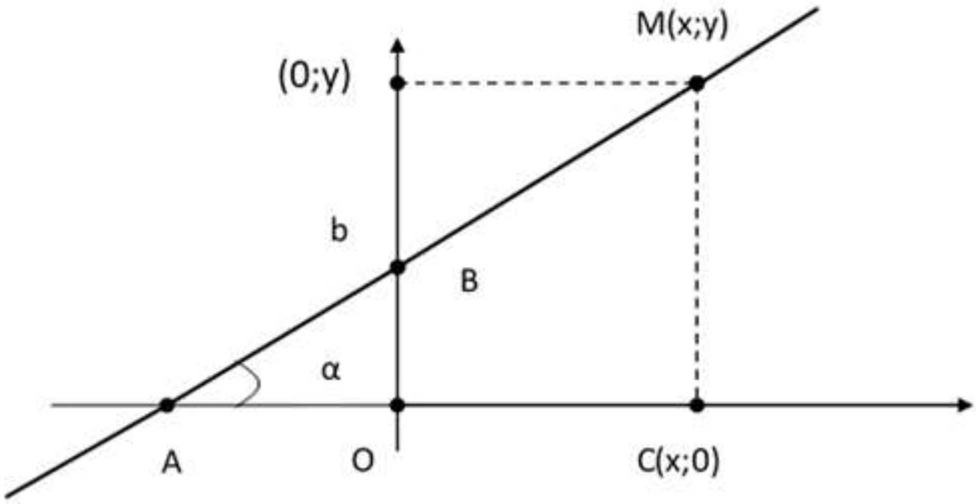
**Tayanch iboralar:** burchak koeffitsiyenti, parallellik, perpendukulyarlik, to‘g‘ri chiziqlar dastasi, normal, normal vektor, bissektrisa.

## 5.1. To‘g‘ri chiziqning burchak koeffitsiyentli tenglamasi. To‘g‘ri chiziq orasidagi burchak.

Bizda $XOY$ dekart koordinatalar sistemasida to‘g‘ri chiziqning quyidagi parametrlari berilgan bo‘lsin.

1) To‘g‘ri chiziq bilan absissa o‘qining musbat yo‘nalishi orasidagi burchak $\alpha$;

2) To‘g‘ri chiziq bilan ordinata o‘qining kesishish nuqtasi(to‘g‘ri chiziqning ordinata o‘qidan ajratgan kesmasi) koordinatalari $(0; b)$.




**5.1.1-chizma**

Berilganlardan foydalanib, to‘g‘ri chiziq tenglamasini keltirib chiqaramiz, hamda $tg\alpha = \frac{OB}{AO}$, $OB = b$ ekanligidan $AO = \frac{b}{tg\alpha}$ natijaga erishiladi.

1. $Ox$ o‘qining musbat yo‘nalishi bilan to‘g‘ri chiziq orasidagi burchak $\alpha$ va $B(0; b)$ nuqta to‘g‘ri chiziqning ordinata o‘qi bilan kesishgan nuqtasi

2. Ixtiyoriy $M(x; y)$ nuqta tanlaymiz va $\triangle ABO$ va $\triangle AMC$ uchburchaklar o‘xshashligidan

$$\frac{OB}{MC} = \frac{AO}{AC} \Longrightarrow b\left(\frac{b}{tg\alpha} + x\right) = \frac{yb}{tg\alpha}$$

$$y = xtg\alpha + b \text{ va } tg\alpha = k$$

deb belgilash kiritsak,

$$y = kx + b \qquad (5.1)$$

***to‘g‘ri chiziq tenglamasi*** kelib chiqadi.

To‘g‘ri chiziqning tenglamasini $k$ va $b$ lar bo‘yicha tahlil qilamiz:

1. $k > 0$ bo‘lsa, $tg\alpha > 0$ bo‘ladi. Bunda $\alpha \in (0; \frac{\pi}{2}) \Longrightarrow \alpha$ o‘tkir burchak;
2. $k < 0$ bo‘lsa, $tg\alpha < 0$, bo‘ladi. Bunda $\alpha \in (\frac{\pi}{2}; \pi) \Longrightarrow \alpha$ o‘tmas burchak;
3. $b > 0$ bo‘lsa, to‘g‘ri chizig‘imiz ordinata o‘qini musbat tomoni bilan kesishadi;
4. $b < 0$ bo‘lsa, to‘g‘ri chizig‘imiz ordinata o‘qini manfiy tomoni bilan kesishadi.

$k$ va $b$ larni o‘zaro kombinatsiyasidan quyidagilar kelib chiqadi:

1. $k > 0,\ b > 0$ bo‘lsa, to‘g‘ri chizig‘imiz koordinatalar sistemasining $I, II$ va $III$ choragidan o‘tadi;
2. $k > 0,\ b < 0$ bo‘lsa, to‘g‘ri chizig‘imiz koordinatalar sistemasining $I, III$ va $IV$ choragidan o‘tadi;
3. $k < 0,\ b > 0$ bo‘lsa, to‘g‘ri chizig‘imiz koordinatalar sistemasining $I, II$ va $IV$ choragidan o‘tadi;
4. $k < 0,\ b < 0$ bo‘lsa, to‘g‘ri chizig‘imiz koordinatalar sistemasining $II, III$ va $IV$ choragidan o‘tadi.

**1-Misol.** Umumiy tenglamasi $\boldsymbol{4x - 6y + 3 = 0}$ bo‘lgan to‘g‘ri chiziqning burchak koeffitsiyentli tenglamasini toping.

**Y**

**e**

Berilgan to‘g‘ri chiziq tenglamasidan burchak koeffitsiyenti va boshlang‘ich ordinatasini topamiz:

$$\boldsymbol{4x - 6y + 3 = 0 \Rightarrow 6y = 4x + 3 \Rightarrow y = \frac{2}{3}x + \frac{1}{2}}$$

hosil bo‘ladi. Bu yerda

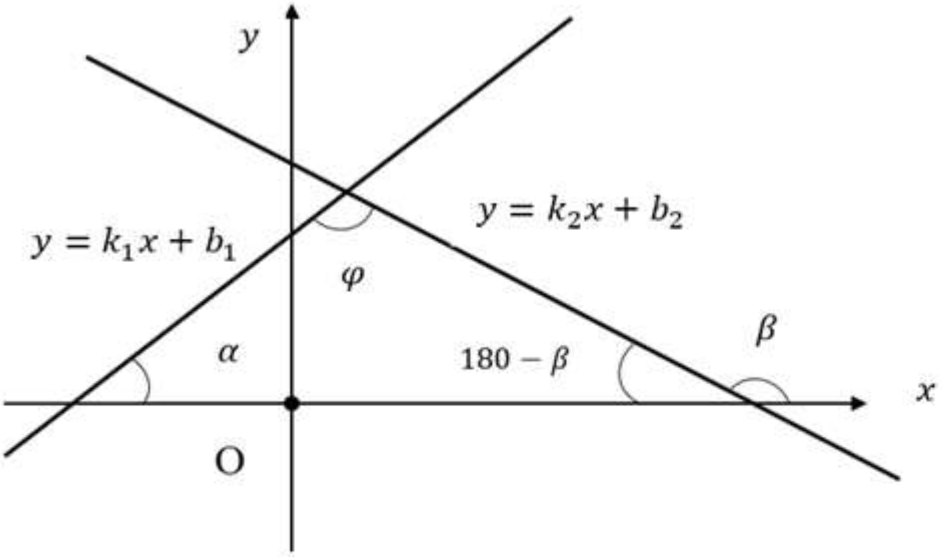
$$\boldsymbol{k = \frac{2}{3}, \quad b = \frac{1}{2}}$$

ga teng bo‘ladi.

**Ikki to‘g‘ri chiziq orasidagi burchak.**

Dekart kordinatalar tekisligida $y = k_1x + b_1$ va $y = k_2x + b_2$ tenglamalar bilan ikkita to‘g‘ri chiziq berilgan bo‘lsin.

Biz ikki to‘g‘ri chiziq orasidagi $\varphi$ burchakni topish uchun:
1) to‘g‘ri chiziqning $Ox$ o‘qining musbat yo‘nalishi orasidagi burchakni $\alpha$ desak, bundan $tg\alpha = k_1$.
2) to‘g‘ri chiziq $Ox$ o‘qining musbat yo‘nalishi orasidagi burchakni $\beta$ desak, $tg\beta = k_2$ kelib chiqadi.




**5.1.2-chizma**

Uchburchakning ichki burchaklari yig‘indisi formulasidan

$$\alpha + 180 - \beta + \varphi = 180 \implies \varphi = \beta - \alpha \implies tg\varphi = tg(\beta - \alpha)$$

$$tg\varphi = \frac{tg\beta - tg\alpha}{1 + tg\beta \cdot tg\alpha}$$

kelib chiqadi. Yuqoridagi belgilashlardan foydalansak,

$$tg\varphi = \frac{k_2 - k_1}{1 + k_1 \cdot k_2} \qquad (5.2)$$

***ikki to‘g‘ri chiziqning burchak koeffitsiyenti*** topish formulasi kelib chiqadi.

**2-Misol.** Dekart koordinatalar tekisligida $y = x + 4$ va $y = 2x - 7$ tenglamalar 2 ta to‘g‘ri chiziq berilgan bo‘lsa, ular orasidagi burchakni toping.

**Yechish:** Bizga berilgan $k_1 = 1$ va $k_2 = 2$ ekanligidan, yuqorida berilgan (5.2) formuladan foydalanib,

$$tg\varphi = \frac{k_2 - k_1}{1 + k_1 \cdot k_2} = \frac{2 - 1}{1 + 1 \cdot 2} = \frac{1}{3}$$

ikkita to‘g‘ri chiziq orasidagi burchak $\varphi = arctg\frac{1}{3}$ tengligi kelib chiqadi.

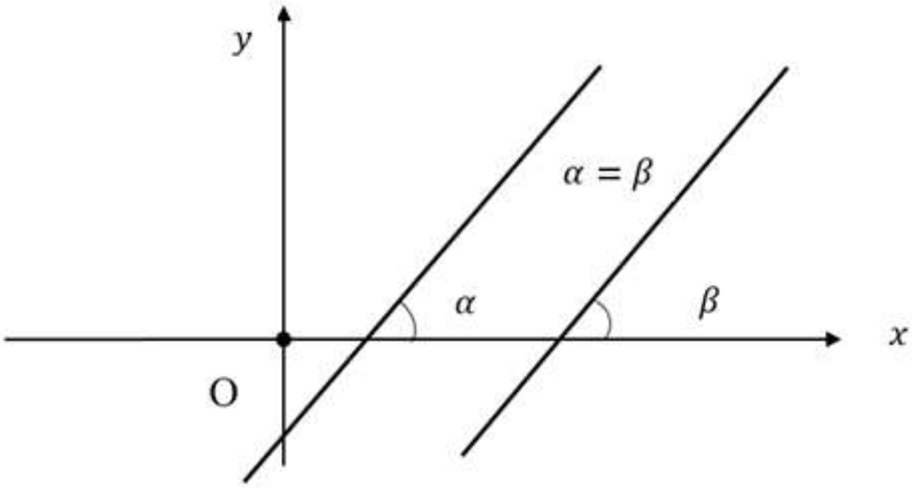
**To‘g‘ri chiziqning parallellik alomatlari.**

**1-usul.** $y = k_1x + b_1$ to‘g‘ri chizig‘imiz $y = k_2x + b_2$ ga parallel, ya’ni $\varphi = 0$ bo‘lganda, $tg\varphi = 0$ bo‘lib, $\frac{k_2 - k_1}{1 + k_1 \cdot k_2}$=0 bo‘ladi. Bu yerda $k_2 - k_1$=0 bo‘lsa, $k_1 = k_2$ bo‘lishi kelib chiqadi.

**2-usul.** $\alpha = \beta$ bo‘lsa, $tg\alpha = tg\beta$ bo‘ladi. Bundan

$$k_1 = k_2 \tag{5.3}$$

ekanligi kelib chiqadi. To‘g‘ri chiziqlar parallel bo‘lishi uchun ularning burchak koeffitsiyenti teng bo‘lishi kerak.




**5.1.3-chizma**

**To‘g‘ri chiziqning perpendikulyarlik alomatlari.**

$y = k_1x + b_1$ to‘g‘ri chiziq $y = k_2x + b_2$ ga perpendikulyar bo‘lsa, $tg\varphi = tg90^0$ bo‘ladi. $tg90^0$ mavjud bo‘lmasligi va aniqlanmagan bo‘lishi kerak. Bu uchun $1 + k_1 \cdot k_2 = 0$ bo‘lishi kerakligidan $k_1 \cdot k_2 = -1$ bo‘ladi. Bundan kelib chiqadiki,

$$k_2 = -\frac{1}{k_1} \tag{5.4}$$

to‘g‘ri chiziqlar perpendikulyar bo‘lishi uchun ularning burchak koeffitsiyentlari ham teskari ham qarama- qarshi ishorali bo‘lishi kerak.

**3-Misol.** $y = 4x - 2$ to‘g‘ri chiziqqa parallel va perpendikulyar to‘g‘ri chiziqlarni toping.

**Yechish:** 1) $y = 4x - 2$ to‘g‘ri chiziqqa parallel to‘g‘ri chiziqni topish uchun yuqoridagi (5.3) formuladan foydalanib, $y = 4x + c$ ekanligi kelib chiqadi.

2) $y = 4x - 2$ to‘g‘ri chiziqqa perpendikulyar to‘g‘ri chiziqni topish uchun yuqoridagi (5.4) formuladan foydalanib, $y = -\frac{1}{4}x + c$ ekanligi kelib chiqadi.

**Berilgan nuqtadan o‘tib berilgan to‘g‘ri chiziqqa parallel bo‘lgan to‘g‘ri chiziq tenglamasi.**

$XOY$ tekisikdagi $l: y = kx + b$ to‘g‘ri chiziq va koordinatalari $M(x_0; y_0)$ nuqtalar berilgan bo‘lsin. $M$ nuqtadan o‘tib $l$ to‘g‘ri chiziqqa parallel bo‘lgan to‘g‘ri chiziq tenglamasini tuzaylik.

**1-usul.** $y = kx + b$ to‘g‘ri chiziqqa parallel bo‘lgan $y = kx + c$ to‘g‘ri chiziq $M(x_1; y_1)$ nuqtadan o‘tishi uchun $y_1 = kx_1 + c$ tenglik o‘rinli bo‘lishi kerak.

$c = y_1 - kx_1$ bu tenglikdan

$$y = kx + (y_1 - kx_1) \tag{5.5}$$

kelib chiqadi.

**2-usul.** $M(x_1; y_1)$ nuqtadan o‘tuvchi to‘g‘ri chiziqlar dastasi

$$y - y_1 = d(x - x_1),$$

$$y = dx + y_1 - dx_1 \tag{5.6}$$

ko‘rinishida ifodalaniladi.

Bu to‘g‘ri chiziq $y = kx + b$ bilan parallel bo‘lishi uchun $d = k$ bo‘lishi kerak. Bundan kelib chiqib,

$$y = kx + y_1 - kx_1$$

ko‘rinishidagi tenglamamiz ***berilgan nuqtadan o‘tib berilgan to‘g‘ri chiziqqa parallel bo‘lgan to‘g‘ri chiziq tenglamasi*** deyiladi.

**4-Misol.** Berilgan $M(2; -1)$ nuqtadan $y = 0{,}3x - 7$ to‘g‘ri chiziqqa parallel bo‘lgan to‘g‘ri chiziq tenglamasi tuzing.

**1-usul.** Bu to‘g‘ri chiziq $y = kx + b$ bilan parallel bo‘lishi uchun $N = k$ bo‘lsa, $y = kx + y_0 - kx_0$ ekanligidan $y = 0{,}3x + c$ formuladan

$$0{,}3 \cdot 2 + c = -1 \implies 0{,}6 + c = -1 \implies c = -1{,}6$$

$y = 0{,}3x - 1{,}6$ kelib chiqadi.

**2-usul:** Bu to‘g‘ri chiziq $y = kx + b$ bilan parallel bo‘lishi uchun $N = k$ bo‘lsa, $y = kx + y_0 - kx_0$ ekanligidan

$$y + 1 = k(x - 2) \implies y = kx - 2k - 1 \implies k = 0{,}3$$

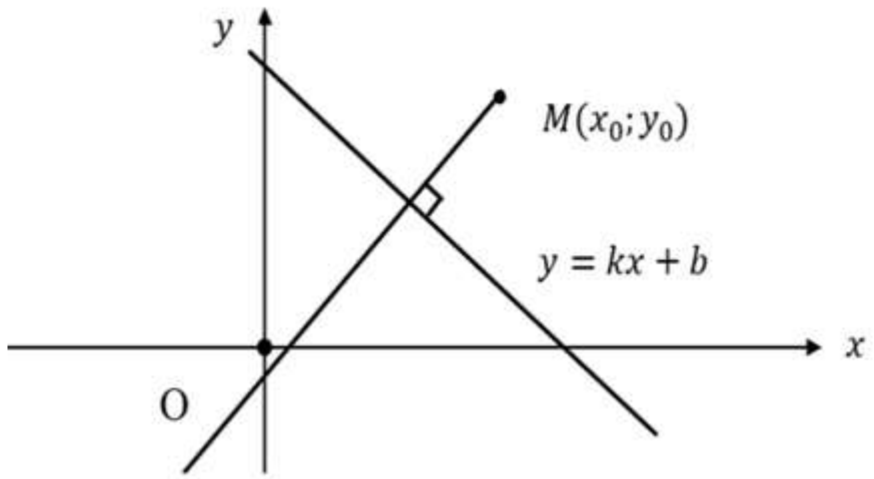
$$y = 0{,}3x - 2 \cdot 0{,}3 - 1$$

$$y = 0{,}3x - 1{,}6$$

kelib chiqadi. $M(2; -1)$ nuqtadan $y = 0{,}3x - 7$ to‘g‘ri chiziqqa parallel bo‘lgan to‘g‘ri chiziq tenglamasi $y = 0{,}3x - 1{,}6$ ko‘rinishida bo‘ladi.

**Berilgan nuqtadan o‘tib berilgan to‘g‘ri chiziqqa perpendikulyar bo‘lgan to‘g‘ri chiziq tenglamasi.**

Bizga dekart koordinatalar tekisligida $y = kx + b$ to‘g‘ri chiziq va $M(x_0; y_0)$ nuqta berilgan bo‘lsin. Bu to‘g‘ri chiziqqa perpendikulyar va $M$ nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzamiz.




**5.1.4-chizma**

**1-usul.** Berilgan to‘g‘ri chiziqqa perpendikulyar bo‘lgan $y = -\frac{1}{k}x_0 + c$ tenglamamizdan $c$ ni topsak, $c = y_0 + \frac{1}{k}x_0$

$$y = -\frac{1}{k}x + y_0 + \frac{1}{k}x_0$$

ko‘rinishda bo‘ladi.

**2-usul.** $y - y_0 = d(x - x_0)$ tenglamadan $d$ ni topsak, $y = dx + y_0 - dx_0$ va $d = -\frac{1}{k}$ ekanligi kelib chiqadi hamda

$$y = -\frac{1}{k}x + y_0 + \frac{1}{k}x_0. \tag{5.7}$$

to‘g‘ri chiziq tenglamasini topdik.

**5-Misol.** Berilgan $M(2;-3)$ nuqtadan o‘tib berilgan $y=0{,}5x-$ $-2$ to‘g‘ri chiziqqa perpendikulyar bo‘lgan to‘g‘ri chiziq tenglamasi toping.

**Yechish:** Yuqorida berilgan (5.7) formuladan, $y=0{,}5x-2$ to‘g‘ri chiziq tenglamasiga perpendikulyar bo‘lgan $y=-2x+c$ tenglamasidan, $c$ va $d$ ni topamiz.

$$-3=-2\cdot 2+c \;\Rightarrow\; c=1$$

$$(y+3)=d(x-2)$$

$$y+3=dx-2d \;\Rightarrow\; y=dx-2d-3 \;\Rightarrow\; d=-2.$$

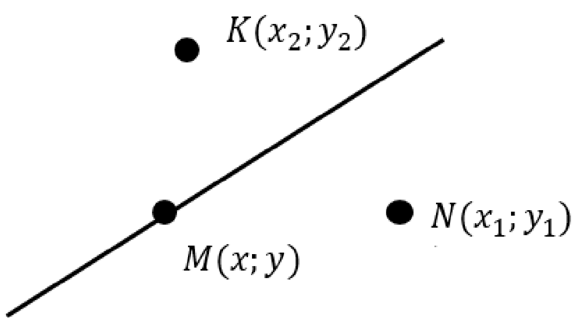
$M(2;-3)$ nuqtadan o‘tib berilgan $y=0{,}5x-2$ to‘g‘ri chiziqqa perpendikulyar bo‘lgan to‘g‘ri chiziq tenglamasi $y=-2x+1$ bo‘ladi.

### 5.2. To‘g‘ri chiziqning umumiy tenglamasi. Berilgan ikki nuqtadan o‘tuvchi tenglamasi. To‘g‘ri chiziqning koordinata o‘qlaridan ajratgan kesmalar bo‘yicha tenglamasi.

**To‘g‘ri chiziqning umumiy tenglamasi.**

Bizda ma’lumki tekislikdagi ixtiyoriy ikkita nuqtadan bir xil uzoqlikda yotgan nuqtalarning geometrik o‘rni to‘g‘ri chiziqni ifodalaydi.

Bizga $N(x_1;y_1)$ va $K(x_2;y_2)$ nuqtalar berilgan bo‘lsin.




**5.2.1-chizma**

Berilgan ikki nuqtadan teng uzoqlikda yotgan to‘g‘ri chiziqda $M(x;y)$ nuqta olamiz. Bu yerda $|KN|=|NM|$ekanligidan $|NM|=\sqrt{(x-x_1)^2(y-y_1)^2}$ va $|KN|=\sqrt{(x-x_2)^2(y-y_2)^2}$ teng bo‘ladi. Bu tengliklardan

$$\sqrt{(x-x_1)^2+(y-y_1)^2}=\sqrt{(x-x_2)^2+(y-y_2)^2}$$
$$x^2-2xx_1+{x_1}^2+y^2-2yy_1+{y_1}^2=$$
$$=x^2-2xx_2+{x_2}^2+y^2-2yy_2+{y_2}^2$$
$$2xx_2-2xx_1+{x_1}^2-{x_2}^2+2yy_2-2yy_1+{y_1}^2-{y_2}^2=0$$
$$(2x_2-2x_1)x+(2y_2-2y_1)y+{x_1}^2-{x_2}^2+{y_1}^2-{y_2}^2=0$$
kelib chiqib, $2x_2-2x_1=A$, $2y_2-2y_1=B$ va ${x_1}^2-{x_2}^2+$ $+{y_1}^2-{y_2}^2=C$ belgilash keritsak,
$$Ax+By+C=0 \qquad (A^2+B^2\neq 0) \tag{5.8}$$
***to‘g‘ri chiziqning umumiy tenglamasi*** hosil bo‘ladi.

Endi ayrim xususiy hollarni ko‘ramiz:

1. $A=0$ va $B\neq 0$ bo‘lsa, $By+C=0$ bo‘ladi va bundan $y=-\frac{C}{B}$ ekanligi kelib chiqadi. Bu to‘g‘ri chiziq tenglamasi absissa o‘qiga parallel va $(0;-\frac{C}{B})$ nuqtadan o‘tadi;
2. $A\neq 0$ va $B=0$ bo‘lsa, $Ax+C=0$ bo‘ladi va bundan $x=-\frac{C}{A}$ ekanligi kelib chiqadi. Bu to‘g‘ri chiziq tenglamasi ordinata o‘qiga parallel va $(-\frac{C}{A};0)$ nuqtadan o‘tadi;
3. $A\neq 0,\ B\neq 0$ va $C=0$ bo‘lsa, $Ax+By=0$ bo‘ladi va berilgan to‘g‘ri chizig‘imiz koordinata boshidan o‘tadi.

**6-Misol.** Berilgan $N_1(2;3)$ va $N_2(4;-2)$ nuqtalardan teng uzoqlikda joylashgan to‘g‘ri chiziq tenglamasini toping.

**Yechish:** Yuqorida berilgan (5.8) formulada

$|N_1M|=|N_2M|$ ekanligidan
$$\sqrt{(x-2)^2+(y-3)^2}=\sqrt{(x-4)^2+(y+2)^2}$$
$$(x^2-4x+4)+(y^2-6y+9)=(x^2-8x+16)+(y^2+4y+4)$$
$$-4x+4-6y+9=-8x+16+4y+4$$
$$4x+13=10y+20$$
ekanligi ma’lum bo‘ldi. Bundan kelib chiqib, $N_1$ va $N_2$ nuqtalardan teng uzoqlikda joylashgan to‘g‘ri chiziq tenglamasi $4x-10y-7=0$ ko‘rinishda bo‘ladi.

## Tekislikda berilgan ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi.

Dekart koordinatalar sistemasida $M_1(x_1; y_1)$ va $M_2(x_2; y_2)$ nuqtalar berilgan bo‘lsin. Bu nuqtalarning har biridan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzamiz.

Tuzmoqchi bo‘lgan to‘g‘ri chizig‘imizni $Ax + By + C = 0$ ko‘rinishida izlaymiz. Bu to‘g‘ri chiziq $M_1(x_1; y_1)$ nuqtadan o‘tishi uchun tenglamalar sistemasidagi $(2^*)$ tenglamani, $M_2(x_2; y_2)$ nuqtadan o‘tishi uchun esa $(3^*)$ tenglamani qanoatlantirishi kerak.

$$\begin{cases} Ax + By + C = 0 & (1^*) \\ Ax_1 + By_1 + C = 0 & (2^*) \\ Ax_2 + By_2 + C = 0 & (3^*) \end{cases}$$

$$(1^*) - (2^*) \Rightarrow \ A(x - x_1) + B(y - y_1) = 0,$$

$$(3^*) - (2^*) \Rightarrow \ A(x_2 - x_1) + B(y_2 - y_1) = 0$$

$$A(x - x_1) = -B(y - y_1),$$

$$A(x_2 - x_1) = -B(y_2 - y_1).$$

hosil bo‘ladi.

$$\begin{cases} A(x - x_1) = -B(y - y_1) \\ A(x_2 - x_1) = -B(y_2 - y_1) \end{cases}$$

bundan

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} \qquad (5.9)$$

kelib chiqadi. Bu tenglama esa tekislikda $M_1$ va $M_2$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasi kelib chiqadi.

**7-Misol.** $A(-3; 5)$ va $B(2; 1)$ nuqtalar berilgan bo‘lsin. Bu nuqtalarning har biridan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.

**Yechish.** Yuqoridagi (5.9) formuladan foydalanib,

$$\frac{x + 3}{2 + 3} = \frac{y - 5}{1 - 5} \Rightarrow \ -4(x + 3) = 5(y - 5) \ \Rightarrow -4x - 12 = 5y - 25$$

$$-4x - 5y + 13 = 0 \ \Rightarrow 4x + 5y - 13 = 0$$

$A$ va $B$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasini keltirib chiqardik.

**To‘g‘ri chiziqning koordinata o‘qlaridan ajratgan kesmalar bo‘yicha tenglamasi.**

Bizda dekart koordinatalar sistemasida to‘g‘ri chiziq berilgan bo‘lsin. To‘g‘ri chizig‘imiz absissa o‘qidan $a$ uzunlikdagi kesmani, ordinata o‘qidan esa $b$ uzunlikdagi kesmani ajratgan bo‘lsin. Demak, to‘g‘ri chizig‘imiz koordinatalari $A(a;0)$ va $B(0;b)$ bo‘lgan nuqtalardan o‘tar ekan. Bu uchun bizga berilgan $A(a;0)$ va $B(0;b)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasini topish yetarli bo‘ladi.

Berilganlardan foydalansak,

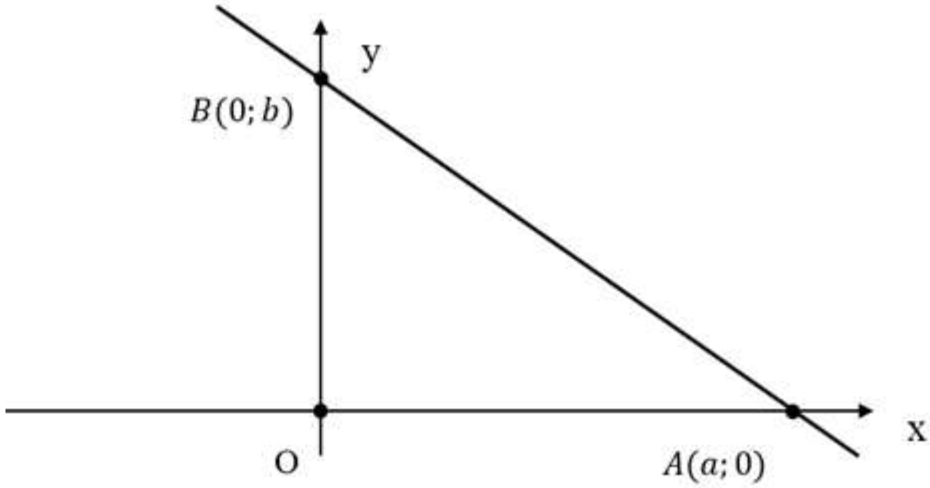
$$\frac{x-a}{0-a} = \frac{y-0}{b-0} \Rightarrow b(x-a) = -a(y-0) \Rightarrow bx - ab = -ay$$

$$bx + ay = ab$$

hosil bo‘ladi. Oxirgi tengligimizni ikkala tomonini $ab$ bo‘lib yuborsak

$$\frac{x}{a} + \frac{y}{b} = 1 \tag{5.10}$$

tenglama kelib chiqadi.




**5.2.2-chizma**

Ushbu tenglama absissa o‘qini $(a;0)$ va ordinata o‘qini $(0;b)$ nuqtada kesib o‘tuvchi to‘g‘ri chiziq tenglamasi bo‘ladi va to‘g‘ri chiziqning koordinata o‘qlaridan ajratgan kesmalar bo‘yicha tenglamasi deyiladi.

**8-Misol.** Berilgan $\frac{x}{2} - \frac{y}{3} = 1$ to‘g‘ri chizig‘imiz dekart koordinatalar sistemasini qaysi nuqtalarini kesib o‘tadi.

**Yechish.** (5.10) formuladan foydalangan holda, berilgan tenglama absissa o‘qini $(2; 0)$ va ordinata o‘qini $(0; -3)$ nuqtada kesib o‘tuvchi to‘g‘ri chiziq tenglamasi hosil bo‘ladi.

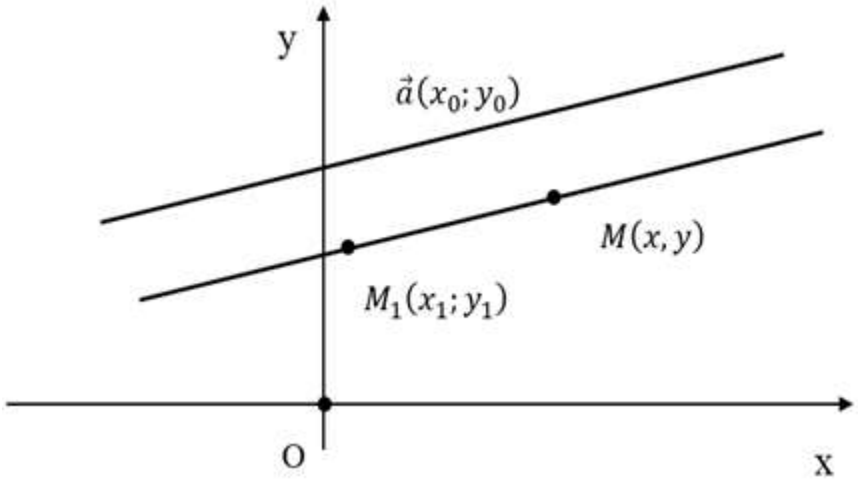
**Berilgan nuqtadan o‘tib, berilgan vektor bo‘yicha yo‘nalgan to‘g‘ri chiziq tenglamasi.**

Bizga dekart koordinatalar sistemasida $\vec{a}(x_0; y_0)$ vektor va $M_1(x_1, y_1)$ nuqta berilgan bo‘lsin.

$M_1$ nuqtadan o‘tib $\vec{a}$ vektor bo‘yicha yo‘nalgan to‘g‘ri chiziq tenglamasini tuzamiz. Bu to‘g‘ri chiziq tenglamasini tuzish uchun berilgan to‘g‘ri chiziqda yotgan ixtiyoriy $M$ nuqtasini olamiz. Bundan $x$ va $y$ bog‘liqligini ko‘rsatib, to‘g‘ri chiziq tenglamasini tuzamiz.

Bu yerda $\overrightarrow{M_1M} = \{x - x_1;\ y - y_1\}$, $\vec{a} = \{x_0;\ y_0\}$. $\overrightarrow{M_1M}$ va $\vec{a}$ vektorlar kollinearligidan

$$\frac{x - x_1}{x_0} = \frac{y - y_1}{y_0}$$




**5.2.3-chizma**

yoki

$$y_0x - x_0y + x_0y_1 - xy_0 = 0 \qquad (5.11)$$

ko‘rinishida bo‘ladi.

**9-Misol.** $\vec{a}(1; 2)$ vektor va $N(3; -4)$ nuqta berilgan. $N$ nuqtadan o‘tib, $\vec{a}$ vektor bilan bir xil yo‘nalgan to‘g‘ri chiziq tenglamasini toping.

**Yechish.** Berilgan (5.11) formuladan

$$\frac{x-3}{1}=\frac{y+4}{2} \Rightarrow 2x-6=y+4$$

ekanligi kelib chiqadi.

Agar tenglama $y=2x-10$ ko‘rinishda bo‘lsa, bu tenglama to‘g‘ri chiziqning burchak koeffitsiyentli tenglamasi. To‘g‘ri chiziqning umumiy tenglamasi esa, ushbu $2x-y-10=0$ ko‘rinishida bo‘ladi.

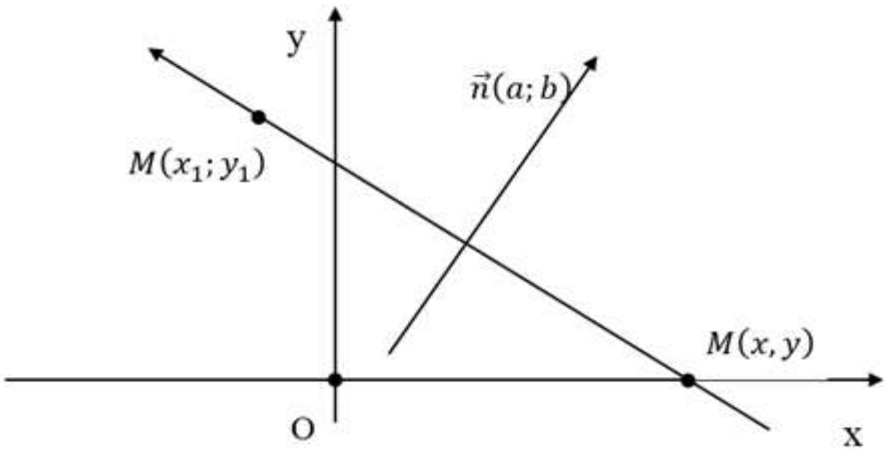
**Berilgan nuqtadan o‘tib, berilgan vektorga perpendikulyar to‘g‘ri chiziq tenglamasi.**

Dekart koordinatalar sistemasida $\vec{n}(a;b)$ vektor va to‘g‘ri chiziqda yotgan $M_1(x_1;y_1)$ nuqta berilgan bo‘lsin. Buning uchun to‘g‘ri chiziq ustidan ixtiyoriy $M(x,y)$ nuqta olib, $\overrightarrow{M_1M}=$ $=\{x-x_1;\ y-y_1\}$ vektorni yasaymiz. $\vec{n}$ va $\overrightarrow{M_1M}$ vektorlar perpendikulyar bo‘lishi uchun skalyar ko‘paytmasi 0 ga teng bo‘lishi kerak.

Ya’ni, $\overrightarrow{M_1M}=\{x-x_1;\ y-y_1\}$ va $\vec{n}(a;b)$, $\overrightarrow{M_1M}\cdot\vec{n}=0.$

$$a(x-x_1)+b(y-y_1)=0$$

$$ax+by-ax_1-by_1=0$$




**5.2.4-chizma**

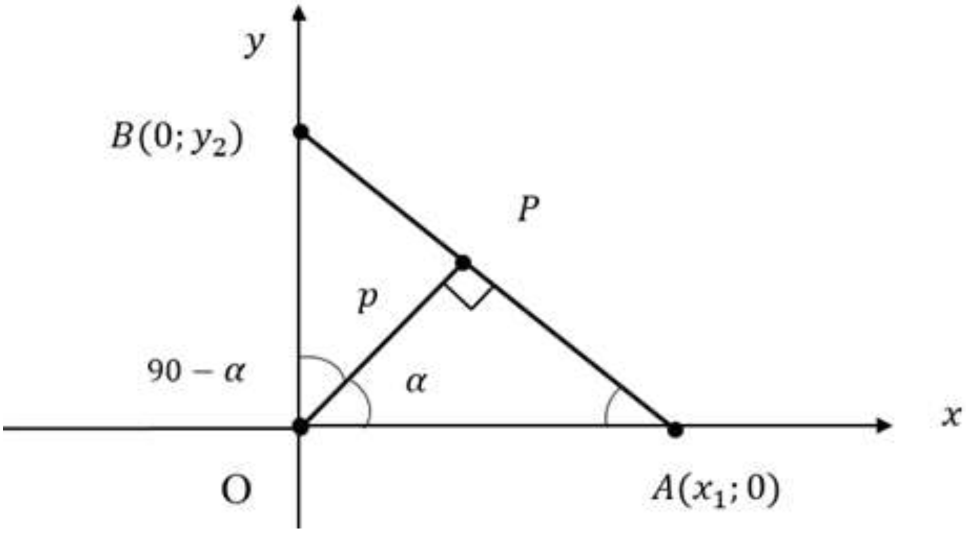
$$ax+by-(ax_1+by_1)=0 \tag{5.12}$$

to‘g‘ri chiziq tenglamasi $\vec{n}(a;b)$ vektorga perpendikulyar bo‘ladi va bu $\vec{n}(a;b)$ vektor to‘g‘ri chiziqning ***normal vektori*** deyiladi.

**5.3. To‘g‘ri chiziqning normal tenglamasi. To‘g‘ri chiziq tenglamasini normal holda keltirish. Berilgan nuqtadan to‘g‘ri chiziqqacha bo‘lgan masofa.**

**To‘g‘ri chiziqning normal tenglamasi**

Dekart koordinatalar sistemasida koordinata boshidan to‘g‘ri chiziqqacha bo‘lgan masofa $p$ berilgan bo‘lsin. Koordinata boshidan to‘g‘ri chiziqqacha tushirilgan perpendikulyar bilan absissa o‘qining musbat yo‘nalishi orasidagi burchak $\boldsymbol{\alpha}$ berilgan bo‘lsin. Berilganlardan foydalanib to‘g‘ri chiziq tenglamasini keltirib chiqaraylik.




**5.3.1-chizma**

$\triangle\, OPA$ dan $\cos\alpha = \frac{p}{x_1} \Rightarrow x_1 = \frac{p}{cos\alpha}$ topamiz.

$\triangle\, OBP$ dan esa $\cos(90^0-\alpha) = \frac{p}{y_2} \Rightarrow y_2 = \frac{p}{sin\alpha}$ ekanligi ma’lum bo‘ladi.

Demak, $A(\frac{P}{cos\alpha};0)$ va $B(0;\frac{P}{sin\alpha})$ nuqtalarning koordinatalaridan foydalanib, berilgan ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzsak

$$\frac{x-\frac{p}{cos\alpha}}{0-\frac{p}{cos\alpha}} = \frac{y-0}{\frac{p}{sin\alpha}-0}$$

$$\frac{p}{sin\alpha}\left(x-\frac{p}{cos\alpha}\right)=-\frac{p}{cos\alpha}y$$
$$\frac{x}{sin\alpha}-\frac{p}{sin\alpha cos\alpha}+\frac{y}{cos\alpha}=0$$
kelib chiqadi va tenglikni ikkala tomonini $sin\alpha cos\alpha$ ga ko‘paytirsak,
$$xcos\alpha+ysin\alpha-p=0 \qquad (5.13)$$
to‘g‘ri chiziqning ***normal tenglamasi*** kelib chiqadi.

Normal tenglama quyidagi xossalarga ega:

1. $x$ va $y$ o‘zgaruvchi koeffitsiyentlari oldidagi qiymatlarining kvadratlari yig‘indisi 1 ga teng. $cos^2\alpha+sin^2\alpha=1$
2. $p>0$ ya’ni, koordinata boshidan to‘g‘ri chiziqqacha bo‘lgan masofa.

**10-Misol.** Koordinata boshidan to‘g‘ri chiziqqacha bo‘lgan masofa 5 ga va absissa o‘qining musbat yo‘nalishi bilan $30^0$ burchak tashkil qiladi. Berilganlardan foydalanib to‘g‘ri chiziqning normal tenglamasini tuzing.

**Yechish.** Berilganlarni yuqoridagi (5.13) formulaga qo‘ysak,
$xcos30^0+ysin30^0-5=0 \Rightarrow \frac{\sqrt{3}}{2}x+\frac{1}{2}y-5=0$ bo‘ladi.

**To‘g‘ri chiziq tenglamasini normal holda keltirish.**

Bizga $ax+by+c=0$ umumiy hol bilan biror to‘g‘ri chiziq tenglamasi berilgan bo‘lsin. Bu tenglamani normal holga keltirish uchun biz tenglamani ikkala tomonini $M\neq 0$ soniga ko‘paytiramiz,
$$aMx+bMy+cM=0$$
normal tenglamaning xossasiga ko‘ra
$$a^2M^2+b^2M^2=1 \Rightarrow M \text{ ni topib olamiz va}$$
$$M^2=\frac{1}{a^2+b^2} \Rightarrow \mathrm{M}=\pm\frac{1}{\sqrt{a^2+b^2}} \Rightarrow$$
$$\pm\frac{ax}{\sqrt{a^2+b^2}}\pm\frac{by}{\sqrt{a^2+b^2}}\pm\frac{c}{\sqrt{a^2+b^2}}=0 \qquad (5.14)$$
natijani olamiz. Bu yerda $+$ yoki $-$ ishorasi ozod hadga qarab olinadi.

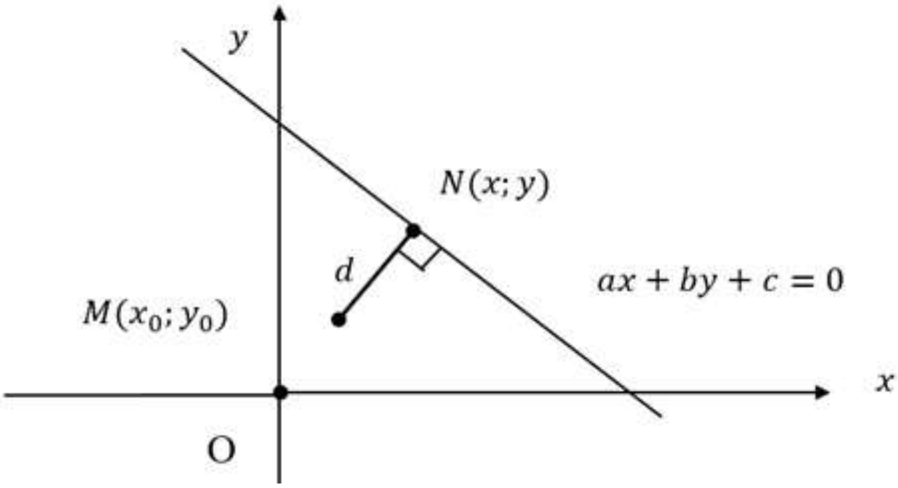
**11-Misol.** $3x-4y-6=0$ to‘g‘ri chiziq tenglamasini normal holga keltiring.

**Yechish.** Normal tenglamaning xossalaridan,

$\mathrm{M} = \pm\frac{1}{\sqrt{9+16}} = \pm\frac{1}{5}$ ekanligi topamiz. (5.14) formuladan foydalanib, to‘g‘ri chiziq tenglamasini $\frac{3}{5}x - \frac{4}{5}y - \frac{6}{5} = 0$ normal holga keltirdik.

**Berilgan nuqtadan to‘g‘ri chiziqqacha bo‘lgan masofa .**

Dekart koordinatalar sistemasida biror bir $ax + by + c = 0$ to‘g‘ri chiziq tenglamasi va $M(x_0; y_0)$ nuqta berilgan bo‘lsin. $M(x_0; y_0)$ nuqtadan to‘g‘ri chiziqqacha bo‘lgan $d$ masofani topishimiz kerak. Bu $M(x_0; y_0)$ nuqta to‘g‘ri chiziqqa perpendikulyar bo‘lishi zarur.




**5.3.2-chizma**

$ax + by + c = 0$ ga perpendikulyar $M(x_0; y_0)$ nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi

$$bx - ay + c_1 = 0$$

$$bx_0 - ay_0 + c_1 = 0$$

$$c_1 = ay_0 - bx_0$$

$$\begin{cases} bx - ay + ay_0 - bx_0 = 0 \\ ax + by + c = 0 \end{cases}$$

tenglamalar sistemasidagi birinchi tengligimizni $b$ ga, ikkinchi tengligimizni $a$ ga ko‘paytirib qo‘shsak, quyidagi tenglama hosil bo‘ladi.

$$(a^2 + b^2)x - aby_0 - bx_0 = 0$$

Natijada,

$$x = \frac{b^2x_0 - aby_0 - ac}{a^2 + b^2}$$

tenglik hosil bo‘ladi. $y$ ni topish uchun tenglamalar sistemasidagi birinchi tengligimizni $-a$ ga, ikkinchi tengligimizni $b$ ga ko‘paytirib qo‘shsak, quyidagi tenglama hosil bo‘ladi.

$$y = \frac{a^2y_0 - abx_0 - bc}{a^2 + b^2}$$

$N\left(\frac{b^2x_0-aby_0-ac}{a^2+b^2}; \frac{a^2y_0-abx_0-bc}{a^2+b^2}\right)$ nuqta bilan $M(x_0; y_0)$ nuqtagacha bo‘lgan masofa quyidagicha topamiz:

$$|MN| = \sqrt{\left(\frac{b^2x_0 - aby_0 - ac}{a^2 + b^2} - x_0\right)^2 + \left(\frac{a^2y_0 - abx_0 - bc}{a^2 + b^2} - y_0\right)^2} =$$

$$= \sqrt{-\left(\frac{a^2x_0 - aby_0 - ac}{a^2 + b^2}\right)^2 + \left(\frac{aby_0 - b^2y_0 - bc}{a^2 + b^2}\right)^2} =$$

$$= \sqrt{a^2\left(\frac{ax_0 + by_0 + c}{a^2 + b^2}\right)^2 + b^2\left(\frac{ax_0 + by_0 + c}{a^2 + b^2}\right)^2} =$$

$$= \sqrt{\frac{(a^2 + b^2)(ax_0 + by_0 + c)^2}{(a^2 + b^2)^2}} =$$

$$= \sqrt{\frac{(ax_0 + by_0 + c)^2}{a^2 + b^2}} = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$

$M(x_0; y_0)$ nuqtadan $ax + by + c = 0$ to‘g‘ri chiziqqacha bo‘lgan masofa

$$d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}} \tag{5.15}$$

formula orqali topiladi. Agar to‘g‘ri chiziqning normal tenglamasidan foydalansak, bu formula quyidagicha oson isbot qilinadi.

Berilgan $ax + by + c = 0$ to‘g‘ri chiziqni normal holga va $M(x_0; y_0)$ nuqtadan o‘tuvchi parallel bo‘lgan to‘g‘ri chiziq tenglamasini ham normal holga keltiramiz. Koordinata boshidan

$N(x;y)$ nuqtagacha bo‘lgan masofani $p_1$, $M(x_0;y_0)$ nuqtagacha bo‘lgan masofani esa $p_2$ bilan belgilab ayirib tashlasak yetarli bo‘ladi. Ya’ni,

$$\pm\frac{a}{\sqrt{a^2+b^2}}x \pm \frac{b}{\sqrt{a^2+b^2}}y \pm \frac{c}{\sqrt{a^2+b^2}} = 0$$

$$\pm\frac{a}{\sqrt{a^2+b^2}}x_0 \pm \frac{b}{\sqrt{a^2+b^2}}y_0 \pm c_1 = 0 \Rightarrow$$

$$c_1 = \mp\frac{a}{\sqrt{a^2+b^2}}x_0 \mp \frac{b}{\sqrt{a^2+b^2}}y_0 \Rightarrow$$

$$\pm\frac{a}{\sqrt{a^2+b^2}}x \pm \frac{b}{\sqrt{a^2+b^2}}y \mp \frac{a}{\sqrt{a^2+b^2}}x_0 \mp \frac{b}{\sqrt{a^2+b^2}}y_0 = 0$$

$$p_1 = \left|\mp\frac{c}{\sqrt{a^2+b^2}}\right|, \ p_2 = \left|\pm\frac{a}{\sqrt{a^2+b^2}}x_0 \pm \frac{b}{\sqrt{a^2+b^2}}y_0\right|$$

bo‘lsa, $d = |p_2 - p_1| = \left|\frac{\pm ax_0 \pm by_0 \pm c}{\sqrt{a^2+b^2}}\right| = \frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}$ ekanligidan (5.15) formula keltirib chiqariladi.

**12-Misol.** Koordinatalari $N(3;-2)$ bo‘lgan nuqtadan $y = 4x - 1$ to‘g‘ri chiziqqacha bo‘lgan masofani toping.

**Yechish.** Berilgan $y = 4x - 1$ to‘g‘ri chiziq tenglamasini umumiy $4x - y - 1 = 0$ ko‘rinishga keltirib, (5.15) formulaga qo‘yamiz.

$$\mathrm{d} = \frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}} = \frac{|4\cdot 3-(-2)-1|}{\sqrt{16+1}} = \frac{13}{\sqrt{17}} = \frac{13\sqrt{17}}{17}.$$

Berilgan nuqtadan to‘g‘ri chiziqqacha bo‘lgan masofa $\frac{13\sqrt{17}}{17}$ ga teng.

**5.4. Tekislikda to‘g‘ri chiziqlarga doir aralash masalalar. Uchburchakni uchlariga ko‘ra yuzini topish.**

Uchlari $A(x_1;y_1)$, $B(x_2;y_2)$ va $C(x_3;y_3)$ nuqtalarda bo‘lgan uchburchakning yuzini topamiz. Buning uchun “Uchburchakning yuzi uning biror tomoni va shu tomonga tushirilgan balandlik ko‘paytmasining yarmiga teng” deyilgan qoidadan foydalanamiz.

Demak, $S = \frac{1}{2}|BC|\cdot|h_{BC}|$,$BC$ tomon uzunligi

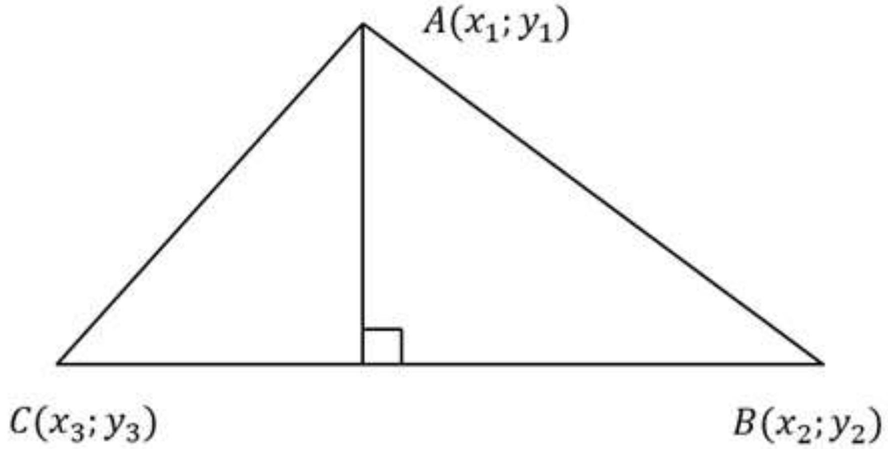
$$|BC| = \sqrt{(x_3-x_2)^2+(y_3-y_2)^2}$$

$h_{BC}$ ni topish uchun $A$ nuqtadan $BC$ to‘g‘ri chiziqqacha bo‘lgan masofani topish yetarli. Buning uchun

$$BC: \quad \frac{x_1 - x_2}{x_3 - x_2} = \frac{y_1 - y_2}{y_3 - y_2}$$

tomon tenglamasini umumiy holga keltiramiz.

$$x_1(y_3 - y_2) - y_1(x_3 - x_2) - x_2(y_3 - y_2) + y_2(x_3 - x_2) = 0$$




**5.4.1-chizma**

$A(x_1; y_1)$ nuqtadan $BC$ tomongacha bo‘lgan masofa esa

$$h_{BC} = \frac{|x_1(y_3 - y_2) - y_1(x_3 - x_2) - x_2(y_3 - y_2) + y_2(x_3 - x_2)|}{\sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}}$$

$$S = \frac{1}{2}\sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2} \cdot$$

$$\cdot \frac{|x_1(y_3 - y_2) - y_1(x_3 - x_2) - x_2(y_3 - y_2) + y_2(x_3 - x_2)|}{\sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}} =$$

$$= \frac{1}{2}|x_1y_3 - x_1y_2 - x_3y_1 + x_2y_1 - x_2y_3 + x_2y_2 + x_3y_2 - x_2y_2| =$$

$$= \frac{1}{2}|x_1y_3 - x_1y_2 - x_3y_1 + x_2y_1 - x_2y_3 + x_3y_2| = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

hosil bo‘ladi.

Demak,

$$S = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \tag{5.16}$$

uchburchakni uchlariga ko‘ra yuzini topish formulasini keltirib chiqardik.

**13-Misol.** Uchburchakning uchlari $A(4;2)$, $B(-3;3)$ va $C(-2;-5)$ berilgan bo‘lsa uning yuzini toping.

**Yechish.** Uchburchakni uchlariga ko‘ra yuzini topish formulasidan foydalanamiz.

$$S=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}=\frac{1}{2}\begin{vmatrix} 4 & 2 & 1 \\ -3 & 3 & 1 \\ -2 & -5 & 1 \end{vmatrix}=$$

$$=\frac{1}{2}\big(4\cdot 3+2\cdot(-2)+(-5)\cdot(-3)-3\cdot(-2)-4\cdot(-5)-2\cdot(-3)\big)$$

$$=\frac{1}{2}(12-4+15+6+20+6)=\frac{1}{2}\cdot 55=27\frac{1}{2}$$

Uchburchakni yuzi $27\frac{1}{2}$ ga teng.

## Mustaqil yechish uchun topshiriqlar.

### 5.1. To‘g‘ri chiziqning burchak koeffitsiyentli tenglamasi. To‘g‘ri chiziq orasidagi burchakga doir misollar.

**5.1.1.** To‘g‘ri chiziq tenglamasini burchak koeffitsiyentli tenglama ko‘rinishiga keltiring.

1) $3x+2y-9=0$; 2) $5x-7y+8=0$;

3) $2x-6y+11=0$; 4) $4x+9y-13=0$.

**5.1.2.** Dekart koordinatalar sistemasida quyidagi tenglamalar bilan berilgan to‘g‘ri chiziqlarni yasang:

1) $y=3x+4$; 2) $y=0{,}5x+2$; 3) $y=\frac{3}{4}x-5$;

4) $x+2=0$; 4) $3x+2y-9=0$; 6) $2y-5x+2=0$.

**5.1.3.** $N(2;3)$ nuqtadan o‘tib, burchak koeffitsiyenti $-5$ ga teng bo‘lgan to‘g‘ri chiziq tenglamasini tuzing.

**5.1.4.** Quyidagi to‘g‘ri chiziqlar qaysi koordinata choraklaridan o‘tadi.

1) $y=2x-5$; 2) $y=5x+7$;

3) $y=-3x+4$; 4) $y=-7x-3$.

**5.1.5.** Quyidagi

1) $2x+3y=0$, $x-y+5=0$;

2) $x - 3y + 2 = 0, \quad 2x + y = 0;$
3) $2x + 5y - 3 = 0, \quad 5x + 2y - 6 = 0;$
4) $3x + 4y - 12 = 0, \quad 5x - 12y + 60 = 0$
to‘g‘ri chiziqlar orasidagi burchakning tangensi topilsin.

**5.1.6.** Dekart koordinatalar sistemasida $N(3;-2)$ nuqtadan o‘tib, koordinata o‘qlariga parallel (perpendikulyar) bo‘lgan to‘g‘ri chiziqlar tenglamalari tuzilsin.

**5.1.7.** $A(2;3)$ va $B(-1;0)$ nuqtalar berilgan. $B$ nuqtadan o‘tuvchi va $AB$ kesmaga perpendikulyar bo‘lgan to‘g‘ri chiziq tenglamasini tuzing.

**5.1.8.** Uchburchakning $A(-1;2)$, $B(3;-1)$, $C(0;4)$ uchlaridan o‘tuvchi va ular qarshisida yotgan tomonlarga parallel (perpendikulyar) to‘g‘ri chiziqlarning burchak koeffitsiyentli tenglamalari tuzilsin.

**5.1.9.** $3x - y = 0$, $x + 4y - 2 = 0$ to‘g‘ri chiziqlarning kesishish nuqtasidan o‘tib, $2x + 7y = 0$ to‘g‘ri chiziqqa perpendikulyar bo‘lgan to‘g‘ri chiziq tenglamasi tuzilsin.

**5.1.10.** Dekart koordinatalar tekisligida $y = 3x - 4$ va $y = -2x + 3$ to‘g‘ri chiziqlar berilgan bo‘lsa, ular orasidagi burchakni toping.

**5.1.11.** $x - 3y + 1 = 0$ to‘g‘ri chiziqda $M(-3;1)$, $N(5;4)$ nuqtalardan teng uzoqlikda joylashgan nuqta topilsin.

**5.1.12.** Ikki to‘g‘ri chiziqning burchak koeffitsiyentlari $k_1 = \frac{1}{3}$, $k_2 = -\frac{1}{2}$ ma’lum, ular orasidagi burchakni toping.

**5.1.13.** Berilgan $K(3;1)$ nuqtadan o‘tib, $2x + 3y - 1 = 0$ to‘g‘ri chiziqqa $45^0$ burchak ostida og‘ishgan to‘g‘ri chiziqlar tenglamalari tuzilsin.

**5.1.14.** Koordinatalar boshidan o‘tib, $5x - 6y + 2 = 0$ to‘g‘ri chiziq bilan tashkil qilgan burchagining tangensi $\pm\frac{7}{6}$ ga teng bo‘lgan to‘g‘ri chiziqlar tenglamalari tuzilsin.

**5.1.15.** Ikkita $A(3;3)$, $B(0;2)$ nuqta berilgan. $x + y - 4 = 0$ to‘g‘ri chiziqdagi $AB$ kesma $45^0$ burchak ostida ko‘rinadigan nuqtani toping.

**5.1.16.** $m$ va $n$ ning qanday qiymatida ushbu ikki to‘g‘ri chiziq
$mx + 8y + n = 0, \qquad 2x + my - 1 = 0$

1) Parallel; 2) ustma-ust; 3) perpendikulyar bo‘ladi .

**5.1.17**. Ordinata o‘qiga parallel va $P(3;5)$ nuqtadan 7 birlik masofada joylashgan to‘g‘ri chiziqlarning tenglamasi tuzilsin.

**5.1.18.** $A(2;-5)$, $B(0;-3)$ nuqtalardan o‘tgan to‘g‘ri chiziq $Ox$ o‘qi bilan qanday burchak tashkil qiladi?

**5.1.19.** Berilgan ikki $C(1;4)$, $D(3;5)$ nuqtadan o‘tgan to‘g‘ri chiziq $Ox$ o‘qiga qanday burchak ostida og‘adi?

**5.1.20.** Koordinatalar boshidan o‘tib, absissa o‘qiga $150^0$ burchak ostida og‘ishgan to‘g‘ri chiziq tenglamasi tuzilsin.

**5.1.21.** Uchburchakning uchlari $A(5;-4)$, $B(-1;3)$, $C(-3;-2)$ nuqtalarda bo‘lsa, uning har bir uchidan o‘tib qarshisidagi tomonga parallel bo‘lgan to‘g‘ri chiziqning tenglamasini tuzing.

**5.1.22.** $x+y-2=0$; $5x+y-14=0$ to‘g‘ri chiziqlardan mos ravishda $A$, $B$ nuqtalar olingan bo‘lib, $AB$ to‘g‘ri chiziqning burchak koeffitsiyenti 3 ga teng va $AB$ kesmaning uzunligi $\sqrt{10}$ ga teng bo‘lsa, $A$ va $B$ nuqtalarning koordinatalari topilsin.

**5.1.23.** $y=4x+1$ va $y=x-3$ to‘g‘ri chiziqlar orasidagi burchak tangensini toping.

**5.1.24.** Berilgan 1) $A(2;1)$, 2) $B(-3;4)$, 3) $C(5;-2)$, 4) $D(-2;3)$ nuqtadan o‘tib $y=3x-2$ to‘g‘ri chiziqqa parallel bo‘lgan to‘g‘ri chiziq tenglamasini tuzing.

**5.1.25.** Berilgan 1) $M(-4;3)$, 2) $N(1;5)$, 3) $P(3;-4)$, 4) $Q(-3;-2)$ nuqtadan o‘tib, $y=-2x+5$ to‘g‘ri chiziqqa perpendikulyar bo‘lgan to‘g‘ri chiziq tenglamasini tuzing.

**5.1.26.** $B(x;y)$ nuqtadan o‘tib, koordinata o‘qlari bilan $S$ yuzali uchburchak hosil qiluvchi to‘g‘ri chiziq tenglamasini tuzing:

1) $B(5;-5)$, $S=50$ kv birlik;

2) $B(12;6)$, $S=150$ kv birlik;

3) $B(8;6)$, $S=12$ kv birlik.

**5.1.27.** $2x+5y=0$ to‘g‘ri chiziqqa parallel va koordinata o‘qlaridan yuzasi 5 ga teng bo‘lgan uchburchak ajratuvchi to‘g‘ri chiziqning umumiy tenglamasi tuzilsin.

**5.1.28.** Uchburchak tomonlari $2x + y - 7 = 0\ (AB)$, $3x - 4y - 5 = 0\ (BC)$, $5x - 3y - 1 = 0\ (CA)$, tenglamalar bilan berilgan. $A$ uchidagi ichki burchagining kosinusini toping.

**5.1.29.** Ikkita parallel $x - y + 5 = 0$, $x - y - 2 = 0$ to‘g‘ri chiziqlar orasidagi kesmasi 5 ga teng bo‘lgan va $N(2; -1)$ nuqtadan o‘tadigan to‘g‘ri chiziqning umumiy tenglamasini tuzing.

**5.1.30**. Uchburchak tomonlarining tenglamalari $x + 2y = 0$, $3x - y = 0$, $x + y - 1 = 0$ berilgan. Uchburchak ichki burchaklarining tangenslari topilsin.

### 5.2. To‘g‘ri chiziqning umumiy tenglamasi. Berilgan ikki nuqtadan o‘tuvchi tenglamasi. To‘g‘ri chiziqning koordinata o‘qlaridan ajratgan kesmalar bo‘yicha tenglamasiga doir misollar.

**5.2.1.** Quyidagi $K_1(3; 1)$, $K_2(2; 3)$, $K_3(6; 3)$, $K_4(-3; -3)$, $K_5(3; -1)$, $K_6(-2; 1)$ nuqtalardan qaysi biri $2x - 3y - 3 = 0$ to‘g‘ri chiziqqa tegishli va qaysilari tegishli emas.

**5.2.2.** $3x - 2y - 6 = 0$ to‘g‘ri chiziqqa tegishli nuqtalar $P_1, P_2, P_3, P_4$, va $P_5$; uning absissalari quyidagicha bo‘lsa: 4; 0; 2; $-2$ va $-6$. Bu nuqtalarning ordinatalarini toping.

**5.2.3.** $x - 3y + 2 = 0$ to‘g‘ri chiziqqa tegishli nuqtalar $Q_1, Q_2, Q_3, Q_4$, va $Q_5$ uning ordinatalari quyidagicha bo‘lsa: 4; 0; 2; $-2$ va $-6$. Bu nuqtalarning absissalarini toping.

**5.2.4.** To‘g‘ri chiziq tenglamasi $5x + 3y - 3 = 0$ berilgan:

1) to‘g‘ri chiziqqa parallel;

2) to‘g‘ri chiziqqa perpendikulyar bo‘lgan to‘g‘ri chiziq tenglamalarini tuzing.

**5.2.5.** $K_1(2; -3)$ nuqtadan o‘tib, quyidagi to‘g‘ri chiziqlarga parallel bo‘lgan to‘g‘ri chiziqning umumiy tenglamasini tuzing:

1) $3x - 7y + 3 = 0$; 2) $2x + 9y - 11 = 0$; 3) $3x - 7y + 3 = 0$;
4) $x + 9y - 11 = 0$; 5) $16x - 24y - 7 = 0$; 6) $2x + 3 = 0$.

**5.2.6.** $M(7; 4)$ nuqtadan o‘tuvchi va $3x - 2y + 4 = 0$ to‘g‘ri chiziqqa parallel (perpendikulyar) to‘g‘ri chiziqning umumiy tenglamasini tuzing.

**5.2.7.** $x+y+7=0$ to‘g‘ri chiziqqa parallel (perpendikulyar) va $M(-8;1)$ nuqtadan o‘tuvchi to‘g‘ri chiziqning umumiy tenglamasini tuzing.

**5.2.8.** $M(2;5)$ nuqtadan o‘tuvchi va $P(-1;2)$, $Q(5;4)$ nuqtalardan teng uzoqlikdagi to‘g‘ri chiziq tenglamasi tuzilsin.

**5.2.9.** Parallel bo‘lgan $x+y-1=0$, $x+y-13=0$ to‘g‘ri chiziqlardan teng uzoqlikda joylashgan va ularga parallel bo‘lgan to‘g‘ri chiziq tenglamasi tuzilsin.

**5.2.10.** $ABC$ uchburchakning $AB$, $BC$ va $AC$ tomonlarining tenglamasi berilgan: $4x+3y-5=0$, $x-3y+10=0$, $x-2=0$.
Uning uchlarining koordinatalarini toping.

**5.2.11.** $A(-3;1)$, $B(2;-3)$, $C(5;-4)$, $D(-2;4)$, $E(-1;3)$ va $F(-5;-1)$ nuqtalar berilgan bo‘lsin.

a) $A$ va $B$ dan o‘tuvchi to‘g‘ri chiziqning; b) $B$ va $C$ dan o‘tuvchi to‘g‘ri chiziqning; c) $A$ va $C$ dan o‘tuvchi to‘g‘ri chiziqning; d) $B$ va $E$ dan o‘tuvchi to‘g‘ri chiziqning; e) $C$ va $F$ dan o‘tuvchi to‘g‘ri chiziqning; f) $D$ va $F$ dan o‘tuvchi to‘g‘ri chiziqning

1) burchak koeffitsiyentli; 2) umumiy; 3) koordinata o‘qlaridan ajratgan kesmalar bo‘yicha tenglamalari tuzilsin.

**5.2.12.** Umumiy tenglama bilan berilgan ikki to‘g‘ri chiziq orasidagi burchakni toping:

1) $5x-y+7=0$, $3x+2y=0$; 3) $x-2y-4=0$, $2x-4y=0$;
2) $3x-2y+7=0$, $2x+3y-3=0$; 4) $3x+2y-1=0$, $5x-$
$-2y+3=0$.

**5.2.13.** $2x+3y+4=0$ to‘g‘ri chiziq berilgan. $M_1(2;1)$ nuqtadan o‘tib, berilgan to‘g‘ri chiziq bilan $45^0$ burchak hosil qiladigan to‘g‘ri chiziqning umumiy tenglamasini tuzing.

**5.2.14.** $x-4y-12=0$ tenglama va koordinata o‘qlari bilan chegaralangan uchburchakning yuzini toping .

**5.2.15.** Koordinatalar sistemasida berilgan $2x-y+5=0$ to‘g‘ri chiziq, boshi $N(5;4)$ nuqtada va oxiri $M(2;1)$ nuqtada joylashgan kesmani qanday nisbatda bo‘ladi?

**5.2.16.** To‘rtta $M_1(5;3)$, $M_2(1;2)$, $M_3(3;0)$, $M_4(2;4)$ nuqta berilgan. $M_1M_2$ va $M_3M_4$ to‘g‘ri chiziqlarning o‘zaro perpendikulyarligi va ularning kesishgan nuqtasi ham $M_1$ va $M_2$ ham $M_3$ va $M_4$ nuqtalar orasida yotishi isbotlansin.

**5.2.17.** Quyidagi hollarning har birida $M_1M_2$ kesmaning $l: 2x - y + +5 = 0$ to‘g‘ri chiziqqa nisbatan vaziyatini aniqlang:

1) $M_1(2;3)$, $M_2(0;-1)$; 2) $M_1(1;1)$, $M_2(3;5)$;

3) $M_1(4;3), M_2(-2;2)$; 4) $M_1(0;2)$, $M_2(5;0)$;

5) $M_1(-6;4)$, $M_2(-2;4)$

va natijalarni chizmada tekshiring.

**5.2.18.** Uchlari $A(3;1)$, $B(-2;4)$, $C(1;0)$ nuqtalardagi uchburchakning $x - 7y + 5 = 0$ to‘g‘ri chiziqqa nisbatan vaziyatini aniqlang.

**5.2.19.** Koordinatalar boshidan va ikki $2x + y - 3 = 0$, $7x - 4y + +2 = 0$ to‘g‘ri chiziqning kesishgan nuqtasi orqali o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.

**5.2.20.** $7x - y + 3 = 0$, $3x + 5y - 4 = 0$ to‘g‘ri chiziqlarning kesishgan nuqtasi va $A(2;-1)$ nuqtadan o‘tadigan to‘g‘ri chiziq tenglamasini tuzing.

**5.2.21.** $3x - 5y + 2 = 0$, $5x - 2y + 4 = 0$ to‘g‘ri chiziqlarning kesishgan nuqtasidan o‘tadigan va $2x - y + 4 = 0$ to‘g‘ri chiziqqa parallel to‘g‘ri chiziqning umumiy tenglamasini tuzing.

**5.2.22.** Koordinatalar sistemasida $2x - 6y + 3 = 0$, $5x + y - 2 = 0$ to‘g‘ri chiziqlarning kesishish nuqtasidan o‘tuvchi va koordinata o‘qlariga parallel bo‘lgan to‘g‘ri chiziqning umumiy tenglamasini tuzing.

**5.2.23.** $x + y - 6 = 0$, $2x + y - 13 = 0$ to‘g‘ri chiziqlarning kesishgan nuqtasidan o‘tib, koordinata o‘qlaridan teng kesmalar ajratadigan to‘g‘ri chiziqning umumiy tenglamasini tuzing.

**5.2.24.** Ikki juft $2x - y = 0$, $x + 4y - 2 = 0$ va $x + 2y = 0$, $3x - -7y + 4 = 0$ to‘g‘ri chiziqlarning kesishgan nuqtalaridan o‘tadigan to‘g‘ri chiziqning umumiy tenglamasini tuzing.

**5.2.25.** Ikki $3x - y = 0$, $x + 4y - 2 = 0$ to‘g‘ri chiziqlarning kesishgan nuqtasidan o‘tuvchi va $2x + 7y = 0$ to‘g‘ri chiziqqa perpendikulyar bo‘lgan to‘g‘ri chiziqning umumiy tenglamasini tuzing.
**5.2.26.** Koordinatalar sistemasida $Ox$ o‘qida 3 ga teng kesma ajratgan va $M(-5; 3)$ nuqtadan o‘tadigan to‘g‘ri chiziqning umumiy tenglamasini tuzing.
**5.2.27.** Koordinatalar sistemasida koordinata o‘qlaridan 3 va 5 ga teng kesmalar ajratgan to‘g‘ri chiziqning umumiy tenglamasini tuzing.
**5.2.28.** Koordinata o‘qlaridan teng kesmalar ajratib, $M(-4; 10)$ nuqta orqali o‘tgan to‘g‘ri chiziqning umumiy tenglamasini tuzing.
**5.2.29.** Koordinatalar sistemasida $C(2; -1)$ nuqtadan o‘tgan, koordinata o‘qlari orasidagi kesmasi shu nuqtada teng ikkiga bo‘linadigan to‘g‘ri chiziqning tenglamasi tuzilsin.
**5.2.30.** Dekart koordinatalar sistemasida koordinata o‘qlari va $x +$ $+2y - 6 = 0$ to‘g‘ri chiziq bilan chegaralangan uchburchak yuzasi topilsin.

## 5.3. To‘g‘ri chiziqning normal tenglamasi. To‘g‘ri chiziq tenglamasini normal holda keltirish. Berilgan nuqtadan to‘g‘ri chiziqqacha bo‘lgan masofaga doir misollar.

**5.3.1.** Berilgan:
1) $5x + 12y - 7 = 0$; 2) $4x - 3y + 9 = 0$;
3) $\frac{x}{6} - \frac{y}{8} = 1$; 4) $\frac{x}{12} + \frac{y}{5} = 1$
to‘g‘ri chiziqlarni normal holga keltiring.
**5.3.2.** Koordinatalari $M(3; -4)$ bo‘lgan nuqtadan $y = 3x + 10$ to‘g‘ri chiziqqacha bo‘lgan masofani toping.
**5.3.3.** $A(3; 1)$, $B(2; -4)$, $C(5; -1)$, $D(0; -3)$ va $O(0; 0)$ nuqtalardan $3x + 4y = 0$ to‘g‘ri chiziqqacha bo‘lgan masofalar topilsin.
**5.3.4.** $M(1; 0)$, $N(-1; 2)$ nuqtalardan $3x - y + 4 = 0$ to‘g‘ri chiziqqacha bo‘lgan masofalar topilsin.
**5.3.5.** $A(2; 1)$, $B(-3; 4)$, $C(5; -2)$ va $D(-2; 3)$ nuqtalardan:
1) $3x - y + 9 = 0$; 2) $2x + 4y - 7 = 0$;
3) $7x + 3y - 1 = 0$; 4) $5x - 2y + 4 = 0$

to‘g‘ri chiziqlargacha bo‘lgan masofani toping.

**5.3.6.** Quyidagi berilgan:

1) $3x-4y+8=0, \quad 3x-4y-19=0;$

2) $6x+y-11=0, \;\; 6x+y+1=0;$

3) $x-7y+3=0, \quad x-7y-13=0$

Parallel to‘g‘ri chiziqlar orasidagi masofani toping.

**5.3.7.** Uchburchak tomonlarining tenglamalari: $3x-4y-3=0$, $5x+12y+2=0$, $3x+4y+390=0$ berilgan. Uchburchak balandliklarining uzunliklari topilsin.

**5.3.8.** $5x+12y-1=0$ to‘g‘ri chiziqqa parallel va undan 5 birlik masofada joylashgan to‘g‘ri chiziq tenglamasi tuzilsin.

**5.3.9.** $7x-2y+4=0$ to‘g‘ri chiziqqa parallel va undan $\sqrt{53}$ birlik masofadan o‘tuvchi to‘g‘ri chiziq tenglamasi tuzilsin.

**5.3.10.** Ushbu ikki $3x-7y+2=0$, $3x-7y+3=0$ to‘g‘ri chiziqning parallelligi isbotlansin va ular orasidagi $d$ masofa topilsin.

**5.3.11.** Ikki $4x-3y+20=0$, $3x+4y-60=0$ to‘g‘ri chiziqning har biridan 5 birlik masofada joylashgan nuqtani toping.

**5.3.12.** $P(-8;12)$ nuqtaning $A(2;-3)$ va $B(-5;1)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqdagi proyeksiyasini toping .

**5.3.13.** Ikki $3x+y+10=0$, $4x+5y+6=0$ to‘g‘ri chiziqlarning kesishgan nuqtasidan o‘tuvchi va koordinatalar boshidan 4 birlik masofada joylashgan to‘g‘ri chiziq tenglamasi tuzilsin.

**5.3.14.** Burchak koeffitsiyenti $k=-\frac{1}{2}$ bo‘lgan va koordinata boshidan $\sqrt{5}$ birlik masofada joylashgan to‘g‘ri chiziq tenglamasi tuzilsin.

**5.3.15.** Koordinata boshidan o‘tib, $M(3;-2)$ nuqtadan 1 birlik masofada joylashgan to‘g‘ri chiziq tenglamasi tuzilsin.

**5.3.16.** $K(3;-1)$ nuqtadan o‘tib, $E(2;-3)$ nuqtadan $\frac{9}{\sqrt{17}}$ birlik masofada joylashgan to‘g‘ri chiziq tenglamasi tuzilsin.

**5.3.17.** $\vec{a}(-4;2)$ vektorga parallel, $M(3;-5)$ nuqtadan o‘tuvchi to‘g‘ri chiziqning parametrik tenglamalari yozilsin.

**5.3.18.** Dekart koordinatalar sistemasida $M(-6;-4)$ nuqtadan o‘tgan va burchak koeffitsiyenti $-\frac{3}{7}$ ga teng bo‘lgan to‘g‘ri chiziqning parametrik tenglamalari tuzilsin.
**5.3.19.** Dekart koordinatalar sistemasida $Ox$, $Oy$ o‘qlarda mos ravishda 3 va $-5$ ga teng kesmalar ajratgan to‘g‘ri chiziqning parametrik tenglamalari tuzilsin.
**5.3.20.** $M(2;3)$, $N(-1;4)$ nuqtalargacha bo‘lgan masofalar yig‘indisi 8 ga teng bo‘ladigan $7x+3y-14=0$ to‘g‘ri chiziq nuqtalarini toping.
**5.3.21.** $M_2(8;-9)$ nuqtaning $A(3;-4)$ va $B(-1;-2)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqqa nisbatan simmetrik nuqtasi $M_1$ ni toping .
**5.3.22.** Uchburchakning $A(4;6)$, $B(-4;0)$, $C(-1;-4)$ uchlari berilgan. Uning $A$ uchidan $BC$ tomoniga tushirilgan balandlik tenglamasini tuzing.
**5.3.23.** $N(-5;6)$ nuqtaning $7x-13y-105=0$ to‘g‘ri chiziqdagi proyeksiyasi topilsin.
**5.3.24.** $M(-2;9)$ nuqtaga $2x-3y+18=0$ to‘g‘ri chiziqqa nisbatan simmetrik bo‘lgan nuqta topilsin.
**5.3.25.** Koordinatalar sistemasida uchburchak tomonlarining tenglamalari $3x-y+4=0$, $2x-y+1=0$, $x-2y=0$ bo‘lsa, $2x-y+3=0$ to‘g‘ri chiziqning uchburchakka nisbatan vaziyatini aniqlang.
**5.3.26.** $M(4;1)$, $N(8;-3)$ nuqtalardan va $5x+12y=0$ to‘g‘ri chiziqdan teng uzoqlikda joylashgan nuqtani toping.
**5.3.27.** Uchburchakning bir uchi $B(2;-7)$ va boshqa-boshqa uchlaridan chiquvchi balandligi tenglamasi $3x+y+11=0$ hamda medianasi tenglamasi $x+2y+7=0$ berilgan bo‘lsa, uning tomonlarining umumiy tenglamalarini tuzing.
**5.3.28.** Uchburchakning $A(2;6)$, $B(5;-2)$ va $C(-1;-2)$ uchlari berilgan, uning balandliklari uzuniklarini toping.
**5.3.29.** Koordinata boshidan to‘g‘ri chiziqqacha bo‘lgan masofa 3 ga va absissa o‘qining musbat yo‘nalishi bilan $45^0$ burchak tashkil qiladi.

Berilganlardan foydalanib, to‘g‘ri chiziqning normal tenglamasini tuzing.

**5.3.30.** Ikkita parallel $x + y - 3 = 0,\ x + y - 10 = 0$ to‘g‘ri chiziqlar orasidagi kesmasi 5 ga teng bo‘lgan va $M(2;5)$, $N(3;1)$ nuqtalardan bir xil uzoqlikdagi to‘g‘ri chiziqning umumiy tenglamasini tuzing.

## 5.4. Tekislikda to‘g‘ri chiziqlarga doir aralash masalalar.

**5.4.1.** Uchburchakning tomonlari tenglamalari berilgan:
$x + 5y - 7 = 0,\ 3x - 2y - 4 = 0$ va $7x + y + 19 = 0$. Uning yuzini toping.

**5.4.2.** Uchburchakning yuzi $S = 8$ kv bir., uning ikki $A(1;-2)$ va $B(2;3)$ uchi koordinatalari berilgan bo‘lib, uchinchi uchi $2x + y = 0$ to‘g‘ri chiziqda yotgan bo‘lsa, $C$ uchining koordinatasini toping .

**5.4.3.** Uchburchakning yuzi $S = 15$ kv birlik, uning ikki uchi $A(2;-3)$ va $B(3;2)$ nuqta; uchburchakning og‘irlik markazi $3x - y -$ $-8 = 0$ to‘g‘ri chiziqda yotadi. Uchinchi uchi $C$ ning koordinatasini toping.

**5.4.4.** Uchburchakning $A(-6;2)$, $B(2;-2)$ uchlari va balandliklarining kesishish nuqtasi $H(1;2)$ berilgan. Uchinchi $C$ uchining koordinatalari topilsin.

**5.4.5.** Uchburchakning bitta $A(3;-4)$ uchi va ikkita balandliklarinining tenglamalari $7x - 2y - 1 = 0$, $2x - 7y - 6 = 0$ berilgan. Uchburchak tomonlarining tenglamalarini tuzing.

**5.4.6.** Koordinatalar sistemasida $N_1(0;0)$, $N_2(2;1)$, $N_3(-3;1)$, $N_4(3;-1)$, $N_5(4;2)$, $N_6(-1;1)$, $N_7(-6;4)$, $N_8(1;-1)$ nuqtalarning $2x + 3y = 0$ to‘g‘ri chiziqqa nisbatan vaziyati aniqlansin.

**5.4.7.** Parallelogramning ikki tomon tenglamasi: $8x + 3y + 1 = 0$, $2x + y - 1 = 0$ va uning bir diagonalining tenglamasi $3x + 2y + 3 =$ $= 0$ berilgan. Parallelogramning uchlarining koordinatalarini toping .

**5.4.8.** To‘g‘ri to‘rtburchakning ikki tomon tenglamasi berilgan:
$2x - 3y + 5 = 0$, $3x + 2y - 7 = 0$ va uning $A(2;-3)$ uchi berilgan. To‘g‘ri to‘rtburchakning qolgan ikki tomonining tenglamasini tuzing .

**5.4.9.** To‘g‘ri to‘rtburchakning ikki tomon to‘g‘ri chiziq tenglamasi $x - 2y = 0$, $x - 2y + 15 = 0$ va uning bir diogonalining tenglamasi ham $7x + y - 15 = 0$ berilgan. To‘g‘ri to‘rtburchakning uchlarining koordinatasini toping.
**5.4.10.** To‘g‘ri to‘rtburchakni ikki tomoni $5x + y - 7 = 0,\ 5x + 2y - -36 = 0$ bo‘lgan to‘g‘ri chiziq tenglamasi va uning diagonalining $3x + 7y - 10 = 0$ tenglamasi berilgan. To‘rtburchakning qolgan ikki tomoni va ikkinchi diagonalining tenglamasini tuzing.
**5.4.11.** $3x - y - 1 = 0$ to‘g‘ri chiziqdan shunday $M$ nuqta topingki, $A(4;1)$ va $B(0;4)$ nuqtalargacha bo‘lgan masofalar ayirmasi eng katta bo‘lsin.
**5.4.12.** Uchburchakning $M_1(2;1)$, $M_2(-1;-1)$ va $M_3(3;2)$ uchlari berilgan bo‘lsa, uning balandliklari tenglamasini tuzing .
**5.4.13.** Uchburchakning tomonlarining tenglamasi berilgan bo‘lsa, $4x - y - 7 = 0,\quad x + 3y - 31 = 0,\quad x + 5y - 7 = 0.$ Uning balandliklari kesishgan nuqtasini toping.
**5.4.14.** $m$ ning qanday qiymatida ushbu ikki to‘g‘ri chiziq

$$(m-1)x + my - 5 = 0,\ \ mx + (2m-1)y + 7 = 0$$

kesishgan nuqtasi absissa o‘qida yotadi.
**5.4.15.** Koordinatalar sistemasida ikkita $A(-3;1)$, $B(5;4)$ nuqta va $x - 2y = 0$ to‘g‘ri chiziq berilgan. Bu to‘g‘ri chiziq $AB$ kesma bilan $B$ uchi davomida kesishganligi isbotlansin.
**5.4.16.** Koordinatalar sistemasida ushbu $5x - y - 5 = 0$ to‘g‘ri chiziq $3x - 2y - 6 = 0$ to‘g‘ri chiziqning koordinata o‘qlari orasida joylashgan kesmasi bilan kesishishini isbotlang.
**5.4.17.** Kvadratning qarama-qarshi $A(-1;3)$ va $C(6;2)$ uchlari berilgan. Uning tomonlari tenglamalarini tuzing.
**5.4.18.** Ikkita parallel $2x - 5y + 6 = 0$, $2x - 5y - 7 = 0$ to‘g‘ri chiziq tekislikni uch sohaga ajratadi; bu to‘g‘ri chiziqlar orasidagi sohaga va undan tashqaridagi ikki sohaga bo‘ladi. $A(2;1)$, $B(3;2)$, $C(1;1)$, $D(2;8)$, $E(7;1)$ va $F(-4;6)$ nuqtalarning qaysi sohaga tegishliligini aniqlang.

**5.4.19.** Uchburchak tomonlarining tenglamalari: $2x - y + 2 = 0$, $2x + y = 0$, $x + y - 4 = 0$. Bu uchburchakka nisbatan $A(3; 1)$, $B(7; -6)$, $C(-1; 1)$, $D(3; 2)$ nuqtalarning vaziyatini aniqlang.

**5.4.20.** Quyidagi to‘g‘ri chiziqlardan tuzilgan burchak bissektrisalarining tenglamalari tuzilsin:

1) $3x - y + 5 = 0$, $3x + y - 4 = 0$;

2) $3x - 4y + 2 = 0$, $5x + 12y - 3 = 0$;

3) $x - y = 0$, $x + y = 0$;

4) $x + 2y = 0$, $3x + 4y = 0$.

**5.4.21.** $x + y - 12 = 0$ to‘g‘ri chiziqda $x + y - 5 = 0$, $7x - y + 11 = 0$ to‘g‘ri chiziqlardan teng uzoqlikda joylashgan nuqtalar topilsin.

**5.4.22.** $C(1; 1)$, $D(2; 3)$ nuqtalardan mos ravishda 2 va 4 birlik masofada joylashgan to‘g‘ri chiziq tenglamasi tuzilsin.

**5.4.23.** Kvadratning simmetriya markazi $(-1; 0)$ nuqtada joylashgan: uning bir tomonining tenglamasi: $x + 3y - 5 = 0$ qolgan uchta tomonining tenglamalari tuzilsin.

**5.4.24.** Koordinata o‘qlari va $3x - 4y - 5 = 0$ to‘g‘ri chiziq bilan chegaralangan uchburchakka ichki chizilgan doiraning markazi topilsin.

**5.4.25.** Uchlari $A(4; 4)$, $B(-6; -1)$, $C(-2; -4)$ nuqtalardagi uchburchak berilgan. Uchburchakning $C$ uchidagi ichki burchak bissektrisasi tenglamasi tuzilsin.

**5.4.26.** Teng yonli trapetsiya asoslari mos ravishda $10$ va $6$ ga teng, yon tomonlari esa asosi bilan $60^0$ li burchak hosil qiladi. $Ox$ o‘q sifatida katta asos, $Oy$ o‘q sifatida trapetsiyaning simmetriya o‘qi olinib, $Oy$ o‘qining musbat yo‘nalishi kichik asos bilan kesishadigan nur yo‘nalishida bo‘lsa, trapetsiya tomonlarining tenglamalari tuzilsin.

**5.4.27.** Tomonlari: $2x + 3y - 13 = 0$, $x + 2y - 7 = 0$, $x + y - 5 = 0$ tenglamalar bilan berilgan uchburchakning yuzi topilsin.

**5.4.28.** $A(3;5)$, $B(-1;-2)$ nuqtalar berilgan. $ABC$ uchburchakning yuzi 1 ga teng bo‘ladigan $7x-6y+1=0$ to‘g‘ri chiziqdagi $C$ nuqtani toping.
**5.4.29.** $A(3;0)$, $B(-1;-2)$, $C(-3;1)$ va $D(7;2)$ nuqtalar berilgan. $5x-2y-95=0$ to‘g‘ri chiziqda, $MAB$ va $MCD$ uchburchaklar tengdosh (yuzlari teng) bo‘ladigan $M$ nuqtani toping.
**5.4.30.** Uchburchakning uchlari $A(0;7)$, $B(-2;3)$, uchinchi uchi $x-$ $-7=0$ to‘g‘ri chiziqda yotadi, yuzi 3 ga teng. Tomonlarining tenglamalarini tuzing.

# 6-MAVZU: FAZODA TEKISLIK VA TO‘G‘RI CHIZIQ

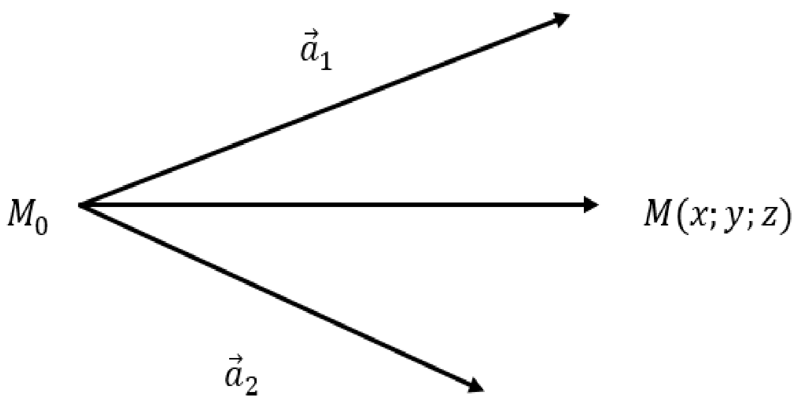
**Reja:**
**1. Fazoda tekislikning ba’zi tenglamalari.**
**2. Fazoda to‘g‘ri chiziq tenglamalari.**
**3. Fazoda tekislik va to‘g‘ri chiziq orasidagi munosabatlar.**

**Tayanch iboralar:** fazo, parallellik, perpendukulyarlik, kanonik, parametrik, normal.

## 6.1. Fazoda tekislikning ba’zi tenglamalari.

**Ikkita kollinear bo‘lmagan vektorlar va berilgan nuqtadan o‘tuvchi tekislik tenglamasi.**

Fazoda $\vec{a}_1(l_1;m_1;n_1)$ va $\vec{a}_2(l_2;m_2;n_2)$ kollinear bo‘lmagan vektorlar va $M_0(x_0;y_0;z_0)$ nuqta berilgan bo‘lsin. $\vec{a}_1$ va $\vec{a}_2$ vektorlardan hamda $M_0$ nuqtadan o‘tuvchi tekislik tenglamasini tuzaylik. Buning uchun $\vec{a}_1$ va $\vec{a}_2$ vektorlar boshini $M_0$ nuqtaga keltirib qo‘yamiz.




**6.1.1-chizma**

tuzmoqchi bo'lgan tekisligimizdan ixtiyoriy $M(x;y;z)$ nuqtani olamiz. Uchinchi $\overrightarrow{M_0M} = (x - x_0;\ y - y_0; z - z_0)$ vektorni tuzamiz.

$\vec{a}_1$, $\vec{a}_2$ va $\overrightarrow{M_0M}$ vektorlar komplanarligidan ularning aralash ko'paytmasi nolga teng, ya'ni

$$\overrightarrow{M_1M} \cdot \vec{a}_1 \cdot \vec{a}_2 = 0 \qquad (6.1)$$

bo'lishi kerak. Bundan

$$\begin{vmatrix} x - x_0 & y - y_0 & z - z_0 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

kelib chiqadi.

**1-Misol.** $\vec{a}(-3;2;1)$, $\vec{b}(2;3;-2)$ vektorlardan va $M_0(2;1;3)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing.

**Yechish:** Yuqorida berilgan $(6.1)$ formuladan foydalanib,

$$\begin{vmatrix} x - 2 & y - 1 & z - 3 \\ -3 & 2 & 1 \\ 2 & 3 & -2 \end{vmatrix} = 0$$

$$-4(x-2) - 9(z-3) + 2(y-1) - 4(z-3) -$$

$$-3(x-2) - 6(y-1) = 0$$

$$-7(x-2) - 4(y-1) - 13(z-3) = 0$$

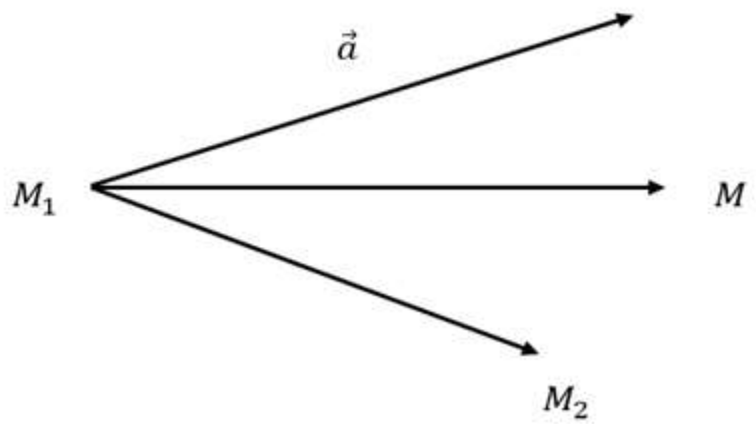
$$-7x + 14 - 4y + 4 - 13z + 39 = 0$$

$$7x + 4y + 13z - 57 = 0$$

to'g'ri chiziq tenglamasi $7x + 4y + 13z - 57 = 0$ ko'rinishida bo'ladi.

**Fazoda berilgan vektordan va berilgan ikki nuqtadan o'tuvchi tekislik tenglamasi.**

Fazoda $\vec{a}(l;m;n)$ koordinatali vektor va $M_1(x_1;y_1;z_1)$ va $M_2(x_2;y_2;z_2)$ nuqtalar berilgan bo'lsin. $\vec{a}$ vektordan hamda $M_1$ va $M_2$ nuqtalardan o'tuvchi $\alpha$ tekislik tenglamasini tuzamiz. Buning uchun $\vec{a}$ vektorning boshini $M_1(x_1;y_1;z_1)$ nuqtaga keltirib qo'yamiz. Tuzmoqchi bo'lgan tekisligimizdan ixtiyoriy $M(x;y;z)$ nuqtani olib, $\overrightarrow{M_1M}$ va $\overrightarrow{M_1M_2}$ vektorlarni yasaymiz.




**6.1.2-chizma**

$$\overrightarrow{M_1M} = (x - x_1;\ y - y_1; z - z_1),$$
$$\overrightarrow{M_1M_2} = (x_2 - x_1;\ y_2 - y_1; z_2 - z_1),$$
$$\vec{a}(l;m;n).$$

$\overrightarrow{M_1M}$ , $\overrightarrow{M_1M_2}$ va $\vec{a}$ vektorlar bir tekislikda yotishidan ularning aralash ko'paytmasi 0 ga teng bo'ladi.

$$\overrightarrow{M_1M} \cdot \overrightarrow{M_1M_2} \cdot \vec{a} = 0 \qquad (6.2)$$

bo'lishi kerak. Bundan

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l & m & n \end{vmatrix} = 0$$

kelib chiqadi.

**2-Misol.** $\vec{a}(2;3;-1)$ vektor $M_1(-2;5;4)$ va $M_2(0;0;0)$ nuqtadan o'tuvchi tekislik tenglamasini tuzing.

**Yechish:** Berilgan vektor va ikki nuqtadan o'tuvchi tekislik tenglamasi

$$\overrightarrow{M_1M}\cdot\overrightarrow{M_1M_2}\cdot\vec{a}=0$$

$$\begin{vmatrix} x+2 & y-5 & z-4 \\ 2 & -5 & -4 \\ 2 & 3 & -1 \end{vmatrix}=0$$

$$5(x+2)+6(z-4)-8(y-5)+10(z-4)+12(x+2)+$$
$$+2(y-5)=0$$
$$17(x+2)-6(y-5)+16(z-4)=0$$
$$17x+34-6y+30+16z-64=0$$
$$17x-6y+16z=0$$

$17x-6y+16z=0$ ko‘rinishida bo‘ladi.

**Fazoda berilgan uch nuqtadan o‘tuvchi tekislik tenglamasi.**

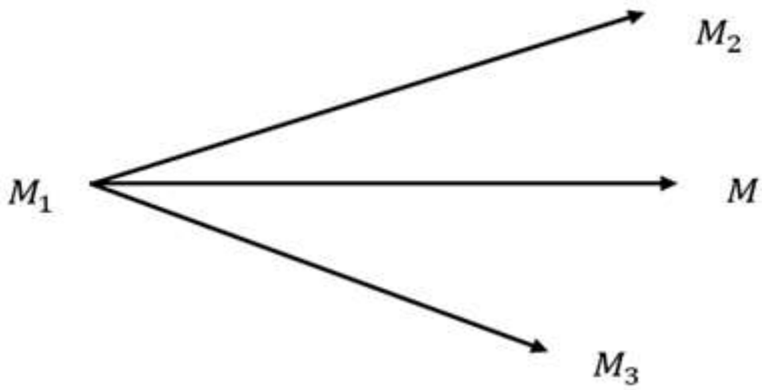
Bizga $M_1(x_1;y_1;z_1)$, $M_2(x_2;y_2;z_2)$ va $M_3(x_3;y_3;z_3)$ nuqtalar berilgan bo‘lsin. Bu nuqtalardan o‘tuvchi tekislik tenglamasini tuzish uchun tekislikdan yana bir $M(x;y;z)$ nuqta olamiz va $\overrightarrow{M_1M}$, $\overrightarrow{M_1M_2}$ va $\overrightarrow{M_1M_3}$ vektorlarni yasaymiz.

Bu $\overrightarrow{M_1M}=(x-x_1;y-y_1;z-z_1)$,

$$\overrightarrow{M_1M_3}=(x_3-x_1;\ y_3-y_1;z_3-z_1)$$

va $\overrightarrow{M_1M_2}=(x_2-x_1;\ y_2-y_1;z_2-z_1)$ vektorlar bitta tekislikda yotadi. Bundan kelib chiqadiki, aralash ko‘paytmasi 0 ga teng bo‘lishi kerak.

$$\overrightarrow{M_1M}\cdot\ \overrightarrow{M_1M_2}\cdot\ \overrightarrow{M_1M_3}=0 \qquad (6.3)$$




**6.1.3-chizma**

hamda,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

ko‘rinishida bo‘ladi, bu tenglamaning

$$\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0$$

tenglamaga teng kuchli ekanligini ko‘rish murakkab emas.

**3-Misol.** $M_1(2;3;-2)$ va $M_2(5;6;7)$ va $M_3(1;-2;1)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.

**Yechish:** Berilgan uch nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi

$$\begin{vmatrix} x-2 & y-3 & z+2 \\ 3 & 3 & 9 \\ -1 & -5 & 3 \end{vmatrix} = 0$$

$$9(x-2)-15(z+2)-9(y-3)+3(z+2)+$$

$$+45(x-2)-9(y-3)=0$$

$$54(x-2)-18(y-3)-12(z+2)=0$$

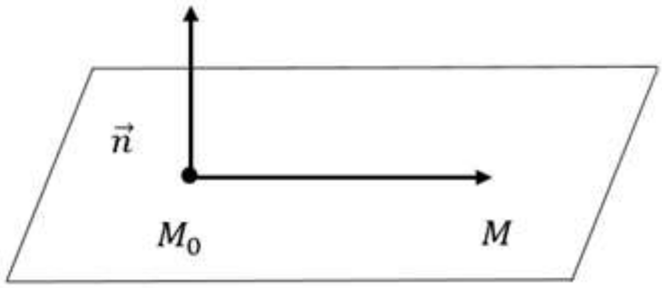
$$54x-108-18y+54-12z-24=0$$

$$54x-18y-12z-78=0$$

tenglikni ikkala tomonini 6 ga bo‘lib yuborsak, $9x-3y-2z-13=$ $=0$ ko‘rinishida bo‘ladi.

**Berilgan nuqtadan o‘tuvchi berilgan vektorga perpendikulyar bo‘lgan tekislik tenglamasi.**

Fazoda koordinatalari $M_0(x_0;y_0;z_0)$ nuqta va $\vec{n}(A;B;C)$ vektor berilgan bo‘lsin. $M_0$ nuqtadan o‘tib, $\vec{n}$ vektorga perpendikulyar bo‘lgan tekislik tenglamasini tuzamiz. Buning uchun tekislikdan ixtiyoriy $M(x;y;z)$ nuqtani olamiz. $\vec{n}$ vektorning boshini $M_0(x_0;y_0;z_0)$ nuqtaga keltirib qo‘yamiz.




**6.1.4-chizma**

$\overrightarrow{M_0M} \perp \vec{n}$ ekanligidan, $\overrightarrow{M_0M} \cdot \vec{n} = 0$ kelib chiqadi hamda ushbu ko‘rinishda

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$
$$Ax + By + Cz - (Ax_0 + By_0 + Cz_0) = 0$$

bo‘ladi. Tekislikga perpendikulyar bo‘lgan $\vec{n}$ vektor tekislikning ***normal vektori*** deyiladi.

**Tekislikning normal tenglamasi**

Faraz qilaylik, fazoda tekislikning koordinata o‘qlari bilan uchrashgan nuqtalari $A$, $B$ va $C$ bo‘lsin. Tekislikning koordinata o‘qlariga nisbatan o‘rni aniq bo‘lishi uchun koordinatalar boshidan unga perpendikulyar qilib tushirilgan $OP$ ning uzunligi va uning koordinata o‘qlari bilan tashkil qilgan burchaklari ma’lum bo‘lsa kifoya qiladi. Faraz qilaylik, $OP = p$ va $OP$ ning $Ox$, $Oy$, $Oz$ o‘qlarining musbat yo‘nalishi bilan tashkil qilgan burchaklari tartib bilan $\alpha, \beta, \gamma$ bo‘lsin.

Tekislikdagi biror $M$ nuqtaning koordinatalari: $x = OR,\ y = RN$, $z = MN$ bo‘lsin. $M$ va $P$ nuqtalarni o‘zaro tutashtirish natijasida $ORNMP$ siniq chiziq hosil bo‘ladi va bu siniq chiziqning tutashtiruvchisi $OP$ bo‘ladi. Siniq chiziqning o‘qdagi proyeksiyasi to‘g‘risidagi teorema bo‘yicha haligi siniq chiziqning $OP$ ga proyeksiyasi quyidagicha bo‘ladi:

$$ORcos\alpha + NRcos\beta + NMcos\gamma + MPcos\frac{\pi}{2} = p,$$

yoki shaklga asosan

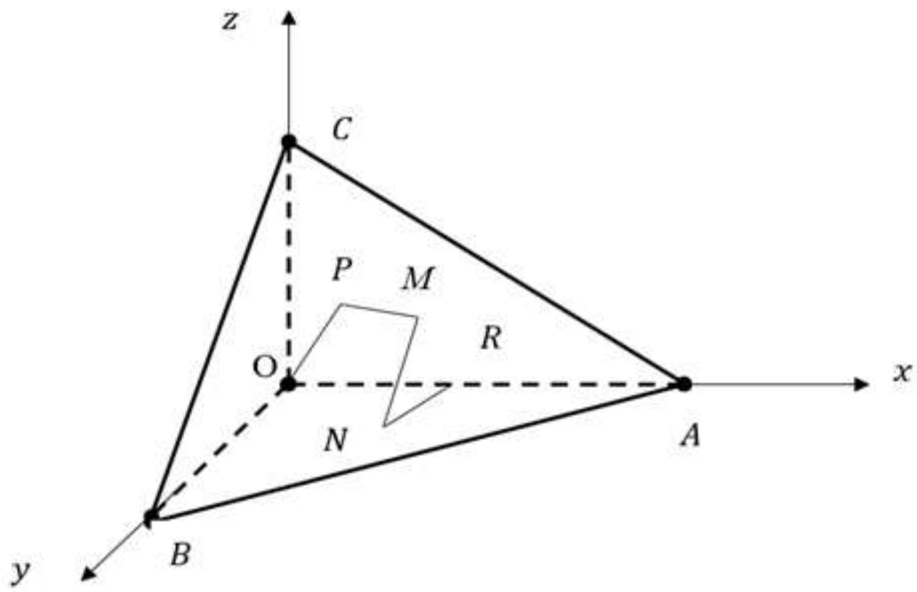
$$OR = x, \qquad NR = y, \qquad NM = z, \qquad cos\frac{\pi}{2} = 0$$

bo‘lgani uchun tekislikning tenglamasi quyidagicha bo‘ladi:

$$xcos\alpha + ycos\beta + zcos\gamma - p = 0 \qquad (6.4)$$

chunki $M$ nuqta tekislikning qaysi yerida bo‘lsada bu tenglama o‘z kuchini saqlaydi.

Bu tenglama tekislikning ***normal tenglamasi*** deyiladi. Bu tenglama $x, y, z$ ga nisbatan birinchi darajali. Demak: har bir tekislik o‘zgaruvchi $x, y, z$ koordinatalarga nisbatan birinchi darajali tenglama bilan ifoda qilinadi.




**6.1.5-chizma**

**4-Misol.** Tekislikning $2x - y + 2z - 5 = 0$ umumiy tenglamasidan normal tenglamasiga o‘ting.

**Yechish:** Normallashtiruvchi $\mu$ ko‘paytuvchini topamiz va berilgan umumiy tenglamani unga ko‘paytirib, normal tenglamani topamiz:

$$\mu = \frac{1}{\sqrt{2^2 + (-1)^2 + 2^2}} = \frac{1}{3} \Rightarrow \frac{2}{3}x - \frac{1}{3}y + \frac{2}{3}z - \frac{5}{3} = 0.$$

Bunda ozod had $D = -5 < 0$ bo‘lgani uchun $\mu$ ishorasi musbat qilib olindi va normal tenglamada

$$cos\alpha = \frac{2}{3}, \qquad cos\beta = -\frac{1}{3}, \qquad cos\gamma = \frac{2}{3}, \quad p = \frac{5}{3}$$

bo‘ladi.

**Tekislikning umumiy tenglamasi.**

Ixtiyoriy tekislik tenglamasi $Ax + By + Cz + D = 0(A^2 + B^2 + +C^2 \neq 0)$ ko‘rinishida tasvirlash mumkin. Buni isbotlash uchun fazoda berilgan $M_1(x_1; y_1; z_1)$, $M_2(x_2; y_2; z_2)$ va $M_3(x_3; y_3; z_3)$ nuqtalardan o‘tuvchi tekilik tenglamasini qarasak, ushbu

$$\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0$$

ko‘rinishida bo‘ladi. Determinantni satr bo‘yicha yoyib hisoblaymiz va nolga tenglashtiramiz:

$$\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = (-1)^{1+1} \cdot x \begin{vmatrix} y_1 & z_1 & 1 \\ y_2 & z_2 & 1 \\ y_3 & z_2 & 1 \end{vmatrix} +$$

$$+(-1)^{1+2} \cdot y \begin{vmatrix} x_1 & z_1 & 1 \\ x_2 & z_2 & 1 \\ x_3 & z_3 & 1 \end{vmatrix} + (-1)^{1+3} \cdot z \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} +$$

$$+(-1)^{1+4} \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = 0$$

$$A = \begin{vmatrix} y_1 & z_1 & 1 \\ y_2 & z_2 & 1 \\ y_3 & z_2 & 1 \end{vmatrix}, \quad B = -\begin{vmatrix} x_1 & z_1 & 1 \\ x_2 & z_2 & 1 \\ x_3 & z_3 & 1 \end{vmatrix},$$

$$C = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}, \quad D = -\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}.$$

$$Ax + By + Cz + D = 0 \tag{6.5}$$

hosil bo‘ladi.

Istalgan $Ax + By + Cz + D = 0$ ko‘rinishidagi tenglama ***tekislik tenglamasi*** bo‘ladi.

Ixtiyoriy $M_0(x_0; y_0; z_0)$ nuqta berilgan bo‘lsin. $Ax + By + Cz + +D = 0$ va $Ax_0 + By_0 + Cz_0 + D = 0$ tekislik tenglamasi berilgan bo‘lsa, tekislik tenglamalarini bir – biridan ayirsak

$$A(x-x_0)+B(y-y_0)+C(z-z_0)+D-D=0 \Longrightarrow$$
$$A(x-x_0)+B(y-y_0)+C(z-z_0)=0 \Longrightarrow$$
$$\begin{vmatrix} x-x_0 & y-y_0 & z-z_0 \\ -B & A & 0 \\ -C & 0 & A \end{vmatrix}=0 \qquad (6.6)$$
natija hosil bo‘ladi. Bu tenglama $M_0(x_0;y_0;z_0)$ nuqtadan va $\vec{a}(-B;A;0)$ va $\vec{b}(-C;0;A)$ vektorlardan o‘tuvchi tekislik tenglamasi hisoblanadi.

### **Umumiy tenglamaning xususiy hollari**.

$Ax+By+Cz+D=0$ tekislik tenglamasi berilgan bo‘lsa uning xususiy hollarini keltiramiz($A^2+B^2+C^2\neq 0$).

1) $A=0,\ B\neq 0,\ C\neq 0,\ D\neq 0$ bo‘lsa, bu tekislik tenglamamiz $Ox$ o‘qiga parallel $Oyz$ tekisligini $By+Cz+D=0$ to‘g‘ri chiziq bo‘yicha kesib o‘tadi.

2) $B=0,\ A\neq 0,\ C\neq 0,\ D\neq 0$ bo‘lsa, bu tekislik tenglamamiz $Oy$ o‘qiga parallel $Oxz$ tekisligini $Ax+Cz+D=0$ to‘g‘ri chiziq bo‘yicha kesib o‘tadi.

3) $C=0,\ A\neq 0,\ B\neq 0,\ D\neq 0$ bo‘lsa, bu tekislik tenglamamiz $Oz$ o‘qiga parallel $Oxy$ tekisligini $Ax+By+D=0$ to‘g‘ri chiziq bo‘yicha kesib o‘tadi.

4) $D=0,\ A\neq 0, B\neq 0,\ C\neq 0$ bo‘lsa, koordinata boshidan o‘tuvchi tekislik tenglamasi hosil bo‘ladi.

5) $B=C=0,\ A\neq D\neq 0$ bo‘lsa, $Ax+D=0$ tekislik tenglamamiz $Ox$ o‘qiga perpendikulyar ($Oyz$ tekisligiga paralel) $x=-\frac{D}{A}$ nuqtadan o‘tuvchi tekislik tenglamasi, $D=0$ bo‘lsa $x=0$ va $Oyz$ tekisligi bilan ustma –ust tushadi.

6) $A=C=0,\ B\neq D\neq 0$ bo‘lsa, $By+D=0$ tekislik tenglamamiz $Oy$ o‘qiga perpendikulyar ($Oxz$ tekisligiga paralel) $y=-\frac{D}{B}$ nuqtadan o‘tuvchi tekislik tenglamasi, $D=0$ bo‘lsa $y=0$ va $Oxz$ tekisligi bilan ustma – ust tushadi.

7) $A = B = 0,\ C \neq 0$ bo‘lsa, $Cz + D = 0$ tekislik tenglamamiz $Oz$ o‘qiga perpendikulyar ($Oz$ tekisligiga paralel) $z = -\frac{D}{C}$ nuqtadan o‘tuvchi tekislik tenglamasi, $D = 0$ bo‘lsa $z = 0$ va $Oxy$ tekisligi bilan ustma –ust tushadi.

**5-Misol.** Berilgan uchta $M_1(1; -2; 3)$, $M_2(4; -1; 2)$ va $M_3(2; -3; 3)$ nuqtalardan o‘tuvchi tekislik tenglamasini toping.

**Yechish:** Berilgan (6.5) va (6.6) formulalardan foydalanib,

$$\begin{vmatrix} x & y & z & 1 \\ 1 & -2 & 3 & 1 \\ 4 & -1 & 2 & 1 \\ 2 & -3 & 3 & 1 \end{vmatrix} = 0$$

$$(-1)^{1+1} \cdot x \begin{vmatrix} -2 & 3 & 1 \\ -1 & 2 & 1 \\ -3 & 3 & 1 \end{vmatrix} + (-1)^{1+2} \cdot y \begin{vmatrix} 1 & 3 & 1 \\ 4 & 2 & 1 \\ 2 & 3 & 1 \end{vmatrix} +$$

$$+(-1)^{1+3} \cdot z \begin{vmatrix} 1 & -2 & 1 \\ 4 & -1 & 1 \\ 2 & -3 & 1 \end{vmatrix} + (-1)^{1+4} \begin{vmatrix} 1 & -2 & 3 \\ 4 & -1 & 2 \\ 2 & -3 & 3 \end{vmatrix} = 0$$

$$A = \begin{vmatrix} -2 & 3 & 1 \\ -1 & 2 & 1 \\ -3 & 3 & 1 \end{vmatrix}, \quad B = -\begin{vmatrix} 1 & 3 & 1 \\ 4 & 2 & 1 \\ 2 & 3 & 1 \end{vmatrix},$$

$$C = \begin{vmatrix} 1 & -2 & 1 \\ 4 & -1 & 1 \\ 2 & -3 & 1 \end{vmatrix}, \quad D = -\begin{vmatrix} 1 & -2 & 3 \\ 4 & -1 & 2 \\ 2 & -3 & 3 \end{vmatrix}.$$

$$A = -1,\ B = -1,\ C = -4,\ \ D = 11.$$

$$-x - y - 4z + 11 = 0 \Longrightarrow\ x + y + 4z - 11 = 0$$

**Tekislikning koordinata o‘qlaridan ajratgan kesmalar bo‘yicha tenglamasi.**

1.Tekislikning fazodagi o‘rni aniq bo‘lishi uchun uning koordinata o‘qlaridan kesgan kesmalari ma’lum bo‘lishi kifoya qiladi. Faraz qilaylik, tekislikning koordinata o‘qlaridan kesgan kesmalari:

$$OA = a, \quad OB = b, \quad OC = c$$

bo‘lsin. Ya’ni tekislik absissa o‘qini $A(a; 0; 0)$ nuqtada, ordinata o‘qini $B(0; b; 0)$, aplikata o‘qini esa $C(0; 0; c)$ nuqtalarda kesib o‘tadi.

Uni quyidagi ikki usul bilan isbot qilamiz:

**1-usul:** Buning uchun tekislikning berilgan uchta nuqtadan o‘tuvchi tenglamasidan foydalanamiz. Ma’lumki, bu tekislik $A(a;0;0)$, $B(0;b;0)$ va $C(0;0;c)$ nuqtalardan o‘tadi.

$$\begin{vmatrix} x & y & z & 1 \\ a & 0 & 0 & 1 \\ 0 & b & 0 & 1 \\ 0 & 0 & c & 1 \end{vmatrix} = 0$$

$$(-1)^{1+1} \cdot x \begin{vmatrix} 0 & 0 & 1 \\ b & 0 & 1 \\ 0 & c & 1 \end{vmatrix} + (-1)^{1+2} \cdot y \begin{vmatrix} a & 0 & 1 \\ 0 & 0 & 1 \\ 0 & c & 1 \end{vmatrix} +$$

$$+(-1)^{1+3} \cdot z \begin{vmatrix} a & 0 & 1 \\ 0 & b & 1 \\ 0 & 0 & 1 \end{vmatrix} + (-1)^{1+4} \begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix} = 0$$

$$x \cdot bc + y \cdot ac + z \cdot ab - abc = 0$$

tengligimizni ikkala tomonini $abc$ ga bo‘lib yuborsak, quyidagi

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \qquad (6.7)$$

natijaga erishamiz.

**2-usul:** Koordinatalar boshidan tekislikka perpendikulyar qilib $\mathrm{OP} = \mathrm{p}$ ni o‘tkazamiz. Faraz qilaylik OP ning koordinata o‘qlarining musbat yo‘nalishlari bilan tashkil qilgan burchaklari $\alpha$, $\beta$, $\gamma$ bo‘lsin. Shaklga muvofiq $a$, $b$ va $c$ dan har birining OP dagi proyeksiyasi OP ning o‘zi, ya’ni $p$ bo‘ladi. Shuning uchun

$$p = acos\alpha, \quad p = bcos\beta, \quad p = ccos\gamma,$$

yoki bulardan:

$$cos\alpha = \frac{p}{a}, \qquad cos\beta = \frac{p}{b}, \qquad cos\gamma = \frac{p}{c};$$

bular tekislikning ushbu

$$xcos\alpha + ycos\beta + zcos\gamma - p = 0$$

tenglamasiga qo‘yilsa:

$$\frac{p}{a}x + \frac{p}{b}y + \frac{p}{c}z - p = 0$$

yoki tenglamani ikkala tomonini $p$ bo‘lib, so‘ngra ozod hadini o‘ng tomonga o‘tkazsak, tenglamaning odatdagi ko‘rinishi quyidagicha bo‘ladi:

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

Tenglamadagi $a$, $b$, $c$ ning qiymatlari algebraik bo‘lib, ular musbat va manfiy bo‘lishlari mumkin.

Tengsizlikning umumiy tenglamasi bo‘lgan ushbu (6.5) tenglamaning koeffitsiyentlaridan hech biri nolga teng bo‘lmagan holda u tenglamani hamma vaqt (6.7) shaklga keltirish mumkin.

Buning uchun tenglamaning ozod hadi bo‘lgan $D$ ni o‘ng tomoniga o‘tkazib, so‘ngra tenglamaning ikkala tomonini $-D$ ga bo‘lamiz:

$$-\frac{Ax}{D}-\frac{By}{D}-\frac{Cz}{D}=1.$$

yoki

$$-\frac{x}{\frac{D}{A}}-\frac{y}{\frac{D}{B}}-\frac{z}{\frac{D}{C}}=1,$$

demak,

$$a=-\frac{D}{A}, \qquad b=-\frac{D}{B}, \qquad c=-\frac{D}{C}$$

bo‘ladi.

**6-Misol.** Umumiy $2x+3y-5z-7=0$ tenglamasi bilan berilgan tekislikning kesmalardagi tenglamasini toping.

**Yechish:** Umumiy tenglamani $D=7$ soniga bo‘lib, (6.7) tenglamada

$$a=\frac{D}{A}=\frac{7}{2}, \ \ b=\frac{D}{B}=\frac{7}{3}, \ \ c=-\frac{D}{C}=-\frac{7}{5}.$$

ekanligini topamiz. Bundan berilgan tekislikning kesmalardagi tenglamasi

$$\frac{2x}{7}+\frac{3y}{7}-\frac{5z}{7}=1$$

yoki

$$\frac{x}{7/2}+\frac{y}{7/3}-\frac{z}{7/5}=1$$

ekanligi kelib chiqadi.

### 6.2. Fazoda to‘g‘ri chiziq tenglamalari.

**Berilgan vektorga parallel va berilgan nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi.**

Fazoda $M_0(x_0; y_0; z_0)$ nuqtadan o‘tuvchi $\vec{a}(l; m; n)$ vektorga parallel bo‘lgan to‘g‘ri chiziq tenglamasini tuzaylik. Buning uchun tuzmoqchi bo‘lgan to‘g‘ri chizig‘imiz tenglamasini qanoatlantiradigan ixtiyoriy $M(x; y; z)$ nuqta olamiz va $\overrightarrow{M_0M}(x - x_0; y - y_0; z - z_0)$ vektorni yasaymiz.

$\overrightarrow{M_0M}(x - x_0; y - y_0; z - z_0)$ vektor va $\vec{a}$ vektorimizning kollinearligidan

$$\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n} \tag{6.8}$$

kelib chiqadi va bu tenglamamiz to‘g‘ri chiziqning berilgan nuqtadan o‘tib berilgan vektorga parallel tenglamasi yoki kanonik tenglamasi deyiladi. $\vec{a}(l; m; n)$ vektor (6.8) to‘g‘ri chiziqning yo‘naltiruvchi vektori deyiladi.

**7-Misol.** $N(3; -2; 4)$ nuqtadan o‘tuvchi $\vec{a}(-2; 4; -3)$ vektorga parallel bo‘lgan to‘g‘ri chiziq tenglamasini tuzing.

**Yechish:** Yuqoridagi (6.8) formuladan foydalanib,

$$\frac{x - 3}{-2} = \frac{y - (-2)}{4} = \frac{z - 4}{-3} \Rightarrow \frac{x - 3}{-2} = \frac{y + 2}{4} = \frac{z - 4}{-3}$$

berilgan nuqtadan o‘tib berilgan vektorga parallel tenglamasini keltirib chiqardik.

**Fazoda berilgan ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi.**

Fazoda $M_1(x_1; y_1; z_1)$ va $M_2(x_2; y_2; z_2)$ nuqtalar berilgan bo‘lsin. Bu nuqtalardan o‘tuvchi tekislik tenglamasini tuzaylik. Buning uchun izlanayotgan to‘g‘ra chizig‘imiz $M(x; y; z)$ nuqtani olamiz. $\overrightarrow{M_1M}$ va $\overrightarrow{M_2M}$ vektorlarni tuzamiz.

$$\overrightarrow{M_1M} = (x - x_1; y - y_1; z - z_1),$$
$$\overrightarrow{M_2M} = (x - x_2; y - y_2; z - z_2)$$

bu vektorlar bir to‘g‘ri chiziqda yotadi, ya’ni ular kollinear. Bundan

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1} \qquad (6.9)$$

kelib chiqadi.

Bu tenglamalar berilgan $M_1(x_1;y_1;z_1)$ va $M_2(x_2;y_2;z_2)$ nuqtalardan o‘tgan to‘g‘ri chiziqni ifoda qiladi, chunki bular $x$, $y$, $z$ ga nisbatan birinchi darajali bo‘lib, har ikki nuqtaning koordinatalarini qanoatlantiradi.

**8-Misol.** $M_1(-1;3;-5)$ va $M_2(2;1;0)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasi toping.

**Yechish:** Yuqoridagi (6.9) formuladan foydalanib,

$$\frac{x+1}{2+1}=\frac{y-3}{1-3}=\frac{z+5}{0+5} \Rightarrow \frac{x+1}{3}=\frac{y-3}{-2}=\frac{z+5}{5}$$

fazoda berilgan $M_1$ va $M_2$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasini topdik.

**To‘g‘ri chiziqning parametrik tenglamasi.**

Fazoda $\frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{n}$ tenglama bilan to‘g‘ri chiziq berilgan bo‘lsin. Biz uni

$$\frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{n}=t$$

ko‘rinishida yozsak:

$$\begin{cases} x=lt+x_0 \\ y=mt+y_0 \\ z=nt+z_0 \end{cases} \qquad (6.10)$$

ko‘rinishga keladi. Bu tenglama to‘g‘ri chiziqning parametrik tenglamasi deyiladi.

**9-Misol.** $M(2;-3;5)$ va $N(-1;4;-3)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq parametrik tenglamasi toping.

**Yechish:** Yuqoridagi (6.10) formuladan foydalanib,

$$\frac{x+1}{3}=\frac{y-4}{-7}=\frac{z+3}{8}$$

tenglamani tuzamiz. Bu to‘g‘ri chiziqning kanonik tenglamasi. Bu tenglamani parametrik ko‘rinishga

$$\frac{x+1}{3}=\frac{y-4}{-7}=\frac{z+3}{8}=t\Rightarrow$$

$$\begin{cases}\dfrac{x+1}{3}=t\\ \dfrac{y-4}{-7}=t\\ \dfrac{z+3}{8}=t\end{cases}\Rightarrow\ \begin{cases}x=3t-1\\ y=-7t+4\\ z=8t-3\end{cases}$$

keltirdik.

## 6.3. Fazoda tekislik va to‘g‘ri chiziq orasidagi munosabatlar. Fazoda berilgan ikki to‘g‘ri chiziq orasidagi burchak.

Faraz qilaylik, berilgan to‘g‘ri chiziqlarning tenglamalari

$$\begin{cases}\dfrac{x-x_1}{l_1}=\dfrac{y-y_1}{m_1}=\dfrac{z-z_1}{n_1}\\ \dfrac{x-x_2}{l_2}=\dfrac{y-y_2}{m_2}=\dfrac{z-z_2}{n_2}\end{cases}$$

bo‘lsin. Ular orasidagi burchakni topmoqchimiz. Ma’lumki, birinchi to‘g‘ri chiziqning yo‘naltiruvchi vektori $\vec{a}_1(l_1;m_1;n_1)$, ikkinchi to‘g‘ri chiziqning yo‘naltiruvchi vektori $\vec{a}_2(l_2;m_2;n_2)$ bo‘ladi. Bu to‘g‘ri chiziqlar orasidagi burchak $\vec{a}_1$ va $\vec{a}_2$ vektorlar orasidagi burchakka teng bo‘ladi va

$$cos\varphi=\frac{\vec{a}_1\,\vec{a}_2}{|\vec{a}_1|\cdot|\vec{a}_2|}$$

$$cos\varphi=\frac{l_1l_2+m_1m_2+n_1n_2}{\sqrt{{l_1}^2+{m_1}^2+{n_1}^2}\cdot\sqrt{{l_2}^2+{m_2}^2+{n_2}^2}}\qquad(6.11)$$

bu formula yordamida berilgan ikki to‘g‘ri chiziq orasidagi burchak topiladi.

**10-Misol.** $\frac{x-1}{1}=\frac{y}{-4}=\frac{z+3}{1}$, $\frac{x}{2}=\frac{y+2}{-2}=\frac{z}{-1}$ tenglamalar bilan berilgan to‘g‘ri chiziqlar orasidagi $\varphi$ burchakni toping.

**Yechish.** Berilganlardan $l_1=1$, $m_1=-4$, $n_1=1$, $l_2=2$, $m_2=-2$, $n_2=-1$. Bularni (6.11) qo‘ysak:

$$cos\varphi = \frac{1 \cdot 2 + (-4) \cdot (-2) + 1 \cdot (-1)}{\sqrt{1^2 + 4^2 + 1^2} \cdot \sqrt{2^2 + 2^2 + 1^2}} = \frac{9}{\sqrt{18} \cdot \sqrt{9}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

to‘g‘ri chiziqlar orasidagi burchak $45^0$ ga teng ekanligi ma’lum bo‘ldi.

**Fazoda tekisliklar orasidagi burchak.**

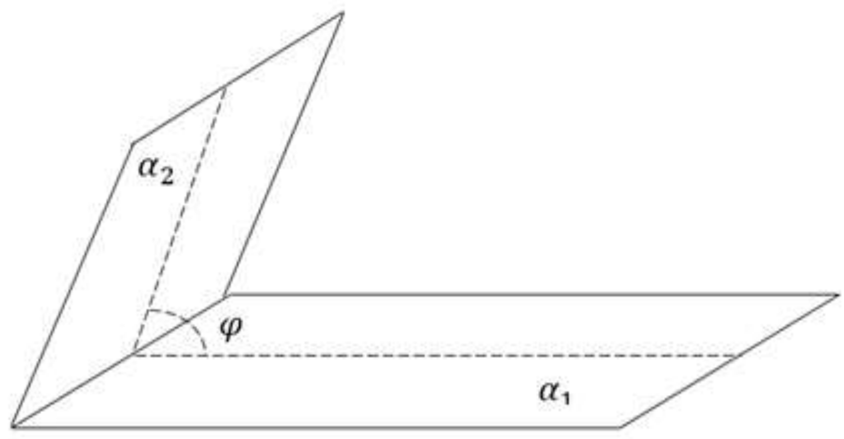
Fazoda bizga

$$\begin{cases} A_1x + B_1y + C_1z + D_1 = 0 & (\alpha_1) \\ A_2x + B_2y + C_2z + D_2 = 0 & (\alpha_2) \end{cases}$$

tenglamalar bilan berilgan tekisliklar orasidagi burchakni topish uchun $\alpha_1$ tekislikning normal vektori $\vec{n}_1(A_1; B_1; C_1)$, $\alpha_2$ tekislikning normal vektori $\vec{n}_2(A_2; B_2; C_2)$ bo‘ladi.Bundan,

$$cos\varphi = \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{{A_1}^2 + {B_1}^2 + {C_1}^2} \cdot \sqrt{{A_2}^2 + {B_2}^2 + {C_2}^2}} \qquad (6.12)$$

tekisliklar orasidagi burchak $(6.12)$ formula orqali topiladi.




**6.3.1-chizma**

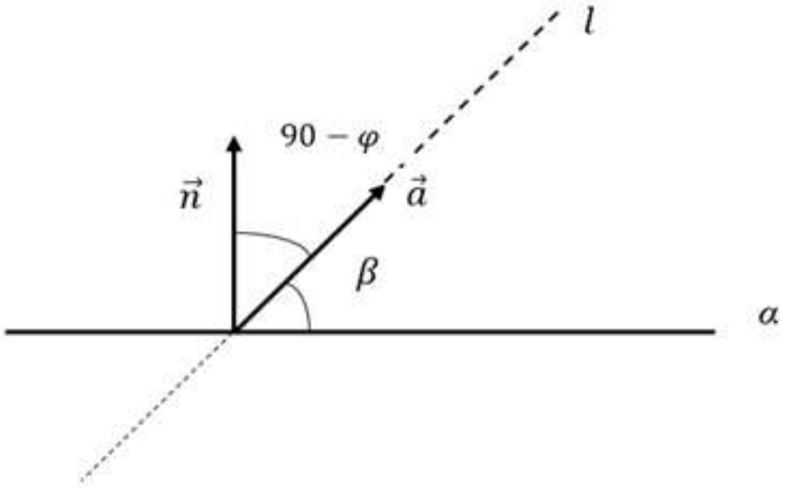
**Berilgan to‘g‘ri chiziq va tekislik orasidagi burchak.**

Fazoda $Ax + By + Cz + D = 0$ tekislik va $\frac{x-x_0}{l} = \frac{y-y_0}{m} = \frac{z-z_0}{n}$ to‘g‘ri chiziq berilgan bo‘lsin. Tekislikning normal vektori $\vec{n}(A; B; C)$ va to‘g‘ri chiziqning yo‘naltiruvchi vektori $\vec{a}(l; m; n)$ bo‘lsa,

$$\cos(90 - \varphi) = \frac{Al + Bm + Cn}{\sqrt{A^2 + B^2 + C^2} \cdot \sqrt{l^2 + m^2 + n^2}} \Rightarrow$$

$$\sin\varphi = \frac{Al + Bm + Cn}{\sqrt{A^2 + B^2 + C^2} \cdot \sqrt{l^2 + m^2 + n^2}} \qquad (6.13)$$

tenglamasi hosil bo‘ladi.




**6.3.2-chizma**

**11-Misol.** Kanonik tenglamasi

$$\frac{x+1}{1} = \frac{y-2}{\sqrt{2}} = \frac{z-1}{1}$$

bo‘lgan $l$ to‘g‘ri chiziq va umumiy tenglamasi $x + \sqrt{2}y - z + 1 = 0$ bo‘lgan $P$ tekislik orasidagi burchakni toping.

**Yechish.** Yuqorida berilgan (6.13) formuladan foydalanib,

$$sin\varphi = \frac{1 \cdot 1 + \sqrt{2} \cdot \sqrt{2} + 1 \cdot (-1)}{\sqrt{1^2 + \left(\sqrt{2}\right)^2 + 1^2} \cdot \sqrt{1^2 + \left(\sqrt{2}\right)^2 + (-1)^2}} = \frac{2}{2 \cdot 2} = \frac{1}{2}$$

tekislik va to‘g‘ri chiziq orasidagi burchak $30^0$ ga teng ekanligi ma’lum bo‘ldi.

**To‘g‘ri chiziq va tekislikning fazoda joylashishi.**

Fazoda $Ax + By + Cz + D = 0$ tekislik va parametrik ko‘rinishdagi

$$\begin{cases} x = lt + x_0 \\ y = mt + y_0 \\ x = nt + z_0 \end{cases}$$

to‘g‘ri chiziq berilgan bo‘lsin.

Bularni fazoda qanday joylashganini o‘rganaylik. Buning uchun

$$A(lt+x_0)+B(mt+y_0)+C(nt+z_0)+D=0 \Rightarrow$$

$$(Al+Bm+Cn)t=-(Ax_0+By_0+Cz_0+D) \qquad (6.14)$$

bo‘ladi.

Keltirib chiqarilgan tenglamamizni xususiy hollarda qaraymiz:

1. $Al+Bm+Cn \neq 0$ bo‘lsa, to‘g‘ri chiziq va tekislik bitta nuqtada kesishadi.

$$t=-\frac{Ax_0+By_0+Cz_0+D}{Al+Bm+Cn}$$

2. $\begin{cases} Al+Bm+Cn=0 \\ Ax_0+By_0+Cz_0+D=0 \end{cases}$ to‘g‘ri chiziq tekislikning ustida yotgan bo‘ladi.

3. $\begin{cases} Al+Bm+Cn=0 \\ Ax_0+By_0+Cz_0+D\neq 0 \end{cases}$ to‘g‘ri chiziqlar va tekislik parallel bo‘ladi.

**12-Misol.** $4x-3y+2z-4=0$ tekislik bilan $A(-3;4;2)$, $B(1;2;3)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqning kesishish nuqtasini toping.

**Yechish.** $A(-3;4;2)$ va $B(1;2;3)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqning parametrik tenglamasini tuzamiz va bu to‘g‘ri chiziq bilan tekislikni kesishish nuqtasini topamiz.

$$\frac{x+3}{1+3}=\frac{y-4}{2-4}=\frac{z-2}{3-2} \Rightarrow \frac{x+3}{2}=\frac{y-4}{-2}=\frac{z-2}{1}=k \Rightarrow$$

$$\begin{cases} x=4k-3 \\ y=-2k+4 \\ z=k+2 \end{cases} \Rightarrow 4(4k-3)-3(-2k+4)+2(k+2)-4=0 \Rightarrow$$

$$24k=24 \Rightarrow k=1 \Rightarrow \begin{cases} x=1 \\ y=2 \\ z=3 \end{cases}$$

$M(1;2;3)$ nuqtada kesishar ekan.

**13-Misol.** Fazoda $\frac{x-5}{2}=\frac{y+1}{1}=\frac{z}{3}$ va $\frac{x-1}{5}=\frac{y+1}{2}=\frac{z-3}{1}$ to‘g‘ri chiziqlar orasidagi vaziyatni aniqlang.

**Yechish.** Berilgan to‘g‘ri chiziqlar tenglamalarini ixtiyoriy $t$ va $k$ sonlariga tenglashtirib,

$$\begin{cases} \dfrac{x-5}{2}=\dfrac{y+1}{1}=\dfrac{z}{3}=\mathrm{t} \\ A\dfrac{x-1}{5}=\dfrac{y+1}{2}=\dfrac{z-3}{1}=k \end{cases} \Rightarrow \begin{cases} x=2t+5 \\ y=t-1 \\ z=3t \end{cases} \text{va} \begin{cases} x=5k+1 \\ y=2k-1 \\ z=k+3 \end{cases}$$

oxirgi ikkala tenglamalar sistemasidagi noma’lumlarni tenglashtirsak, quyidagiga

$$\begin{cases} 2t+5=5k+1 \\ t-1=2k-1 \end{cases} \Rightarrow \begin{cases} 2t-5k=-4 \\ t-2k=0 \end{cases} \Rightarrow 4k-5k=-4 \Rightarrow k=4,$$
$$t=8$$

ega bo‘lamiz. Uni tenglamalar sistemasiga eltib qo‘ysak $3\cdot 8 \neq 4+3$ ekanligidan bizga berilgan to‘g‘ri chiziq tenglamalari o‘zaro ayqashligi ma’lum bo‘ladi.

## Mustaqil yechish uchun topshiriqlar.

### 6.1. Fazoda tekislikning ba’zi tenglamalariga doir misollar.

**6.1.1**. Koordinatalar sistemasida $M_1(2;3;1)$, $M_2(3;1;4)$, $M_3(2;1;5)$ nuqtalar va $\vec{a}_1(4;1;2)$, $\vec{a}_2(-2;3;5)$, $\vec{a}_3(5;-1;3)$ vektorlar berilgan bo‘lsin.

1) $M_1$ nuqtadan o‘tib, $\vec{a}_1$ va $\vec{a}_2$ vektorlarga;
2) $M_1$ nuqtadan o‘tib, $\vec{a}_1$ va $\vec{a}_3$ vektorlarga;
3) $M_2$ nuqtadan o‘tib, $\vec{a}_1$ va $\vec{a}_2$ vektorlarga;
4) $M_2$ nuqtadan o‘tib, $\vec{a}_2$ va $\vec{a}_3$ vektorlarga;
5) $M_3$ nuqtadan o‘tib, $\vec{a}_1$ va $\vec{a}_2$ vektorlarga;
6) $M_3$ nuqtadan o‘tib, $\vec{a}_1$ va $\vec{a}_3$ vektorlarga parallel bo‘lgan tekislik tenglamasini tuzing.

**6.1.2.** Koordinatalar sistemasida $N_1(2;3;-1)$, $N_2(-2;4;1)$, $N_3(3;2;-1)$ nuqtalar va $\vec{a}_1(-3;-1;2)$, $\vec{a}_2(1;-3;-5)$, $\vec{a}_3(1;2;3)$ vektorlar berilgan bo‘lsin.

1) $N_1$ va $N_2$ nuqtalardan o‘tib, $\vec{a}_1$ vektorga;
2) $N_1$ va $N_3$ nuqtalardan o‘tib, $\vec{a}_1$ vektorga;
3) $N_1$ va $N_3$ nuqtalardan o‘tib, $\vec{a}_2$ vektorga;
4) $N_2$ va $N_3$ nuqtalardan o‘tib, $\vec{a}_2$ vektorga;

5) $N_1$ va $N_2$ nuqtalardan o‘tib, $\vec{a}_3$ vektorga;
6) $N_2$ va $N_3$ nuqtalardan o‘tib, $\vec{a}_3$ vektorga parallel bo‘lgan tekislik tenglamasini tuzing.

**6.1.3.** Koordinatalar sistemasida $M_1(3;7;-2)$, $M_2(4;1;3)$, $M_3(5;3;-1)$, $M_4(3;-5;1)$ va $M_3(2;3;-5)$ nuqtalar berilgan bo‘lsin.
1) $M_1$ , $M_2$ va $M_3$ nuqtalardan;
2) $M_1$ , $M_2$ va $M_4$ nuqtalardan;
3) $M_1$ , $M_3$ va $M_5$ nuqtalardan;
4) $M_2$ , $M_3$ va $M_4$ nuqtalardan;
5) $M_2$ , $M_4$ va $M_5$ nuqtalardan;
6) $M_3$ , $M_4$ va $M_5$ nuqtalardan
o‘tuvchi tekislik tenglamasini tuzing.

**6.1.4.** Koordinatalar sistemasida $N_1(-3;2;1)$ va $N_2(2;-4;3)$ berilgan bo‘lsin.
1) $N_1$ nuqtadan $Oxy$ tekisligiga;
2) $N_1$ nuqtadan $Oyz$ tekisligiga;
3) $N_1$ nuqtadan $Oxz$ tekisligiga;
4) $N_2$ nuqtadan $Oxy$ tekisligiga;
5) $N_2$ nuqtadan $Oyz$ tekisligiga;
6) $N_2$ nuqtadan $Oxz$ tekisligiga
parallel bo‘lgan tekislik tenglamasini tuzing.

**6.1.5.** Koordinatalar sistemasida $M_1(4;3;-1)$, $M_2(-2;4;5)$ va $M_3(3;-2;2)$ berilgan bo‘lsin.
1) $M_1$ nuqtadan va $Ox$ o‘qidan o‘tuvchi;
2) $M_1$ nuqtadan va $Oz$ o‘qidan o‘tuvchi;
3) $M_2$ nuqtadan va $Ox$ o‘qidan o‘tuvchi;
4) $M_2$ nuqtadan va $Oy$ o‘qidan o‘tuvchi;
5) $M_3$ nuqtadan va $Oy$ o‘qidan o‘tuvchi;
6) $M_3$ nuqtadan va $Oz$ o‘qidan o‘tuvchi tekislik tenglamasini tuzing.

**6.1.6.** Koordinatalar sistemasida $N_1(1;-2;3)$ va $N_2(2;4;-3)$ berilgan bo‘lsin.
1) $N_1$ nuqtadan o‘tib, $Ox$ o‘qiga;

2) $N_1$ nuqtadan o‘tib, $Oy$ o‘qiga;
3) $N_1$ nuqtadan o‘tib, $Oz$ o‘qiga;
4) $N_2$ nuqtadan o‘tib, $Ox$ o‘qiga;
5) $N_2$ nuqtadan o‘tib, $Oy$ o‘qiga;
6) $N_2$ nuqtadan o‘tib, $Oy$ o‘qiga
perpendikulyar bo‘lgan tekislik tenglamasini tuzing.

**6.1.7.** Koordinatalar sistemasida $M_1(3;2;-1)$, $M_2(1;3;5)$ va $M_3(2;-4;3)$ berilgan bo‘lsin.
1) $M_1$va $M_2$ nuqtalardan o‘tib, $Ox$ o‘qiga;
2) $M_2$va $M_3$ nuqtalardan o‘tib, $Ox$ o‘qiga;
3) $M_1$va $M_2$ nuqtalardan o‘tib, $Oy$ o‘qiga;
4) $M_1$va $M_3$ nuqtalardan o‘tib, $Oy$ o‘qiga;
5) $M_1$va $M_2$ nuqtalardan o‘tib, $Oz$ o‘qiga;
6) $M_2$va $M_3$ nuqtalardan o‘tib, $Oz$ o‘qiga parallel bo‘lgan tekislik tenglamasini tuzing.

**6.1.8**. Koordinatalar sistemasida uchta nuqtadan o‘tgan tekislikning kanonik tenglamasi tuzilsin.
1) $M_1(2;3;1)$, $M_2(3;1;4)$, $M_3(2;1;5)$;
2) $M_1(2;0;-1)$, $M_2(-2;4;1)$, $M_3(0;2;-1)$;
3) $M_1(3;7;-2)$, $M_2(4;1;3)$, $M_3(5;3;-1)$;
4) $M_1(4;3;-1)$, $M_2(-2;4;5)$ va $M_3(3;-2;2)$.

**6.1.9.** $Ox$ va $Oy$ o‘qlaridan mos ravishda 5 va $-7$ ga teng kesmalar ajratadigan va $N(1;1;2)$ nuqtadan o‘tuvchi tekislik tenglamasi tuzilsin.

**6.1.10.** $A(3;5;-7)$ nuqtadan o‘tuvchi va koordinata o‘qlaridan teng kesmalar ajratadigan tekislik tenglamasi tuzilsin.

**6.1.11.** $A(3;5;1)$ va $B(7;7;8)$ nuqtalardan o‘tib, $Ox$, $Oy$ o‘qlaridan teng kesmalar ajratgan tekislik tenglamasi yozilsin.

**6.1.12.** Koordinatalar sistemasi o‘qlaridan mos ravishda 3, 5, $-7$ ga teng kesmalar ajratadigan tekislik tenglamasi tuzilsin.

**6.1.13.** Koordinatalar sistemasida $x-y+7z-4=0$ tekislikning koordinata o‘qlaridan ajratgan kesmalari aniqlansin.

**6.1.14**. Uchi $A(2;1;0)$, $B(1;3;5)$, $C(6;3;4)$, $D(0;-7;8)$ nuqtalarda bo‘lgan tetraedr berilgan. $AB$ qirradan va $CD$ qirraning o‘rtasidan o‘tuvchi tekislik tenglamasi tuzilsin.

**6.1.15**. Quyidagi hollarning har biri uchun tekislikning parametrik tenglamalariga ko‘ra umumiy tenglamasi yozilsin:

1) $x = 2 + 3u - 4v; y = 4 - v;\ z = 2 + 3u;$

2) $x = u + v;\ y = u - v;\ z = 5 + 6u - 4v.$

**6.1.16**. Quyidagi tekisliklar juftlarining qaysilari parallel, kesishadi yoki ustma-ust tushishi aniqlansin:

1) $2x + 3y + 4z - 12 = 0, \quad 3x - 6y + 1 = 0;$

2) $3x - 4y + 6z + 9 = 0, \quad 6x - 8y - 10z + 15 = 0;$

3) $3x - 2y - 3z + 5 = 0, \quad 9x - 6y - 9z - 5 = 0$

**6.1.17**. $A(-3; 3; 5)$, $B(0; -7; -14)$, $C(6; 5; 1)$, $D(-3; -5; 2)$, $E(4; -7; 10)$, $F(2; 6; 1)$ nuqtalarning $2x - 3y + 4z - 5 = 0$ tekislikka nisbatan vaziyatini aniqlang.

**6.1.18**. $A(3; 5; 1)$, $B(2; -6; 3)$ nuqtalar berilgan, $AB$ kesmani $2x - -3y + 6z - 1 = 0$ tekislik qanday nisbatda bo‘ladi?

**6.1.19**. $2x - 3y - 4z - 24 = 0$ tekislikning koordinata o‘qlari bilan kesishish nuqtasini toping.

**6.1.20.** Koordinata boshidan $3x - 4y - 24z + 12 = 0$ tekislikning koordinata o‘qlari bilan kesishgan nuqtasigacha bo‘lgan masofasini toping.

**6.1.21.** $Oxy$ tekisligi va $5x - 6y + 3z + 120 = 0$ tekislik kesishishidan hosil bo‘lgan uchburchakning yuzini toping.

**6.1.22.** $2x - 3y + 6z - 12 = 0$ tekislik bilan va koordinatalar tekisliklari bilan chegaralangan piramida hajmini toping.

**6.1.23**. Tekislik $M_1(6; -10; 1)$ nuqtadan o‘tadi va absissa o‘qida $a = -3$ va aplikata o‘qida esa $c = 2$ kesmani kesib o‘tadi. Bu tekislik uchun kesmalardagi tenglamasini tuzing.

**6.1.24.** Quyidagi tekisliklar tenglamasining qaysi biri normal ekanligini aniqlang:

1) $\frac{1}{3}x - \frac{2}{3}y - \frac{2}{3}z - 5 = 0;$ 2) $\frac{2}{3}x + \frac{1}{3}y - \frac{1}{3}z - 3 = 0;$

3) $\frac{6}{7}x - \frac{3}{7}y + \frac{2}{7}z + 5 = 0;$ 4) $-\frac{6}{7}x + \frac{3}{7}y - \frac{2}{7}z - 5 = 0;$

5) $\frac{6}{7}x + \frac{4}{5}z - 3 = 0$; 6) $-\frac{5}{13}y + \frac{12}{13}z + 1 = 0$;
7) $\frac{5}{13}x + \frac{12}{13}z - 1 = 0$; 8) $\frac{4}{5}x - \frac{3}{5}y + 5 = 0$.

**6.1.25.** Quyidagi tekislik tenglamalarini normal ko‘rinishga keltiring:
1) $2x - 2y + 2z - 18 = 0$;
2) $x - y - \sqrt{2}z + 16 = 0$;
3) $4x - 6y - 12z - 11 = 0$;
4) $-4x - 4y + 2z + 1 = 0$;
5) $5y - 12z + 26 = 0$;
6) $3x - 4y - 1 = 0$.

**6.1.26.** $P(-1;1;-2)$ nuqtadan $M_1(1;-1;1)$, $M_2(-2;1;3)$ va $M_3(4;-5;-2)$ nuqtalardan o‘tadigan tekislikgacha $d$ masofani aniqlang.

**6.1.27.** Quyidagi holatlarda parallel tekisliklar orasidagi masofani hisoblang:
1) $x - 2y - 2z - 12 = 0$, $x - 2y - 2z - 6 = 0$;
2) $2x - 3y + 6z - 14 = 0$, $4x - 6y + 12z + 21 = 0$.
3) $2x - y + 2z + 9 = 0$, $4x - 2y + 4z - 21 = 0$.

**6.1.28**. Quyidagi holatlarda 2 parallel tekisliklardan bir xil uzoqlashgan nuqtalarning geometrik o‘rni tenglamasini tuzing:
1) $4x - y - 2z - 3 = 0$, $4x - y - 2z - 5 = 0$;
2) $3x + 2y - z + 3 = 0$, $3x + 2y - z - 1 = 0$;
3) $5x - 3y + 2z + 3 = 0$, $10x - 6y + 2z + 7 = 0$.

**6.1.29**. $2x - 3y + z - 3 + \lambda(x + 3y + 2z + 1) = 0$ tekislik tenglamasi berilgan, $\lambda$ ning qanday qiymatlarida:
1) $M_1(1;-2;3)$ nuqtadan o‘tuvchi; 2) $Ox$ o‘qiga parallel;
3) $Oy$ o‘qiga parallel; 4) $Oz$ o‘qiga parallel bo‘ladi.

**6.1.30.** $\begin{cases} 3x - y + 2z + 9 = 0 \\ x + y - 3 = 0 \end{cases}$ to‘g‘ri chiziqdan o‘tuvchi va:
1) $M_1(4;-2;-3)$ nuqtani qanoatlantiruvchi;
2) $Ox$ o‘qiga parallel bo‘lgan;
3) $Oy$ o‘qiga parallel bo‘lgan;

4) $Oz$ o‘qiga parallel bo‘lgan tekislik tenglamasini tuzing.

## 6.2. Fazoda to‘g‘ri chiziq tenglamalariga doir misollar.

**6.2.1**. Koordinatalar sistemasida $\vec{a}_1(4;1;2)$, $\vec{a}_2(-2;3;5)$,$\vec{a}_3(5;-1;3)$ vektorlar va $M_1(2;3;1)$, $M_2(3;1;4)$ nuqtalar berilgan bo‘lsin.

1) $\vec{a}_1$ vektorga parallel va $M_1$ nuqtadan o‘tuvchi;
2) $\vec{a}_1$ vektorga parallel va $M_2$ nuqtadan o‘tuvchi;
3) $\vec{a}_2$ vektorga parallel va $M_1$ nuqtadan o‘tuvchi;
4) $\vec{a}_2$ vektorga parallel va $M_2$ nuqtadan o‘tuvchi;
5) $\vec{a}_3$ vektorga parallel va $M_1$ nuqtadan o‘tuvchi;
6) $\vec{a}_3$ vektorga parallel va $M_2$ nuqtadan o‘tuvchi to‘g‘ri chiziqning;
a) kanonik; b) parametrik tenglamalarini tuzing.

**6.2.2**. Koordinatalar sistemasida $A(2;3;1)$, $B(3;1;4)$, $C(2;1;5)$ va $D(0;2;-1)$ nuqtalardan

1) $A$ va $B$; 2) $A$ va $C$; 3) $A$ va $D$;
4) $B$ va $C$; 5) $B$ va $D$; 6) $C$ va $D$ o‘tuvchi to‘g‘ri chiziqning:
a) kanonik; b) parametrik tenglamalarini tuzing.

**6.2.3**. Quyida berilgan to‘g‘ri chiziqning kanonik tenglamasini tuzing.

1) $\begin{cases} 3x-2y+5z-4=0 \\ 2x+3y+z+12=0 \end{cases}$;

2) $\begin{cases} x+2y-2z-2=0 \\ 3x-2y+6z+9=0 \end{cases}$;

3) $\begin{cases} 12x-6y+5z+1=0 \\ 2x+2y-7z-12=0 \end{cases}$;

4) $\begin{cases} 4x-y-2z-3=0 \\ 3x+2y-z+3=0 \end{cases}$.

**6.2.4**. Koordinatalar sistemasida $A(2;-3;1)$, $B(3;1;-4)$ va $C(-2;1;5)$ nuqtalar berilgan bo‘lsin.

1) $A$ nuqtadan o‘tib, $Oxy$ o‘qiga;
2) $A$ nuqtadan o‘tib, $Oyz$ o‘qiga;
3) $B$ nuqtadan o‘tib, $Oxz$ o‘qiga;
4) $B$ nuqtadan o‘tib, $Oyz$ o‘qiga;
5) $C$ nuqtadan o‘tib, $Oxy$ o‘qiga;

6) $C$ nuqtadan o‘tib, $Oxz$ o‘qiga
parallel bo‘lgan to‘g ‘ri chiziq tenglamasini tuzing.

**6.2.5**. $M_1(-1; 4; -3)$ va $M_2(-8; 2; 5)$ nuqtalar berilgan bo‘lsin.
1) $M_1$ nuqtadan o‘tib, $Oxy$ tekisligiga;
2) $M_1$ nuqtadan o‘tib, $Oxz$ tekisligiga;
3) $M_1$ nuqtadan o‘tib, $Oyz$ tekisligiga;
4) $M_2$ nuqtadan o‘tib, $Oxy$ tekisligiga;
5) $M_2$ nuqtadan o‘tib, $Oxz$ tekisligiga;
6) $M_2$ nuqtadan o‘tib, $Oyz$ tekisligiga perpendikulyar bo‘lgan to‘g‘ri chiziq tenglamasini tuzing.

**6.2.6**. Quyidagi nuqtalardan qaysilari bitta to‘g‘ri chiziqda yotadi?
1) $A(3; 0; 1)$, $B(0; 2; 4)$, $C(-3; 4; 7)$;
2) $C(1; 2; 3)$, $D(10; 8; 4)$, $E(3; 0; 2)$;
3) $M(2; 6; 4)$, $N(5; 7; 1)$, $K(3; -7; 2)$.

**6.2.7**. Ushbu $A(5; 8; 15)$, $B(-1; -1; -3)$, $C(5; 7; 1)$, $D(0; \frac{1}{2}; 0)$, $E(0; 0; 1)$ nuqtalardan qaysilari $x = 1 + 2t,\ y = 2 + 3t,\ z = 3 + 6t$ to‘g‘ri chiziqda yotadi?

**6.2.8**. Fazoda to‘g‘ri chiziqlarning parametrik tenglamalari tuzilsin:
1) $x - 2y + 4z = 0,\ 3x - 2y + 5z = 0;$
2) $x + y - z + 5 = 0,\ 2x - y + 2z - 2 = 0.$

**6.2.9**. 1) $M(3; 5; 1)$ nuqtadan o‘tib, $x = 2 + 4t,\ \ y = -3t,\ \ z = -3$ to‘g‘ri chiziqqa parallel;
2)$N(0; -5; 4)$ nuqtadan o‘tib, $x + 2y + 6 = 0$, $z = 5$ to‘g‘ri chiziqqa parallel to‘g‘ri chiziq tenglamalari yozilsin.

**6.2.10**. Quyidagi hollarning har birida to‘g‘ri chiziqning $Oxy$ tekislikka proyeksiyasi topilsin.
1) $5x + 8y - 3z + 9 = 0,\ \ 2x - 4y + z - 1 = 0;$
2) $\frac{x-3}{-5} = \frac{y-4}{6} = \frac{z-6}{8}$.

**6.2.11**. Quyidagi to‘g‘ri chiziqlarning koordinata tekisliklari bilan kesishish nuqtalari topilsin:
1) $6x + 2y - z - 9 = 0,\ \ 3x + 2y + 2z - 12 = 0;$

2) $x = 6 + 2t,\ y = -2 + 4t,\ z = -5t.$

**6.2.12**. To‘g‘ri chiziqning ikkita koordinata tekisliklar bilan kesishish nuqtalari $M(0; y_1; z_1)$, $N(x_2; 0; z_2)$ berilgan. Shu to‘g‘ri chiziqning uchinchi koordinata tekisligi bilan kesishish nuqtasini toping.

**6.2.13.** Quyidagi to‘g‘ri chiziqlarni

1) $\begin{cases} 2x + y - z - 3 = 0 \\ x + y + z - 1 = 0 \end{cases}$;

2) $\begin{cases} 2x - y + 3z - 5 = 0 \\ x + 2y - z + 2 = 0 \end{cases}$;

3) $\begin{cases} x + 4y - 6z - 3 = 0 \\ 2x - y + 3z + 4 = 0 \end{cases}$

koordinata o‘qlari bilan kesishish nuqtasini toping.

**6.2.14.** $\vec{a}(2; -1; -2)$ vektorga parallel bo‘lgan va

$\begin{cases} 2x - y + 3z - 5 = 0 \\ x + 2y - z + 2 = 0 \end{cases}$

to ‘g‘ri chiziqdan o‘tuvchi tekislik tenglamasini tuzing.

**6.2.15**. $\vec{a}(7; 9; 17)$ vektorga parallel bo‘lgan va

$\begin{cases} 5x - 2y - z - 3 = 0 \\ x + 3y - 2z + 5 = 0 \end{cases}$

to‘g‘ri chiziqdan o‘tuvchi tekislik tenglamasini tuzing.

**6.2.16**. Fazoda $N(2; 3; 1)$ nuqtadan o‘tib $\begin{cases} x + y = 0 \\ x + 3y - 1 = 0 \end{cases}$ va $\begin{cases} x - y + z + 4 = 0 \\ y + z - 2 = 0 \end{cases}$ to‘g‘ri chiziqlar bilan kesishadigan to‘g‘ri chiziq tenglamasi tuzilsin.

**6.2.17**. Quyida berilgan to‘g‘ri chiziqlarning qaysi berilgan tekislikda yotadi, qaysi birida unga parallel, qaysi birida u bilan kesishadi? Agar ular kesishsa kesishish nuqtasi topilsin.

1) $\frac{x-12}{4} = \frac{y-9}{3} = \frac{z-1}{1}$, $\quad 3x + 5y - z - 2 = 0$

2) $\frac{x+1}{2} = \frac{y-3}{4} = \frac{z}{3}$, $\quad 3x - 3y + 2z - 5 = 0$

3) $\frac{x-13}{7} = \frac{y-1}{2} = \frac{z-4}{3}$, $\quad x + 2y - 4z + 1 = 0$

4) $\frac{x-7}{5}=\frac{y-4}{1}=\frac{z-5}{4}$, $\qquad 3x-y+2z-5=0.$

**6.2.18.** To‘g‘ri chiziqlarning yo‘naltiruvchi kosinuslari topilsin:

1) $\frac{x-1}{4}=\frac{y-5}{-3}=\frac{z+2}{12}$; $\qquad$ 2) $\frac{x}{12}=\frac{y-7}{9}=\frac{z+3}{20}$

**6.2.19.** $\frac{x-1}{3}=\frac{y+2}{6}=\frac{z-5}{2}$ va $\frac{x}{2}=\frac{y-3}{9}=\frac{z+1}{6}$ to‘g‘ri chiziqlar orasidagi burchak topilsin.

**6.2.20**. Ikki $\begin{cases} 3x-4y-2z=0 \\ 2x+y-2z=0 \end{cases}$ va $\begin{cases} 4x+y-6z-2=0 \\ y-3z+2=0 \end{cases}$ to‘g‘ri chiziqlar orasidagi burchak topilsin.

**6.2.21**. To‘g‘ri chiziqlar orasidagi burchaklar kosinuslari topilsin:

1) $\begin{cases} x=3+t \\ y=7-2t \\ z=4+3t \end{cases}$ va $\begin{cases} x=2+5t \\ y=1-t \\ z=1 \end{cases}$;

2) $\begin{cases} 3x+y-z+1=0 \\ 3x-y+z=0 \end{cases}$ va $\begin{cases} x-y+1=0 \\ 2x+2y-5z+1=0 \end{cases}$

**6.2.22**. Kanonik tenglamalari $\frac{x-1}{\alpha}=\frac{y-3}{-3}=\frac{z+4}{\sqrt{2}}$, $\frac{x+2}{2}=\frac{y+13}{\alpha}=\frac{z-6}{\sqrt{2}}$ bo‘lgan to‘g‘ri chiziqlar α parametrning qanday qiymatida o‘zaro perpendikular bo‘ladi?

**6.2.23**. $\begin{cases} x=5+6t \\ y=1-3t \\ z=2+t \end{cases}$ to‘g‘ri chiziq bilan $7x+2y-3z+5=0$ tekislik orasidagi burchak topilsin.

**6.2.24.** $x+y-z=0$, $2x-3y+z=0$ to‘g‘ri chiziq bilan $3x+$ $+5y-4z+2=0$ tekislik orasidagi burchak topilsin.

**6.2.25**. $2x+y-z+4=0$, $x+y=0$ to‘g‘ri chiziqning $Oxy$ tekislikdagi proyeksiyasi tenglamasi tuzilsin.

**6.2.26**. $2x-y+z-8=0$, $4x+3y-z+14=0$ tekisliklarning kesishish chizig‘ida $2x+3y-6z-10=0$ tekislikdan 7 masofada joylashgan nuqtalar topilsin.

**6.2.27**. Quyida berilgan to‘g‘ri chiziqlar juftlaridan qaysilari ayqash, qaysilari kesishadi, qaysilari parallel yoki ustma-ust tushadi? Agar to‘g‘ri chiziqlar parallel bo‘lsa, ular orqali o‘tgan tekislik tenglamasi tuzilsin, agar ular kesishsa, ularni o‘z ichiga olgan tekislik tenglamasi tuzilsin va ularning kesishish nuqtasi topilsin:

1) $\begin{cases} x = 1 + 2t \\ y = 7 + t \\ z = 3 + 4t \end{cases}$ va $\begin{cases} x = 6 + t \\ y = -1 - 2t; \\ z = -2 + t \end{cases}$

2) $\begin{cases} x = 1 + 2t \\ y = 2 - 2t \\ z = -t \end{cases}$ va $\begin{cases} x = -2t \\ y = -5 + 3t \\ z = 4 \end{cases}$

3) $\begin{cases} x = 9t \\ y = 5t \\ z = -3 + t \end{cases}$ va $\begin{cases} 2x - 3y - 3z - 9 = 0 \\ x - 2y + z + 3 = 0 \end{cases}$

4) $\begin{cases} x = t \\ y = -8 - 4t \\ z = -3 - 3t \end{cases}$ va $\begin{cases} x + y - z = 0 \\ 2x - y + 2z = 0 \end{cases}$

**6.2.28**. $l$ to‘g‘ri chiziqning

$$\frac{x+1}{3} = \frac{y-2}{n} = \frac{z+3}{-2}$$

kanonik tenglamasidagi $n$ parametr qanday qiymat qabul qilganda u umumiy tenglamasi $x - 3y + 6z + 7 = 0$ bo‘lgan tekislikka parallel bo‘ladi?

**6.2.29**. $l$ to‘g‘ri chiziqning

$$\frac{x-2}{m} = \frac{y+1}{4} = \frac{z-5}{-3}$$

kanonik tenglamasidagi $m$ va $P$ tekislikning $3x - 2y + Cz + 1 = 0$ umumiy tenglamasidagi $C$ parametrlarning qanday qiymatida ular o‘zaro perpendikulyar bo‘ladilar?

**6.2.30**. Fazoda $M(3; -1; -4)$ nuqtadan o‘tib, $Oy$ o‘qni kesib o‘tadigan va $y + 2z = 0$ tekislikka kollinear(parallel) bo‘lgan to‘g‘ri chiziq tenglamalari tuzilsin.

## 6.3. Fazoda tekislik va to‘g‘ri chiziq orasidagi munosabatlarga doir misollar.

**6.3.1**. Tekisliklar orasidagi burchaklarning kosinuslari topilsin:

1) $2x - y + 3z = 0, \qquad x + 4y - 6z = 0;$

2) $x + 3y - 4z + 5 = 0, \qquad 2x + 2y + 2z - 7 = 0.$

**6.3.2**. $N(1;3;5)$ nuqtadan $2x + y + z - 1 = 0$; $3x + y + 2z - 3 = 0$ tekisliklarning kesishish chizig‘iga tushirilgan perpendikulyarning asosi topilsin.

**6.3.3**. $A(3;-2;1)$, $B(6;0;5)$ nuqtalar berilgan. $B$ nuqtadan o‘tib, $AB$ to‘g‘ri chiziqqa perpendikulyar bo‘lgan tekislik tenglamasi tuzilsin.

**6.3.4**. Quyidagi hollarning har birida tekisliklarning o‘zaro vaziyati aniqlansin:

1) $2x - 4y + 5z - 21 = 0$; $x - 3y + 18 = 0$; $6x + y + z - 30 = 0$;
2) $x + 2y - 3z = 0$; $3x + 6y - 9z + 10 = 0$; $2x + 4y - 6z - 1 = 0$;
3) $3x - y + 2z + 1 = 0$; $7x + 2y + z = 0$; $15x + 8y - z - 2 = 0$;
4) $5x - 2y + 4 = 0$; $3x + z - 5 = 0$; $8x - 2y + z + 7 = 0$;

**6.3.5**. Koordinatalar boshidan va $2x + 5y - 6z + 4 = 0$, $3y + 2z + {}$ $+6 = 0$ tekisliklarning kesishish chizig‘idan o‘tuvchi tekislik tenglamasi tuzilsin.

**6.3.6**. Fazoda $M(-3;1;0)$ nuqtadan va $x + 2y - z + 4 = 0$, $3x -$ $-y + 2z - 1 = 0$ to ‘g‘ri chiziq orqali o‘tuvchi tekislik tenglamasi tuzilsin.

**6.3.7**. $x + 2y + 3z - 4 = 0$, $3x + z - 5 = 0$ tekisliklarning kesishishi chizig‘idan o‘tuvchi va $Oy$, $Oz$ o‘qlaridan teng kesmalar ajratuvchi tekislik tenglamasi tuzilsin.

**6.3.8**. $2x - z = 0$, $x + y - z + 5 = 0$ tekisliklarning kesishish chizig‘idan o‘tib, $7x - y + 4z - 3 = 0$ tekislikka perpendikulyar bo‘lgan tekislik tenglamasi tuzilsin.

**6.3.9**. Quyidagi: $x - y = 0$, $x + y - 2z + 1 = 0$, $2x + z - 4 = 0$ tenglamalar bilan berilgan tekisliklarning kesishish nuqtasidan va

1) $Ox$ o‘qi orqali o‘tadigan;
2) $Oy$ o‘qi orqali o‘tadigan;
3) $Oz$ o‘qi orqali o‘tadigan;
4) $Oxy$ tekisligiga parallel;
5) $Oxz$ tekisligiga parallel;
6) $Oyz$ tekisligiga parallel;

7) koordinatalar boshidan va $M(2;1;7)$ nuqtadan o‘tadigan tekislik tenglamasi tuzilsin.

**6.3.10**. $x = 3 + 5t,\ \ y = -1 + t,\ \ z = 4 + t$ to‘g‘ri chiziqning $2x - -2y + 3z - 5 = 0$ tekislikdagi proyeksiyasining tenglamasi tuzilsin.

**6.3.11.** $M(3;-2;4)$ nuqtadan $5x + 3y - 7z + 1 = 0$ tekislikka tushirilgan perpendikulyar to‘g‘ri chiziq tenglamasi tuzilsin.

**6.3.12**. $M(1;2;-3)$ nuqtaning $6x - y + 3z - 41 = 0$ tekisllikka nisbatan simmetrik nuqtasi topilsin.

**6.3.13**. Koordinata boshidan ushbu $\frac{x+2}{4} = \frac{y-3}{5} = \frac{z-1}{-2}$ to‘g‘ri chiziqqa perpendikulyar tekislik o‘tkazilsin.

**6.3.14**. $N(4;3;10)$ nuqtaga $x = 1 + 2t,\ \ y = 2 + 4t,\ \ z = 3 + 5t$ to‘g‘ri chiziqqa nisbatan simmetrik bo‘lgan nuqta topilsin.

**6.3.15**. $N(3;2;1)$ nuqtadan $Ox$, $Oy$ va $Oz$ o‘qlariga tushirilgan perpendikulyar tekislikning tenglamasi tuzilsin.

**6.3.16**. $M(-1;0;4)$ nuqtadan $x = 1 + t,\ y = 2t,\ z = 4 - t$ to‘g‘ri chiziqqa tushirilgan perpendikulyar tekislikning tenglamasi tuzilsin.

**6.3.17**. Quyidagi har bir tekisliklarga nisbatan $Q(2;-1;1)$ nuqta va koordinata boshi bir tomonda yoki har xil tomonda yotganligini aniqlang:

1) $5x - 3y + z - 18 = 0$;
2) $2x + 7y + 3z + 1 = 0$;
3) $x + 5y + 12z - 1 = 0$;
4) $2x - y + z + 11 = 0$.

**6.3.18.** $3x - 4y - 2z + 5 = 0$ tekislik $M_1(3;-2;1)$ va $M_2(-2;5;2)$ nuqtalar bilan chegaralangan kesmani kesib o‘tishini isbotlang.

**6.3.19**. $5x - 2y + z - 1 = 0$ tekislik $M_1(1;4;-3)$ va $M_2(2;5;0)$ nuqtalar bilan chegaralangan kesmani kesib o‘tmasligini isbotlang.

**6.3.20.** $x - 2y + z + 5 = 0$ tekislikka perpendikulyar bo‘lgan va

$$\begin{cases} 2x - y + 3z - 5 = 0 \\ x + 2y - z + 2 = 0 \end{cases}$$

to‘g‘ri chiziqdan o‘tuvchi tekislik tenglamasini tuzing.

**6.3.21**. $x + 19y - 7z - 11 = 0$ tekislikka perpendikulyar bo‘lgan va

$$\begin{cases} 5x - y - 2z - 3 = 0 \\ 3x - 2y - 5z + 2 = 0 \end{cases}$$

to ‘g‘ri chiziqdan o‘tuvchi tekislik tenglamasini tuzing.

**6.3.22**. $\begin{cases} 2x + y - z + 1 = 0 \\ x + y + 2z + 1 = 0 \end{cases}$ to‘g‘ri chiziqdan o‘tuvchi va 1) $M_1(2; 5; -3)$; 2) $M_2(3; -2; -2)$ nuqtalarni qanoatlantiruvchi tekislik tenglamasini tuzing.

**6.3.23.** $\begin{cases} 3x - y + 2z + 9 = 0 \\ x + y - 3 = 0 \end{cases}$ to‘g‘ri chiziqdan o‘tuvchi va:

1) $M_1(4; -2; -3)$ nuqtani qanoatlantiruvchi;

2) $Ox$ o‘qiga parallel bo‘lgan;

3) $Oy$ o‘qiga parallel bo‘lgan;

4) $Oz$ o‘qiga parallel bo‘lgan tekislik tenglamasini tuzing.

**6.3.24**. $M(-3; 1; 0)$ nuqtadan va $x + 2y - z + 4 = 0$, $3x - y + 2z - -1 = 0$ to‘g‘ri chiziqdan o‘tuvchi tekislik tenglamasi tuzilsin.

**6.3.25**. $Oxy$ tekislikka perpendikulyar bo‘lgan va $x = t$, $y = -4 + t$, $z = 3 - t$ va $x = 1 - 2t$, $y = -3 + t$, $z = 4 - 5t$ dan iborat to‘g‘ri chiziqlarni kesib o‘tadigan to‘g‘ri chiziq tenglamalari tuzilsin.

**6.3.26**. $M(0; 1; 1)$ nuqtadan o‘tib, $y + 1 = 0$, $x + 2z - 7 = 0$ to‘g‘ri chiziq bilan to‘g‘ri burchak hosil qilgan va $x - 1 = 0$, $z + 1 = 0$ to‘g‘ri chiziqni to‘g‘ri burchak ostida kesib o‘tadigan to‘g‘ri chiziq tenglamalari tuzilsin.

**6.3.27**. Koordinatalar boshidan va $M(1; 2; 3)$ nuqtadan o‘tib, $x - y + +2z - 4 = 0$ tekislikka perpendikulyar tekislik tenglamasi tuzilsin.

**6.3.28**. $M_0(x_0;\ y_0; z_0)$ nuqtadan $Ax + By + Cz + D = 0$ tekislikka tushirilgan perpendikulyarning tenglamasi tuzilsin.

**6.3.29.** Quyidagi tekisliklarning koordinata o‘qlari bilan hosil bo‘lgan normal $\alpha$, $\beta$ va $\gamma$ burchaklarini va koordinata boshigacha bo‘lgan $p$ masofani toping:

1) $x + \sqrt{2}y + z - 10 = 0$;

2) $x - y - \sqrt{2}z + 16 = 0$;

3) $x + z - 6 = 0$;

4) $y - z + 2 = 0$;

5) $\sqrt{3}x + y + 10 = 0$.

**6.3.30**. Uchta tekislik berilgan:

$A_1x + B_1y + C_1z + D_1 = 0$

$A_2x + B_2y + C_2z + D_2 = 0$

$A_3x + B_3y + C_3z + D_3 = 0$

Ularning:

1) bitta umumiy nuqtaga ega bo‘lishi uchun;
2) bitta to‘g‘ri chiziqdan o‘tishi uchun;
3) juft-juft olganda parallel bo‘lishi uchun;
4) Prizma hosil qilishi uchun, ya’ni ikkita tekislikning kesishish chizig‘i uchinchi tekislikka parallel bo‘lishi uchun;
5) Ikkita tekislik o‘zaro parallel, uchinchi tekislik esa ularni kesib o‘tishi uchun qanday zarur va yetarli shartlar bajarilishi kerak?

# 7-MAVZU: TEKISLIKDA IKKINCHI TARTIBLI CHIZIQLAR.

**Reja:**

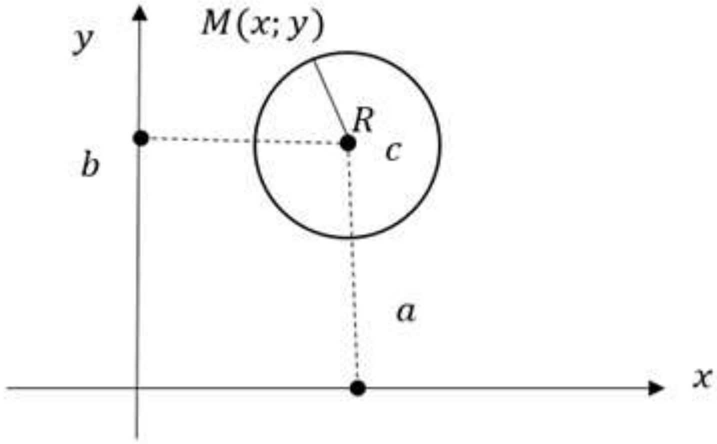
**1. Ellips.**
**2. Giperbola.**
**3. Parabola.**

**Tayanch iboralar**: ellips, giperbola, parabola, diametr, vatar, fokus, parametr, asimptota, direkrtisa, ekssentrisitet.

## 7.1.Ellips.

**Ta’rif.** Tekislikda berilgan nuqtadan bir xil uzoqlikdagi nuqtalarning geometrik o‘rniga aylana deyiladi.

Aylananing ta’rifidan foydalanib uning tenglamasini keltirib chiqaramiz. Bizga Dekart koordinatalar sistemasi berilgan bo‘lsin. Koordinatalar sistemasida $C(a;b)$ nuqta berilgan bo‘lsin. $C(a;b)$ nuqtadan bir xil $(R)$ uzoqlikdagi $M(x;y)$ nuqtalar to‘plamiga aylana deyilar ekan.




**7.1.1-chizma**

$|CM| = R$ aylana tenglamasi bo‘ladi.

$|CM| = \sqrt{(x-a)^2 + (y-b)^2}$ ekanligi kelib chiqadi.

$$\sqrt{(x-a)^2 + (y-b)^2} = R \Longrightarrow$$

$$(x-a)^2 + (y-b)^2 = r^2 \qquad (7.1)$$

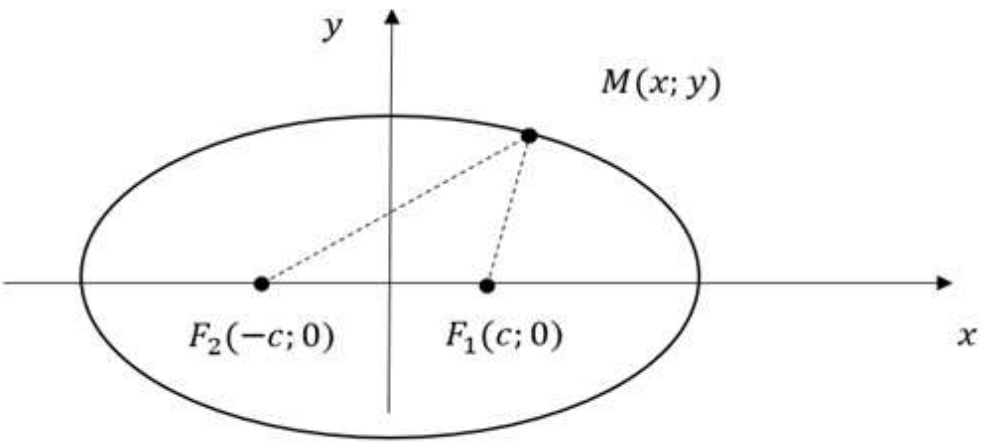
(7.1) tenglama markazi $C(a; b)$ nuqtada radiusi $R$ ga teng bo‘lgan tenglamasini keltirib chiqardik.

Xususiy hollarda markazi koordinatalar boshida bo‘lgan, ya’ni $O(0; 0)$ da bo‘lsa, $x^2 + y^2 = R^2$ bo‘ladi. Bu tenglama markazi koordinatalar boshida radiusi $R$ ga teng bo‘lgan aylana tenglamasi.

**Ta’rif.** Tekislikda qo‘zg‘almaydigan ikki nuqtagacha masofalarning yig‘indisi o‘zgarmas bo‘lgan nuqtalarning geometrik o‘rni ***ellips*** deyiladi.

Bizga qo‘zg‘almas ikkita nuqta berilgan bo‘lsin. Mana shu qo‘zg‘almas ikki nuqtaga ***fokus*** deyiladi.

Tekislikda ikkita $F_1$ va $F_2$ nuqta berilgan bo‘lsin. $F_1$ va $F_2$ nuqtalardan to‘g‘ri chiziq o‘tkazamiz va to‘g‘ri chiziqqa yo‘nalish berib uni absissa o‘qi deymiz. $F_1$ va $F_2$ nuqtalarning o‘rtasidan ordinata o‘qini o‘tkazamiz.




**7.1.2-chizma**

$|F_1F_2| = 2c$ ga teng bo‘lsin, bundan kelib chiqadiki $F_1(c;0)$, $F_2(-c;0)$ bo‘ladi. Ellepsning ta’rifini qanoatlantiruvchi $M(x;y)$ nuqta bo‘lsin.

$$|F_1M| + |F_2M| = 2a \text{ bo‘ladi.}$$

$$|F_1M| = \sqrt{(x-c)^2 + (y-0)^2},\ |F_2M| = \sqrt{(x+c)^2 + (y-0)^2} \Longrightarrow$$

$$\sqrt{(x-c)^2 + y^2} + \sqrt{(x+c)^2 + y^2} = 2a$$

$$\sqrt{(x-c)^2 + y^2} = 2a - \sqrt{(x+c)^2 + y^2}$$

$$x^2 - 2xc + c^2 + y^2 =$$

$$= 4a^2 - 4a\sqrt{(x+c)^2 + y^2} + x^2 + 2xc + c^2 + y^2$$

$$-2xc = 4a^2 - 4a\sqrt{(x+c)^2 + y^2} + 2xc$$

$$4a^2 - 4a\sqrt{(x+c)^2 + y^2} + 4xc = 0$$

$$a^2 - a\sqrt{(x+c)^2 + y^2} + xc = 0$$

$$a^2 + xc = a\sqrt{(x+c)^2 + y^2}$$

$$a^4 + 2a^2xc + x^2c^2 = a^2(x^2 + 2xc + c^2 + y^2)$$

$$a^4 + 2a^2xc + x^2c^2 = a^2x^2 + 2a^2xc + a^2c^2 + a^2y^2$$

$$a^4 + x^2c^2 = a^2x^2 + a^2c^2 + a^2y^2$$

$$a^4 + x^2c^2 - a^2x^2 - a^2c^2 - a^2y^2 = 0$$

$$a^4 + x^2c^2 - a^2x^2 - a^2c^2 - a^2y^2 = 0$$

$$(a^2 - c^2)x^2 + a^2y^2 = a^2(a^2 - c^2)$$

$$a^2 + b^2 = c^2 \Longrightarrow\ b^2 = a^2 - c^2 \Longrightarrow b^2x^2 + a^2y^2 = a^2b^2 \Longrightarrow$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad (7.2)$$

ekanligi kelib chiqadi.

Fokuslar orasidagi masofani katta o‘qqa nisbatiga ekssentrisitet deyiladi.

$$\varepsilon=\frac{2c}{2a} \Longrightarrow \varepsilon=\frac{c}{a} \qquad (7.3)$$

Ellipsning kichik o‘qiga parallel va uning markazidan $a/\varepsilon$ masofadan o‘tuvchi parallel to‘g‘ri chiziqlar ellipsning ***direktrisalari*** deyiladi.

$$x=\pm\frac{c}{\varepsilon}=\pm\frac{a}{c/a}=\pm\frac{a^2}{c}$$

**1-Misol.** $x^2+4y^2=4$ tenglama ellipsni ifodalashini ko‘rsating va uning barcha xarakteristikalarini toping.

**Yechish:** Dastlab berilgan tenglamani ikkala tomonini 4 soniga bo‘lamiz:

$$\frac{x^2}{4}+\frac{y^2}{1}=1$$

bu yerdan berilgan tenglama yarim o‘qlari $a=2$ va $b=1$ bo‘lgan ellipsni ifodalashini ko‘ramiz. Unda $c^2=a^2-b^2=3$ bo‘lgani uchun qaralayotgan ellipsning fokuslari $F_1(-\sqrt{3};0)$ va $F_2(\sqrt{3};0)$ nuqtalarda joylashganligini ko‘ramiz. Bu natijalardan foydalanib, ellipsning ekssentrisiteti va direktrisalarini topamiz:

$$\varepsilon=\frac{c}{a}=\frac{\sqrt{3}}{2}; \quad x=\pm\frac{a}{\varepsilon}=\pm\frac{\sqrt{3}}{2}\cdot\frac{1}{2}=\pm\frac{\sqrt{3}}{4}.$$

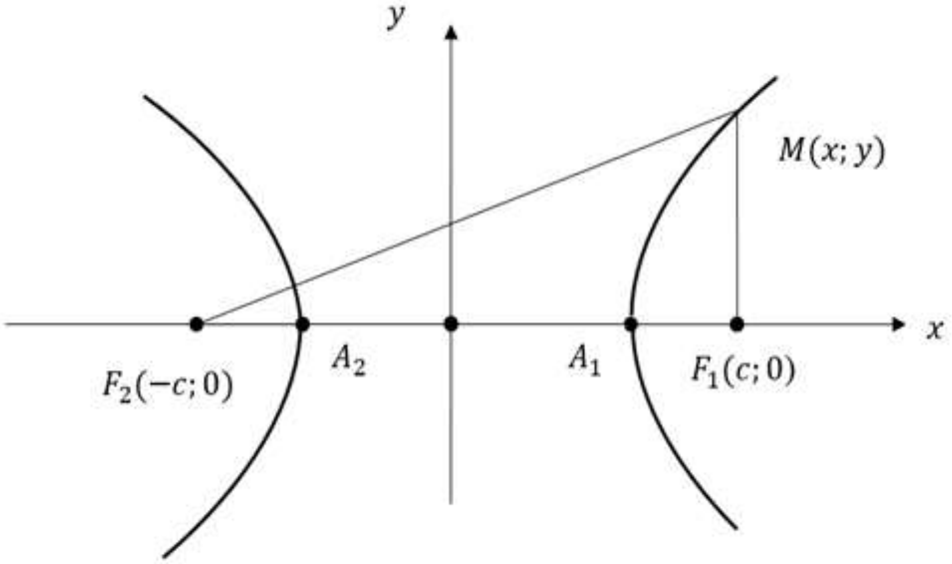
Ellipsga tegishli $M(x;y)$ nuqtaning fokal radiuslari

$$r_1=a+\varepsilon x=2+\frac{\sqrt{3}}{2}x,\ r_2=a+\varepsilon x=2+\frac{\sqrt{3}}{2}x$$

formulalar bilan topiladi.

### 7.2. Giperbola.

Fokus deb ataladigan ikki nuqtagacha bo‘lgan masofalarining ayirmasi o‘zgarmas songa teng bo‘lgan nuqtalarning geometrik o‘rniga ***giperbola*** deyiladi. Fokuslar - $F_1, F_2$, ular orasidagi masofa $|F_1F_2| = 2c$. Fokuslar yotgan to‘g‘ri chiziqqa yo‘nalish berib absissa o‘qi deylik.




**7.2.1-chizma**

Absissa o‘qini 2 ta fokusdan o‘tkazaylik. Fokuslarning o‘rtasidan absissa o‘qiga perpendikulyar qilib ***ordinata*** o‘qini o‘tkazaylik.

$$|F_1M| - |F_2M| = 2a$$

$$|F_1M| = \sqrt{(x-c)^2 + y^2},\ |F_2M| = \sqrt{(x+c)^2 + y^2}$$

$$\left|\sqrt{(x-c)^2 + y^2} - \sqrt{(x+c)^2 + y^2}\right| = 2a$$

$$\sqrt{(x-c)^2 + y^2} = \pm 2a + \sqrt{(x+c)^2 + y^2}$$

$$x^2 - 2xc + c^2 + y^2 =$$

$$= x^2 + 2xc + c^2 + y^2 \pm 4a\sqrt{(x+c)^2 + y^2} + 4a^2$$

$$\pm 4a\sqrt{x^2 + 2xc + c^2 + y^2} = 4a^2 + 4xc$$

$$a^2x^2 + 2a^2xc + a^2c^2 + a^2y^2 = a^4 + 2a^2xc + x^2c^2$$

$$x^2(a^2 - c^2) - a^2(a^2 - c^2) + a^2y^2 = 0$$

$$x^2(a^2 - c^2) - a^2y^2 = a^2(a^2 - c^2) \Rightarrow c > 0 \Rightarrow\ c^2 - a^2 = b^2$$

$$\Delta MF_1F_2 \Rightarrow\ F_1M - F_2M = 2a\ \Rightarrow\ |F_1M - F_2M| < |F_1F_2| \Rightarrow$$

$$2a < 2c \Rightarrow a < c$$
$$x^2b^2 - a^2y^2 = a^2b^2 \Rightarrow$$
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad (7.4)$$

ekanligi kelib chiqadi.

Bu yerda $2a$ −haqiqiy o‘q, $2b-$ mavhum o‘q, $2c-$ fokuslar orasidagi masofa.

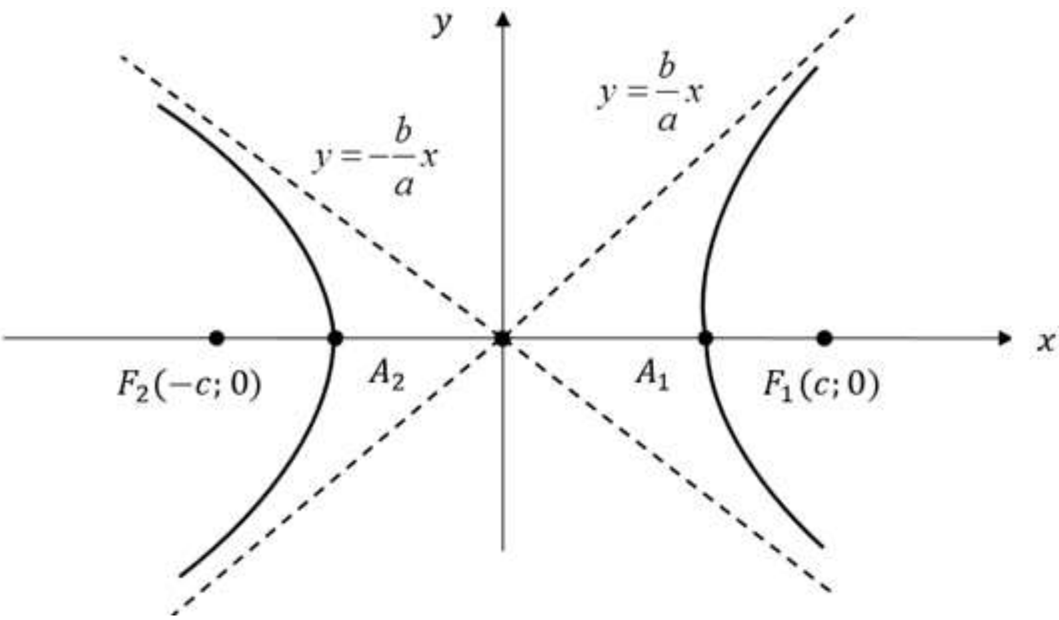
Giperbolaning ekssentrisiteti deb, giperbola fokuslari orasidagi masofaning haqiqiy o‘qqa nisbatiga aytiladi.

$$\varepsilon = \frac{2c}{2a} = \frac{c}{a} \Rightarrow c^2 - a^2 = b^2 \Rightarrow c > a \Rightarrow \varepsilon > 1$$

Giperbolaning mavhum o‘qiga parallel va uning markazidan

$$x = \frac{c}{\varepsilon} = \frac{a}{c/a} = \frac{a^2}{c}$$

masofada yotuvchi ikki to‘g‘ri chiziqqa ***direktrisalari*** deyiladi.




**7.2.2-chizma**

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow d: x = \pm\frac{c}{\varepsilon} = \pm\frac{a^2}{c}$$
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1 \Rightarrow \ y = \pm\frac{b^2}{c} \Rightarrow \frac{c^2}{c} < a \Rightarrow a < c.$$

$$y = \pm \frac{b}{a} x \tag{7.5}$$

***asimptota*** deyiladi.

$a = b$ bo‘lsa, ***teng yonli geperbola*** deyiladi.

**Fokal radiuslar.** Ellips uchun

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \implies \frac{y^2}{b^2} = 1 - \frac{x^2}{a^2} \implies$$

$$y^2 = b^2\left(1 - \frac{x^2}{a^2}\right) \implies a^2 - c^2 = b^2$$

$$|MF_1| = r_1 = \sqrt{(x-c)^2 + y^2} = \sqrt{(x-c)^2 + b^2 - \frac{b^2x^2}{a^2}} =$$

$$= \sqrt{x^2 - 2xc + c^2 + b^2 - \frac{b^2x^2}{a^2}} = \sqrt{x^2 - 2xc + c^2 + b^2 - \frac{b^2x^2}{a^2}} =$$

$$= \sqrt{\frac{a^2 - b^2}{a^2} x^2 - 2xc + c^2 + b^2} = \sqrt{\frac{c^2}{a^2} x^2 - 2xc + a^2} =$$

$$= \left|\frac{c}{a} x - a\right| = |\varepsilon x - a|.$$

$$|MF_2| = r_2 = \sqrt{(x+c)^2 + y^2} = \left|\frac{c}{a} x + a\right| = |\varepsilon x + a| \implies$$

$$0 < c < a$$

$$r_1 = a - \frac{c}{a} x, \quad r_2 = a + \frac{c}{a} x \implies r_1 = r_2 = 2a$$

Ellips yoki giperbola uchun fokal radiusi degani, uning biror nuqtasidan fokuslarigacha bo‘lgan masofalar.

**Giperbola uchun**

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1 \implies \frac{x^2}{a^2} + 1 = \frac{y^2}{b^2} \implies y^2 = b^2\left(1 + \frac{x^2}{a^2}\right)$$

$$|MF_1| = r_1 = \sqrt{(x-c)^2 + y^2} = \sqrt{x^2 - 2xc + c^2 + \left(\frac{x^2}{a^2} + 1\right) b^2} =$$

$$= \sqrt{x^2 - 2xc + c^2 + b^2 + \frac{b^2x^2}{a^2}} =$$

$$= \sqrt{\frac{(a^2 - b^2)}{a^2}x^2 - \frac{a^2}{a^2}2xc + \frac{c^2}{a^2} + \frac{b^2}{a^2}b^2 + a^2b^2} = \left|\frac{c}{a}x - a\right|.$$

$$|MF_2| = r_2 = \sqrt{(x + c)^2 + y^2} = \left|\frac{c}{a}x + a\right| \;\Rightarrow c^2 - a^2 = b^2 \Rightarrow$$

$$c < a < 0$$

$$r_1 = \left|\frac{c}{a}x - a\right| = \frac{c}{a}x - a,\quad r_2 = \frac{c}{a}x + a \qquad (7.6)$$

bo‘ladi.

**2-Misol.** Quyidagi kanonik tenglamasi bilan berilgan giperbolaning barcha xarakteristikalarini toping:

$$\frac{x^2}{16} - \frac{y^2}{9} = 1.$$

Bu giperbolaning absissasi $x = 8$, ordinatasi $y > 0$ bo‘lgan $M$ nuqtasining fokal radiuslarini aniqlang.

**Yechish:** Berilgan tenglamani (7.4) kanonik tenglama bilan taqqoslab, giperbolaning haqiqiy va mavhum yarim o‘qlari $a = 4$, $b = 3$ ekanligini ko‘ramiz.Bu holda $c^2 = a^2 + b^2 = 16 + 9 = 25 \Rightarrow c = 5$ bo‘lgani uchun giperbolaning fokuslari $F_1(-5; 0)$ va $F_2(5; 0)$ nuqtalarda joylashganligini aniqlaymiz. Berilgan giperbolaning asimptotalari

$$y = \pm\frac{b}{a}x = \pm\frac{3}{4}x = \pm 0{,}75x,$$

ekssentrisiteti $\varepsilon = c/a = 5/4 = 1{,}25$, direktrisalarining tenglamasi esa $x = \pm a/\varepsilon = \pm 4/1{,}25 = \pm 3{,}2$ bo‘ladi. Endi giperbolaning berilgan $M(8; 0)$ nuqtasining fokal radiuslarini topamiz. Bu nuqta giperbolaning o‘ng shoxida joylashgan va shu sababli (7.6) formulani $" + "$ ishora bilan qaraymiz:

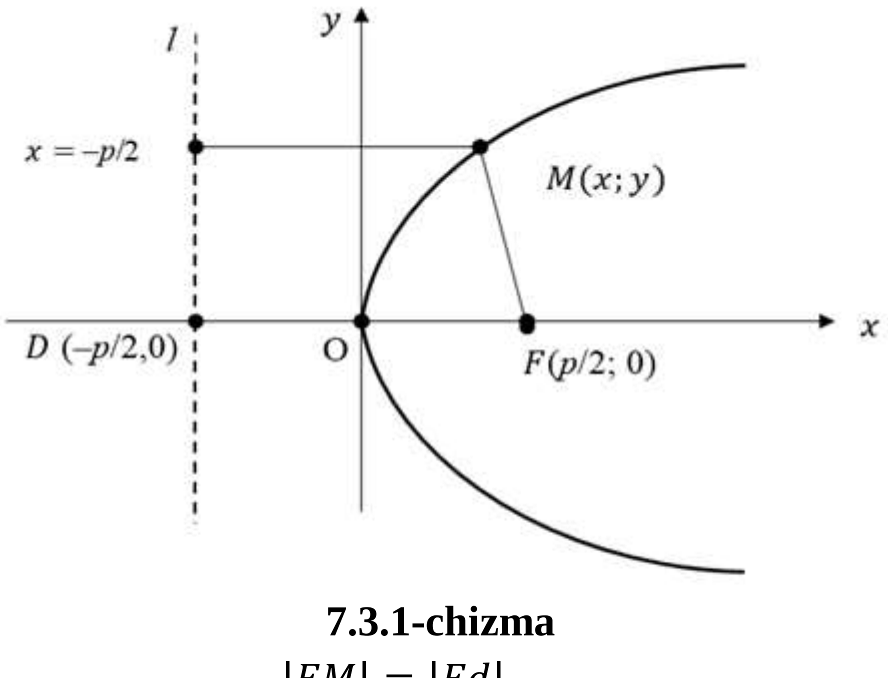
$$r_1 = a + \varepsilon x = 4 + 1{,}25 \cdot 8 = 14,$$

$$r_2 = -a + \varepsilon x = -4 + 1{,}25 \cdot 8 = 6$$

### 1.3.Parabola.

Berilgan nuqtadan va berilgan to‘g‘ri chiziqdan bir xil uzoqlikda joylashgan nuqtalarning geometrik o‘rniga ***parabola*** deyiladi.

Ta’rifdan foydalanib, parabolaning kanonik tenglamasini keltirib chiqaraylik. Bizga $F$ nuqta va $l$ to‘g‘ri chiziq berilgan ekan. $F$ nuqtadan o‘tib $l$ to‘g‘ri chiziqqa perpendikulyar qilib $Ox$ o‘qini olaylik. $l$ to‘g‘ri chiziqqa parallel va $F$ nuqta bilan $l$ to‘g‘ri chiziqni o‘rtasidan $Oy$ o‘qini o‘tkazaylik. Kanonik tenglamasini topmoqchi bo‘lgan parabola ustidan ixtiyoriy $M(x;y)$ nuqta olaylik. $F$ nuqtadan $l$ to‘g‘ri chiziqqacha bo‘lgan masofa $P$ bo‘lsin. U holda $F$ nuqtaning koordinatalari $F(\frac{p}{2};0)$ va $l$ to‘g‘ri chiziqning tenglamasi $x = -\frac{p}{2}$ bo‘ladi. $F$ nuqtadan $M$ nuqtagacha bo‘lgan masofa $|FM| = \sqrt{\left(x - \frac{p}{2}\right)^2 + y^2}$ bo‘ladi. $M$ nuqtadan $l$ to‘g‘ri chiziqqacha bo‘lgan masofa esa $d = \left|x + \frac{p}{2}\right|$ bo‘ladi.




**7.3.1-chizma**

$$|FM| = |Fd|$$

$$\sqrt{\left(x - \frac{p}{2}\right)^2 + y^2} = \left|x + \frac{p}{2}\right| \Rightarrow x^2 - 2x\frac{p}{2} + \frac{p^2}{4} + y^2 =$$

$$= x^2 + 2x\frac{p}{2} + \frac{p^2}{4} \Rightarrow y^2 - xp = xp \Rightarrow$$

$$y^2 = 2px \qquad (7.7)$$

tenglamamiz ***parabola tenglamasi*** hisoblanadi.

**3-Misol.** $Ox$ o‘qi parabolaning simmetriya o‘qi bo‘lib, uning uchi koordinatalar boshida yotadi. Parabola uchidan fokusigacha bo‘lgan masofa 4 birlikka teng. Parabola va uning direktrisasi tenglamasini toping.

**Yechish:** Dastlab, masala shartiga asosan, parabolaning $p$ parametrini topamiz:

$$|OF| = 4 \Rightarrow p/2 = 4 \Rightarrow p = 8.$$

Unda, (7.7) formulaga asosan, parabola tenglamasini topamiz:

$$y^2 = 2px \Rightarrow y^2 = 2 \cdot 8x = 16x.$$

Bu yerdan direktrisa tenglamasi $x = -p/2 \Rightarrow x = -4$ ekanligini ko‘ramiz.

Shuni ta’kidlab o‘tish kerakki, $y = ax^2 + bx + c \ (a \neq 0)$ kvadrat uchhadning grafigi uchi koordinatalari

$$x_0 = -\frac{b}{2a}, \quad y_0 = \frac{4ac - b^2}{4a}$$

bo‘lgan $M_0(x_0; y_0)$ nuqtada, simmetriya o‘qi esa $Oy$ o‘qiga parallel va $x = -b/2a$ tenglamaga ega bo‘lgan vertikal to‘g‘ri chiziqdan tashkil topgan paraboladan iboratdir. Agar $a > 0$ bo‘lsa, parabola yuqoriga, $a < 0$ bo‘lsa, pastga yo‘nalgan bo‘ladi.

## Mustaqil yechish uchun topshiriqlar.

### 7.1. Ellipsga doir misollar.

**7.1.1**. Markazi $C(x; y)$ va radiusi $R$ ga teng bo‘lgan aylana tenglamasini tuzing.

1) $C(2; -3)$, $R = 5$; 2) $C(-5; 4)$, $R = 2$;
3) $C(7; 1)$, $R = 3$; 4) $C(-2; 9)$, $R = 4$;
5) $C(-4; 6)$, $R = 7$; 6) $C(6; -3)$, $R = 6$;

**7.1.2**. Quyidagi har bir holda aylananing kanonik tenglamasini tuzing. Markazi va radiusini aniqlang.

1) $x^2 + y^2 - 6x = 0;$
2) $x^2 + y^2 + 6x - 8y = 0;$
3) $x^2 + y^2 - 10x + 24y - 56 = 0;$
4) $3x^2 + 3y^2 + 6x - 4y - 1 = 0;$
5) $x^2 + y^2 - 2x + 4y = 0;$
6) $3x^2 + 3y^2 - 2x + 7y + 1 = 0;$
**7.1.3**. 1) $x^2 + y^2 - 1 = 0;$ 2) $x^2 + y^2 - 10x + 24y - 56 = 0;$
3) $9x^2 + 9y^2 - 3 = 0;$ 4) $3x^2 + 3y^2 + 6x - 4y - 1 = 0;$
aylanalarga nisbatan $A(3;1)$, $B(1;0)$, $C(-2;0)$ va $D(-2;1)$ nuqtalarning vaziyatini aniqlang.
**7.1.4**. Koordinatalari quyidagi:
1) $(x-1)^2 + (y-3)^2 \geq 25;$ 2) $16 \leq (x-1)^2 + (y+3)^2 \leq 25;$
3) $(x-1)^2 + (y-2)^2 \leq 25,$ 4)$(x-4)^2 + (y-6)^2 \leq 9;$
5) $x^2 + y^2 - 6x \leq 0, y \geq 0;$ 6) $x^2 + y^2 - 4x \leq 0, |x| \geq 1$
tengsizliklarni qanoatlantiruvchi nuqtalar tekislikda qanday joylashadi?
**7.1.5**. $Ox$ o‘qiga $M(6;0)$ nuqtada urinuvchi va $N(9;9)$ nuqta orqali o‘tadigan aylananing tenglamasi tuzilsin.
**7.1.6**. Markazi $C(2;3)$ nuqtada yotadigan va $x - 2y + 1 = 0$ to‘g‘ri chiziqqa urinadigan aylananing tenglamasi tuzilsin.
**7.1.7**. $A(-4;1)$, $B(3;2)$, $C(-2;-5)$ , $D(5;0)$ va $E(3;-6)$ nuqtalar berilgan.
1) $A$, $B$, $C$ nuqtalardan; 2) $A$, $B$, $D$ nuqtalardan;
3) $A$, $B$, $E$ nuqtalardan; 4) $B$, $C$, $D$ nuqtalardan;
5) $B$, $C$, $E$ nuqtalardan; 6) $C$, $D$, $E$ nuqtalardan
o‘tuvchi aylana tenglamasini tuzing.
**7.1.8**. Berilgan $A(2;7)$, $B(-2;1)$ nuqtalar orqali o‘tadigan va radiusi $r = \sqrt{26}$ bo‘lgan aylananing tenglamasini tuzing.
**7.1.9**. Quyidagilardan foydalanib aylana tenglamasini tuzing:
1) markazi koordinata boshida va radiusi $R = 3$ bo‘lgan;
2) markazi $C(2;-3)$ nuqtada va radiusi $R = 7$ bo‘lgan;
3) markazi koordinata boshida va $C(6;-8)$ nuqtadan o‘tuvchi;
4) markazi $A(2;6)$ nuqtada va $C(-1;2)$ nuqtadan o‘tuvchi;

5) $A(3;2)$ va $B(-1;6)$ nuqtalar diametrning uchlari bo‘lgan;
6) markazi koordinata boshida va $3x-4y+20=0$ to‘g‘ri chiziqqa urinuvchi;
7) markazi $C(1;-1)$ nuqtada va $5x-12y+9=0$ to‘g‘ri chiziqqa urinuvchi;
8) $A(3;1)$ va $B(-1;3)$ nuqtalardan o‘tadigan va markazi $3x-y-$ $-2=0$ to‘g‘ri chiziqda yotuvchi;
9) $A(1;1)$, $B(1;-1)$ va $C(2;0)$ uchta nuqtadan o‘tuvchi;
10) $M_1(-1;5)$, $M_2(-2;-2)$ va $M_3(5;5)$ uchta nuqtadan o‘tuvchi.

**7.1.10**. Ikki parallel to‘g‘ri chiziq tenglamasi $2x+y-5=0$, $2x+$ $+y+15=0$ va bu to‘g‘ri chiziqlarning biri bilan $A(2;1)$ nuqtada urinuvchi aylana berilgan. Aylananing tenglamasini tuzing .

**7.1.11**. Ikki aylana markazidan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing:
1) $(x-3)^2+y^2=9$ va $(x+2)^2+(y-1)^2=1$;
2) $(x+2)^2+(y-1)^2=16$ va $(x+2)^2+(y+5)^2=25$;
3) $x^2+y^2-4x+6y=0$ va $x^2+y^2-6x=0$;
4) $x^2+y^2-x+2y=0$ va $x^2+y^2+5x+2y-1=0$;

**7.1.12**. $A(1;-1)$ nuqta va ikki $x^2+y^2+2x-2y-23=0$, $x^2+$ $+y^2-6x+12y-35=0$ aylananing kesishgan nuqtasi orqali o‘tuvchi aylana tenglamasini tuzing.

**7.1.13**. $x^2+y^2+3x-y=0$, $3x^2+3y^2+2x+y=0$ aylananing kesishgan nuqtalari orqali o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.

**7.1.14**. Quyidagi malumotlarga ko‘ra ellipsning kanonik tenglamasi tuzilsin:
1) yarim o‘qlari mos ravishda 5 va 4ga teng;
2) katta o‘qi 10, fokuslari orasidagi masofa 8 ga teng;
3) katta o‘qi 26 va ekssentrisiteti $\varepsilon=\frac{12}{13}$.

**7.1.15**. $\frac{x^2}{25}+\frac{y^2}{16}=1$ ellips fokuslarining koordinatalari topilsin.

**7.1.16**. $\frac{x^2}{25}+\frac{y^2}{169}=1$ ellips fokuslarining koordinatalari topilsin.

**7.1.17.** $\frac{x^2}{9}+\frac{y^2}{25}=1$ ellipsga nisbatan quyidagi nuqtalarning: 1) $A_1(1;2)$; 2) $A_2(-1;3)$; 3) $A_3(6;1)$; 4) $A_4(-1;7)$ vaziyati aniqlansin.
**7.1.18**. O‘qlari koordinata o‘qlari bilan ustma-ust tushuvchi va $P(2;2)$, $Q(3;1)$ nuqtalar orqali o‘tuvchi ellips tenglamasi tuzilsin.
**7.1.19**. Katta o‘qi 2 birlikka teng, fokuslari $F_1(0;1)$, $F_2(1;0)$ nuqtalarda bo‘lgan ellipsning tenglamasi tuzilsin.
**7.1.20**. Ellips fokuslarining biridan katta o‘qi uchlarigacha masofalar mos ravishda 7 va 1 ga teng. Bu ellipsning tenglamasini tuzing.
**7.1.21**. $\frac{x^2}{36}+\frac{y^2}{20}=1$ ellips direktrisalarining tenglamalarini yozing.
**7.1.22**. Ellipsning direktrisalari $x=\pm 8$ to‘g‘ri chiziqlar, uning kichik o‘qi 8 ga teng ekanligi ma’lum bo‘lsa, ellips tenglamasini tuzing.
**7.1.23**. $\frac{x^2}{100}+\frac{y^2}{36}=1$ elliipsda o‘ng fokusigacha masofa chap fokusigacha bo‘lgan masofasiga nisbatan 4 marta katta bo‘lgan nuqta topilsin.
**7.1.24**. Quyidagilarni bilgan holda fokusi absissa o‘qida yotib, koordinata boshiga nisbatan simmetrik bo‘lgan ellips tenglamasini tuzing:
1) uning yarim o‘qlari 5 va 2 ga teng;
2) uning katta o‘qi 10 ga teng, fokuslar orasidagi masofa esa $2c=8$;
3) uning kichik o‘qi 24 ga teng, fokuslar orasidagi masofa esa $2c=10$;
4) fokuslar orasidagi masofa $2c=6$ ekssentrisiteti $\varepsilon=\frac{3}{5}$;
5) uning katta o‘qi 20 ga teng va ekssentrisiteti $\varepsilon=\frac{3}{5}$;
6) uning kichik o‘qi 10 ga teng , ekssentrisiteti $\varepsilon=\frac{12}{13}$;
7) direktrisalar orasidagi masofa 5 ga va fokuslar orasidagi masofa $2c=4$ ga teng;
8) uning katta o‘qi 8 ga teng, direktrisalar orasidagi masofa 16 ga teng;
9) uning kichik o‘qi 6 ga teng va direktrisalar orasidagi masofa 13 ga teng;
10) direktrisalar orasidagi masofa 32 ga va ekssentrisiteti $\varepsilon=\frac{1}{2}$.

**7.1.25**. Quyidagilarni bilgan holda fokusi ordinata o‘qida yotib, koordinata boshiga nisbatan simmetrik bo‘lgan ellips tenglamasini tuzing:

1) uning yarim o‘qlari mos ravishda 7 va 2 ga teng;

2) uning katta o‘qi 10 ga teng , fokuslar orasidagi masofa esa $2c = 8$;

3) fokuslar orasidagi masofa $2c = 24$ va ekssentrisiteti $\varepsilon = \frac{12}{13}$;

4) uning kichik o‘qi 16 va ekssentrisiteti $\varepsilon = \frac{3}{5}$;

5) fokuslar orasidagi masofa $2c = 6$ va direktrisalar orasidagi masofa $16\frac{2}{3}$ ga teng;

6) direktrisalar orasidagi masofa $10\frac{2}{3}$ ga va ekssentrisiteti $\varepsilon = \frac{3}{4}$ ga teng;

**7.1.26**. Ellipsning yarim o‘qlarini toping:

1) $\frac{x^2}{16}+\frac{y^2}{9}=1$; 2) $\frac{x^2}{4}+y^2=1$; 3) $x^2 + 25y^2 = 25$;

4) $x^2 + 5y^2 = 15$; 5) $4x^2 + 9y^2 = 25$; 6) $9x^2 + 25y^2 = 1$;

7) $x^2 + 4y^2 = 1$; 8) $16x^2 + y^2 = 16$; 9) $25x^2 + 9y^2 = 1$;

10) $x^2 + y^2 = 1$.

**7.1.27**. Ellips tenglamasi $9x^2 + 25y^2 = 225$ berilgan bo‘lsa, quyidagilarni toping: 1) yarim o‘qlarini; 2)fokuslarni; 3) ekssentrisiteti; 4) direktrisa tenglamalarini tuzing.

**7.1.28**. Ellips tenglamasi $9x^2 + 5y^2 = 45$ berilgan bo‘lsa, quyidagilarni toping:

1) yarim o‘qlarini; 2) fokuslarni; 3) ekssentrisiteti; 4) direktrisa tenglamalarini tuzing .

**7.1.29.** Quyidagi nuqtalardan qaysi birlari ushbu $8x^2 + 5y^2 = 77$ ellipsda yotadi:

1) $A_1(-2; 3)$; 2) $A_2(2; -2)$; 3) $A_3(2; -4)$; 4) $A_4(-1; 3)$;

5) $A_5(-4; -3)$; 6) $A_6(3; -1)$; 7) $A_7(3; -2)$; 8) $A_8(2; 1)$;

9) $A_9(0; 15)$; 10) $A_{10}(0; -16)$

shulardan qaysilari ichki, qaysilari tashqi nuqtalar?

**7.1.30.** Fokuslari absissa o‘qida joylashgan bo‘lib, koordinata boshiga nisbatan simmetrrik bo‘lgan ellipsning tenglamasini tuzing agar quyidagilar berilgan bo‘lsa:

1) ellipsdan $M_1(-2\sqrt{5}; 2)$ nuqta va uning kichik yarim o‘qi $b = 3$;

2) ellipsdan $M_1(2; -2)$ nuqta va uning katta yarim o‘qi $a = 4$;

3) ellipsdan $M_1(4; -\sqrt{3})$ nuqta va $M_2(2\sqrt{2}; 3)$ nuqta;

4) ellipsdan $M_1(\sqrt{15}; -1)$ nuqta va fokuslar orasidagi masofa $2c = 8$;

5) ellipsdan $M_1(2; -\frac{5}{3})$ nuqta va uning ekssentrisiteti $\varepsilon = \frac{2}{3}$;

6) ellipsdan $M_1(8; 12)$ nuqta va chap fokusgacha bo‘lgan masofa $r_1 = 20$ ga teng;

7) ellipsdan $M_1(-\sqrt{5}; 2)$ nuqta va uning direktrisalari orasidagi masofa 10 ga teng.

## 7.2. Giperbolaga doir misollar.

**7.2.1**. $x^2 - y^2 = 1$ giperbolaga nisbatan $A(4; 1)$, $B(1; -2)$, $C(\sqrt{2}; 1)$ nuqtalarning vaziyati aniqlansin.

**7.2.2**. Quyidagi malumotlarga ko‘ra:

1) haqiqiy o‘qi $a = 5$ mavhum o‘qi $b = 3$;

2) fokuslari orasidagi masofa 10 ga, haqiqiy o‘qi esa 8 ga teng giperbolaning kanonik tenglamasi tuzilsin.

**7.2.3**. Quyidagi ma’lumotlarga ko‘ra:

1) ekssentrisiteti $\varepsilon = \frac{12}{13}$ haqiqiy o‘qi 48 ga teng;

2) haqiqiy o‘qi 16 ga, asimptotasi bilan absissa o‘qi orasidagi $\varphi$ burchak tangensi $\frac{3}{4}$ ga teng.

3) giperbolaning kanonik tenglamasi tuzilsin.

**7.2.4**. Teng tomonli giperbolaning ekssentrisiteti hisoblansin.

**7.2.5**. Giperbola asimptotalarining tenglamalari $y = \pm\frac{5}{12}x$ va giperbolada yotuvchi $M(24; 5)$ nuqta berilgan. Giperbola tenglamasi tuzilsin.

**7.2.6**. $\frac{x^2}{25} - \frac{y^2}{144} = 1$ giperbolaning fokuslarini aniqlang.

**7.2.7**. $\frac{x^2}{225}-\frac{y^2}{64}=-1$ giperbolaning fokuslarini aniqlang.

**7.2.8**. Quyidagi ma’lumotlarga ko‘ra:

1) direktrisalari orasidagi masofa $\frac{32}{5}$ ga teng va ekssentrisiteti $\varepsilon=\frac{5}{4}$;

2) asimptotalari orasidagi burchak $60^0$ ga teng va $c=2\sqrt{3}$ giperbolaning kanonik tenglamasi tuzilsin.

**7.2.9**. $\frac{x^2}{49}+\frac{y^2}{24}=1$ ellips bilan fokusdosh va ekssentrisiteti $\varepsilon=\frac{5}{4}$ bo‘lgan giperbolaning tenglamasi yozilsin.

**7.2.10**. $\frac{x^2}{9}-\frac{y^2}{16}=1$ giperbola berilgan:

1) fokuslarining koordinatalari;
2) ekssentrisiteti;
3) asimptotalarining va direktrisalarining tenglamalari;
4) qo‘shma giperbolaning tenglamasi va uning ekssentrisiteti hisoblansin.

**7.2.11**. Giperbola haqida quyidagilar ma’lum bo‘lsa, uning yarim o‘qlari hisoblansin:

1) fokuslari orasidagi masofa 8 ga va direktrisalari orasidagi masofa 6 ga teng;
2) direktrisalari $x=\pm 3\sqrt{2}$ tenglamalar bilan berilgan va asimptotalari orasidagi burchak to‘g‘ri burchak;
3) asimptotalari $y=\pm 2$ tenglamalar bilan berilgan va fokuslari markazdan 5 birlik masofada;
4) asimptotalari $y=\pm\frac{5}{3}x$ tenglamalar bilan berilgan va giperbola $N(6;9)$ nuqtadan o‘tadi.

**7.2.12**. Teng tomonli giperbola $x^2-y^2=8$ berilgan. Unga fokusdosh bo‘lib, $M(-5;3)$ nuqtadan o‘tuvchi giperbolaning tenglamasi topilsin.

**7.2.13.** Fokusi absissa o‘qida joylashgan va koordinata boshiga nisbatan simmetrik bo‘lgan giperbola tenglamasini tuzing, agar quyidagilar ma’lum bo‘lsa:

1) uning o‘qlari $2a=10$ va $2b=8$;

2) fokuslar orasidagi masofa $2a = 10$ va mavhum o‘qi $2b = 8$;
3) fokuslar orasidagi masofa $2c = 6$ va ekssentrisiteti $\varepsilon = \frac{3}{2}$;
4) haqiqiy o‘qi $2a = 16$ va ekssentrisiteti $\varepsilon = \frac{5}{4}$;
5) asimptota tenglamasi $y = \pm\frac{4}{3}x$ va fokuslar orasidagi masofa $2c = 20$;
6) direktrisalar orasidagi masofa 22 ga teng va fokuslar orasidagi masofa $2c = 26$;
7) direktrisalar orasidagi masofa $\frac{32}{5}$ ga teng va mavhum o‘qi $2b = 6$;
8) direktrisalar orasidagi masofa $\frac{8}{3}$ ga teng va ekssentrisiteti $\varepsilon = \frac{3}{2}$;
9) asimptota tenglamasi $y = \pm\frac{3}{4}x$ va direktrisalar orasidagi masofa $12\frac{4}{5}$ ga teng.

**7.2.14**. Fokusi ordinata o‘qida joylashgan va koordinata boshiga nisbatan simmetrik bo‘lgan giperbola tenglamasini tuzing, agar quyidagilar ma’lum bo‘lsa:
1) uning yarim o‘qlari $a = 6$, $b = 18$;
2) fokuslar orasidagi masofa $2c = 10$ va ekssentrisiteti $\varepsilon = \frac{5}{3}$;
3) asimptota tenglamasi $y = \pm\frac{12}{5}x$ va uchlari orasidagi masofa 48;
4) direktrisalar orasidagi masofa $7\frac{1}{7}$ va ekssentrisiteti $\varepsilon = \frac{7}{5}$;
5) asimptota tenglamasi $y = \pm\frac{4}{3}x$ va direktrisalar orasidagi masofa $6\frac{2}{5}$ ga teng .

**7.2.15**. Quyida berilgan giperbolalarni $a$ va $b$ yarim o‘qlarini toping:
1) $\frac{x^2}{9} - \frac{y^2}{4} = 1$; 2) $\frac{x^2}{16} - y^2 = 1$; 3) $x^2 - 4y^2 = 16$;
4) $x^2 - y^2 = 1$; 5) $4x^2 - 9y^2 = 25$; 6) $25x^2 - 16y^2 = 1$;
7) $9x^2 - 16y^2 = 1$.

**7.2.16**. Giperbola tenglamasi berilgan $16x^2 - 9y^2 = 144$ bo‘lsa, quyidagilarni:
1) yarim o‘qlari $a$ va $b$; 2) fokuslarini; 3) ekssentrisiteti;

4) asimptota tenglamasini toping;  5) direktrisa tenglamasini toping.

**7.2.17**. Giperbola tenglamasi berilgan $16x^2 - 9y^2 = -144$ bo‘lsa, quyidagilarni:

1) yarim o‘qlari $a$ va $b$;  2) fokuslarini;  3) ekssentrisiteti;

4) asimptota tenglamasini toping; 5) direktrisa tenglamasini toping.

**7.2.18**. $\frac{x^2}{4} - \frac{y^2}{9} = 1$ giperbola asimtotasi va $9x + 2y - 24 = 0$ to‘g‘ri chiziq bilan chegaralangan uchburchakning yuzini toping.

**7.2.19**. Quyidagi tenglamalar qanday chiziqlarni ifodalashini aniqlang:

1) $y = +\frac{2}{3}\sqrt{x^2 - 9}$;  2) $y = -3\sqrt{x^2 + 1}$;

3) $y = -\frac{4}{3}\sqrt{x^2 + 9}$;  4) $y = +\frac{2}{5}\sqrt{x^2 + 25}$.

va bu chiziqlarni chizmasini chizing.

**7.2.20**. $\frac{x^2}{80} - \frac{y^2}{20} = 1$ giperbolaning $M_1(10; -\sqrt{5})$ nuqtasi berilgan. Fakal radiusi $M_1$ nuqta bo‘lgan to‘g‘ri chiziq tenglamasini tuzing.

**7.2.21**. $\frac{x^2}{16} - \frac{y^2}{9} = 1$ giperbolani $M_1(-5; \frac{9}{4})$ nuqta qanoatlantirishi ko‘rinib turibdi, $M_1$ nuqtaning fakal radiusini toping.

**7.2.22**. $\frac{x^2}{64} - \frac{y^2}{36} = 1$ giperboladagi nuqtadan o‘ng fokusgacha bo‘lgan masofa 4,5 ga teng bo‘lsa shu nuqtani koordinatasini toping.

**7.2.23**. $\frac{x^2}{9} - \frac{y^2}{16} = 1$ giperboladagi nuqtadan chap fokusgacha bo‘lgan masofa 7 ga teng bo‘lsa, shu nuqtani koordinatasini toping.

**7.2.24**. Fokuslari absissa o‘qida yotib koordinata boshiga nisbatan simmetrik bo‘lgan giperbola tenglamasini tuzing, quyidagilar berilgan bo‘lsa:

1) giperbolaning $M_1(6; -1)$ va $M_2(-8; 2\sqrt{2})$ nuqtalari;

2) giperbolaning $M_1(-5; 3)$ nuqtasi va ekssentrisiteti $\varepsilon = \sqrt{2}$;

3) giperbolaning $M_1(\frac{9}{2}; -1)$ nuqtasi va $y = +\frac{2}{3}x$ asimtota tenglamasi;

4) giperbolaning $M_1(-3; \frac{5}{2})$ nuqtasi va $y = +\frac{4}{3}$ direktrisa tenglamasi;

5) $y=+\frac{3}{4}x$ asimtota tenglamasi va $x=\pm\frac{16}{5}$ direktrisa tenglamasi;

**7.2.25**. Teng tomonli giperbolaning ekssentrisitetini hisoblang.

**7.2.26**. $\frac{x^2}{25}+\frac{y^2}{9}=1$ ellipsning fokusi giperbolaning fokusi bilan ustma - ust bo‘ladi. Agar giperbolaning ekssentrisiteti $\varepsilon=2$ ga teng bo‘lsa, giperbolaning tenglamasini tuzing.

**7.2.27**. Fokusi $\frac{x^2}{100}+\frac{y^2}{64}=1$ ellipsning uchida yotuvchi, direktrisasi esa ellipsning fokusidan o‘tuvchi giperbola tenglamasini tuzing.

**7.2.28**. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ giperbolaning fokusidan uning asimptotasigacha bo‘lgan masofa $b$ ga teng bo‘lishini isbotlang.

**7.2.29**. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ giperbolaning ixtiyoriy nuqtasidan uning ikki asimptotasigacha bo‘lgan masofalar ko‘paytmasi har doim $\frac{a^2b^2}{a^2+b^2}$ ga teng bo‘lishini isbotlang.

**7.2.30**. Agar yarim o‘qlari $a$ va $b$, markazi $C(x_0;y_0)$ nuqta va fokusi quyidagi chiziqlarda:

1) $Ox$ o‘qiga parallel;

2) $Oy$ o‘qiga parallel

bo‘lsa, giperbola tenglamasini tuzing.

### 7.3. Parabolaga doir misollar.

**7.3.1.** Quyidagi nuqtalardan qaysilari $y^2=6x$ parabolaga tegishli:

1) $A(-2;4)$; 2) $B(1;5)$; 3) $C(3;1)$; 4) $A(-2;4)$.

**7.3.2.** Quyidagi parabolalardan qaysilarining fokusi a) $F_1(3;0)$, b) $F_2(-3;0)$, c)$F_3(0;3)$ va d)$F_4(0;-3)$ nuqtadan o‘tadi:

1) $y^2=3x$; 2) $y^2=-3x$; 3) $x^2=3y$; 4) $x^2=-3y$;

5) $y^2=6x$; 6) $y^2=-6x$; 7) $x^2=6y$; 8) $x^2=-6y$;

9) $y^2=12x$; 10) $y^2=-12x$; 11) $x^2=12y$; 12) $x^2=-12y$.

**7.3.3.** Quyidagi parabolalardan qaysilarining direktrisa tenglamasi

a) $x=5$, b) $x=-5$, c) $y=5$ va d) $y=-5$:

1) $y^2=5x$; 2) $y^2=-5x$; 3) $x^2=5y$; 4) $x^2=-5y$;

5) $y^2 = 10x$;    6) $y^2 = -10x$;  7) $x^2 = 10y$;    8) $x^2 = -10y$;
9) $y^2 = 20x$;    10) $y^2 = -20x$;  11) $x^2 = 20y$; 12) $x^2 = -20y$.

**7.3.4.** $y^2 = 4x$ parabola fokusining koordinatalarini aniqlang.

**7.3.5**. $x^2 = 4y$ parabola fokusining koordinatalarini aniqlang.

**7.3.6**. $y^2 = -8x$ parabola fokusining koordinatalarini aniqlang.

**7.3.7**. $y^2 = 6x$ parabola direktrisasi tenglamasini tuzing.

**7.3.8**. Parabolaning fokusidan uchigacha bo‘lgan masofa 3 ga teng, uning kanonik tenglamasini tuzing.

**7.3.9**. Parabolaning fokusidan direktrisasigacha bo‘lgan masofa 2 ga teng, uning kanonik tenglamasini tuzing.

**7.3.10**. Parabolaning fokusi $F(3;0)$ nuqtada va $x = -1$ direktrisasining tenglamasi bo‘lsa, parabola tenglamasini tuzing.

**7.3.11**. Parabolaning uchidan fokusigacha bo‘lgan masofa 3 ga teng va parabola $Ox$ o‘qiga nisbatan simmetrik bo‘lib, $Oy$ o‘qiga urinsa parabola tenglamasini tuzing.

**7.3.12.** Fokusi $M(5;0)$ nuqtada bo‘lib, ordinatalar o‘qi direktrisa bo‘lsa, parabola tenglamasini tuzing.

**7.3.13.** Parabola $Ox$ o‘qiga nisbatan simmetrik bo‘lib, $M(1;-4)$ nuqtadan va koordinatalar boshidan o‘tadigan parabola tenglamasini tuzing.

**7.3.14.** Parabolaning fokusi $M(0;2)$ nuqtada va uchi koordiniatalar boshida yotsa, parabola tenglamasini tuzing.

**7.3.15.** Parabola $Oy$ o‘qiga nisbatan simmetrik bo‘lib, $M(6;-2)$ nuqtadan va koordinatalar boshidan o‘tadi, parabola tenglamasini tuzing.

**7.3.16**. $y^2 = 8x$ paraboladagi fokal radius vektori 20 ga teng bo‘lgan nuqta topilsin.

**7.3.17**. $Ox$ o‘qiga nisbatan simmetrik, $A(9;6)$ nuqtadan va uchi koordinatalar boshidan o‘tuvchi parabola tenglamasini tuzing.

**7.3.18.** $Ox$ o‘qiga nisbatan simmetrik, $B(-1;3)$ nuqtadan va uchi koordinatalar boshidan o‘tuvchi parabola tenglamasini tuzing.

**7.3.19.** $Oy$ o‘qiga nisbatan simmetrik, $C(1;1)$ nuqtadan va uchi koordinatalar boshidan o‘tuvchi parabola tenglamasini tuzing .

**7.3.20.** $Oy$ o‘qiga nisbatan simmetrik, $D(4;-8)$ nuqtadan va uchi koordinatalar boshidan o‘tuvchi parabola tenglamasini tuzing .

**7.3.21**. Koordinata boshidan o‘tib, $Oy$ o‘qiga simmetrik va fokusi $F(0;-3)$ nuqtada bo‘lgan parabola tenglamasini tuzing.

**7.3.22**. $y^2 = 24x$ parabola tenglamasidan $F$ fokusini va direktrisa tenglamasini toping.

**7.3.23**. $y^2 = -24x$ parabola tenglamasidan $F$ fokusini va direktrisa tenglamasini toping.

**7.3.24**. $x^2 = -24y$ parabola tenglamasidan $F$ fokusini va direktrisa tenglamasini toping.

**7.3.25**. $y^2 = 20x$ parabola tenglamasi berilgan, agar $M$ nuqtaning absissasi 7 ga teng bo‘lsa, $M$ fokal radiusni toping.

**7.3.26**. $y^2 = 12x$ parabola tenglamasi berilgan, agar $M$ nuqtaning ordinatasi 6 ga teng bo‘lsa, $M$ fokal radiusni toping.

**7.3.27**. $y^2 = 16x$ parabola tenglamasi berilgan. Fokal radius 13 ga teng bo‘ladigan $M$ nuqtani toping .

**7.3.28**. $x^2 = 16y$ parabola tenglamasi berilgan. Fokal radius 13 ga teng bo‘ladigan $M$ nuqtani toping .

**7.3.29**. $x^2 = -16y$ parabola tenglamasi berilgan. Fokal radius 13 ga teng bo‘ladigan $M$ nuqtani toping .

**7.3.30**. Agar $F(-7;0)$ fokus va direktirisa tenglamasi $x - 7 = 0$ berilgan bo‘lsa, parabola tenglamasini tuzing .

## 8-MAVZU: TEKILIKDA IKKINCHI TARTIBLI CHIZIQLARNING QUTB KOORDINATALAR SISTEMASIDAGI TENGLAMALARI.

**Reja:**

**1.Qutb koordinatalar. Konus kesimlari.**

**2.Qutb koordinatalardagi tenglamalar.**

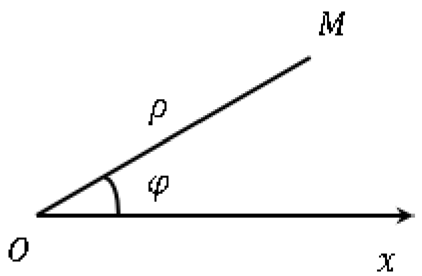
**3.Konus kesimlarining dekart koordinatalardagi kanonik ko‘rinishli tenglamalari.**

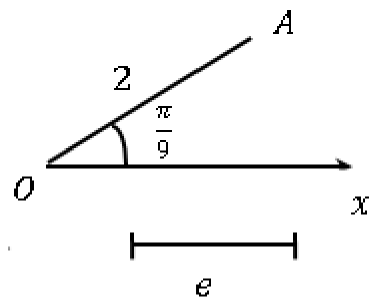
**Tayanch iboralar**: qutb, ellips, giperbola, parabola, diametr, vatar, fokus, parametr, direktrisa, asimptota, ekssentrisitet.

### 8.1. Qutb koordinatalar.

Qutb koordinatalar sistemasining asosiy elementlari va undan chiquvchi nur, ya’ni qutb $O$ va ***qutb o‘qi*** $Ox$ dir (8.1.1- chizma).

$M$ nuqtaning tekislikdagi o‘rni bu nuqtaning qutbdan bo‘lgan masofasi – radius – vektori $\rho$ va radius – vektorning qutb o‘qi bilan tashkil etgan qutb burchgi $\varphi$ bilan aniqlanadi.



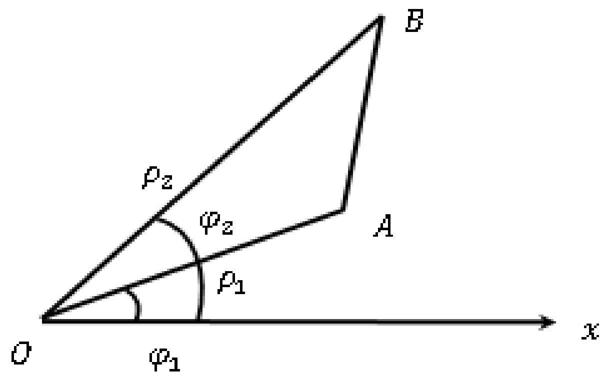

**8.1.1-chizma** **8.1.2-chizma**

Ikki koordinata $(\rho, \varphi)$ birgina nuqtani aniqlaydi. 8.1.2 - chizmada $A$ nuqta $\rho = 2,\ \varphi = \frac{\pi}{9}$ koordinatalari bo‘yicha yasalgan.

Agar biz tekislikdagi nuqtalar va qo‘sh koordinatalar $(\rho, \varphi)$ orasidagi o‘zaro qiymatli moslikni o‘rnatmoqchi bo‘lsak, u holda $\rho$ ga faqat musbat qiymatlar, $\varphi$ ga esa $0$ bilan $2\pi$ orasidagi qiymatlar berish kifoya ( nurni soat strelkasiga qarshi aylantirganda musbat burchaklar hosil bo‘ladi). Agar bu cheklashlarga rioya qilinmasa, u holda birgina nuqtaning o‘zi $(\rho; \varphi + 2\pi n)$ yoki $(-\rho; \varphi(2n+1)\pi)$ koordinatalar bilan aniqlanadi, $n$ – ixtiyoriy butun son. Qutb koordinatalar sistemasida berilgan ikkita nuqta $A(\rho_1, \varphi_1)$ va $B(\rho_2, \varphi_2)$ orasidagi masofa

$$AB = \sqrt{\rho_1^2 + \rho_2^2 - 2\rho_1\rho_2 \cdot \cos(\varphi_2 - \varphi_1)} \qquad (8.1)$$

formula bilan hisoblanadi.




**8.1.3-chizma**

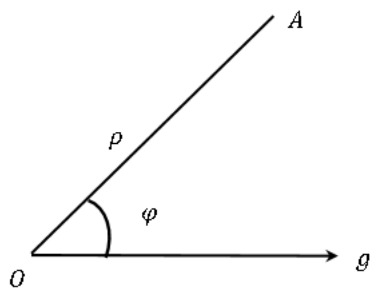
Dekart koordinatalari bilan ish ko‘rganimiz singari, egri chiziqning qutb koordinatalaridagi tenglamasi haqida gapirish mumkin, chunonchi, agar egri chiziqdagi har bir nuqtaning qutb koordinatalari ushbu

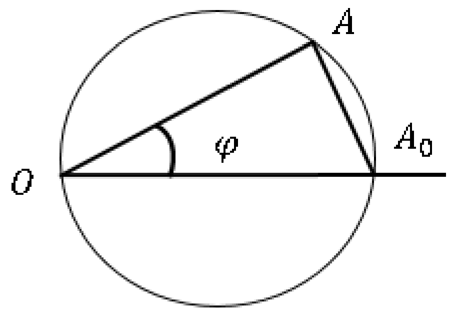
$$F(\rho;\varphi)=0$$

tenglamani qanoatlantirsa, bu tenglama egri chiziqning ***qutb koordinatalardagi tenglamasi*** deb ataladi. Aksincha, $\rho$, $\varphi$ sonlarning bu tenglamani qanoatlantiradigan istalgan jufti egri chiziq nuqtalaridan birining qutb koordinatalari bo‘ladi.

Misol tariqasida qutbdan o‘tgan va markazi qutb o‘qidagi $R$ radiusli aylananing qutb koordinatalaridagi tenglamasini tuzaylik. To‘g‘ri burchakli $OAA_0$ uchburchakdan $OA=OA_0cos\varphi$ ni hosil qilamiz (8.1.5- chizma). Bu yerdan aylana tenglamasini hosil qilamiz:

$$\rho=2Rcos\varphi.$$



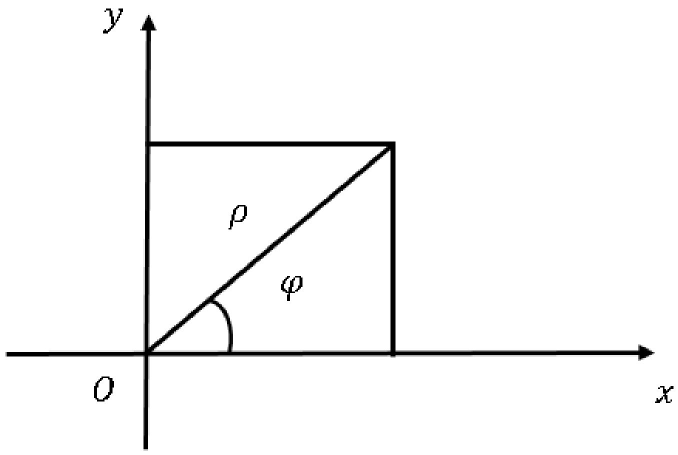




**8.1.4-chizma** **8.1.5-chizma**

$\rho\varphi$ tekislikda dekart koordinatalari sistemasi $xy$ ni kiritamiz, buning uchun qutb $O$ ni dekart koordinatalari sistemasining boshi, qutb o‘qini musbat yarim o‘q $x$ sifatida va musbat yarim $y$ ning musbat yo‘nalishini shunday tanlab olamizki, burchaklarni hisoblash uchun tanlab olingan yo‘nalish bilan muvofiq holda qutb o‘qi bilan $+\frac{\pi}{2}$ burchakni hosil qilsin.

Nuqtaning qutb va dekart koordinatalari orasida quyidagicha bog‘lanishning mavjudligi ravshan:

$$x = \rho cos\varphi,\ y = \rho sin\varphi \qquad (8.2)$$




**8.1.6-chizma**

Bu egri chiziqning qutb koordinatalar sistemasidagi tenglamasini bilgan holda, uning dekart koordinatalaridagi tenglamasini va aksincha, hosil qilish imkonini beradi.

Misol tariqasida ixtiyoriy to‘g‘ri chiziqning qutb sistemasidagi tenglamasini tuzaylik. To‘g‘ri chiziqning dekart koordinatalaridagi tenglamasi

$$ax + by + c = 0 \qquad c < 0.$$

Bu tenglama $\rho$ bilan $\varphi$ ni $x$ va $y$ o‘rniga (8.2) formula bo‘yicha kiritsak, natijada:

$$\rho(a\ cos\varphi + b\ sin\varphi) + c = 0.$$

So‘ngra ushbularni faraz qilsak:

$$\frac{a}{\sqrt{a^2+b^2}} = cos\alpha, \qquad \frac{b}{\sqrt{a^2+b^2}} = sin\alpha, \qquad \frac{c}{\sqrt{a^2+b^2}} = -\rho_0$$

to‘g‘ri chiziqning ushbu ko‘rinishidagi tenglamani hosil qilamiz:

$$\rho\cos(\alpha - \varphi) = \rho_0.$$

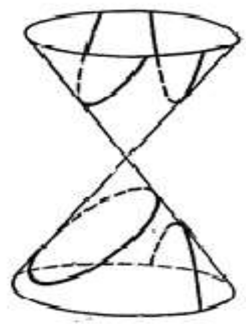
## 8.2. Konus kesimlari. Qutb koordinatalardagi tenglamalar.

Doiraviy konusni uning uchidan o‘tmaydigan tekislik bilan kesish natijasida hosil qilingan egri chiziq ***konus kesimi*** deyiladi (8.1.7-chizma). Konus kesimlari qator ajoyib xossalarga ega. Ularning biri quyidagidan iborat.

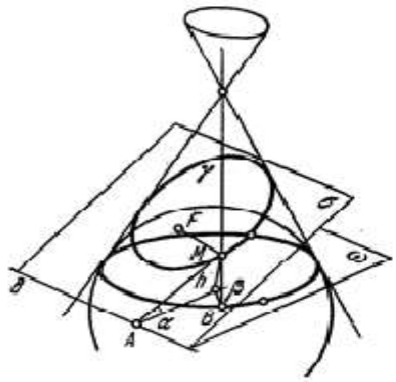
Aylanadan boshqa har qanday konus kesimi shunday nuqtalarning geometrik o‘rnidan iboratki, ularning berilgan $F$ nuqta va berilgan $\delta$ to‘g‘ri chiziqqacha masofalarning nisbati o‘zgarmasdir.

$F$ nuqta konus kesimining fokusi, $\delta$ to‘g‘ri chiziq esa ***direktrisasi*** deyiladi. Bu xossani isbot qilaylik. Aytaylik, $\sigma$ tekislikning konus bilan kesishgan egri chizig‘i $\gamma$ bo‘lsin(8.1.8 – chizma).

Konus ichiga $\sigma$ tekislikka urinadigan sfera chizaylik; sferaning tekislikka urinish nuqtasini $F$ orqali belgilaylik. $\omega$ bilan sferaning konusga urinish aylanasini belgilaylik. $\gamma$ egri chiziqda ixtiyoriy $P$ nuqta olamiz. Bu $P$ nuqta orqali konusning yasovchisini o‘tkazib, uning $\omega$ tekislik bilan kesishgan nuqtasini $B$ orqali belgilaymiz. Nihoyat, $P$ nuqtadan $\sigma$, $\omega$ tekisliklarning kesishgan to‘g‘ri chizig‘i $\delta$ ga perpendikulyar tushiramiz.



**8.1.7-chizma**



**8.1.8-chizma**

Ana shu $\gamma$ chiziqning $F$ nuqta bilan $\delta$ to‘g‘ri chiziqqa nisbatan yuqorida aytilgan xossaga egaligini isbot qilish talab qilinadi. Haqiqatdan ham, $FP = BP$, chunki bu kesmalar sferaga bitta nuqtadan o‘tkazilgan urinmalardir. So‘ngra $h(P)$ bilan $P$ nuqtadan $\omega$ tekislikkacha masofani belgilasak, u holda:

$$AP = \frac{h(P)}{sin\alpha}\ , \qquad BP = \frac{h(P)}{sin\beta}\ .$$

bunda $\alpha$ bilan $\omega, \delta$ tekisliklar orasidagi burchak belgilangan.

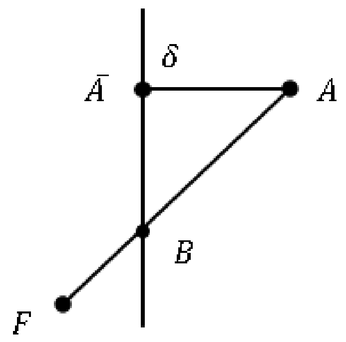
Bulardan ushbu tengliklarni hosil qilamiz:

$$\frac{AP}{FP} = \frac{AP}{BP} = \frac{sin\beta}{sin\alpha}$$

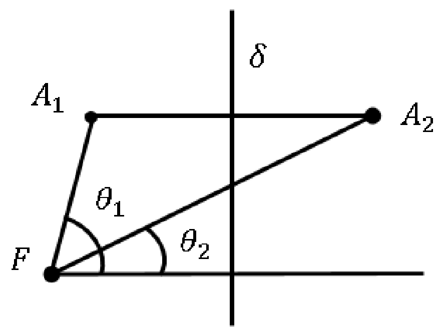
ya'ni $\frac{AP}{FP}$ nisbat $P$ nuqtaga bog'liq emas degan xulosaga kelamiz. Aytganimiz isbot qilindi.

Konus kesimidagi nuqtaning fokus va direktrisagacha masofalarining nisbati $\lambda$ ning qiymatiga qarab egri chiziq ***ellips*** ($\lambda <$ 1), ***parabola*** ($\lambda$ =1), ***giperbola*** ($\lambda > 1$) deb ataladi. $\lambda$ son konus kesimining ***ekssentrisiteti*** deyiladi.

$F$ – konus kesimining fokusi, $\delta$ – uning direktrisasi bo'lsin (8.1.9-chizma). Ellips bilan parabola ($\lambda \leq 1$) holi uchun egri chiziqning hamma nuqtalari direktrisadan bir tarafda joylashadi, chunonchi: $F$ fokus qayerdan joy olsa, bu nuqtalar ham o'sha yerdan joy oladi.




**8.1.9-chizma**




**8.1.10-chizma**

Haqiqatdan ham, direktrisaning ikkinchi tarifidagi nuqtalar uchun:

$$\frac{AF}{A\bar{A}} > \frac{AB}{A\bar{A}} \geq 1.$$

Girepbola bilan ish ko'rganda esa ($\lambda > 1$) direktrisaning ikkala tarafidan joylashga nuqtalar mavjud. Giperbola ikki tarmoqdan iborat bo'lib, direktrisa ularni bir – biridan ajratib turadi.

$\rho\varphi$ koordinatalar sistemasining qutbi sifatida konus kesimining fokusini qabul qilib, qutb o‘qini esa shunday o‘tkazamizki, u direktrisaga perpendikulyar bo‘lsin va uning bilan kesishadigan bo‘lsin. Koordinatalarning ana shunday qutb sistemasidan konus kesimining tenglamasini tuzamiz.

Fokusdan direktrisagacha masofa $P$ bo‘lsin. Konus kesimidagi ixtiyoriy $A$ nuqtadan fokusgacha masofa $\rho$ ga va direktrisagacha masofa esa $A$ va $F$ nuqtalarning direktrisadan bir tarafda yoki turli tarafda bo‘lishiga qarab $P-\rho cos\varphi$ yoki $\rho cos\varphi - P$ ga teng. Bulardan konus kesimining tenglamasini hosil qilamiz: ellips bilan parabola uchun:

$$\frac{\rho}{P-\rho cos\varphi}=\lambda \tag{8.3}$$

va giperbola uchun:

$$\frac{\rho}{P-\rho cos\varphi}=\pm\lambda \tag{8.4}$$

$("+"$ ishora giperbolaning bir tarmog‘i, $"-"$ ishora esa ikkinchi tarmog‘iga mos keladi).

$(8.3), (8.4)$ tenglamalarni $\rho$ ga nisbatan yechib, ushbuni hosil qilamiz:

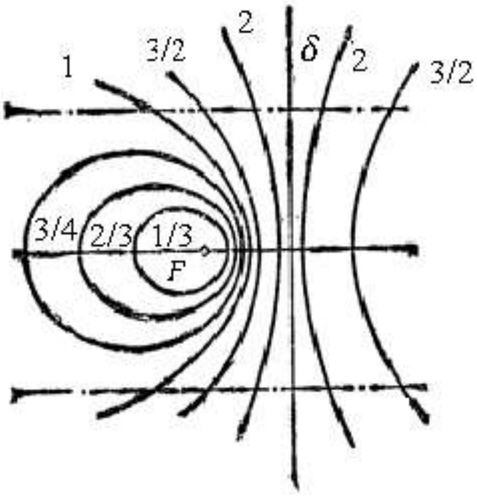
$$\rho=\frac{\lambda P}{1+\lambda cos\varphi}$$

bu ***ellips bilan parabola tenglamasi*** va

$$\rho=\frac{\pm\lambda P}{1\pm\lambda cos\varphi}$$

***giperbola tenglamasidir***.

Ekssentrisitetning qabul qilgan qiymatlariga qarab konus kesimining ko‘rinishi 8.1.11 - chizmada ko‘rsatilgan.




**8.1.11-chizma**

## 8.3. Konus kesimlarining dekart koordinatalardagi kanonik ko‘rinishli tenglamalari.

Yuqorida biz konus kesimlarning $\rho\varphi$ qutb koordinatalaridagi tenglamalarini hosil qilgan edik. Endi dekart koordinatalar sistemasiga o‘tamiz, buning uchun qutb $O$ ni koordinatalar boshi va qutb o‘qini musbat yarim o‘q $x$ sifatida qabul qilamiz.

(8.3) va (8.4) tenglamalardan istalgan konus kesimi uchun ushbuni hosil qilamiz:

$$\rho^2 = \lambda^2(P - \rho\, cos\varphi)^2.$$

Bundan esa qutb va dekart koordinatalar orasidagi bog‘lanishni aniqlovchi formulalarini nazarga olsak:

$$x^2 + y^2 = \lambda^2(P - x)^2$$

yoki

$$(1 - \lambda^2)x^2 + 2P\lambda^2 x + y^2 - \lambda^2 P^2 = 0 \qquad (8.5)$$

ni hosil qilamiz.

Koordinatalar boshini $x$ o‘qi bo‘ylab kerakligicha siljitish natijasida bu tenglama ancha soddalashadi.

Avval ellips bilan giperbolaga to‘g‘ri kelgan holni ko‘zdan kechiraylik. Bu holda (8.5) tenglamani quyidagicha yozish mumkin:

$$(1 - \lambda^2)\left(x + \frac{P\lambda^2}{1 - \lambda^2}\right) + y^2 - \frac{P^2\lambda^2}{1 - \lambda^2} = 0.$$

Endi ushbu formulalar bo‘yicha yangi $x', y'$ koordinatalarni kiritaylik:

$$x + \frac{P\lambda^2}{1-\lambda^2} = x', \qquad y = y',$$

bu esa koordinatalar boshini $\left(-\frac{P\lambda^2}{1-\lambda^2}, 0\right)$ nuqtaga ko‘chirishga mos keladi. Egri chiziq tenglamasi bu holda ushbu ko‘rinishni oladi:

$$(1-\lambda^2){x'}^2 + {y'}^2 - \frac{P^2\lambda^2}{1-\lambda^2} = 0$$

yoki qisqalik uchun

$$\frac{P^2\lambda^2}{(1-\lambda^2)^2} = a^2, \quad \frac{P^2\lambda^2}{|1-\lambda^2|} = b^2$$

deb faraz qilsak, ushbu tenglamalarni hosil qilamiz;

***ellips uchun:***

$$\frac{{x'}^2}{a^2} + \frac{{y'}^2}{b^2} - 1 = 0,$$

***giperbola uchun:***

$$\frac{{x'}^2}{a^2} - \frac{{y'}^2}{b^2} - 1 = 0.$$

$a, b$ parametrlar ellips (giperbola)ning yarim o‘qlari deyiladi.

Parabola ($\lambda = 1$) olgan holda (8.5) tenglama ushbu ko‘rinishda qabul qilinadi:

$$2Px + y^2 - P^2 = 0$$

yoki

$$y^2 - 2P\left(-x + \frac{P}{2}\right) = 0;$$

yangi

$$x' = -x + \frac{P}{2}, \qquad y' = y$$

koordinatalarni kiritish natijasida tenglama

$${y'}^2 - 2Px' = 0$$

ko‘rinishga keltiriladi.

Konus kesimlarining $x'$, $y'$ koordinatalarga nisbatan hosil qilinadigan tenglamalari ***kanonik tenglamalar*** diyiladi.

**1-misol.** Berilgan $A\left(2;\frac{\pi}{12}\right)$ va $B\left(1;\frac{5\pi}{12}\right)$ nuqtalar orasidagi masofani toping.

**Yechish:** Berilgan $A$ va $B$ nuqtalar orasidagi masofani (8.1) formuladan foydalanib topamiz:

$$|AB| = \sqrt{\rho_1^2 + \rho_2^2 - 2\rho_1\rho_2 \cdot \cos(\varphi_2 - \varphi_1)} =$$

$$= \sqrt{2^2 + 1^2 - 2 \cdot 2 \cdot 1 \cdot \cos\left(\frac{5\pi}{12} - \frac{\pi}{12}\right)} =$$

$$= \sqrt{5 - 4 \cdot \cos\frac{\pi}{3}} = \sqrt{5 - 4 \cdot \frac{1}{2}} = \sqrt{3}$$

Qutb koordinatalar sistemasida $A$ va $B$ nuqtalar orasidagi masofa $\sqrt{3}$ ga teng ekan.

**Mustaqil yechish uchun topshiriqlar.**

**8.1.1.** Qutb koordinatalari quyidagi qiymatlarga ega bo‘lgan nuqtalar yasalsin:
1)$\left(3;\frac{\pi}{6}\right)$; 2)$\left(1;\frac{5\pi}{3}\right)$; 3)$\left(5;\frac{7\pi}{6}\right)$; 4)$\left(0{,}5;\frac{\pi}{2}\right)$; 5)$\left(2{,}5;\frac{2\pi}{3}\right)$; 6)$(6;\pi)$; 7)$\left(3;\frac{\pi}{3}\right)$; 8)$\left(\sqrt{3};-\frac{\pi}{6}\right)$; 9)$\left(-2;\frac{\pi}{4}\right)$.

**8.1.2.** Qutb koordinatalari quyidagi tenglamalardan birini qanoatlantirgan nuqtalar qanday joylashgan:
1) $\rho = 1$; 2) $\rho = 5$; 3) $\rho = a$; 4) $\varphi = \frac{\pi}{6}$; 5) $\varphi = \frac{\pi}{3}$;
6) $\varphi = \frac{\pi}{2}$; 7) $\varphi = const$.

**8.1.3.** a) Qutbga nisbatan, b) qutb o‘qiga nisbatan, ushbu
1)$\left(1;\frac{\pi}{4}\right)$; 2) $\left(3;\frac{2\pi}{3}\right)$; 3) $\left(\frac{2}{3};-\frac{\pi}{6}\right)$; 4) $M(\rho;\varphi)$ nuqtalarga simmetrik bo‘lgan nuqtalarning qutb koordinatalari topilsin.

**8.1.4.** Tomoni $a$ ga teng bo‘lgan muntazam oltiburchak uchlarining qutb koordinatalari aniqlansin; oltiburchakning uchlaridan biri qutb, shu uchidan o‘tuvchi tomoni qutb o‘qi deb olinsin.

**8.1.5.** Qutb burchaklari $0^0$, $15^0$, $30^0$, $45^0$, $60^0$, $75^0$, $90^0$ ga teng bo‘lgan, mos radius – vektorlari $\rho = a \cdot sin2\varphi$ tenglamadan hisoblanuvchi nuqtalar yasalsin. Olingan nuqtalar uzluksiz egri chiziq bilan tutashtirilsin.

**8.1.6.** Berilgan ikki nuqta orasidagi masofa hisoblansin:

1)$A\left(2;\frac{\pi}{12}\right)$ va $B\left(1;\frac{5\pi}{12}\right)$; 2) $C\left(4;\frac{\pi}{5}\right)$ va $D\left(6;\frac{6\pi}{5}\right)$; 3) $E\left(3;\frac{11\pi}{18}\right)$ va $F\left(4;\frac{\pi}{9}\right)$.

**8.1.7.** Qutb koordinatalar sistemasida uchburchakning uchlari berilgan: $A\left(5;\frac{\pi}{2}\right)$, $B\left(8;\frac{5\pi}{6}\right)$, $C\left(3;\frac{7\pi}{6}\right)$. Bu uchburchakning muntazam ekanligi tekshirilsin.

**8.1.8.** Qutb o‘qiga joylashgan va $A\left(4\sqrt{2};\frac{\pi}{4}\right)$ nuqtadan 5 birlik masofada yotgan nuqta topilsin.

**8.1.9.** Uchlaridan biri qutb bilan ustma –ust tushgan uchburchakning yuzini hisoblash uchun formula chiqarilsin.

**8.1.10.** Uchlaridan biri qutbda, qolgan ikki uchining qutb koordinatalari $\left(4;\frac{\pi}{9}\right)$ va $\left(1;\frac{5\pi}{18}\right)$ bo‘lgan uchburchakning yuzi hisoblansin.

**8.1.11.** Qutb koordinatalar sistemasida o‘zining $A\left(9;\frac{\pi}{10}\right)$, $B\left(12;\frac{4\pi}{15}\right)$ va $C\left(10;\frac{3\pi}{5}\right)$ uchlari bilan berilgan uchburchakning yuzi hisoblansin.

**8.1.12.** Qutb koordinatalar sistemasiga nisbatan, radiusi $a$ ga teng va markazi:

1) qutbda; 2) $(a;0)$; 3) $(\rho_1;\varphi_1)$ nuqtada bo‘lgan aylananing tenglamasi tuzilsin.

**8.1.13.** Qutb koordinatalar sistemasiga nisbatan markazi qutb bilan va fokal o‘qi qutb o‘qi bilan ustma – ust tushgan ellipsning tenglamasi tuzilsin.

**8.1.14.** $\rho = \frac{288}{16-7cos^2\varphi}$ ellipsning uzunligi 10 birlikka teng bo‘lgan diametri fokal o‘qqa qanday burchak ostida og‘ishgan?

**8.1.15.** Ellipsning fokal o‘qini qutb o‘qi deb va qutbni 1) ellipsning chap fokusiga joylashtirib; 2) ellipsning o‘ng fokusiga joylashtirib, ellipsning tenglamasini tuzing.

**8.1.16.** $\rho = \frac{3\sqrt{2}}{2-cos\varphi}$ ellips yarim o‘qlarining uzunligi va ikkala fokus orasidagi masofa hisoblansin.

**8.1.17.** Markazi qutb bilan va haqiqiy o‘qi qutb o‘qi bilan ustma – ust tushgan giperbolaning tenglamasi tuzilsin.

**8.1.18.** $\rho = \frac{48}{4cos^2\varphi-1}$ giperbolaning asimptotalari orasidagi burchak hisoblansin.

**8.1.19.** Giperbolaning fokal o‘qini qutb o‘qi qabul qilinib va qutbni giperbolaning o‘ng fokusida olib, uning tenglamasi tuzilsin.

**8.1.20.** $\rho = \frac{2}{1-\sqrt{2}cos\varphi}$ giperbola asimptotalarining va direktrisalarining tenglamalari tuzilsin.

**8.1.21.** Parabolaning o‘qini qutb o‘qi va uchini qutb deb olib, uning tenglamasi tuzilsin.

**8.1.22.** $\rho = \frac{8cos\varphi}{sin^2\varphi}$ parabolada shunday nuqta topilsinki, uning parabola dipektrisasidan bo‘lgan masofa shu nuqtaning radius – vektoriga teng bo‘lsin.

**8.1.23.** Fokusi qutb bilan ustma – ust tushgan va o‘qi qutb o‘qidan iborat bo‘lgan parabolaning tenglamasi tuzilsin.

**8.1.24.** $\rho = \frac{p}{1-cos\varphi}$ parabolada shunday nuqta topilsinki, u

1) eng kichik radius – vektorga;

2) parabolaning parametriga teng bo‘lgan radius – vektorga ega bo‘lsin.

**8.1.25.** Parabolaning ixtiyoriy fokal vatarining uchlaridan uning o‘qiga tushirilgan perpendikulyarning ko‘paytmasi o‘zgarmas miqdor ekanligini isbotlang.

**8.1.26.** To‘g‘ri burchakli koordinatalar sistemasiga nisbatan quyidagi egri chiziqlarning eng sodda tenglamalari tuzilsin:

1) $\rho = \frac{25}{13-12cos\varphi}$; 2) $\rho = \frac{1}{3-3cos\varphi}$;

3) $\rho = \frac{9}{4-5cos\varphi}$; 4) $\rho = \frac{4}{\sqrt{5}-cos\varphi}$.

**8.1.27.** $P(2;-1)$ nuqta polyarasining $x^2+6xy+y^2+6x+2y-$ $-1=0$ egri chiziqqa nisbatan tenglamasi tuzilsin.

**8.1.28.** Quyidagi nuqtalarning polyarasi topilsin:

1) $(-3;5)$ nuqtaning $4x^2+2xy-y^2+6x+2y+3=0$ egri chiziqqa nisbatan;

2) $(0;1)$ nuqtaning $6x^2-xy-2y^2+4y=0$ egri chiziqqa nisbatan.

**8.1.29.** Quyidagi nuqtalarning

1) $(1;-2)$ nuqtaning $2x^2-4xy+5y^2-8x+6=0$ egri chiziqqa nisbatan;

2) $(0;0)$ nuqtaning $x^2-2xy+2y^2-4x-6y+3=0$ egri chiziqqa nisbatan polyarasini toping.

**8.1.30.** $18x-17y-41=0$ to‘g‘ri chiziqning $2x^2-xy-3y^2-$ $-x-6y-15=0$ egri chiziqqa nisbatan qutbi topilsin.

# 9-MAVZU: TEKISLIKDA IKKINCHI TARTIBLI CHIZIQLARNING UMUMIY TENGLAMALARI.

**Reja:**

**1. Koordinata boshini ko‘chirish yordamida ikkinchi tartibli chiziqlarni sinflarga ajratish.**

**2. Markaziy va nomarkaziy chiziqlar.**

**Tayanch iboralar**: ellips, giperbola, parabola, diametr, vatar, fokus, parametr, asimptota, direktrisa, notrivial.

## 9.1. Koordinata boshini ko‘chirish yordamida ikkinchi tartibli chiziqlarni sinflarga ajratish.

Tekislikda dekart koordinatalar sistemasida

$$a_{11}x^2 + 2a_{12}xy + a_{22}y^2 + 2a_{13}x + 2a_{23}y + a_{33} = 0 \quad (9.1)$$

tenglama bilan berilgan ikkinchi tartibli chiziqni tekshirish bilan shug'ullanamiz. Bu ishni koordinatalar sistemasini o'zgartirish va (9.1) tenglamani soddalashtirish yordamida amalga oshiramiz. Birinchi navbatda parallel ko'chirishda (9.1) tenglama koeffitsiyentlarini qanday o'zgarishini tekshiramiz. Buning uchun

$$x' = x - x_0, \quad y' = y - y_0 \quad (9.2)$$

formulalar yordamida almashtirishlarni bajaramiz. Bu holda koordinata o'qlarining yo'nalishlari o'zgarmaydi, faqat koordinata boshi $O'(x_0; y_0)$ nuqtaga ko'chadi. Bu formulalardan $x, y$ larni topib va (9.1) ga qo'yib,

$$a_{11}(x' + x_0)^2 + 2a_{12}(x' + x_0)(y' + y_0) + a_{22}(\mathrm{y}' + y_0)^2 +$$
$$+2a_{13}(x' + x_0) + 2a_{23}(y' + y_0) + a_{33} = 0 \quad (9.3)$$

tenglamani hosil qilamiz. Bu tenglamadan

$$a_{11}({x'}^2 + 2x_0x' + x_0^2) + 2a_{12}(x'y' + x'y_0 + y'x_0 + x_0y_0) +$$
$$+a_{22}({y'}^2 + y_0y' + y_0^2) + 2a_{13}x' + 2a_{13}x_0 + 2a_{23}y' +$$
$$+2a_{23}y_0 + a_{33} = 0$$
$$a_{11}{x'}^2 + 2a_{12}x'y' + a_{22}{y'}^2 + (2a_{11}x_0 + 2a_{12}y_0 + 2a_{13})x' +$$
$$+(2a_{12}x_0 + 2a_{22}y_0 + 2a_{23})y' + a_{11}x_0^2 + 2a_{12}x_0y_0 + a_{22}y_0^2 +$$
$$+2a_{13}x_0 + 2a_{23}y_0 + a_{33} = 0 \quad (9.4)$$

kelib chiqadi.

Bu formulalardan ko'rinib turibdiki, parallel ko'chirishda ikkinchi darajali hadlar oldidagi koeffitsiyentlar o'zgarmaydi.
Agar $O'(x_0; y_0)$ nuqtaning koordinatalari

$$\begin{cases} a_{11}x + a_{12}y + a_{13} = 0 \\ a_{21}x + a_{22}y + a_{23} = 0 \end{cases} \quad (9.5)$$

sistemani qanoatlantirsa, (9.3) tenglamada birinchi darajali hadlar qatnashmaydi.

Bundan tashqari, agar $O'(x_0; y_0)$ nuqtaning koordinatalari (9.5) sistemani qanoatlantirsa, $O'(x_0; y_0)$ nuqta ikkinchi tartibli chiziq uchun simmetriya markazi bo'ladi. Haqiqatan ham bu holda koordinatalar markazini $O'(x_0;y_0)$ nuqtaga ko‘chirsak, tenglamada birinchi darajali hadlar qatnashmaydi. Shuning uchun yangi koordinatalar sistemasida

$$F(x';y') = F(-x';-y')$$

tenglik o‘rinli bo‘ladi. Demak, $O'(x_0;y_0)$ nuqta chiziq uchun simmetriya markazidir. Va aksincha, agar birorta $A$ nuqta chiziq uchun simmetriya markazi bo‘lsa uning koordinatalari (9.5) sistemani qanoatlantirishini ko‘rsatamiz. Koordinata boshini $A$ nuqtaga joylashtirib, yangi $x, y$ koordinatalar sistemasini kiritamiz. Agar $F(x;y)$ nuqta chiziqqa tegishli bo‘lsa,

$$F(x;y) = 0$$

tenglik o‘rinli bo‘ladi. Koordinata boshi simmetriya markazi bo‘lgani uchun $F(-x;-y) = 0$ tenglik ham o‘rinli bo‘ladi. Yuqoridagilarni hisobga olsak quyidagi ta’rifning geometrik ma’nosi yaxshi tushunarli bo‘ladi.

***1-ta’rif.*** Tekislikdagi $M_0(x_0;y_0)$ nuqtaning koordinatalari (9.5) sistemani qanoatlantirsa, u (9.1) tenglama bilan berilgan ikkkinchi tartibli chiziqning markazi deyiladi.

Tabiiyki, (9.5) sistema yagona yechimga ega bo‘lishi, cheksiz ko‘p yechimga ega bo‘lishi yoki umuman yechimga ega bo‘lmasligi mumkin.

Agar, $a_{11}a_{22} - a_{21}^2 \neq 0$

munosabat o‘rinli bo‘lsa, (9.5) sistema yagona yechimga ega bo‘ladi. Agar,

$$\frac{a_{11}}{a_{21}} = \frac{a_{12}}{a_{22}} = \frac{a_{13}}{a_{23}}$$

munosabat o‘rinli bo‘lsa sistema cheksiz ko‘p yechimga,

$$\frac{a_{11}}{a_{21}} = \frac{a_{12}}{a_{22}} \neq \frac{a_{13}}{a_{23}}$$

munosabat bajarilsa sistema yechimga ega emas. Bularni e’tiborga olib, biz ikkinchi tartibli chiziqlarni uchta sinfga ajratamiz:

a) yagona markazga ega bo‘lgan chiziqlar;

b) cheksiz ko‘p markazga ega bo‘lgan chiziqlar;

d) markazga ega bo‘lmagan chiziqlar.

### 9.2. Markaziy va nomarkaziy chiziqlar.

Biz quyidagi determinantlami kiritamiz

$$\delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{vmatrix}, \ \Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{01} & a_{02} & a_{33} \end{vmatrix}$$

bu yerda $a_{21} = a_{12}, \ a_{31} = a_{13}, \quad a_{32} = a_{23}$ belgilashlar kiritilgan. Yagona markazga ega chiziqlar uchun $\delta \neq 0$, yagona markazga ega bo‘lmagan chiziqlar uchun $\delta = 0$. Chiziqlar cheksiz ko‘p markazga ega bo‘lishi uchun $\Delta = 0$ tenglik bajarilshi kerak.

Uchinchi tartibli determinantni

$$\Delta = a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} - a_{23} \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} + a_{33} \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

ko‘rinishda yozib olsak, oxirgi determinant $\delta$ ga tengdir. Agar $\delta = 0$ bo‘lsa, birorta $k$ soni uchun

$$\frac{a_{11}}{a_{21}} = \frac{a_{12}}{a_{22}} = k, \quad \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} = k \begin{vmatrix} a_{12} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

munosabat bajariladi. Bu tenglikni hisobga olib

$$\Delta = (a_{13} - k a_{23}) \begin{vmatrix} a_{12} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

tenglikni hosil qilamiz. Agar $\Delta = 0$ tenglik ham bajarilsa

$$a_{13} - k a_{23} = 0 \text{ yoki } \begin{vmatrix} a_{12} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} = 0$$

tengliklardan kamida bittasi o‘rinli bo‘ladi. Bu tengliklarning birinchisi o‘rinli bo‘lsa, $\frac{a_{11}}{a_{12}} = \frac{a_{12}}{a_{22}} = k$ munosabatdan $\frac{a_{11}}{a_{12}} = \frac{a_{12}}{a_{22}} = \frac{a_{13}}{a_{23}}$ munosobat kelib chiqadi. Agar, $\begin{vmatrix} a_{12} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} = 0$ bo‘lsa, $\frac{a_{11}}{a_{12}} = \frac{a_{12}}{a_{22}} = k$ va $\frac{a_{12}}{a_{22}} = \frac{a_{13}}{a_{23}}$ tengliklardan

$$\frac{a_{11}}{a_{12}} = \frac{a_{12}}{a_{22}} = \frac{a_{13}}{a_{23}} = k$$

munosobat kelib chiqadi. Demak, $\delta = 0$ va $\Delta = 0$ tengliklarning bir vaqtda bajarilishi

$$\frac{a_{11}}{a_{12}} = \frac{a_{12}}{a_{22}} = \frac{a_{13}}{a_{23}} = k$$

shartga teng kuchlidir. Natijada biz quyidagi tasdiqni hosil qilamiz:

***1-tasdiq.*** Ikkinchi tartibli chiziq

a) $\delta \neq 0$ bo‘lsa yagona markazga ega,

b) $\delta = 0$ va $\Delta = 0$ bo‘lsa, cheksiz ko‘p markazga ega va markazlar to‘plami bitta to‘g‘ri chiziqni tashkil etadi;

c) $\delta = 0$ va $\Delta = 0$ bo‘lsa markazga ega emas.

***2-tasdiq.*** Yagona markazga ega bo‘lgan ikkinchi tartibli chiziq markazi unga tegishli bo‘lishi uchun $\Delta = 0$ tenglikning bajarilishi zarur va yetarlidir.

***Isbot.*** Ikkinchi tartibli chiziq markazi $M_0(x_0; y_0)$ nuqtada bo‘lib, u chiziqqa tegishli bo‘lsa

$$\begin{cases} a_{11}x_0 + a_{12}y_0 + a_{13} = 0 \\ a_{21}x_0 + a_{22}y_0 + a_{23} = 0 \end{cases} \tag{9.6}$$

va

$$a_{11}x_0^2 + 2a_{12}x_0y_0 + a_{22}y_0^2 + 2a_{13}x_0 + 2a_{23}y_0 + a_{33} = 0 \tag{9.7}$$

tengliklar bajariladi. Yuqoridagi (9.6) tenglikning birinchisini $x_0$ ga, ikkinchisini $y_0$ ga ko‘paytirib, (9.7) tenglikdan ayirsak,

$$a_{31}x_0 + a_{32}y_0 + a_{33} = 0$$

tenglikni hosil qilamiz. Demak, $(x_0; y_0; 1)$ uchlik

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = 0 \\ a_{21}x + a_{22}y + a_{23}z = 0 \\ a_{31}x + a_{32}y + a_{33}z = 0 \end{cases} \tag{9.8}$$

bir jinsli sistemaning notrivial yechimidir. Bu esa $\Delta = 0$ shartga teng kuchlidir. Aksincha $\Delta = 0$ bo‘lsa, (9.8) sistema notrivial $(x_0; y_0; z_0)$ yechimga egadir. Bu uchlikda $z_0 \neq 0$, chunki $\delta \neq 0$. Biz $z_0 = 1$ deb hisoblay olamiz, chunki $\delta \neq 0$ bo‘lganligi uchun har bir $z_0$ uchun $(x_0; y_0)$ juftlik mavjud. Yuqoridagi (9.8) sistemada $z_0 = 1$ bo‘lganda $(x_0; y_0)$ juftlik markaz koordinatalari ekanligi kelib chiqadi. Bundan tashqari (9.8) sistemadan foydalanib,

$$a_{11}x_0^2 + 2a_{12}x_0y_0 + a_{22}y_0^2 + 2a_{13}x_0 + 2a_{23}y_0 + a_{33} = 0$$

tenglikni olish mumkin.

**1-misol.** Quyidagi tenglama bilan berilgan chiziqning turi va joylashishi aniqlansin.

$$6xy + 8y^2 - 12x - 26y + 11 = 0$$

**Yechish:**

$$I_2 = \begin{vmatrix} 0 & 3 \\ 3 & 8 \end{vmatrix} = -9 < 0,$$

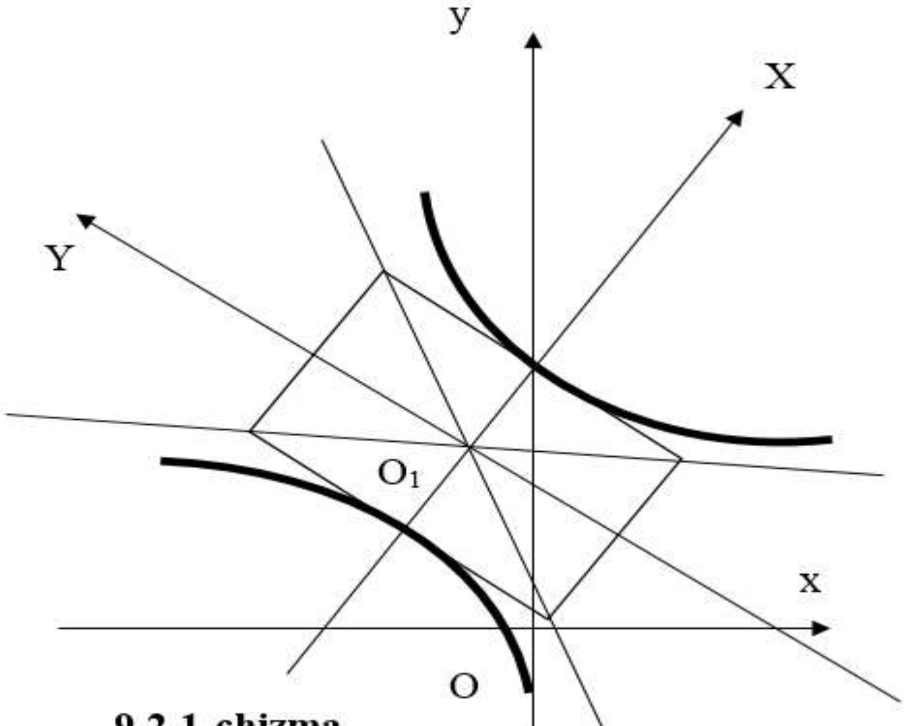
demak bu chiziq - birinchi guruhga tegishli:

$$K_3 = \begin{vmatrix} 0 & 3 & -6 \\ 3 & 8 & -13 \\ -6 & -13 & 11 \end{vmatrix} = 81 \neq 0,$$

chiziq giperboladan iborat:

$$I_1 = 0 + 8 = 8.$$

Chiziqning xarakteristik tenglamasi: $\lambda^2 - 8\lambda - 9 = 0$.




**9.2.1-chizma**

Xarakteristik tenglamaning yechimlari:

$$\lambda_1 = 9, \lambda_2 = -1,$$

Almashtirishdan so‘ng tenglama $9X^2 - Y^2 + \dfrac{81}{-9} = 0$ ko‘rinishga keladi.

Kanonik tenglamasi esa:

$$\frac{X^2}{1} - \frac{Y^2}{9} = 1.$$

Markazi quyidagi

$$\begin{cases} 3y - 6 = 0, \\ 3x + 8y - 13 = 0 \end{cases}$$

tenglamalardan topiladi. $O'(-1;2)$ - nuqta chiziq markazi. $O'X$ o‘qning burchak koeffitsiyenti $k=\frac{9}{3}=3$.

**2-misol.** Quyidagi

$$x^2-4xy+4y^2+4x-3y-7=0$$

chiziqning shakli va joylashishi, fokusi va direktrisalari aniqlansin :

**Yechish:**

$$I_2=\begin{vmatrix}1 & -2\\ -2 & 4\end{vmatrix}=0, \qquad K_3=\begin{vmatrix}1 & -2 & 2\\ -2 & 4 & -\frac{3}{2}\\ 2 & -\frac{3}{2} & -7\end{vmatrix}=-\frac{25}{4}.$$

Demak, berilgan chiziq - parabola:

$$I_1=1+4=5.$$

Parametri:

$$p=\sqrt{\frac{25}{4\cdot 5^3}}=\frac{1}{2\sqrt{5}}.$$

Kanonik tenglamasi:

$$Y^2=\frac{1}{\sqrt{5}}X.$$

O‘qining tenglamasi:

$$x-2y-\frac{1\cdot 2-2\cdot(-\frac{3}{2})}{1+4}=0$$

yoki

$$x-2y+1=0.$$

Parabola uchining koordinatalarini topish uchun tenglamalar:

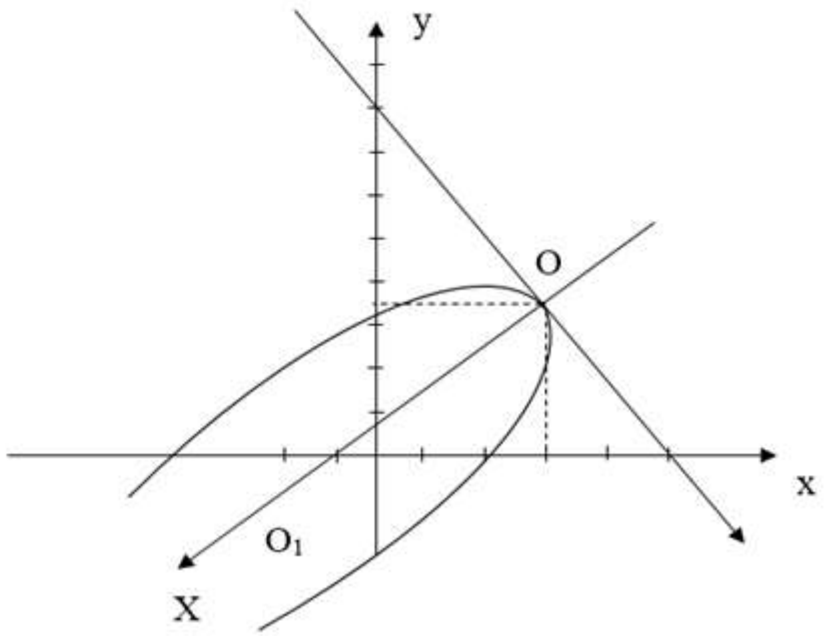
$$\begin{cases} x-2y+1=0 \\ x^2-4xy+4y^2+4x-3y-7=0, \end{cases}$$

$$\begin{cases} x-2y=-1 \\ (x-2y)^2+4x-3y-7=0, \end{cases} \quad .$$

$$\begin{cases} x-2y+1=0 \\ 4x-3y-6=0. \end{cases}$$

Natijada, parabola uchi $O'(3;2)$ nuqtada, botiqlik tomoniga yo‘nalgan o‘q vektori esa

$$\left\{\begin{vmatrix} -2 & 4 \\ 2 & -\frac{3}{2} \end{vmatrix}, -\begin{vmatrix} 1 & -2 \\ 2 & -\frac{3}{2} \end{vmatrix}\right\} = \{-5, -\frac{5}{2}\} \downarrow\downarrow (-2, -1)$$




**9.2.2-chizma**

koordinatalarga ega (9.2.2-chizma). ( $X=\sqrt{5}$ bo‘lganda $Y=\pm 1$ teng ekanini bilish foydali).

Berilgan chiziq parametri $p=\frac{1}{2\sqrt{5}}$ ga teng parabolani ifodalaydi. Parabola uchi $O'(3;2)$ nuqtada. Parabola o‘qining musbat yo‘nalishi $(-2;-1)$ vektor bilan aniqlanadi. $Ox$ va $O'x$ o‘qlar orasidagi burchakni $\varphi$ desak:

$$\cos\varphi = -\frac{2}{\sqrt{5}},\ \sin\varphi = -\frac{1}{\sqrt{5}};$$

Demak almashtirish formulalari:

$$x = \frac{-2X + Y}{\sqrt{5}} + 3,\quad y = \frac{-X - 2Y}{\sqrt{5}} + 2$$

bundan

$$X = \frac{-2x - y + 8}{\sqrt{5}},\qquad Y = \frac{x - 2y + 1}{\sqrt{5}}$$

Kanonik sistemada fokus koordinatalari: $X = \frac{1}{4\sqrt{5}},\quad Y = 0,$

boshlang‘ich sistemada esa: $x = 2{,}9;\quad y = 1{,}95$

kanonik sistemada direktrisa tenglamasi: $X = -\frac{1}{4\sqrt{5}}$

boshlang‘ich sistemada esa:

$$\frac{-2x - y + 8}{\sqrt{5}} = -\frac{1}{4\sqrt{5}} \qquad \text{yoki} \qquad 8x + 4y - 33 = 0.$$

**Mustaqil yechish uchun topshiriqlar.**

**9.1.1.** Beshta nuqtadan o‘tuvchi ikkinchi tartibli chiziqning tenglamasi tuzilsin: $(0;0)$, $(0;1)$, $(1;0)$, $(2;-5)$, $(-5;2)$.

**9.1.2.** Quyidagi egri chiziqlarning markazlari topilsin:

1) $5x^2 + 8xy + 5y^2 - 18x - 18y + 11 = 0;$

2) $2xy - 4x + 2y + 11 = 0;$

3) $4x^2 + 4xy + y^2 - 10x - 5y + 6 = 0;$

4) $x^2 - 2xy + y^2 - 3x + 2y - 11 = 0.$

**9.1.3.** 1) Parallelogrammga tashqi chizilgan ikkinchi tartibli chiziqning markaziy chiziq ekanligi va markazi parallellogram diagonallarining kesishish nuqtasidan iboratligi isbotlansin.

2) Parallelogrammga ichki chizilgan ikkinchi tartibli chiziqning hamisha markaziy ekanligi va markaz parallellogram diagonallarining kesishish nuqtasida ekanligi ko‘rsatilsin.

**9.1.4.** Uchburchakka ichki chizilgan ikkinchi tartibli chiziq markazi uchburchakning og‘irlik markazi bo‘lsa, bu chiziqning ellips ekanligi isbotlansin.

**9.1.5.** To‘g‘ri burchakli koordinatalar sistemasida quyidagi:
1) $5x^2 + 8xy + 5y^2 - 18x - 18y + 9 = 0;$
2) $2xy - 4x + 2y - 3 = 0;$
3) $x^2 - 4xy + 4y^2 - 5x + 10y + 6 = 0;$
4) $2x^2 + 3xy - 2y^2 + 5x - 2 = 0;$
5) $x^2 - 4xy + 4y^2 - 5x + 6 = 0.$
ikkinchi tartibli chiziqlarning markaziy yoki nomarkaziy chiziq ekanligini aniqlang.
**9.1.6.** Quyidagi beshta nuqtadan o‘tuvchi ikkinchi tartibli egri chiziqning tenglamasi tuzilsin:
$(0; 0)$, $(0; 2)$, $(-1; 0)$, $(-2; 1)$, $(-1; 3)$.
**9.1.7.** Quyidagi nuqtalardan qanday ikkinchi tartibli egri chiziq o‘tkazish mumkin: $(0; 0)$, $(0; 3)$, $(6; 0)$, $(2; 2)$ va $(-2; 1)$.
**9.1.8.** To‘rtta nuqta berilgan: $(0; 15)$, $(3; 0)$, $(5; 0)$ va $(2; 3)$. Bular orqali parabola tipidagi egri chiziq o‘tkazilsin.
*Ko‘rsatma.* Parabola tipidagi egri chiziq to‘rtta shart bilan aniqlanadi, chunki uning koeffitsiyentlari orasida $a_{11}a_{22} - {a_{12}}^2 = 0$ munosabat mavjud bo‘lishi kerak; demak, parabolik egri chiziqning tenglamasi faqat to‘rtta erkli parametrga ega.
**9.1.9.** Agar koordinatalar boshi $O'(1; 0)$ nuqtaga ko‘chirilsa,
$$x^2 - 4xy + 3y^2 - 2x + 1 = 0$$
egri chiziqning tenglamasi qanday shaklni oladi?
**9.1.10.** $xy - 6x + 2y + 3 = 0$ egri chiziq berilgan. Koordinatalar boshi $(-2; 6)$ nuqtaga ko‘chirilgandan so‘ng bu egri chiziqning almashingan tenglamasi topilsin.
**9.1.11.** $x^2 + 6x - 8y + 1 = 0$ egri chiziqning koordinatalar boshi $(-3; -1)$ nuqtaga ko‘chirilgandan so‘ng almashingan tenglamasi topilsin.
**9.1.12.** Quyidagi egri chiziqlarning markazlari topilsin:
1) $x^2 - 2xy + 2y^2 - 4x - 6y + 3 = 0;$
2) $3x^2 + 2xy + 3y^2 + 4x + 4y - 4 = 0;$
3) $2x^2 - 3xy - y^2 + 3x + 2y = 0;$

4) $x^2 - 2xy + y^2 - 4x - 6y + 3 = 0$;
5) $x^2 + 2xy + y^2 + 2x + 2y - 4 = 0$;
6) $2x^2 - 4xy + 5y^2 - 8x + 6 = 0$.

**9.1.13.** $a$ va $b$ parametrlarning qanday qiymatlarida

$$x^2 + 6xy + ay^2 + 3x + by - 4 = 0.$$

tenglama:

a) markaziy egri chiziqni;
b) parabola tipidagi egri chiziqni;
c) markazlar chizig‘iga ega bo‘lgan egri chiziqni ifodalaydi?

**9.1.14.** Quyidagi egri chiziqlarning markazlari topilsin:
1) $5x^2 - 3xy + y^2 + 4 = 0$;
2) $3x^2 - 2xy + 4 = 0$;
3) $7xy - 3 = 0$;
4) $9x^2 - 12xy + 4y^2 - 1 = 0$;
5) $a_{11}x^2 + 2a_{12}xy + a_{22}y^2 + a_{33} = 0$.

**9.1.15.** Koordinatalar boshi $2x^2 - 6xy + 5y^2 - 2x + 2y - 10 = 0$ egri chiziqning markaziga ko‘chirilsa, uning tenglamasi qanday ko‘rinishni oladi?

**9.1.16.** Koordinatalar boshini ko‘chirishdan foydalanib, quyidagi egri chiziqlarning tenglamalari soddalashtirilsin:
a) $7x^2 + 4xy + 4y^2 - 40x - 32y + 5 = 0$;
b) $x^2 - 2xy + 2x + 2y + 1 = 0$;
c) $6x^2 - 4xy + 9y^2 - 4x - 32y - 6 = 0$.

**9.1.17.** Umumiy $(x_0; y_0)$ markazga ega bo‘lgan hamma ikkinchi tartibli egri chiziqlarning umumiy tenglamasi tuzilsin.

**9.1.18.** Ikkinchi tartibli egri chiziqning koordinatalar boshidan va $A(0;1)$, $B(1;0)$ nuqtalardan o‘tadi. Bundan tashqari uning $C(2;3)$ markazi ma’lum. Shu egri chiziqning tenglamasi tuzilsin.

**9.1.19.** $x^2 + 2xy - y^2 - 2ax + 4ay + 1 = 0$ egri chiziqlar markazlarining geometrik o‘rni topilsin, bunda $a-$ o‘zgaruvchi parametr.

**9.1.20.** To‘rtta $(0;0)$, $(2;0)$, $(0;1)$ va $(1;2)$ nuqtalardan o‘tuvchi hamma ikkinchi tartibli markaziy egri chiziqlar markazlarining geometrik o‘rni topilsin.

# 10-MAVZU: IKKINCHI TARTIBLI CHIZIQ VA TO‘G‘RI CHIZIQNING O‘ZARO VAZIYATI.

**Reja:**

**1. Ikkinchi tartibli egri chiziqning to‘g‘ri chiziq bilan kesilishi.**

**2. Egri chiziqning diametrlari. Bosh o‘qlar. Asimptotalar. Egri chiziqning qo‘shma yo‘nalishlarga nisbatan tuzilgan tenglamasi; egri chiziqning asimptotalarga nisbatan tenglamasi.**

**3. Ikkinchi tartibli chiziqlarning urinma tenglamalari.**

**Tayanch iboralar**: ellips, giperbola, parabola, diametr, vatar, fokus, urinma, asimptota, direktrisa, qo‘shma diametr.

## 10.1. Ikkinchi tartibli egri chiziqning to‘g‘ri chiziq bilan kesilishi.

Ikkinchi tartibli

$$a_{11}x^2 + 2a_{12}xy + a_{22}y^2 + 2a_{13}x + 2a_{23}y + a_{33} = 0 \quad (10.1)$$

egri chiziqning

$$Ax + By + C = 0 \quad (10.2)$$

to‘g‘ri chiziq bilan kesilish nuqtalarining koordinatalari (10.1) va (10.2) tenglamalarni birgalikda yechib aniqlanadi.

Bu tenglamalar sistemasi, umuman aytganda, ikki qo‘sh ildizga ega bo‘lishi kerak, shuning uchun ikkinchi tartibli egri chiziq to‘g‘ri chiziq bilan ikkita (haqiqiy, mavhum yoki ustma – ust tushgan) nuqtada kesishadi. Agar bu ikki nuqta ustma – ust tushsa, to‘g‘ri chiziq egri chiziqqa shu nuqtadagi ***urinma*** deyiladi.

(10.1) egri chiziqqa $(x';y')$ nuqtadagi urinmaning tenglamasi:

$$(a_{11}x' + a_{12}y' + a_{13})x + (a_{21}x' + a_{22}y' + a_{23})y +$$

$$+(a_{31}x' + a_{32}y' + a_{33}) = 0 \qquad (10.3)$$

Agar berilgan

$$Ax + By + C = 0$$

(10.1) egri chiziqqa urinsa, u holda urinish nuqtasining koordinatalari bu to‘g‘ri chiziq va (10.3) urinma tenglamalari koeffitsiyentlarining proporsionallik, ya’ni:

$$\frac{a_{11}x' + a_{12}y' + a_{13}}{A} = \frac{a_{21}x' + a_{22}y' + a_{23}}{B} =$$

$$= \frac{a_{31}x' + a_{32}y' + a_{33}}{C} \qquad (10.4)$$

shartidan aniqlanadi.

(10.1) egri chiziq va (10.2) to‘g‘ri chiziq tenglamalarining koordinatalaridan birini yo‘qotganda ikkinchi koordinatani aniqlash uchun ikkinchi darajali emas, balki birinchi darajali tenglama hosil bo‘lishi (aniqlanayotgan koordinataning kvadrati oldidagi koeffitsiyent nolga aylanadi), (10.1) egri chiziqning (10.2) to‘g‘ri chiziq bilan kesilishining xususiy holiga ega bo‘lamiz. Bu holda tekislikning chekli qismida (10.1) egri chiziq bilan (10.2) to‘g‘ri chiziqning birgina umumiy nuqtasi bo‘ladi. Ular faqat bir nuqtada kesishadi deymiz. Bu to‘g‘ri chiziqlarning burchak koeffitsiyentlari

$$a_{11} + 2a_{12}k + a_{22}k^2 = 0$$

tenglamadan aniqlanadi. Agar (10.1) va (10.2) tenglamalar birgalikda bo‘la olmasa, ya’ni umumiy chekli ildizlarga ega bo‘lsa, (10.1) egri chiziq (10.2) to‘g‘ri chiziq bilan hech bir umumiy nuqtaga ega emas deymiz. Bu holda (10.1) va (10.2) tenglamalarning koordinatalaridan birini yo‘qotishda faqat aniqlanayotgan koordinataning kvadrati oldidagi koeffitsiyenti ham nolga aylanadi.

### 10.2. Egri chiziqning diametrlari. Bosh o‘qlar. Asimptotalar. Egri chiziqning qo‘shma yo‘nalishlarga nisbatan tuzilgan tenglamasi; egri chiziqning asimptotalarga nisbatan tenglamasi.

Agar ikkinchi tartibli egri chiziqning bir xil yo‘nalishdagi hamma vatarlari o‘tkazilsa, bu vatarlar o‘rtalarining geometrik o‘rni biror to‘g‘ri chiziqdan iborat bo‘ladi. Bu to‘g‘ri chiziq berilgan vatarga ***qo‘shma diametr*** deyiladi. Diametrning tenglamasi:

$$(a_{11}x + a_{12}y + a_{13}) + k(a_{21}x + a_{22}y + a_{33}) = 0, \quad (10.5)$$

yoki

$$F_x + kF_y = 0 \quad (10.6)$$

bu yerda $k$ – berilgan yo‘nalishdagi vatarlarning burchak koeffitsiyenti. $k$ ni o‘zgartirish bilan, ya’ni vatarlar yo‘nalishini o‘zgartirish bilan cheksiz ko‘p diametrlar hosil qilamiz, ularning hammasi egri chiziqning markazidan o‘tadi. Parabolaning hamma diametrlari o‘zaro paralleldir.

Vatarning yo‘nalishi va ularga qo‘shma diametrning yo‘nalishi berilgan egri chiziqqa nisbatan qo‘shma yo‘nalishlar deyiladi. Ikki qo‘shma yo‘nalish orasidagi bog‘lanish quyidagicha bo‘ladi:

$$a_{11} + a_{12}(k + k') + a_{22}kk' = 0. \quad (10.7)$$

***Qo‘shma diametrlar*** deb, shunday ikkita diametrga aytiladiki, ularning har biri ikkinchisiga parallel vatarlarni teng ikkiga bo‘ladi. Parabolaning qo‘shma diametrlari yo‘q, chunki hamma diametrlar bir xil yo‘nalishga ega.

Qo‘shma vatarlarga perpendikulyar bo‘lgan diametrlar egri chiziqning bosh o‘qlari deyiladi; ularning yo‘nalishlari ***bosh yo‘nalishlar*** deyiladi.

To‘g‘ri burchakli koordinatalar sistemasida bosh yo‘nalishlar

$$a_{12}k^2 + (a_{11} - a_{22})k - a_{12} = 0, \quad (10.8)$$

yoki

$$tg2\varphi = \frac{2a_{12}}{a_{11} - a_{22}} \quad (10.9)$$

tenglamadan aniqlanadi, bunda $\varphi$ – bosh yo‘nalishlardan birining $x$ o‘qi yo‘nalishi bilan tashkil etgan burchagi.

Qiyshiq burchakli koordinatalar sistemasida bosh yo‘nalishlar tenglamasi:

$$(a_{12} - a_{22}cos\omega)k^2 + (a_{11} - a_{22})k - (a_{12} - a_{11}cos\omega) = 0. \quad (10.10)$$

Aylanadan tashqari har qanday ikkinchi tartibli egri chiziq ikkita bosh yo‘nalishga ega; aylananing bosh yo‘nalishlari aniq emas(cheksiz ko‘p).

Parabolaning hamma diametrlari uchun burchak koeffitsiyenti

$$k = -\frac{a_{11}}{a_{22}} = \frac{a_{12}}{a_{22}} \qquad (10.11)$$

formula bilan aniqlanadi yoki parabolaning ikkinchi darajali hadlarining koeffitsiyentlari:

$$a_{11} = \alpha^2, \;\; a_{12} = \alpha\beta, \;\; a_{22} = \beta^2$$

bilan belgilansa, u holda burchak koeffitsiyenti

$$k = -\frac{\alpha}{\beta} \qquad (10.12)$$

bo‘ladi.

Parabolaning bosh o‘qi uning diametrlaridan biri bo‘lgani uchun u ham shu yo‘nalishga egadir va to‘g‘ri burchakli koordinatalarda

$$F_x + \frac{\beta}{\alpha} F_y = 0 \qquad (10.13)$$

tenglama bilan bilan ifodalanadi.

Parabolaning ikkinchi bosh yo‘nalishi uning diametriga perpendikulyardir, lekin parabolaning ikkinchi bosh o‘qi yo‘q.

Agar ikki qo‘shma yo‘nalishga nisbatan egri chiziqning tenglamasi tuzilsa, ya’ni koordinata o‘qlari deb, bu egri chiziqqa nisbatan qo‘shma yo‘nalishlarga ega bo‘lgan ikki to‘g‘ri chiziq olinsa, egri chiziq tenglamasiga koordinatalarning ko‘paytmasidan tuzilgan had kirmaydi($a_{12} = 0$). Parabola tenglamasida, bundan tashqari ikkinchi darajali hadlardan biri yo‘qoladi($a_{11} = 0$ yoki $a_{22} = 0$).

Agar markaziy egri chiziqning tenglamasi ikki qo‘shma diametrga(yoki bosh o‘qlarga) nisbatan yozilsa, uning tenglamasi

$$a'_{11}x^2 + a'_{22}y^2 + \frac{\Delta}{\delta} = 0 \qquad (10.14)$$

ko‘rinishni oladi.

Koordinatalar boshini parabolaning uchiga, ya’ni parabolaning bosh o‘qi bilan kesishish nuqtasiga olib ($a'_{33} = 0$), bosh o‘qni absissalar o‘qi ($a'_{23} = 0$, $a'_{12} = 0$ va $a'_{11} = 0$) va parabola uchidan o‘tgan urinmani (u parabola o‘qiga perpendikulyar) ordinatalar o‘qi deb olinsa, parabolaning eng sodda tenglamasi hosil bo‘ladi

$$a'_{22}y^2 + 2a'_{13}x = 0. \qquad (10.15)$$

Koordinata o‘qlari yuqoridagidek olinsa, markaziy egri chiziq

$$a'_{11}x^2 + a'_{22}y^2 + 2a'_{13}x = 0 \qquad (10.16)$$

tenglama bilan ifodalanadi.

Egri chiziqning diametrlaridan o‘z – o‘ziga qo‘shma bo‘lganlariga uning asimptotalari deb qarash mumkin. Asimptotalarning burchak koeffitsiyentlari

$$a_{11} + 2a_{12}k + a_{22}k^2 = 0 \qquad (10.17)$$

tenglamadan aniqlanadi.

Asimptotalar faqat markaziy egri chiziqlarda bo‘lishi mumkin: giperbola ikkita haqiqiy asimptotaga ega, ellips ikkita mavhum asimptotaga ega. Egri chiziq ikkita kesishuvchi to‘g‘ri chiziqlarga ajralsa, u holda asimptotalar bu to‘g‘ri chiziqlar bilan ustma – ust tushadi. Giperbolaning asimptotalari koordinata o‘qlari deb olinsa, u holda bu giperbolaning tenglamasi,

$$2a'_{12}xy + a'_{33} = 0 \qquad (10.18)$$

shaklni oladi.

### 10.3. Ikkinchi tartibli chiziqlarning urinma tenglamalari.

Tekislikdagi $F(x;y) = 0$ oshkormas tenglama bilan berilgan egri chiziqning $M_0(x_0;y_0)$ nuqtasidagi urinma tenglamasi

$$F_x'(x_0;y_0)(x - x_0) - F_y'(x_0;y_0)(y - y_0) = 0 \qquad (10.19)$$

Ellips uchun

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1. \quad F_x'(x) = \frac{2x}{a^2}, \quad F_y'(y) = \frac{2y}{a^2} \Rightarrow$$

$$\frac{2x}{a^2}(x - x_0) + \frac{2y}{a^2}(y - y_0) = 0 \Rightarrow$$

$$-\frac{2xx_0}{a^2} + \frac{2x^2}{a^2} + \frac{2y^2}{a^2} - \frac{2yy_0}{a^2} = 0 \Rightarrow$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{xx_0}{a^2} + \frac{yy_0}{a^2}$$

***ellipsni urinma tenglamasi***

$$\frac{xx_0}{a^2}+\frac{yy_0}{b^2}=1 \qquad (10.20)$$

deyiladi.

Giperbola uchun

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1. \quad F_x{}'(x)=\frac{2x}{a^2}, \quad F_y{}'(y)=\frac{2y}{a^2}\Longrightarrow$$

$$F_x{}'(x_0;y_0)=\frac{2x_0}{a^2}, \quad F_y{}'(x_0;y_0)=-\frac{2y_0}{a^2}\Longrightarrow$$

$$\frac{2x_0}{a^2}(x-x_0)+\frac{2y_0}{a^2}(y-y_0)=0\Longrightarrow \frac{xx_0}{a^2}-\frac{yy_0}{a^2}=\frac{x_0{}^2}{a^2}-\frac{y_0{}^2}{a^2}$$

$$\frac{xx_0}{a^2}-\frac{yy_0}{a^2}=1 \qquad (10.21)$$

***giperbolani urinma tenglamasi*** deyiladi.

**1-misol.** $x^2-2xy+2y^2-4x-6y+3=0$ egri chiziqning shunday ikkita qo‘shma diametrlari topilsinki, ularning biri koordinatalar boshidan o‘tsin.

**Yechish:** $\delta\neq 0$ bo‘lgani uchun, berilgan egri chiziq markaziy egri chiziq hisoblanadi. Uning ixtiyoriy diametrining tenglamasi

$$(x-y-2)+k(-x+2y-3)=0$$

bo‘ladi, bu yerda $k-$ unga qo‘shma diametrning burchak koeffitsiyenti. Izlangan diametr koordinatalar boshidan o‘tganligi uchun uning tenglamasidagi ozod had nolga teng bo‘lishi, ya’ni $-2-$ $-3k=0$ va $k=-\frac{2}{3}$ bo‘lishi kerak. Parametrning bu qiymatini diametrning umumiy tenglamasiga qo‘yib va soddalashtirib, $5x-7y=0$ tenglamani olamiz. Bu izlangan diametrlardan birining tenglamasidir, uning burchak koeffitsiyenti $k'=\frac{5}{7}$, demak, bunga qo‘shma diametrning tenglamasi

$$(x-y-2)+\frac{5}{7}(-x+2y-3)=0,$$

yoki $2x+3y-29=0$ ko‘rinishida bo‘ladi.

**2-misol.** $x^2-2xy+y^2+x-2y+3=0$ parabolaning o‘qi topilsin.

**Yechish:** Berilgan parabolaning hamma diametrlari $k = 1$ burchak koeffitsiyentga ega (10.12 tenglamaga asosan). Parabolaning o‘qi diametriga perpendikulyar vatarga, ya’ni $k_1 = -1$ burchak koeffitsiyentli vatarlarga qo‘shma bo‘lgan diametrdir, bu holatda koordinatalar sistemasi to‘g‘ri burchakli deb faraz qilinadi. Bu parabola har qanday diametrining tenglamasi

$$2x - 2y + 1 + k(-2x + 2y - 2) = 0$$

bo‘ladi: $k = -1$ bo‘lganda parabola o‘qining $4x - 4y + 3 = 0$ tenglamasi hosil bo‘ladi.

**3-misol.** Koordinatalar boshidan o‘tuvchi ikkinchi tartibli egri chiziqning ikki juft qo‘shma diametrlari ma’lum:

$$\begin{cases} x - 3y - 2 = 0 \\ 5x - 5y - 4 = 0 \end{cases} \text{va} \quad \begin{cases} 5y + 3 = 0 \\ 2x - y - 1 = 0. \end{cases}$$

Bu egri chiziqning tenglamasi tuzilsin.

**Yechish:** Qo‘shma diametrlarning burchak koeffitsiyentlari

$$c_{11} + a_{12}(k_1 + k_2) + a_{22}k_1k_2 = 0$$

tenglamani qanoatlantiradi. Berilgan diametrlarning burchak koeffitsiyentlari $k_1 = \frac{1}{3}$ va $k_2 = 1;\ k_1' = 0$ va $k_2' = 2.$ Bu qiymatlarni ko‘rsatilgan tenglamaga qo‘yib, quyidagilarni hosil qilamiz:

$$\begin{cases} 3a_{11}x + 4a_{12} + a_{22} = 0 \\ a_{11} + 2a_{12} = 0 \end{cases}, \qquad a_{11}: a_{12}: a_{22} = 2: -1: -2.$$

Izlangan egri chiziq markazining koordinatalarini ikki diametr tenglamalarini birgalikda yechib aniqlashimiz mumkin:

$$x_0 = \frac{1}{5}; \quad y_0 = \frac{3}{5}.$$

Bu koordinatalar $Fx_0 = 0$ va $Fy_0 = 0$ tenglamalarni, ya’ni berilgan holda $2x_0 - y_0 + a_{13} = 0$ va $-x_0 - 2y_0 + a_{23} = 0$ tenglamalarni qanoatlantirishi kerak; $x_0$ va $y_0$ ning qiymatlarini bu tenglamalarga qo‘yib, $a_{13} = -1$ va $a_{23} = -1$ ni olamiz. Bundan tashqari egri chiziq koordinatalar boshidan o‘tadi, demak, $a_{33} = 0$ va egri chiziqning tenglamasi:

$$2x^2 - 2xy - 2y^2 - 2x - 2y = 0 \text{ yoki } x^2 - xy - y^2 - x - y = 0.$$

## Mustaqil yechish uchun topshiriqlar.

**10.1. Ikkinchi tartibli egri chiziqning to‘g‘ri chiziq bilan kesilishi. Ikkinchi tartibli to‘g‘ri chiziqning diametriga doir misollar.**

**10.1.1.** $5x^2 - 3xy + y^2 - 3x + 2y - 5 = 0$ chiziqning $x - 2y - 1 = = 0$ to‘g‘ri chiziq bilan kesishishidan hosil qilingan vatarning o‘rtasidan o‘tadigan diametr tenglamasi yozilsin.

**10.1.2.** $4xy - 5y^2 + 2x + 6y + 1 = 0$ egri chiziqning $(-4; 2)$ nuqta orqali o‘tadigan diametri tenglamasi yozilsin.

**10.1.3.** $5x^2 - 6xy + 3y^2 - 2x = 0$ egri chiziqning $2x - 3y = 0$ to‘g‘ri chiziqqa parallel bo‘lgan diametri tenglamasi yozilsin.

**10.1.4.** Ikki egri chiziqning umumiy diametri topilsin:
$x^2 - 2xy - y^2 - 2x - 2y = 0,\ \ x^2 - 2xy + y^2 - 2x - 2y = 0.$

**10.1.5.** $5x^2 - 3xy + y^2 - 3x + 2y - 5 = 0$ chiziq va ikkita $A(2; 1)$ va $B(1; 4)$ nuqta berilgan. $A$ nuqtadan o‘tuvchi diametrga qo‘shma bo‘lgan $B$ nuqtadan shunday vatar o‘tkazilsin.

**10.1.6.** $\frac{x^2}{9} - \frac{y^2}{4} = 1$ giperbolaning $N(5; 1)$ nuqtada teng ikkiga bo‘linadigan vatarining tenglamasi tuzilsin.

**10.1.7.** $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ giperbolaning o‘qlari ular teng ikkiga bo‘ladigan vatarlarga perpendikulyar bo‘lgan yagona diametrlari ekanligini tekshiring.

**10.1.8.** $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ giperbolaga ichki chizilgan kvadratning uchlari topilsin va qanday giperbolalarga ichki kvadrat chizish mumkinligi tekshirilsin.

**10.1.9.** $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ellipsning $F(c; 0)$ fokusi orqali katta o‘qiga perpendikulyar bo‘lgan vatar o‘tkazilgan. Bu vatar uzunligini toping.

**10.1.10.** $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ellipsga ichki chizilgan kvadrat tomonining uzunligi hisoblansin.

**10.1.11.** $\frac{x^2}{100}+\frac{y^2}{64}=1$ ellipsning $2x-y+7=0,\quad 2x-y-1=0$ vatarlarining o‘rtalari orqali o‘tadigan to‘g‘ri chiziq tenglamasini tuzing.

**10.1.12.** $\frac{x^2}{25}+\frac{y^2}{16}=1$ ellipsning $M(2;1)$ nuqtada teng ikkiga bo‘linuvchi vatar tenglamasi tuzilsin.

**10.1.13.** $y^2=2px$ parabolaning fokusi orqali uning o‘qiga perpendikulyar bo‘lgan vatar o‘tkazilgan. Bu vatarning uzunligini aniqlang.

**10.1.14.** $y^2=4x$ parabolaning $M(3;1)$ nuqtada teng ikkiga bo‘ladigan vatarini toping.

**10.1.15.** $x+y-3=0$ to‘g‘ri chiziq va $x^2=4y$ parabola kesishgan nuqtani toping.

**10.1.16.** $3x+4y-12=0$ to‘g‘ri chiziq va $y^2=-9x$ parabola kesishgan nuqtani toping.

**10.1.17.** $3x-2y+6=0$ to‘g‘ri chiziq va $y^2=6x$ parabola kesishgan nuqtani toping.

**10.1.18.** $\frac{x^2}{100}+\frac{y^2}{225}=1$ ellips bilan $y^2=24x$ parabolaning kesishish nuqtalarini toping.

**10.1.19.** $\frac{x^2}{20}-\frac{y^2}{5}=-1$ giperbola va $y^2=3x$ parabola kesishgan nuqtalarini toping.

**10.1.20.** Ikki parabolaning kesishish nuqtalarini toping:
$y=x^2-2x+1,\quad x=y^2-6y+7.$

**10.1.21.** $x+2y-7=0$ to‘g‘ri chiziq bilan $x^2+4y^2=25$ ellipsning kesishish nuqtalarini toping.

**10.1.22.** $3x+10y-25=0$ to‘g‘ri chiziq va $\frac{x^2}{25}+\frac{y^2}{4}=1$ tenglama bilan berilgan ellipsning kesishish nuqtalarini toping.

**10.1.23.** $3x-4y-40=0$ to‘g‘ri chiziq va $\frac{x^2}{16}+\frac{y^2}{9}=1$ tenglama bilan berilgan ellipsning kesishish nuqtalarini toping.

**10.1.24.** Agar to‘g‘ri chiziq va ellips quyidagi tenglamalar bilan berilgan bo‘lsa, ularning o‘zaro kesishishi, urinishi yoki umumiy nuqtaga ega emasligini aniqlang:

1) $2x - y - 3 = 0,$ $\frac{x^2}{16}+\frac{y^2}{9}=1$

2) $2x + y - 10 = 0,$ $\frac{x^2}{9}+\frac{y^2}{4}=1$

3) $3x + 2y - 20 = 0,$ $\frac{x^2}{40}+\frac{y^2}{10}=1$

**10.1.25.** $y = -kx + m$ to‘g‘ri chiziq $m$ ning qanday qiymatlarida $\frac{x^2}{20}+\frac{y^2}{5}=1$ ellipsni: 1) kesib o‘tadi; 2) unga urinadi; 3) ushbu ellips tashqarisidan o‘tadi.

**10.1.26.** $\frac{x^2}{20}-\frac{y^2}{5}=1$ giperbola va $2x - y - 10 = 0$ to‘g‘ri chiziq kesishmasining nuqtalarini toping.

**10.1.27.** $\frac{x^2}{25}-\frac{y^2}{16}=1$ giperbola va $4x - 3x - 16 = 0$ to‘g‘ri chiziq kesishmasining nuqtalarini toping.

**10.1.28.** $\frac{x^2}{9}-\frac{y^2}{4}=1$ giperbola va $2x - y + 1 = 0$ to‘g‘ri chiziq kesishmasining nuqtalarini toping.

**10.1.29.** $m$ ning qanday qiymatlarida $y = 5x + m$ to‘g‘ri chiziq:

1) $\frac{x^2}{9}-\frac{y^2}{36}=1$ giperbola bilan kesishadi; 2) urinma bo‘ladi;

3) giperbolaning tashqarisidan o‘tadi.

**10.1.30.** $k$ va $m$ ning qanday qiymatida $y = kx + m$ to‘g‘ri chiziq $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ giperbolaga urinadi.

## 10.2. Egri chiziqning diametrlari. Bosh o‘qlar. Asimptotalar. Egri chiziqning qo‘shma yo‘nalishlarga nisbatan tuzilgan

**tenglamasi; egri chiziqning asimptotalarga nisbatan tenglamasiga doir misollar.**

**10.2.1.** Berilgan to‘rtta $A(1;0)$, $B(3;2)$, $C(0;2)$, $D(0;-2)$ nuqta orqali o‘tadigan; $AB$ va $CD$ vatarlari o‘zaro qo‘shma yo‘nalishlari ekanligini bilgan holda, ikkinchi tartibli chiziqning tenglamasini yozing.

**10.2.2.** $3x^2-2xy+y^2+6x-10=0$, $3x^2-2xy-y^2+6x-$ $-10=0$ ikkinchi tartibli chiziqlar berilgan. Har bir chiziq uchun shunday qo‘shma diametrlar juftini topingki, birinchi juft diametrlar ikkinchi juft diametrlarga parallel bo‘lsin.

**10.2.3.** Giperbolaning asimptotalari topilsin:
$10x^2+21xy+9y^2-41x-39y+4=0.$

**10.2.4.** Quyidagi
1) $x^2-3xy-10y^2+6x-8y=0;$
2) $3x^2+2xy-y^2+8x+10y-14=0;$
3) $3x^2+7xy+4y^2+5x+2y-6=0;$
4) $10xy-2y^2+6x+4y+21=0.$
giperbolalarning asimptotalari topilsin.

**10.2.5.** $x^2-2y^2-5x+4y+6=0$ chiziqning absissa o‘qi bilan kesishish nuqtalaridagi urinmalari tenglamalari tuzilsin.

**10.2.6.** $x^2+xy+y^2+2x+3y-3=0$ chiziqning $3x+3y-5=$ $=0$ to‘g‘ri chiziqqa parallel bo‘lgan urinmalari tenglamalari tuzilsin.

**10.2.7.** $4x^2+4xy+y^2-6x+4y+2=0$ ikkinchi tartibli chiziqning $Oy$ o‘qiga parallel urinmalarining tenglamalari yozilsin.

**10.2.8.** $M(3;4)$ nuqtadan $2x^2-4xy+y^2-2x+6y-3=0$ egri chiziqqa urinmalar o‘tkazilsin.

**10.2.9.** $(\alpha x+\beta y+\gamma)^2+2(Ax+By+C)=0,$ $\begin{vmatrix}\alpha & \beta \\ A & B\end{vmatrix}\neq 0$ tenglama berilgan.
1) Bu tenglamaning parabolani aniqlashi;
2) $\alpha x+\beta y+\gamma$ to‘g‘ri chiziqning diametr ekanligi;
3) $Ax+By+C=0$ to‘g‘ri chiziqning diametr bilan parabolaning kesishish nuqtasidagi urinmasi ekanligi ko‘rsatilsin.

**10.2.10.** Parabola: $(\alpha x+\beta y)^2+2a_{13}x+2a_{23}y+a_{33}=0$, $\begin{vmatrix} \alpha & \beta \\ a_{13} & a_{23} \end{vmatrix} \neq 0$ tenglama bilan berilgan. Parabolaga uning ixtiyoriy $(x_0;y_0)$ nuqtasidan o‘tkazilgan urinma tenglamasini va unga mos kelgan qo‘shma diametr tenglamasini yozing.

**10.2.11.** Ikkinchi tartibli chiziqqa tashqi chizilgan parallelogrammning diagonallari berilgan chiziqning qo‘shma diametrlari ekanligi isbotlansin.

**10.2.12.** $\frac{x^2}{5}-\frac{y^2}{4}=1$ giperbolaga $M(5;-4)$ nuqtada urinadigan to‘g‘ri chiziq tenglamasi yozilsin.

**10.2.13.** $x^2-y^2=8$ giperbolaga $N(3;-1)$ nuqtada urinadigan to‘g‘ri chiziq tenglamasi yozilsin.

**10.2.14.** $x^2-\frac{y^2}{4}=1$ giperbolaga $M(1;4)$ nuqta orqali o‘tadigan urinmalarning tenglamalari yozilsin.

**10.2.15.** Berilgan $\frac{x^2}{9}-\frac{y^2}{36}=1$ giperbolaga:

1) $3x-y-17=0$ to‘g‘ri chiziqqa parallel;

2) $2x+5y+11=0$ to‘g‘ri chiziqqa perpendikulyar qilib o‘tkazilgan urinmalarning tenglamalari tuzilsin.

**10.2.16.** Giperbola asimptotalarining tenglamalari $y=\pm\frac{1}{2}x$ va urinmalardan birining tenglamasi $5x-6y-8=0$ ma’lum bo‘lsa, giperbola tenglamasini tuzing.

**10.2.17.** $\frac{x^2}{20}-\frac{y^2}{5}=1$ giperbolaga urinma va $4x+3y-7=0$ to‘g‘ri chiziqqa perpendikulyar bo‘lgan tenglamani tuzing.

**10.2.18.** $\frac{x^2}{16}-\frac{y^2}{64}=1$ giperbolaga urinma va $10x-3y+9=0$ to‘g‘ri chiziqqa parallel bo‘lgan tenglamani tuzing.

**10.2.19.** $\frac{x^2}{16}-\frac{y^2}{8}=1$ giperbolaga urinma va $2x+4y-5=0$ to‘g‘ri chiziqqa parallel bo‘lgan tenglamani tuzing va ular orasidagi masofa $d$ ni toping.

**10.2.20.** $\frac{x^2}{24}-\frac{y^2}{18}=1$ giperbola va $3x+2y+1=0$ to‘g‘ri chiziqqa eng yaqin bo‘lgan $M_1$ nuqtani toping va $M_1$ nuqtadan to‘g‘ri chiziqqacha bo‘lgan $d$ masofani hisoblang.
**10.2.21.** $N(-1;-7)$ nuqtadan o‘tib, $x^2-y^2=16$ giperbolaga o‘tkazilgan urinma tenglamasini tuzing.
**10.2.22.** $C(1;-10)$ nuqtadan $\frac{x^2}{8}-\frac{y^2}{32}=1$ giperbolaga urinma o‘tkazilgan. $C$ nuqtadan o‘tuvchi tenglamasini tuzing.
**10.2.23.** $\frac{x^2}{25}+\frac{y^2}{16}=1$ ellipsning burchak koeffitsiyenti $k=\frac{2}{3}$ bo‘lgan vatariga qo‘shma diametrini aniqlang.
**10.2.24.** $\frac{x^2}{32}+\frac{y^2}{18}=1$ elipsning $M(4;3)$ nuqtasida o‘tkazilgan urinmasining tenglamasi tuzilsin.
**10.2.25.** $\frac{x^2}{25}+\frac{y^2}{16}=1$ ellipsning $N(10;4)$ nuqta orqali o‘tuvchi urunmalarining tenglamalarini tuzing.
**10.2.26.** $\frac{x^2}{16}+\frac{y^2}{9}=1$ ellipsning $x+y-1=0$ to‘g‘ri chiziqqa parallel bo‘lgan urinmalarini aniqlang.
**10.2.27.** $3x^2+8y^2=45$ ellips markazidan 3 birlik uzoqlikdan o‘tadigan urinmalarining tenglamalari tuzilsin.
**10.2.28.** $\frac{x^2}{6}+\frac{y^2}{3}=1$ ellipsga tashqi chizilgan kvadrat tomonlarining tenglamalarini tuzing.
**10.2.29.** $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellipsning ikkkita qo‘shma diametri orasidagi o‘tkir burchakning o‘zgarish chegaralarini aniqlang.
**10.2.30.** $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellipsning teng qo‘shma radiuslarini toping.
**10.2.31.** $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellipsning fokusi orqali o‘tib, uning qo‘shma diametrlariga parallel bo‘lgan ikki vatarining yig‘indisini toping.
**10.2.32.** Quyidagi tenglamalar giperbola hosil qilishini tekshiring va uning $C$ markazi koordinatasini, yarim o‘qlarini, ekssentrisitetini, asimptota va direktrisa tenglamalarini toping:

1) $16x^2 - 9y^2 - 64x - 54y - 161 = 0$;
2) $9x^2 - 16y^2 + 90x + 32y - 367 = 0$;
3) $16x^2 - 9y^2 - 64x - 18y + 199 = 0$.

**10.2.33.** Giperbolaning bitta diametrini uchlariga o‘tkazilgan urinmalar parallel bo‘lishini isbotlang.

**10.2.34.** $y^2 = 4x$ parabolaning $M(9;6)$ nuqtasida o‘tkazilgan urinmasining tenglamasini tuzing.

**10.2.35.** $y^2 = 2px$ parabolaga o‘tkazilgan urinmaning $x - 3y + 9 = 0$ tenglamasi berilgan. Parabolaning tenglamasini tuzing.

**10.2.36.** Burchak koeffisiyenti $k$ ning qanday qiymatlarida ushbu $y = ax + 2$ to‘g‘ri chiziq:
1) $y^2 = 4x$ parabolaga urinma bo‘ladi;
2) $y^2 = 4x$ parabola bilan kesishadi.

**10.2.37.** $y^2 = 2px$ parabolaga $M_1(x_1; y_1)$ nuqtadan o‘tuvchi urinma tenglamasini tuzing.

**10.2.38.** $y^2 = 2px$ parabolaga $M_1(x_1; y_1)$ nuqtadan o‘tib, $2x + 2y - 3 = 0$ to‘g‘ri chiziqqa parallel bo‘lgan urinma tenglamasini tuzing .

**10.2.39.** $y^2 = 16x$ parabolaga $M_1(x_1; y_1)$ nuqtadan o‘tib, $2x + 4y + 7 = 0$ to‘g‘ri chiziqqa perpendikulyar bo‘lgan urinma tenglamasini tuzing .

**10.2.40.** $y^2 = 2px$ parabolaning o‘qiga $45^0$ li burchak ostida og‘ishgan vatarlariga qo‘shma bo‘lgan diametri topilsin.

## 11-MAVZU: IKKINCHI TARTIBLI CHIZIQLARNING TENGLAMALARINI SODDALASHTIRISH.

**Reja:**

**1. Markazli egri chiziqning tenglamasini soddalashtirish.**
**2. Markazli egri chiziqning kanonik tenglamasini tekshirish.**
**3. Markazsiz egri chiziqning tenglamasini soddalashtirish.**
**4. Umumiy tenglama bilan berilgan ikkinchi tartibli chiziqlarni aniqlash va sinflarga ajratish.**

**Tayanch iboralar**: ellips, giperbola, parabola, diametr, vatar, fokus, urinma, asimptota, direktrisa, invariant, mavhum ellips.

### 11.1. Markazli egri chiziqning tenglamasini soddalashtirish.

Ikkinchi tartibli egri chiziqning markazi koordinatalar boshida bo‘lgan holda uning tenglamasi

$$Ax_1^2 + 2Bx_1y_1 + Cy_1^2 + 2f(a,b) = 0, \qquad (11.1)$$

bunda

$$2f(a,b) = Aa^2 + 2Bab + Cb^2 + 2Da + 2Eb + F. \qquad (11.2)$$

O‘zgaruvchi koordinatalarni $a, b$ orqali faraz qilganda,

$$f_a(a,b) + kf_b(a,b) = 0 \qquad (11.3)$$

bo‘ladi. Izlangan geometrik o‘rinning, ya’ni diametrning tenglamasi shundan iborat

$$f_a(a,b) = Aa + Bb + D, \qquad f_b(a,b) = Ba + Cb + E$$

bo‘ladi. (11.3) ga asosan markazli egri chiziqning $a$ va $b$ koordinatalari ushbu sistema bilan aniqlangan edi:

$$\begin{cases} Aa + Bb + D = 0 \\ Ba + Cb + E = 0 \end{cases} \qquad (11.4)$$

bulardan birinchisini $a$ ga va ikkinchisini $b$ ga ko‘paytirib, so‘ngra ularni qo‘shamiz. Bu chog‘da

$$Aa^2 + 2Bab + Cb^2 + Da + Eb = 0$$

bo‘ladi. Buning ikkala tomoniga $Da + Eb + F$ ni qo‘shib, (11.2) ni e’tiborga olsak,

$$f(a,b) = Da + Eb + F \qquad (11.5)$$

bo‘ladi. Agar bundagi $a$ va $b$ ning o‘rniga ularning ifodalari qo‘yilsa:

$$2f(a,b) = \frac{D^2C + BDE + AE^2 - BDE}{B^2 - AC} + F =$$

$$= \frac{D^2C - 2BDE + AE^2 + B^2F - ACF}{B^2 - AC},$$

yoki

$$2f(a,b) = -\frac{\Delta}{B^2 - AC} \qquad (11.6)$$

Shuning uchun (11.1) tenglamaning ko‘rinishi quyidagicha bo‘ladi:

$$Ax_1^2 + 2Bx_1y_1 + Cy_1^2 = \frac{\Delta}{B^2 - AC}. \tag{11.7}$$

Bu tenglamani yana soddalashtirish maqsadi bilan koordinata o‘qlarining yo‘nalishlarini o‘zgartiramiz, ya’ni biror, hozircha ma‘lum bo‘lmagan, ixtiyoriy burchakka aylantiramiz. Aylantirilgan burchak, ya’ni koordinata o‘qlarining yangi va eski yo‘nalishlar orasidagi burchak $\alpha$ faraz qilinsa va egri chiziqning yangi sistemaga nisbatan o‘zgaruvchi koordinatalari $x$ va $y$ faraz qilinsa, u holda almashtirish formulalari quyidagicha bo‘ladi:

$$\begin{cases} x_1 = xcos\alpha - ysin\alpha \\ y_1 = xsin\alpha + ycos\alpha \end{cases} \tag{11.8}$$

Bularni (11.7) ga qo‘yamiz:

$$A(xcos\alpha - ysin\alpha)^2 + 2B(xcos\alpha - ysin\alpha)(xsin\alpha + ycos\alpha) +$$
$$+C(xsin\alpha + ycos\alpha)^2 = \frac{\Delta}{B^2 - AC}$$

yoki bundagi qavslarni ochib, $x^2$, $xy$ va $y^2$ li hadlari to‘plab olinsa, uning ko‘rinishi bunday bo‘ladi:

$$(Acos^2\alpha + 2Bcos\alpha \cdot sin\alpha + Csin^2\alpha)x^2 + 2(-Acos\alpha \cdot sin\alpha +$$
$$+Bcos^2\alpha - Bsin^2\alpha + Csin\alpha \cdot cos\alpha)xy + (Asin^2\alpha -$$
$$-2Bsin\alpha \cdot cos\alpha + Ccos^2\alpha)y^2 = \frac{\Delta}{B^2 - AC}. \tag{11.9}$$

Bu tenglamaning koeffitsiyentlarini quyidagicha ifoda qilamiz:

$$\begin{cases} A_1 = Acos^2\alpha + 2Bsin\alpha \cdot cos\alpha + Csin^2\alpha \\ B_1 = -Asin\alpha \cdot cos\alpha + Bcos^2\alpha - Bsin^2\alpha + Csin\alpha \cdot cos\alpha \\ C_1 = Asin^2\alpha - 2Bsin\alpha \cdot cos\alpha + Ccos^2\alpha \end{cases} \tag{11.10}$$

Bu holda (11.9) ning ko‘rinishi bunday bo‘ladi:

$$A_1x^2 + 2B_1xy + C_1y^2 = \frac{\Delta}{B^2 - AC}. \tag{11.11}$$

(11.10) dagi ifodalardan birinchisi bilan uchinchisini qo‘shsak:

$$A_1 + C_1 = A + C \tag{11.12}$$

va birinchisidan uchinchisini ayirsak:

$$A_1 - C_1 =$$
$$= A(cos^2\alpha - sin^2\alpha) + 4Bsin\alpha \cdot cos\alpha - C(cos^2\alpha - sin^2\alpha) =$$
$$= (A - C)(cos^2\alpha - sin^2\alpha) + 4Bsin\alpha \cdot cos\alpha;$$
$$cos^2\alpha - sin^2\alpha = cos2\alpha, \quad 2sin\alpha \cdot cos\alpha = sin2\alpha$$
bo‘lgani uchun
$$A_1 - C_1 = (A - C)cos2\alpha + 2Bsin2\alpha,$$
$$B_1 = B(cos^2\alpha - sin^2\alpha) - (A - C)sin\alpha \cdot cos\alpha, \tag{11.13}$$
yoki
$$B_1 = Bcos2\alpha - \frac{1}{2}(A - C)sin2\alpha, \tag{11.14}$$
yoki
$$4B_1^2 = 4B^2cos^2 2\alpha - 4B(A - C)sin2\alpha \cdot cos2\alpha + (A - C)^2 sin^2 2\alpha, \tag{11.15}$$
$$(A_1 - C_1)^2 = (A - C)^2 cos^2 2\alpha + 4B(A - C)sin2\alpha \cdot cos2\alpha + +4B^2 sin^2 2\alpha \tag{11.16}$$
(11.15) va (11.16) ni qo‘shganda
$$(A_1 - C_1)^2 + 4B_1^2 = (A - C)^2 + 4B^2.$$
So‘ngi ifodadan (11.12) ning kvadratini ayirib olamiz:
$$(A_1 - C_1)^2 - (A_1 + C_1)^2 + 4B^2 = (A - C)^2 - (A + C)^2 + 4B^2,$$
yoki
$$-4A_1C_1 + 4B_1^2 = 4AC + 4B^2,$$
yoki
$$B_1^2 - A_1C_1 = B^2 - AC. \tag{11.17}$$
Bajarilgan almashtirish muhim xossaga egadir. Haqiqatda, (11.7) tenglama, to‘g‘ri burchakli koordinatalarda bajarilgan (11.8) almashtirish natijasida (11.11) ga kelib, hamon o‘z ko‘rinishini saqladi. (11.7) tenglamaning chap tomonidagi $x_1$ va $y_1$ ga nisbatan tuzilgan bir jinsli ikkinchi darajali ko‘p hadli xuddi shunga o‘xshash $x$ va $y$ ga nisbatan tuzilgan (11.11) ning chap tomonidagi ko‘p hadiga aylanadi. Ikkinchi tomondan (11.12) va (11.17) ga muvofiq.
$$A + C \text{ va } B^2 - AC \tag{11.18}$$
ifodalar forma va miqdor jihatdan o‘zini saqlab qoldi; umuman bunday xossaga ega bo‘lgan hadi kabi ifodalarni ***invariant*** deyiladi. Shuning

uchun (11.18) ifodalari $Ax_1^2 + 2Bx_1y_1 + Cy_1^2$ bir jinsli ko‘p hadligining (11.8) almashtirish bo‘yicha invariant deyiladi. $\alpha$ burchagi hozirgacha bizda ixtiyoriy edi. Endi uning qiymatini shunday qilib aniqlaymizki, (11.11) tenglamaning $(xy)$ li hadi yo‘q bo‘lsin. Bu esa (11.14) ga asosan

$$B_1 = Bcos2\alpha - \frac{1}{2}(A - C)sin2\alpha = 0$$

bo‘lganda, yoki

$$tg2\alpha = \frac{2B}{A - C} \qquad (11.19)$$

bo‘lgan holda mumkin. Bu chog‘da (11.11) tenglamaning ko‘rinishi bunday bo‘ladi:

$$A_1x^2 + C_1y^2 = \frac{\Delta}{B^2 - AC}. \qquad (11.20)$$

Ikkinchi tomondan $B_1 = 0$ bo‘lganda (11.17) ning ko‘rinishi bunday bo‘ladi:

$$-A_1C_1 = B^2 - AC,$$

yoki

$$A_1C_1 = AC - B^2 \qquad (11.21)$$

(11.12) bilan (11.21) ga asosan (11.20) tenglamaning $A_1$ va $C_1$ koeffitsiyentlari ushbu ikkinchi darajali tenglamaning ildizlaridan iborat:

$$t^2 - (A + C)t + (AC - B^2) = 0,$$

demak,

$$A_1 = \frac{A + C + \sqrt{(A + C^2) - 4(AC - B^2)}}{2} =$$

$$= \frac{A + C + \sqrt{A^2 + 2AC + C^2 - 4AC + 4B^2}}{2} =$$

$$= \frac{A + C + \sqrt{A^2 - 2AC + C^2 + 4B^2}}{2} = \frac{A + C + \sqrt{(A - C)^2 + 4B^2}}{2};$$

$$C_1 = \frac{A + C - \sqrt{(A - C)^2 + 4B^2}}{2}.$$

Shuning bilan natijada markazli egri chiziqning eng sodda yoki kanonik tenglamasining koeffitsiyentlari ushbu formulalar bilan aniqlanadi:

$$\begin{cases} A_1 = \frac{1}{2}(A + C + \sqrt{(A-C)^2 + 4B^2}) \\ C_1 = \frac{1}{2}(A + C - \sqrt{(A-C)^2 + 4B^2}. \end{cases} \qquad (11.22)$$

## 11.2. Markazli egri chiziqning kanonik tenglamasini tekshirish.

Muvofiq markazli egri chiziqning ***kanonik tenglamasi*** bo‘lsa,

$$A_1x^2 + C_1y^2 = \frac{\Delta}{M} \qquad (11.23)$$

bunda

$$M = B^2 - AC. \qquad (11.24)$$

Endi (11.23) tenglamaning qanday egri chiziq ifoda qilishini tekshiramiz. U tenglamaning ikkala tomonini $\frac{\Delta}{M}$ ga bo‘lganda

$$\frac{A_1M}{\Delta}x^2 + \frac{C_1M}{\Delta}y^2 = 1 \qquad (11.25)$$

bo‘ladi. Bu tenglamaning geometrik ma’nosi uning koeffitsiyentlariga bog‘liqdir. Shuning uchun ularning ustida turlicha faraz qilishga to‘g‘ri keladi. $M \neq 0$ bo‘lgani uchun uning ustida ikki xil faraz qilish mumkin: 1) $M < 0$ va 2) $M > 0$. Eng avval birinchi holni tekshiramiz, ya’ni

$$M = B^2 - AC < 0 \qquad (11.26)$$

bo‘lsin. (11.12) va (11.21) tenglamalarni (11.26) ga asosan

$$AC > B^2$$

bu esa $A$ va $C$ ishoralarning bir xilligini ko‘rsatadi. (11.26) ga muvofiq (11.21) dan $A_1C_1 > 0$, bu esa $A_1$ va $C_1$ ishoralarning bir xilligini ko‘rsatadi. Shuning uchun $A_1 + C_1$ va $A + C$ yig‘indilari ham bir xil ishorali bo‘ladi. Ikkinchi tomondan (11.12) ga asosan bu yig‘indilar o‘zaro teng bo‘lgani uchun: $A$, $C$, $A_1$ va $C_1$ koeffitsiyentlarining ishoralari bir xil bo‘ladi. Shuning uchun ulardan birining ishorasiga diqqat qilinsa kifoya bo‘ladi. Masalan, $A$ ni olganda, agar:

a) $A\cdot\Delta < 0$ bo‘lsa, ya’ni $A$ va $\Delta$ ning ishoralari har xil bo‘lsa, u holda (11.23) ga asosan

$$\frac{A_1M}{\Delta} > 0, \qquad \frac{C_1M}{\Delta} > 0.$$

Shuning uchun

$$\frac{A_1M}{\Delta} = \frac{1}{a^2}, \qquad \frac{C_1M}{\Delta} = \frac{1}{b^2}$$

faraz qilinsa, (11.25) tenglamaning ko‘rinishi quyidagicha bo‘ladi:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1. \qquad (11.27)$$

Bu esa yarim o‘qlari $a$ va $b$ dan iborat bo‘lgan ellipsni ifoda qiladi,

b) $A\cdot\Delta > 0$ bo‘lsa, ya’ni $A$ va $\Delta$ ning ishoralari bir xil bo‘lsa, u holda (11.26) ga asosan

$$\frac{A_1M}{\Delta} < 0, \qquad \frac{C_1M}{\Delta} < 0.$$

Shuning uchun bu holda

$$\frac{A_1M}{\Delta} = -\frac{1}{a^2}, \qquad \frac{C_1M}{\Delta} = -\frac{1}{b^2}$$

faraz qilinsa, (11.25) tenglamaning ko‘rinishi quyidagicha bo‘ladi:

$$-\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1. \ \ yoki \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = -1. \qquad (11.28)$$

Bu tenglamada $x$ va $y$ hech qanday haqiqiy qiymatga ega bo‘la olmaydi. Shuning uchun ***mavhim ellipsni*** ifoda qiladi.

Endi $M$ ni musbat faraz qilamiz:

$$M = B^2 - AC > 0. \qquad (11.29)$$

Bu holda (11.21) ga asosan $A_1C_1 < 0$, ya’ni $A_1$ va $C_1$ ning ishoralari har xil bo‘ladi. Shuning uchun bu holda

$$\frac{A_1M}{\Delta} = \pm\frac{1}{a^2}, \qquad \frac{C_1M}{\Delta} = \pm\frac{1}{b^2}$$

faraz qilish mumkin. Bu holda (11.25) tenglamaning ko‘rinishi quyidagicha bo‘ladi:

$$\frac{x^2}{\pm a^2} + \frac{y^2}{\pm b^2} = 1 \ \text{ yoki } \ \frac{x^2}{a^2} - \frac{y^2}{b^2} = \pm 1.$$

Bu esa yarim o‘qlari $a$ va $b$ dan iborat bo‘lgan giperbolani ifoda qiladi.

## 11.3. Markazsiz egri chiziqning tenglamasini soddalashtirish.

Egri chiziqning markazi cheksiz uzoqda bo‘lgan holda

$$M = B^2 - AC = 0 \text{ yoki } AC = B^2 \qquad (11.30)$$

bo‘ladi. Ikkinchi tartibli egri chiziqning umumiy tenglamasini olib uning ikkila tomonini $A$ ga ko‘paytiramiz:

$$A^2x^2 + 2ABxy + ACy^2 + A(2Dx + 2Ey + F) = 0$$

yoki (11.30) ga asosan:

$$(Ax + By)^2 + A(2Dx + 2Ey + F) = 0. \qquad (11.31)$$

Tenglamani soddalashtirish maqsadi bilan koordinata o‘qlarining yo‘nalishlarini o‘zgartiramiz, masalan, uni biror $\alpha$ burchakka aylantiramiz. Bu holda almashtirish formulalari quyidagicha bo‘ladi:

$$\begin{cases} x = x_1cos\alpha - y_1sin\alpha \\ y = x_1sin\alpha + y_1cos\alpha. \end{cases} \qquad (11.32)$$

Bularni (11.31) formulaga qo‘yilsa:

$$[A(x_1cos\alpha - y_1sin\alpha) + B(x_1sin\alpha + y_1cos\alpha)]^2 +$$
$$+A[2D(x_1cos\alpha - y_1sin\alpha) + 2E(x_1sin\alpha + y_1cos\alpha) + F] = 0$$

yoki

$$[(Acos\alpha + Bsin\alpha)x_1 + (Bcos\alpha - Asin\alpha)y_1]^2 +$$
$$+A[2(Dcos\alpha + Esin\alpha)x_1 + 2(Ecos\alpha - Dsin\alpha)y_1 + F] = 0. \qquad (11.33)$$

Hozirgacha $\alpha$ ixtiyoriy burchak edi. Endi uning qiymatini shunday aniqlaymizki,

$$Acos\alpha + Bsin\alpha = 0$$

yoki

$$tg\alpha = -\frac{A}{B} \qquad (11.34)$$

bo‘lsin. Buni e’tiborga olib,

$$\begin{cases} N = (Bcos\alpha - Asin\alpha)^2 \\ P = A(Dcos\alpha + Esin\alpha) \\ Q = A(Ecos\alpha - Dsin\alpha) \\ R = AF \end{cases} \qquad (11.35)$$

faraz qilinsa, (11.33) tenglamaning ko‘rinishi bunday bo‘ladi:

$$Ny_1^2 + 2Px_1 + 2Qy_1 + R = 0 \qquad (11.36)$$

$(11.34)$ ga asosan $tg\alpha$ ma’lum bo‘lgani uchun uning yordami bilan hamma vaqt $(11.35)$ dagi $sin\alpha$ va $cos\alpha$ ni aniqlash mumkin. Demak $(11.36)$ ning hamma koeffitsiyentlari ma’lum bo‘ladi.

$(11.36)$ tenglamani yana soddalashtirish maqsadi bilan koordinatalar boshini biror $(a, b)$ nuqtaga ko‘chiramiz. Bu holda almashtirish formulalari

$$x_1 = x + a, \quad y_1 = y + b$$

bo‘ladi va $(11.36)$ ning ko‘rinishi

$$N(y + b)^2 + 2P(x + a) + 2Q(y + b) + R = 0$$

yoki

$$Ny^2 + 2(Nb + Q)y + 2Px + (Nb^2 + 2Pa + 2Qb + R) = 0 \quad (11.37)$$

bo‘ladi. Nuqtaning $a$ va $b$ koordinatalariga shunday qiymat tayin qilamizki,

$$Nb + Q = 0, \quad Nb^2 + 2Pa + 2Qb + R = 0$$

bo‘lsin. Bu esa

$$b = -\frac{Q}{N} \text{ va } \frac{Q^2}{N} + 2Pa - \frac{2Q^2}{N} + Q = 0$$

yoki

$$2PNa - Q^2 + NR = 0 \text{ yoki } a = \frac{Q^2 - NR}{2PNB}$$

bo‘lgan holda mumkin. Bu chog‘da $(11.32)$ ning ko‘rinishi bunday bo‘ladi:

$$Ny^2 + 2Px = 0 \text{ yoki } y^2 = -2\frac{P}{N}x, \qquad (11.38)$$

yoki

$$-\frac{P}{N} = p$$

faraz qilinsa,

$$y^2 = 2px \qquad (11.39)$$

bo‘ladi va bu ***parabolani*** ifoda qiladi.

$p$ ning qiymatini aniqlash uchun $(11.34)$ dan $sin\alpha$ va $cos\alpha$ ning qiymatlarini aniqlashga to‘g‘ri keladi. Buning uchun $(11.34)$ ni

$$\frac{sin\alpha}{cos\alpha} = -\frac{A}{B} \text{ yoki } \frac{sin\alpha}{-A} = \frac{cos\alpha}{B}$$

kabi yozib, undan ushbu hosila proportsiyani tuzamiz:

$$\frac{sin\alpha}{-A} = \frac{cos\alpha}{B} = \frac{\sqrt{sin^2\alpha + cos^2\alpha}}{\sqrt{A^2+B^2}} = \frac{1}{\sqrt{A^2+B^2}}.$$

Demak

$$sin\alpha = \frac{-A}{\sqrt{A^2+B^2}}, \quad cos\alpha = \frac{B}{\sqrt{A^2+B^2}}. \tag{11.40}$$

(11.36) ga muvofiq

$$P = \frac{ABD - A^2E}{\sqrt{A^2+B^2}}, \quad N = \left(\frac{A^2+B^2}{\sqrt{A^2+B^2}}\right)^2 = A^2 + B^2.$$

Demak

$$p = -\frac{P}{N} = \frac{A(AE-BD)}{(A^2+B^2)\sqrt{A^2+B^2}} = \sqrt{\frac{A^2(AE-BD)^2}{(A^2+B^2)^3}} =$$

$$= \sqrt{\frac{A^2(A^2E^2 - 2ABDE + B^2D^2)}{(A^2+B^2)^3}} =$$

$$= \sqrt{\frac{A^2(A^2E^2 - 2ABDE + ACD^2)}{(A^2+AC)^3}} =$$

$$= \sqrt{-\frac{(-AE^2 + 2BDE - CD^2)}{(A+C)^3}} = \sqrt{-\frac{\Delta}{(A+C)^3}};$$

chunki $B^2 - AC = 0$ bo‘lganda $\Delta = -AE^2 + 2BDE - CD^2$ bo‘ladi. Natijada

$$p = \sqrt{-\frac{\Delta}{(A+C)^3}}. \tag{11.41}$$

$B^2 = AC$ bo‘lgani uchun $A$ va $C$ ning ishoralari bir xil bo‘ladi. $A$ ning ishorasini hamma vaqt musbat qilish mumkin. Shuning uchun $(A+C)$ ni musbat faraz qilib bo‘ladi. Ikkinchi tomondan

$$\Delta = -AE^2 + 2BDE - CD^2 = -(AE^2 + CD^2 - 2BDE) =$$

$$= -\left(AE^2 + CD^2 - 2DE\sqrt{AC}\right) = -\left(E\sqrt{A} - D\sqrt{C}\right)^2 < 0.$$

Shuning uchun parabolaning diskriminanti hamma vaqt manfiy bo‘ladi va radikal ostida musbat son bo‘ladi. Demak, $p -$ hamma vaqt mavjud va musbat sondan iborat.

Shuning bilan, tekshirishimizning natijasini ushbu jadval bilan tasvirlash mumkin:

$$Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0,$$

$$\Delta = \begin{vmatrix} A & B & D \\ B & C & E \\ D & E & F \end{vmatrix},$$

$$M = B^2 - AC.$$

| | $M < 0$ | | $M > 0$ | $M = 0$ |
|---|---|---|---|---|
| $\Delta \neq 0$ | $A \cdot \Delta < 0$ | Ellips | Giperbola | Parabola |
| | $A \cdot \Delta > 0$ | Mavhum ellips | | |
| $\Delta = 0$ | | Ikkita bir – birini kesuvchi mavhum to‘g‘ri chiziq | Ikkita bir – birini kesuvchi haqiqiy to‘g‘ri chiziq | Ikkita parallel to‘g‘ri chiziq |

## 11.4. Umumiy tenglama bilan berilgan ikkinchi tartibli chiziqlarni aniqlash va sinflarga ajratish.

$$\begin{cases} x' = x + a \\ y' = y + a \end{cases}$$

parallel ko‘chirish formulasi.

$$\begin{cases} x = x'cos\alpha - y'sin\alpha \\ y = x'sin\alpha + y'cos\alpha \end{cases}$$

burish formulasi.

$$\begin{cases} x = x'cos\alpha - y'sin\alpha + a \\ y = y'cos\alpha + x'sin\alpha + b \end{cases}$$

parallel ko‘chirish va burish birgalikda harakat deyiladi.

$$a_{11}x^2 + a_{22}y^2 + 2a_{10}x + 2a_{20}y + 2a_{12}xy + a_{00} = 0$$

shu ifoda bilan berilgan tenglama ***ikkinchi tartibli chiziqning umumiy tenglamasi*** deyiladi.

Umumiy tenglama bilan berilgan ikkinchi tartibli chiziqni qanday chiziq ekanligini aniqlash uchun quyidagi ishlarni bajaramiz.

$$\begin{cases} x' = x + a \\ y' = y + a \end{cases}$$

$$a_{11}(x' + a)^2 + a_{22}(y' + b)^2 + 2a_{10}(x' + a) + 2a_{20}(y' + b) +$$
$$+2a_{12}(x' + a)(y' + b) + a_{00} = 0 \Longrightarrow$$
$$a_{11}({x'}^2 + 2ax' + a^2) + a_{22}({y'}^2 + 2by' + b^2) + (2a_{10}x' + 2aa_{10}) +$$
$$+2a_{20}y' + 2a_{20}b + 2a_{12}(x'y' + ay' + bx' + ab) + a_{00} = 0 \Longrightarrow$$
$$a_{11}{x'}^2 + 2aa_{11}x' + a^2a_{11} + a_{22}{y'}^2 + 2y'ba_{22} + a_{22}b^2 +$$
$$+2a_{10}x' + 2aa_{10} + 2a_{20}y' + 2a_{20}b + 2a_{12}x'y' +$$
$$+2a_{12}ay' + 2a_{12}bx' + 2a_{12}ab + a_{00} = 0$$
$$\begin{cases} 2a_{11}a + 2a_{12}b = -2a_{10} \\ 2a_{22}b + 2a_{12}a = -2a_{20} \end{cases}$$
$$a_{11}{x'}^2 + a_{22}{y'}^2 + A_{00} = 0 \Longrightarrow \begin{cases} x = x''cos\alpha - y''sin\alpha \\ y = x''sin\alpha + y''cos\alpha \end{cases}$$

kelib chiqadi.

$$a_{11}(x''cos\alpha - y''sin\alpha)^2 + a_{22}(x''sin\alpha + y''cos\alpha)^2 +$$
$$+2a_{12}(x''cos\alpha - y''sin\alpha)(x''sin\alpha + y''cos\alpha) + A_{00} = 0$$
$$a_{11}({x''}^2cos^2\alpha - 2a_{11}x''cos\alpha y''sin\alpha + {y''}^2sin^2\alpha) +$$
$$+a_{22}({x''}^2sin^2\alpha + 2x''sin\alpha y''cos\alpha + {y''}^2cos^2\alpha) + A_{00} = 0 \Longrightarrow$$
$$a_{11}{x''}^2cos^2\alpha - 2a_{11}x''cos\alpha y''sin\alpha + a_{11}{y''}^2sin^2\alpha +$$
$$+a_{22}{y''}^2cos^2\alpha + 2a_{13}{x''}^2sin\alpha cos\alpha + 2a_{12}{x''}^2{y''}^2cos^2\alpha -$$
$$-2a_{12}x''y''sin^2\alpha - 2a_{12}{y''}^2sin\alpha cos\alpha + A_{00} = 0 \Longrightarrow$$
$$(a_{11}cos^2\alpha + a_{22}sin^2\alpha + 2a_{12}sin\alpha cos\alpha){x''}^2 +$$
$$+(a_{11}sin^2\alpha + a_{22}cos^2\alpha - 2a_{12}sin\alpha cos\alpha){y''}^2 +$$
$$+(-2a_{11}sin\alpha cos\alpha + 2a_{22}sin\alpha cos\alpha + 2a_{12}cos^2\alpha$$
$$- 2a_{12}sin^2\alpha)x''y'' +$$
$$+A_{00} = 0$$
$$(a_{22} - a_{11})sin2\alpha + 2a_{12}cos2\alpha = 0 \Longrightarrow$$

$$tg2\alpha = \frac{2a_{12}}{a_{11} - a_{22}} \tag{11.42}$$

$A_{11}{x'}^2 + A_{22}{y'}^2 + A_{00} = 0$ tenglamani xususiy hollarini qaraymiz:

1) $A_{11} > 0,\ A_{22} > 0, A_{00} < 0$ bo‘lsa, ellips;
2) $A_{11} < 0,\ A_{22} < 0, A_{00} > 0$ bo‘lsa, ellips;
3) $A_{11}\ A_{22} < 0, A_{00} > 0$ bo‘lsa, giperbola;
4) $A_{11}\ A_{22} < 0, A_{00} < 0$ bo‘lsa, giperbola;
5) $A_{11}\ A_{22} < 0, A_{00} = 0$ bo‘lsa, nuqta;
6) $A_{11} = 0,\ \ A_{22}\ A_{00} < 0,\ A_{00} > 0$ bo‘lsa, parallel to‘g‘ri chiziqlar hosil bo‘ladi.

**1-Misol.** Quyidagi tenglamaning geometrik ma’nosini tekshirib, uning kanonik tenglamasi tuzilsin:

$$5x^2 + 4xy + 8y^2 - 32x - 56y + 80 = 0.$$

**Yechish:** Egri chiziqning jinsini aniqlash uchun $\Delta$ va $M$ ni hisoblashga to‘g‘ri keladi:

$$\Delta = \begin{vmatrix} 5 & 2 & -16 \\ 2 & 8 & -28 \\ -16 & -28 & 80 \end{vmatrix} = -1296, \qquad A \cdot \Delta < 0.$$

$$M = B^2 - AC = 4 - 5 \cdot 8 = -36 < 0.$$

Demak, berilgan tenglama haqiqiy ellipsdan iborat. Uning kanonik tenglamasining ko‘rinishi:

$$A_1x^2 + C_1y^2 = \frac{\Delta}{M},$$

$$A_1 = \frac{1}{2}\left[A + C + \sqrt{(A - C)^2 + 4B^2}\right] =$$

$$= \frac{1}{2}\left[5 + 8 + \sqrt{(5 - 8)^2 + 4 \cdot 2^2}\right] = 9;$$

$$C_1 = \frac{1}{2}\left[A + C - \sqrt{(A - C)^2 + 4B^2}\right] =$$

$$= \frac{1}{2}\left[5 + 8 - \sqrt{(5 - 8)^2 + 4 \cdot 2^2}\right] = 4;$$

$$\frac{\Delta}{M} = \frac{-1296}{-36} = 36.$$

Demak, ellipsning kanonik tenglamasi

$$9x^2 + 4y^2 = 36$$

yoki

$$\frac{x^2}{4} + \frac{y^2}{9} = 1.$$

**2-Misol.** $Ox$ o‘qi parabolaning simmetriya o‘qi bo‘lib, uning uchi koordinatalar boshida yotadi. Parabola uchidan fokusigacha bo‘lgan masofa 4 birlikka teng. Parabola va uning direktrisasi tenglamasini toping.

**Yechish:** Dastlab, masala shartiga asosan, parabolaning $p$ parametrini topamiz:

$$|OF| = 4 \Rightarrow p/2 = 4 \Rightarrow p = 8.$$

Unda, (11.39) formulaga asosan, parabola tenglamasini topamiz:

$$y^2 = 2px \quad \Rightarrow y^2 = 2 \cdot 8x = 16x.$$

Bu yerdan direktrisa tenglamasi $x = -p/2 \Rightarrow x = -4$ ekanligini ko‘ramiz.

Shuni ta’kidlab o‘tish kerakki, $y = ax^2 + bx + c \ (a \neq 0)$ kvadrat uchhadning grafigi uchi koordinatalari

$$x_0 = -\frac{b}{2a}, \ y_0 = \frac{4ac - b^2}{4a}$$

bo‘lgan $M_0(x_0; y_0)$ nuqtada, simmetriya o‘qi esa $Oy$ o‘qiga parallel va $x = -b/2a$ tenglamaga ega bo‘lgan vertikal to‘g‘ri chiziqdan tashkil topgan paraboladan iboratdir. Agar $a > 0$ bo‘lsa, parabola yuqoriga, $a < 0$ bo‘lsa, pastga yo‘nalgan bo‘ladi.

### Mustaqil yechish uchun topshiriqlar.

**11.1.1.** Quyidagi giperbolalarning tenglamalari sodda shaklga keltirilsin:

1) $9x^2 - 25y^2 - 18x - 100y - 316 = 0$;

2) $5x^2 - 6y^2 + 10x - 12y - 31 = 0$;

3) $x^2 - 4y^2 + 6x + 5 = 0$.

Markazlarining koordinatalari va o‘qlari topilsin.

**11.1.2.** Quyidagi tenglamalar bilan qanday egri chiziqlar berilganligi tekshirilsin:
1) $x^2 - 2xy + 2y^2 - 4x - 6y + 3 = 0$;
2) $x^2 - 2xy - 2y^2 - 4x - 6y + 3 = 0$;
3) $x^2 - 2xy + y^2 - 4x - 6y + 3 = 0$;
4) $x^2 - 2xy + 2y^2 - 4x - 6y + 29 = 0$;
5) $x^2 - 2xy - 2y^2 - 4x - 6y - \frac{13}{3} = 0$.
**11.1.3.** Quyidagi egri chiziqlarning turlari aniqlansin:
1) $x^2 + 6xy + y^2 + 6x + 2y - 1 = 0$;
2) $3x^2 - 2xy + 3y^2 + 4x + 4y - 4 = 0$;
3) $x^2 - 4xy + 3y^2 + 2x - 2y = 0$;
4) $y^2 + 5xy - 14x^2 = 0$;
5) $x^2 - xy - y^2 - x - y = 0$.
**11.1.4.** Berilgan tenglamalarning chap tomonlarini ko‘paytuvchilarga ajratishdan foydalanib, tenglamalarning geometrik ma’nosi ko‘rsatilsin:
1) $xy - bx - ay + ab = 0$;
2) $x^2 - 2xy + 5x = 0$;
3) $x^2 - 4xy + 4y^2 = 0$;
4) $9x^2 + 30xy + 25y^2 = 0$;
5) $4x^2 - 12xy + 9y^2 - 25 = 0$.
**11.1.5.** $y^2 - xy - 5x + 7y + 10 = 0$ tenglamaning ikki (qo‘sh) to‘g‘ri chiziqni ifodalashi tekshirilsin va bu to‘g‘ri chiziqlardan har birining tenglamasi topilsin.
**11.1.6.** Quyidagi tenglama bilan berilgan ikkita to‘g‘ri chiziqdan har birining tenglamasi topilsin:
1) $21x^2 + xy - 10y^2 = 0$;
2) $x^2 + 2xy + y^2 + 2x + 2y - 4 = 0$;
3) $y^2 - 4xy - 5y^2 + 5x - y = 0$;
4) $4x^2 - 4xy + y^2 + 12x - 6y + 9 = 0$.
**11.1.7.** Egri chiziqlar tekshirilsin:
1) $2x^2 + 3xy - 5y^2 = 0$;

2) $x^2 + 4xy + 4y^2 = 0$;
3) $10x^2 - 7xy + y^2 = 0$;
4) $5x^2 - 4xy + y^2 = 0$.

**11.1.8.** Invariantlardan foydalanib, quyidagi egri chiziq tenglamalari sodda shaklga keltirilsin:
1) $x^2 + 2xy - y^2 + 8x + 4y - 8 = 0$;
2) $7x^2 - 24xy - 38x + 24y + 175 = 0$;
3) $5x^2 + 8xy + 5y^2 - 18x - 18y + 9 = 0$;
4) $5x^2 + 12xy - 22x - 12y - 19 = 0$;
5) $6xy + 8y^2 - 12x - 26y + 11 = 0$.
Bu egri chiziqlarning hammasi to‘g‘ri burchakli koordinatalar sistemasiga nisbatan berilgan.

**11.1.9.** Invariantlardan foydalanib, quyidagi parabolalarning tenglamalari soddalashtirilsin:
1) $x^2 - 2xy + y^2 - 10x - 6y + 25 = 0$;
2) $4x^2 - 4xy + y^2 - 2x - 14y + 7 = 0$;
3) $x^2 - 2xy + y^2 - x - 2y + 3 = 0$;
4) $4x^2 - 4xy + y^2 - x - 2 = 0$.
$\omega = \frac{\pi}{2}$ bo‘lgan hol uchun.

**11.1.10.** Quyidagi egri chiziqlarning tenglamalari soddalashtirilsin:
1) $x^2 - 3xy + y^2 + 1 = 0, \quad \omega = 60^0$;
2) $2x^2 + 2y^2 - 2x - 6y + 1 = 0, \quad \omega = 60^0$;
3) $4x^2 - 4xy + y^2 - 4x - 4y + 7 = 0, \quad \omega = 120^0$.

**11.1.11.** Invariantlardan foydalanib, $8y^2 + 6xy - 12x - 26y + 11 = = 0$ giperbolaning asimptotalariga nisbatan tenglamasi yozilsin. $\omega = 90^0$.

**11.1.12.** To‘g‘ri burchakli koordinatalar sistemasiga nisbatan quyidagi tenglamalar bilan berilgan giperbolalarning asimptotalariga nisbatan yozilgan tenglamasi topilsin:
1) $2x^2 + 3xy - 2y^2 - 8x - 11y = 0$;
2) $4x^2 + 2xy - y^2 + 6x + 2y + 3 = 0$;
3) $y^2 - 2xy - 4x - 2y - 2 = 0$.

**11.1.13.** Biror to‘g‘ri burchakli koordinatalar sistemasiga nisbatan egri chiziq $5x^2 + 2xy - 22x - 12y - 19 = 0$ tenglama bilan ifodalanadi. Bu egri chiziqning o‘z uchiga nisbatan tenglamasi topilsin.

**11.1.14.** Quyidagi tenglamalarning har biri ellipsni ifodalasa, uning markazi bo‘lgan C nuqtaning koordinatasi, yarim o‘qi, ekssentrisiteti va direktrisasi tenglamalarini tuzing:

1) $5x^2 + 9y^2 - 30x + 18y + 9 = 0$;
2) $16x^2 + 25y^2 + 32x - 100y - 284 = 0$;
3) $4x^2 + 3y^2 - 8x + 12y - 32 = 0$.

**11.1.15.** Quyidagi tenglamalar giperbola hosil qilishini tekshirib va uning markazi bo‘lgan C nuqtaning koordinatasini, yarim o‘qlarini, ekssentrisitetini, asimptota va direktrisa tenglamalarini tuzing:

1) $16x^2 - 9y^2 - 64x - 54y - 161 = 0$;
2) $9x^2 - 16y^2 + 90x + 32y - 367 = 0$;
3) $16x^2 - 9y^2 - 64x - 18y + 199 = 0$.

**11.1.16.** Quyidagi chiziqlardan qaysilari: 1) yagona markazga; 2) ko‘p markazlarga; 3) markazga ega emasligini aniqlang.

1) $3x^2 - 4xy - 2y^2 + 3x - 12y - 7 = 0$;
2) $4x^2 + 5xy + 3y^2 - x + 9y - 12 = 0$;
3) $4x^2 - 4xy + y^2 - 6x + 8y + 13 = 0$;
4) $4x^2 - 4xy + y^2 - 12x + 6y - 11 = 0$;
5) $x^2 - 2xy + 4y^2 + 5x - 7y + 12 = 0$;
6) $x^2 - 2xy + y^2 - 6x + 6y - 3 = 0$;
7) $x^2 - 20xy + 25y^2 - 14x + 2y - 15 = 0$;
8) $4x^2 - 6xy - 9y^2 + 3x - 7y + 12 = 0$.

**11.1.17.** Quyidagi berilgan chiziqlar markazga ega bo‘lsa, ularning markaziy nuqtalarini toping:

1) $3x^2 + 5xy + y^2 - 8x - 11y - 7 = 0$;
2) $5x^2 + 4xy + 2y^2 + 20x + 20y - 18 = 0$;
3) $9x^2 - 4xy - 7y^2 - 12 = 0$;
4) $2x^2 - 6xy + 5y^2 + 22x - 36y + 11 = 0$.

**11.1.18.** Quyidagi har bir chiziqning ko‘p markazli bo‘lishini tekshirib, ularning har biri uchun geometrik markazini aniqlaydigan tenglamasini tuzing:

1) $x^2 - 6xy + 9y^2 - 12x + 36y + 20 = 0$;
2) $4x^2 + 4xy + y^2 - 8x - 4y - 21 = 0$;
3) $25x^2 - 10xy + y^2 + 40x - 8y + 7 = 0$.

**11.1.19.** Quyidagi tenglamalar markaziy chiziqni ifodalashini tekshirib, ularning har birini koordinata boshiga ko‘chiruvchi tenglamasini tuzing:

1) $3x^2 - 6xy + 2y^2 - 4x + 2y + 1 = 0$;
2) $6x^2 + 4xy + y^2 + 4x - 2y + 2 = 0$;
3) $4x^2 + 6xy + y^2 - 10x - 10 = 0$;
4) $4x^2 + 2xy + 6y^2 + 6x - 10y + 9 = 0$.

**11.1.20.** *m* va *n* ning qanday qiymatlarida

$$mx^2 + 12xy + 9y^2 + 4x + ny - 13 = 0$$

tenglama quyidagilarni aniqlaydi:

a) markaziy chiziqni;
b) markazsiz chiziqni;
c) ko‘p markazli chiziqlarni.

**11.1.21.** Parallel ko‘chirish yo‘li bilan quyidagi tenglamalarning har birining turini aniqlab, sodda holga keltiring. Qanday geometrik shaklni ifodalashini toping. Eski va yangi koordinatalar sistemasida grafigini chizing.

1) $4x^2 + 9y^2 - 40x + 36y + 100 = 0$;
2) $9x^2 - 16y^2 - 54x - 64y - 127 = 0$;
3) $9x^2 + 4y^2 + 18x - 8y + 49 = 0$;

**11.1.22.** Quyidagi berilgan tenglamalarning har birini eng oddiy shaklga keltirib, ularning turini, qanday geometrik shaklni tasvirlashini, grafiklarning eski va yangi koordinata o‘qlariga nisbatan joylashishini aniqlang:

1) $32x^2 + 52xy - 7y^2 + 180 = 0$;
2) $5x^2 - 6xy + 5y^2 - 32 = 0$;

3) $17x^2 - 12xy + 8y^2 = 0$;
4) $5x^2 + 24xy - 5y^2 = 0$;
5) $5x^2 - 6xy + 5y^2 + 8 = 0$.

**11.1.23.** Quyidagi berilgan tenglamalarning har birini eng oddiy shaklga keltirib, ularning turini, qanday geometrik shaklni tasvirlashini, grafiklarning eski va yangi koordinata o‘qlariga nisbatan joylashishini aniqlang:

1) $14x^2 + 24xy + 21y^2 - 4x + 18y - 139 = 0$;
2) $11x^2 - 20xy - 4y^2 - 20x - 8y + 1 = 0$;
3) $7x^2 + 60xy + 32y^2 - 14x - 60y + 7 = 0$;
4) $50x^2 - 8xy + 35y^2 + 100x - 8y + 67 = 0$;

**11.1.24.** Quyidagi tenglamalarni koordinatalar sistemasini almashtirmasdan, har biri ellipsni ifodalashini va uning yarim o‘qlardagi qiymatlarini toping:

1) $41x^2 + 24xy + 9y^2 + 24x + 18y - 36 = 0$;
2) $8x^2 + 4xy + 5y^2 + 16x + 4y - 28 + 9 = 0$;
3) $13x^2 + 18xy + 37y^2 - 26x - 18y + 3 = 0$;
4) $13x^2 + 10xy + 13y^2 + 46x + 62y + 13 = 0$.

**11.1.25.** Koordinatalar sistemasini almashtirmasdan quyidagi tenglamalar bitta nuqtani ifodalashini isbotlang.

1) $5x^2 - 6xy + 2y^2 - 2x + 2 = 0$;
2) $x^2 + 2xy + 2y^2 + 6y + 9 = 0$;
3) $5x^2 + 4xy + y^2 - 6x - 2y + 2 = 0$;
4) $x^2 - 6xy + 10y^2 + 10x - 32y + 26 = 0$.

**11.1.26.** Koordinatalar sistemasini almashtirmasdan, har biri giperbolani ifodalasa, uning yarim o‘qlarining qiymatini toping:

1) $4x^2 + 24xy + 11y^2 + 64x + 42y + 51 = 0$;
2) $12x^2 + 26xy + 12y^2 - 52x - 48y + 73 = 0$;
3) $3x^2 + 4xy - 12x + 16 = 0$;
4) $x^2 - 6xy - 7y^2 + 10x - 30y + 23 = 0$.

**11.1.27.** Koordinatalar sistemasini almashtirmasdan, quyidagi tenglamalarning har biri kesishgan to‘g‘ri chiziqlar juftligini (degenerat giperbola) belgilashini aniqlang va ularning tenglamalarini toping:
1) $3x^2 + 4xy + y^2 - 2x - 1 = 0$;
2) $3x^2 - 6xy + 8y^2 - 4y - 4 = 0$;
3) $x^2 - 4xy + 3y^2 = 0$;
4) $x^2 + 4xy + 3y^2 - 6x - 12y + 9 = 0$.
**11.1.28.** Quyidagi tenglamalarning har biri parabolik ekanligini aniqlang; ularning har birini eng oddiy shaklga keltiring; ular qanday geometrik tasvirlarni belgilashlarini aniqlang:
1) $9x^2 + 24xy + 16y^2 - 18x + 226y + 209 = 0$;
2) $x^2 - 2xy + y^2 - 12x + 12y - 14 = 0$;
3) $4x^2 + 12xy + 9y^2 - 4x - 6y + 1 = 0$.
**11.1.29.** Koordinatalar sistemasini almashtirmasdan, quyidagi tenglamalarning har biri parabolani aniqlaganligini aniqlang va ushbu parabola parametrini toping:
1) $9x^2 + 24xy + 16y^2 - 120x + 90y = 0$;
2) $9x^2 - 24xy + 16y^2 - 54x - 178y + 181 = 0$;
3) $x^2 - 2xy + y^2 + 6x - 14y + 29 = 0$;
4) $9x^2 - 6xy + y^2 - 50x + 50y - 275 = 0$.
**11.1.30.** Koordinatalar sistemasini almashtirmasdan, quyidagi tenglamalarning har biri bir juft parallel to‘g‘ri chiziqni belgilashini aniqlang va ularning tenglamalarini toping:
1) $4x^2 + 4xy + y^2 - 12x - 6y + 5 = 0$;
2) $4x^2 - 12xy + 9y^2 + 20x - 30y - 11 = 0$;
3) $25x^2 - 10xy + y^2 + 10x - 2y - 15 = 0$.

# 12-MAVZU: IKKINCHI TARTIBLI SIRTLAR.

**Reja:**

**1. Ellipsoid va sfera.**

**2. Fazoda ba’zi sirtlarning tenglamalari va nomlanishi.**

**Tayanch iboralar**: transtendent sirt, ellipsoid, siqma ellipsoid, cho‘ziq ellipsoid, diametral tekislik, bir pallali giperboloid , ikki pallali giperboloid , sfera, konus, elliptik silindr, giperbolik silindr, parabolik silindr, elliptik paraboloid, giperbolik paraboloid .

## 12.1. Ellipsoid va sfera.

Sirtlar, ularning Dekart koordinatalariga nisbatan ifoda qilingan tenglamalarga qarab, tekislikdagi chiziqlar kabi, algebraik va transtendent sirtlarga bo‘linadi. Shuning uchun algebraik sirt deb, shunday sirtga aytiladiki, agarda uni

$$f(x;y;z) = 0$$

ko‘rinishidagi tenglama bilan ifodalash mumkin bo‘lsa va $f(x;y;z)$ esa $x, y, z$ ga nisbatan polinom(ko‘p hadli) bo‘lsa, algebraik bo‘lmagan hamma sirtlarni ***transtendent sirtlar*** deyiladi.

Algebraik sirtlar, o‘z navbatida, turli tartibli sirtlarga bo‘linadi. Agarda $f(x;y;z)$ polinomning darajasi $n$ bo‘lsa, unday sirtlarni $\boldsymbol{n}-$ ***tartibli sirt*** deyiladi.

Dekart o‘zgaruvchi $x, y, z$ koordinatalariga nisbatan ikkinchi darajali algebraik tenglama bilan ifoda qilingan sirt ***ikkinchi tartibli sirt deyiladi.*** Shuning uchun ikkinchi tartibli sirt ifoda qiladigan ikkinchi darajali algebraik tenglamaning umumiy ko‘rinishi quyidagi ko‘rinishda bo‘ladi:

$$A_1x^2 + A_2y^2 + A_3z^2 + B_1xy + B_2xz + B_3yz + C_1x + C_2y +$$
$$+C_3z + F = 0,$$

bunda $A_1, A_2, A_3, B_1, B_2, \dots, F$ koeffitsiyentlar haqiqiy o‘zgarmas sonlardan iborat bo‘lib, xususiy holda ulardan ba’zilari nolga teng bo‘lishi mumkin. Bu tenglamaning umumiyligiga xalal bermay uni quyidagi ko‘rinishda yozish mumkin:

$$A_1x^2 + A_2y^2 + A_3z^2 + 2B_1yz + 2B_2xz + 2B_3xy +$$
$$+2C_1x + 2C_2y + 2C_3z + F = 0. \qquad (12.1)$$

Tenglamani ushbu ko‘rinishda yozsak, uning bilan bog‘langan amallarni bajarish birmuncha qulay bo‘ladi.

Koordinatalar sistemasini alamashtirish yordamida (12.1) tenglamani soddalashtirib, uni

$$A_1x^2 + A_2y^2 + A_3z^2 + F = 0 \qquad (12.2)$$

yoki

$$A_1x^2 + A_2y^2 + 2C_3z = 0 \qquad (12.3)$$

shaklga keltirish mumkin.

(12.2) tenglama bilan ifoda qilingan sirt ikkinchi tartibli ***markazli sirt*** deyiladi va (12.3) tenglama bilan ifoda qilingan sirt ikkinchi tartibli ***markazsiz***(yoki markazi cheksizlikdagi) ***sirt*** deyiladi.

Faraz qilaylik, ikkinchi tartibli markazli sirtning eng sodda tenglamasi berilgan bo‘lsin:

$$A_1x^2 + A_2y^2 + A_3z^2 + F = 0 \qquad (12.4)$$

va bundagi ozod had bo‘lgan $F$ ning ishorasi qolgan koeffitsiyentlarining ishorasiga teskari bo‘lsin. Tenglamaning $F$ koeffitsiyentini o‘ng tomonga o‘tkazib, so‘ngra uning ikkala tomonini $(-F)$ ga bo‘lamiz:

$$A_1x^2 + A_2y^2 + A_3z^2 = -F,$$
$$\frac{A_1x^2}{-F} + \frac{A_2y^2}{-F} + \frac{A_3z^2}{-F} = 1,$$

yoki

$$\frac{x^2}{-\frac{F}{A_1}} + \frac{y^2}{-\frac{F}{A_2}} + \frac{z^2}{-\frac{F}{A_3}} = -1. \qquad (12.5)$$

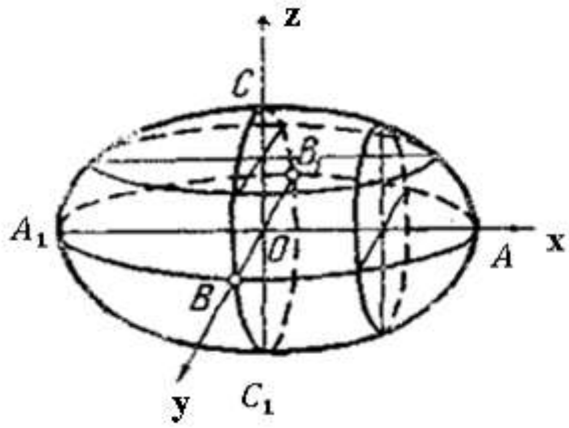
(12.4) tenglamaning koeffitsiyentlari to‘g‘risida qilingan farazga muvofiq $F$ ning ishorasi qolgan koeffitsiyentlarning ishorasiga teskari bo‘lgani uchun, (12.5) tenglamaning chap tomonidagi har bir kasrning maxraji musbat bo‘ladi. Shuning uchun ulardan birinchisini $a^2$, ikkinchisini $b^2$ va uchinchisini $c^2$ deb faqaz qilamiz:

$$-\frac{F}{A_1}=a^2,\ -\frac{F}{A_2}=b^2,\ -\frac{F}{A_3}=c^2, \qquad (12.6)$$

demak, (12.5) tenglamaning ko‘rinishi quyidagicha bo‘ladi:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1. \qquad (12.7)$$

Bu tenglama bilan ifoda qilingan sirt ***ellipsoid*** deyiladi.




**12.1.1-chizma**

Tenglamaning tuzilishiga qaraganda uning chap tomonidagi har bir kasrning qiymati birdan katta bo‘la olmaydi, ya’ni

$$\frac{x^2}{a^2}\le 1, \qquad \frac{y^2}{b^2}\le 1, \qquad \frac{x^2}{c^2}\le 1,$$

yoki

$$x^2\le a^2, \qquad y^2\le b^2, \quad z^2\le c^2,$$

demak,

$$|x|\le a, \quad |y|\le b, \quad |z|\le c.$$

Endi ellipsoidning shaklini tekshiramiz. Buning uchun eng avval uning koordinata o‘qlari bilan uchrashgan nuqtalarini topamiz. Agar (12.7) tenglamada $y=0,\ z=0$ faraz qilinsa, $x=\pm a$ bo‘ladi, ya’ni absissa o‘qi ellipsoidni koordinatalar boshiga nisbatan simmetrik bo‘lgan $A(a;0;0)$ va $A_1(-a;0;0)$ nuqtalarda kesib o‘tadi. Shunga o‘xshash $x=0,\ z=0$ faraz qilinsa, $y=\pm b$ bo‘ladi, ya’ni ordinata o‘qi ellipsoidni koordinatalar boshiga nisbatan simmetrik bo‘lgan $B(0;b;0)$ va $B_1(0;-b;0)$ nuqtalarda kesib o‘tadi; $x=0,\ y=0$ faraz qilinsa, $z=\pm c$ bo‘ladi, ya’ni applikata o‘qi ellipsoidni koordinatalar boshiga nisbatan simmetrik bo‘lgan $C(0;0;c)$ va $C_1(0;0;-c)$ nuqtalarda kesib o‘tadi.

Aniqlangan nuqtalardan $A-$ ellipsoidning $yOz$ tekislikdan eng uzoqlashgan nuqtasi bo‘ladi; shunga o‘xshash qolgan nuqtalar ham tegishli koordinata tekisliklaridan eng uzoqlashgan nuqtalardan iborat. Shuning uchun ularni ellipsoidning boshlari deyiladi va har ikki nuqtalarning orasidagi $2a$, $2b$, $2c$ masofalar ellipsoidning ***o‘qlari*** deyiladi. Ellipsoidning o‘qlari to‘g‘risida qo‘shimcha shart bo‘lmagan holda $a>b>c$ faraz qilinadi. Tekshirishdan chiqarilgan natijalarga qaraganda ellipsoid yopiq sirtdan iborat, chunki u

$$x=\pm a,\ \ y=\pm b,\ \ z=\pm c$$

tekisliklardan yasalgan parallelepipedning ichida bo‘ladi.

Endi ellipsoidning koordinata tekisliklari bilan kesilishidan hosil bo‘lgan shakllarni tekshiramiz. Masalan, $xOy$ tekisligi bilan kesish uchun $z=0$ faraz qilishga to‘g‘ri keladi va bu holda $(12.7)$ ning ko‘rinishi ushbu ko‘rinishida bo‘ladi:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1. \qquad (12.8)$$

Shunga o‘xshash $y=0$ faraz qilinsa,

$$\frac{x^2}{a^2}+\frac{y^2}{c^2}=1 \qquad (12.9)$$

va $x=0$ faraz qilinsa,

$$\frac{x^2}{b^2}+\frac{y^2}{c^2}=1. \qquad (12.10)$$

$(12.8)$, $(12.9)$, $(12.10)$ tenglamalardan har biri ellipsni ifoda qiladi. Demak, ellipsoidning koordinata tekisliklari bilan kesimlari ellipslardan iborat. Bular ellipsoidning ***bosh kesimlari*** deyiladi.

Endi ellipsoidni koordinata tekisliklariga parallel bo‘lgan tekisliklar bilan kesib ko‘ramiz. Masalan, $xOy$ tekislikka parallel bo‘lgan tekislikning tenglamasini birgalikda yechishga to‘g‘ri keladi.

$z=k$ ni $(12.7)$ ga qo‘yilsa:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{k^2}{c^2}=1,$$

yoki

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 - \frac{k^2}{c^2},$$

yoki

$$\frac{x^2}{\frac{a^2(c^2 - k^2)}{c^2}} + \frac{y^2}{\frac{b^2(c^2 - k^2)}{c^2}} = 1,$$

yoki

$$\frac{a^2(c^2 - k^2)}{c^2} = a_1^2, \qquad \frac{b^2(c^2 - k^2)}{c^2} = b_1^2 \qquad (12.11)$$

faraz qilinsa, tenglamani ko‘rinishi quyidagicha bo‘ladi:

$$\frac{x^2}{a_1^2} + \frac{y^2}{b_1^2} = 1. \qquad (12.12)$$

Bu tenglama ellipsni ifoda qiladi. Biroq, bu ellipsning haqiqiy bo‘lishi uchun $|k| \leq c$ bo‘lishi lozim, chunki $(12.11)$ dagi tengliklarga qaraganda $|k| > c$ bo‘lgan holda $a_1$ va $b_1$ mavhum bo‘ladi. Shunga o‘xshash ellipsoidni $yOz$ va $xOz$ tekisliklarga parallel bo‘lgan tekislik bilan kesgan holda ham hamon shu kabi natija kelib chiqadi, ya’ni ellips hosil bo‘ladi.

Ellipsoidning o‘qlaridan ikkitasi o‘zaro teng bo‘lganda, unday ellipsoid ***aylanma ellipsoid*** deyiladi. Masalan, ellipsoidning $(12.7)$ tenglamasida $a = b > c$ faraz qilinsa, u tenglamaning ko‘rinishi

$$\frac{x^2 + y^2}{a^2} + \frac{z^2}{c^2} = 1 \qquad (12.13)$$

bo‘ladi va bu ellipsoid ***siqma ellipsoid*** deyiladi, chunki

$$\frac{x^2}{a^2} + \frac{z^2}{c^2} = 1$$

ellipsning kichik o‘qi atrofida aylanishidan hosil bo‘ladi.

Agar $(12.13)$ da $z = 0$ deb faraz qilinsa,

$$x^2 + y^2 = a^2$$

bo‘ladi, bu esa aylanani ifoda qiladi. Demak, $(12.13)$ aylana ellipsoidning $xOy$ tekisligi bilan kesimi aylanadan iborat. Shunga o‘xshash, $xOy$ tekisligiga parallel bo‘lgan tekislik bilan $(12.13)$ ni kesganda yana aylana hosil bo‘ladi. Agar (12.7) tenglamada $a > b = c$ faraz qilinsa, u tenglamaning ko‘rinishi

$$\frac{x^2}{a^2} + \frac{y^2 + z^2}{b^2} = 1 \qquad (12.14)$$

bo‘ladi va bu ellipsoid ***cho‘ziq ellipsoid*** deyiladi, chunki u

$$\frac{x^2}{a^2} + \frac{z^2}{b^2} = 1$$

ellipsning katta o‘qi atrofida aylanishidan hosil bo‘ladi. Agar (12.14) da $x = 0$ faraz qilinsa, $y^2 + z^2 = b^2$ bo‘ladi, ya’ni cho‘ziq ellipsoidning $yOz$ tekisligiga parallel bo‘lgan tekislik bilan (12.14) ni kesganda, yana aylana hosil bo‘ladi.

Ellipsoidning o‘qlari o‘zaro teng bo‘lgan holda ya’ni $a = b = c$ bo‘lganda (12.7) tenglamaning ko‘rinishi

$$x^2 + y^2 + z^2 = a^2 \qquad (12.15)$$

bo‘ladi. Bu tenglama markazi koordinatalar boshida bo‘lgan radiusi $a$ ga teng bo‘lgan ***sferani*** ifoda etadi.

### 12.2. Fazoda ba’zi sirtlarning tenglamalari va nomlanishi.

Endi bir nechta sirtlarning tenglamalarini va nomlarini keltirib o‘tamiz:

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ - ***bir pallali giperboloid***;

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$ - ***ikki pallali giperboloid***;

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$ - ***konus***;

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ - ***elliptik silindr*** ;

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ - ***giperbolik silindr*** ;

$y^2 = 2px$ - ***parabolik silindr*** ;

$\frac{x^2}{p} + \frac{y^2}{q} = 2z$ - ***elliptik paraboloid***; $(p > 0,\ q > 0)$.

$\frac{x^2}{p} - \frac{y^2}{q} = 2z$ - ***giperbolik paraboloid***; $(p > 0,\ q > 0)$.

Bu sirtlar ham ellipsoid kabi tahlil qilinadi. Tahlilni o‘quvchiga qoldiramiz.

### Mustaqil yechish uchun topshiriqlar.

**12.1.1 - 12.1.38** mashqlarda berilgan sirtlarning:

1) koordinata o‘qlari bilan kesishgan nuqtalarini;

2) koordinata tekisliklari bilan kesishgan nuqtalarining geometrik o‘rnini;

3) koordinata tekisliklariga parallel tekisliklar bilan kesishgan nuqtalarining geometrik o‘rnini aniqlang.

**12.1.1.** $\frac{x^2}{25}+\frac{y^2}{16}+\frac{z^2}{4}=1$ tenglama bilan berilgan sirt.

**12.1.2.** $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{b^2}=1$ tenglama bilan berilgan sirt.

**12.1.3.** $\frac{x^2}{a^2}+\frac{y^2}{a^2}+\frac{z^2}{c^2}=1$ tenglama bilan berilgan sirt.

**12.1.4.** $\frac{x^2}{25}+\frac{y^2}{25}+\frac{z^2}{121}=1$ tenglama bilan berilgan sirt.

**12.1.5.** $\frac{x^2}{49}+\frac{y^2}{16}+\frac{z^2}{16}=1$ tenglama bilan berilgan sirt.

**12.1.6.** $x^2+y^2+z^2=25$ tenglama bilan berilgan sirt.

**12.1.7.** $x^2+y^2+z^2=25$ tenglama bilan berilgan sirt.

**12.1.8.** $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ tenglama bilan berilgan sirt.

**12.1.9.** $\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ tenglama bilan berilgan sirt.

**12.1.10.** $\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$ tenglama bilan berilgan sirt.

**12.1.11.** $\frac{x^2}{25}+\frac{y^2}{16}-\frac{z^2}{9}=1$ tenglama bilan berilgan sirt.

**12.1.12.** $\frac{x^2}{64}-\frac{y^2}{9}+\frac{z^2}{4}=1$ tenglama bilan berilgan sirt.

**12.1.13.** $\frac{x^2}{121}-\frac{y^2}{100}-\frac{z^2}{4}=-1$ tenglama bilan berilgan sirt.

**12.1.14.** $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$ tenglama bilan berilgan sirt.

**12.1.15.** $\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=-1$ tenglama bilan berilgan sirt.

**12.1.16.** $\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ tenglama bilan berilgan sirt.

**12.1.17.** $\frac{x^2}{36} + \frac{y^2}{25} - \frac{z^2}{16} = -1$ (ikki pallali giperboloid) tenglama bilan berilgan sirt.

**12.1.18.** $\frac{x^2}{100} - \frac{y^2}{81} + \frac{z^2}{36} = -1$ tenglama bilan berilgan sirt.

**12.1.19.** $\frac{x^2}{81} - \frac{y^2}{49} - \frac{z^2}{16} = 1$ tenglama bilan berilgan sirt.

**12.1.20.** $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$ (konus) tenglama bilan berilgan sirt.

**12.1.21.** $\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0$ tenglama bilan berilgan sirt.

**12.1.22.** $\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$ tenglama bilan berilgan sirt.

**12.1.23.** $\frac{x^2}{169} + \frac{y^2}{100} - \frac{z^2}{81} = 0$ tenglama bilan berilgan sirt.

**12.1.24.** $x^2 + y^2 - z^2 = 0$ tenglama bilan berilgan sirt.

**12.1.25.** $\frac{x^2}{144} - \frac{y^2}{64} + \frac{z^2}{49} = 0$ tenglama bilan berilgan sirt.

**12.1.26.** $\frac{x^2}{144} - \frac{y^2}{49} - \frac{z^2}{9} = 0$ tenglama bilan berilgan sirt.

**12.1.27.** $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (elliptik silindr) tenglama bilan berilgan sirt.

**12.1.28.** $\frac{x^2}{25} + \frac{y^2}{4} = 1$ tenglama bilan berilgan sirt.

**12.1.29.** $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (giperbolik silindr) tenglama bilan berilgan sirt.

**12.1.30.** $\frac{x^2}{36} - \frac{y^2}{16} = 1$ tenglama bilan berilgan sirt.

**12.1.31.** $\frac{x^2}{25} - \frac{y^2}{9} = -1$ tenglama bilan berilgan sirt.

**12.1.32.** $y^2 = 2px$ (parabolik silindr) tenglama bilan berilgan sirt.

**12.1.33.** $y^2 = 8x$ tenglama bilan berilgan sirt.

**12.1.34.** $y^2 = -2x$ tenglama bilan berilgan sirt.

**12.1.35.** $\frac{x^2}{p} + \frac{y^2}{q} = 2z$ (elliptik paraboloid) tenglama bilan berilgan sirt.

**12.1.36.** $\frac{x^2}{12} + \frac{y^2}{8} = 2z$ tenglama bilan berilgan sirt.

**12.1.37.** $\frac{x^2}{p} - \frac{y^2}{q} = 2z$ (giperbolik paraboloid) tenglama bilan berilgan sirt.

**12.1.38.** $\frac{x^2}{18} - \frac{y^2}{6} = 2z$ tenglama bilan berilgan sirt.

**12.1.39 - 12.1.40** mashqlarda tenglama bilan berilgan sirtlarni simmetriklikka tekshiring.

**12.1.39.** 1) $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{b^2} = 1$; 2) $\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{c^2} = 1$;

3) $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$; 4) $\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$;

5) $\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$; 6) $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$;

7) $\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = -1$; 8) $\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$;

9) $\frac{x^2}{4} + \frac{y^2}{25} - \frac{z^2}{36} = -1$; 10) $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$;

11) $\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0$; 12) $\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$;

13) $x^2 + y^2 + z^2 = a^2$

**12.1.40.** 1) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; 2) $\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$;

3) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$; 4) $y^2 = 2px$;

5) $\frac{x^2}{p} + \frac{y^2}{q} = 2z$ ; 6) $\frac{x^2}{p} - \frac{y^2}{q} = 2z$ ;

7) $\frac{x^2}{81} + \frac{y^2}{49} = 1$; 8) $\frac{x^2}{25} - \frac{y^2}{16} = 1$;

9) $\frac{x^2}{49} - \frac{y^2}{4} = -1$; 10) $y^2 = -4x$;

11) $\frac{x^2}{12} + \frac{y^2}{8} = 6$ ; 12) $\frac{x^2}{16} - \frac{y^2}{10} = 8$

# 13-MAVZU: IKKINCHI TARTIBLI SIRTLARNING TO‘G‘RI CHIZIQLI YASOVCHILARI.

**Reja:**

**1. Ikkinchi tartibli sirtlarning to‘g‘ri chiziqli yasovchilari.**

**2. Ellipsoid va sferaning urinma tekislik tenglamalari.**

**Tayanch iboralar**: bir pallali giperboloid, ikki pallali giperboloid, sfera, konus, elliptik silindr, giperbolik silindr, parabolik silindr, elliptik paraboloid, giperbolik paraboloid .

## 13.1. Ikkinchi tartibli sirtlarning to‘g‘ri chiziqli yasovchilari.

Konus va silindrlar to‘g‘ri chiziqli yasovchilarni o‘z ichiga olgan birdan – bir sirtlardan emas. Ma’lum bo‘lishicha, bir pallali giperboloid bilan giperbolik paraboloid ham shu xossaga ega ekan.

$$\boldsymbol{z = \lambda\left(\frac{x}{a}+\frac{y}{b}\right), \qquad 1 = \frac{1}{\lambda}\left(\frac{x}{a}-\frac{y}{b}\right)} \tag{13.1}$$

tenglamalar bilan berilgan har bir $g_\lambda$ to‘g‘ri chiziq giperbolik paraboloid:

$$z = \frac{x^2}{a^2} - \frac{y^2}{b^2} \tag{13.2}$$

ni ustida yotadi, chunki (13.1) tenglamalarni qanoatlantiruvchi, (13.2) tenglamani qanoatlantiradi, (13.2) esa (13.1) dan hadma – had ko‘paytirish natijasida hosil qilinadi.

Giperbolik paraboloid ustida $g_\lambda$ oiladan tashqari to‘g‘ri chiziqlarning yana bitta $g_\lambda{}'$ oilasi joylashadi:

$$z = \lambda\left(\frac{x}{a}+\frac{y}{b}\right), \qquad 1 = \frac{1}{\lambda}\left(\frac{x}{a}-\frac{y}{b}\right).$$

Shuning singari ***bir pallali giperboloid:***

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} - 1 = 0.$$

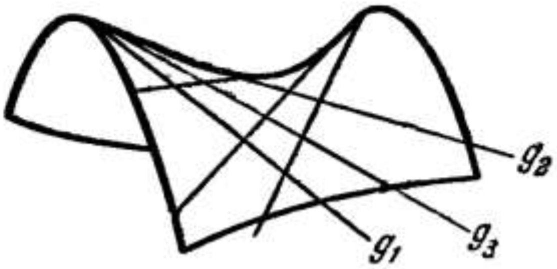
ustida to‘g‘ri chiziqli yasovchilarning ikkita oilasi joylashadi:

$$g_\lambda: \frac{x}{a} - \frac{z}{c} = \lambda\left(1 - \frac{y}{b}\right), \qquad \frac{x}{a} + \frac{z}{c} = \frac{1}{\lambda}\left(1 + \frac{y}{b}\right);$$

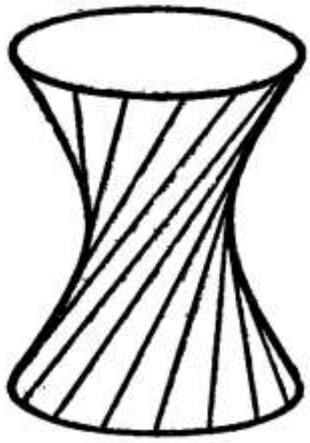
$$g_\lambda{}': \frac{x}{a} - \frac{z}{c} = \lambda\left(1 + \frac{y}{b}\right), \qquad \frac{x}{a} + \frac{z}{c} = \frac{1}{\lambda}\left(1 - \frac{y}{b}\right).$$

Ikkila holda (giperbolik paraboloid va bir pallali giperboloid) ham bitta oilaga qarashli to‘g‘ri chiziqli yasovchilar kesishmaydi, turli oilaga qarashli to‘g‘ri chiziqlar esa kesishadi.

Giperbolik paraboloid bilan bir pallali giperboloidda to‘g‘ri chiziqli yasovchilarning mavjudligi bu sirtlarni hosil qilishning yangi usulini berish imkoniyatini tug‘diradi; bir oilaga qarashli uchta to‘g‘ri chiziqli yasovchini olamiz: $g_1$, $g_2$, $g_3$. Bunday holda ikkinchi oilaga tegishli har bir to‘g‘ri chiziqli yasovchi $g$ yuqoridagi $g_1$, $g_2$, $g_3$ ni kesadi. Demak, sirt berilgan uchta to‘g‘ri chiziqni kesadigan to‘g‘ri chiziqlardan tashkil topadi (13.1.1-chizma).




**13.1.1-chizma**



**13.1.2-chizma**

Bir pallali aylanma giperboloid masalasiga kelganda, uning istalgan to‘g‘ri chiziqli yasovchisining sirt o‘qi atrofida aylantirish natijasida ham hosil bo‘lishini ta’kidlab o‘tamiz (13.1.2-chizma).

Ikkinchi tartibli boshqa sirtlarda ham to‘g‘ri chiziqli yasovchilarning mavjud bo‘lishini pirovardida aytib o‘taylik, biroq bu sirtlarda ular – mavhum. Masalan,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0$$

ellipsoid ustida mavhum to‘g‘ri chiziqlarning

$$g_\lambda: \frac{x}{a} + i\frac{z}{c} = \lambda\left(1 - \frac{y}{b}\right), \qquad \frac{x}{a} - i\frac{z}{c} = \frac{1}{\lambda}\left(1 + \frac{y}{b}\right);$$

$$g_\lambda{}': \frac{x}{a} + i\frac{z}{c} = \lambda\left(1 + \frac{y}{b}\right), \qquad \frac{x}{a} - i\frac{z}{c} = \frac{1}{\lambda}\left(1 - \frac{y}{b}\right).$$

ikkita oilasi joylashdi.

## 13.2. Ellipsoid va sferaning urinma tekislik tenglamalari.

1.Fazoda

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \qquad (13.3)$$

tenglama bilan berilgan ellipsoidni biror ixtiyoriy tekislik bilan kesib ko‘ramiz. Faraz qilaylik, bu tekislikning tenglamasi

$$Ax + By + Cz + D = 0 \qquad (13.4)$$

bo‘lsin. Bu tenglama bilan (13.3) tenglama birgalikda izlangan kesimni ifoda qiladi. Agar bu tenglamalardan $z$ chiqarilsa, izlangan kesimning $xOy$ tekislikdagi proyeksiyasi hosil bo‘ladi. (13.4) dan ($C \neq 0$):

$$Z = -\frac{Ax + By + D}{C},$$

buni ellipsoidning (13.3) tenglamasiga qo‘ysak:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{(Ax + By + D)^2}{C^2c^2} = 1,$$

yoki qavslarni ochib, quyidagi ko‘rinishda ham yozish mumkin:

$$\left(\frac{C^2}{a^2} + \frac{A^2}{c^2}\right)x^2 + \left(\frac{C^2}{b^2} + \frac{B^2}{c^2}\right)y^2 + 2\frac{AB}{c^2}xy + 2\frac{AD}{c^2}x +$$

$$+2\frac{BD}{c^2}y + \frac{D^2}{c^2} - C^2 = 0,$$

yoki

$$\frac{C^2}{a^2} + \frac{A^2}{c^2} = A_1, \quad \frac{C^2}{b^2} + \frac{B^2}{c^2} = C_1,$$

$$\frac{AB}{c^2} = B_1, \quad \frac{AD}{c^2} = D_1, \quad \frac{BD}{c^2} = E_1, \quad \frac{D^2}{c^2} - C^2 = F_1$$

faraz qilinsa:

$$A_1x^2 + 2B_1xy + C_1y^2 + 2D_1x + 2E_1y + F_1 = 0. \qquad (13.5)$$

Bu tenglama $xOy$ tekislikda ikkinchi tartibli chiziqni ifoda qiladi. Bu chiziqning jinsini tekshirish uchun $M = B_1^2 - A_1C_1$ va

$$\Delta = \begin{vmatrix} A_1 & B_1 & D_1 \\ B_1 & C_1 & E_1 \\ D_1 & E_1 & F_1 \end{vmatrix}$$

tuzishga to‘g‘ri keladi. Bizda

$$M = \frac{A^2B^2}{c^2} - \frac{(C^2c^2 + A^2a^2)(C^2c^2 + B^2b^2)}{a^2b^2c^4} =$$

$$= -(A^2a^2 + B^2b^2 + C^2c^2)\frac{C^2}{a^2b^2c^4} < 0; \qquad (13.6)$$

$\Delta$ ni tuzganda uning ifodasi quyidagicha bo‘ladi:

$$\Delta = -\frac{A^2a^2 + B^2b^2 + C^2c^2 - D^2}{a^2b^2c^4} \cdot C^4 \qquad (13.7)$$

(13.6) ga qaraganda hamma vaqt $M < 0$, lekin (13.7) ga qaraganda $\Delta$ ning noldan kichik yoki nolga teng bo‘lishi mumkin. Bunga qarab (13.5) tenglama haqiqiy ellipsni yoki mavhum ellipsni yoki nuqtani ifoda qiladi. (13.7) ning tuzilishiga qaraganda $\Delta$ ning miqdori o‘z navbatida ushbu ifodaga bog‘liq:

$$a^2A^2 + b^2B^2 + c^2C^2 - D^2. \qquad (13.8)$$

Agar bu ifoda musbat bo‘lsa, $\Delta < 0$ bo‘ladi va bu holda izlanayotgan kesim haqiqiy ellipsdan iborat bo‘ladi; shunga o‘xshash agar (13.8) manfiy bo‘lsa, $\Delta > 0$ bo‘ladi va bu holda izlangan kesim mavhum ellipsdan iborat bo‘ladi, agarda (13.8) nolga teng bo‘lsa, bu holda $\Delta = 0$ bo‘ladi va izlangan kesim nuqtaga aylanadi.

Agarda tekislik ellipsoidni kessa, ellips hosil bo‘ladi, yoki tekislik bilan ellipsoidning umumiy nuqtasi bo‘lmaydi, yoki ikkalasining umumiy nuqtasi bo‘ladi.

Tekislik bilan ellipsoidning bir umumiy nuqtasi bo‘lganda, ya’ni tekislik ellipsoidga urunma bo‘lganda

$$a^2A^2 + b^2B^2 + c^2C^2 - D^2 = 0 \qquad (13.9)$$

bo‘ladi. Bu munosabatga asoslanib, ellipsoidga urinma bo‘lgan tekislikning tenglamasini tuzish mumkin. Haqiqatdan ham (13.9) ni ushbu ko‘rinishda yozish mumkin:

$$\left(-\frac{a^2A}{D}\right)^2 : a^2 + \left(-\frac{b^2B}{D}\right)^2 : b^2 + \left(-\frac{c^2C}{D}\right)^2 : c^2 = 1,$$

ya’ni koordinatalari

$$x_1 = -\frac{a^2 A}{D}, \quad y_1 = -\frac{b^2 B}{D}, \quad z_1 = -\frac{c^2 C}{D} \tag{13.10}$$

bo‘lgan nuqta ellipsoid tenglamasini qanoatlantiradi. Ikkinchi tomondan (13.9) ta’minlanganda $(x_1; y_1; z_1)$ nuqtaning koordinatalari (13.4) tekislikning tenglamasini ham qanoatlantiradi. Demak, $(x_1; y_1; z_1)$ nuqta (13.4) tenglamaning ellipsoidga urinish nuqtasi bo‘ladi. Ellipsoidga urinma bo‘lgan (13.4) tekislikning koeffitsiyentlari (13.10) dan aniqlanadi:

$$A = -\frac{Dx_1}{a^2}, \quad B = -\frac{Dy_1}{b^2}, \quad C = -\frac{Dz_1}{c^2},$$

natijada, ellipsoidning $(x_1; y_1; z_1)$ nuqtasidan o‘tgan urinma tekislikning tenglamasi quyidagicha bo‘ladi:

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} + \frac{zz_1}{c^2} = 1. \tag{13.11}$$

2.Fazoda

$$x^2 + y^2 + z^2 = R^2 \tag{13.12}$$

tenglama bilan berilgan sferani biror ixtiyoriy tekislik bilan kesib ko‘ramiz. Faraz qilaylik, bu tekislikning tenglamasi

$$Ax + By + Cz + D = 0 \tag{13.13}$$

bo‘lsin. Bu tenglama bilan (13.12) tenglama birgalikda izlangan kesimni ifoda qiladi. Agar bu tenglamalardan $z$ chiqarilsa, izlangan kesimning $xOy$ tekislikdagi proyeksiyasi hosil bo‘ladi. (13.13) dan $(C \neq 0)$:

$$Z = -\frac{Ax + By + D}{C},$$

buni sferaning (13.12) tenglamasiga qo‘ysak:

$$x^2 + y^2 + \frac{(Ax + By + D)^2}{C^2} = R^2,$$

yoki qavslarni ochib, quyidagi ko‘rinishda ham yozish mumkin:

$$(A^2 + C^2)x^2 + (B^2 + C^2)y^2 + 2ABxy + 2ADx +$$
$$+2BDy + D^2 - C^2R^2 = 0,$$

yoki

$$A^2 + C^2 = A_1, \;\; B^2 + C^2 = C_1,$$
$$AB = B_1, \;\; AD = D_1, \;\; BD = E_1, \;\; D^2 - C^2R^2 = F_1$$
faraz qilinsa:
$$A_1x^2 + 2B_1xy + C_1y^2 + 2D_1x + 2E_1y + F_1 = 0. \qquad (13.14)$$
Bu tenglama $xOy$ tekislikda ikkinchi tartibli chiziqni ifoda qiladi. Bu chiziqning jinsini tekshirish uchun $M = B_1^2 - A_1C_1$ va
$$\Delta = \begin{vmatrix} A_1 & B_1 & D_1 \\ B_1 & C_1 & E_1 \\ D_1 & E_1 & F_1 \end{vmatrix}$$
tuzishga to‘g‘ri keladi. Bizda
$$M = A^2B^2 - (A^2 + C^2)(B^2 + C^2) =$$
$$= -(A^2C^2 + B^2C^2 + C^4) < 0; \qquad (13.15)$$
$\Delta$ ni tuzganda uning ifodasi quyidagicha bo‘ladi:
$$\Delta = -(A^2R^2 + B^2R^2 + C^2R^2 - D^2)C^4 \qquad (13.16)$$
(13.15) ga qaraganda hamma vaqt $M < 0$, lekin (13.16) ga qaraganda $\Delta$ ning noldan kichik yoki nolga teng bo‘lishi mumkin.

(13.16) ning tuzilishiga qaraganda $\Delta$ ning miqdori o‘z navbatida ushbu ifodaga bog‘liq:
$$A^2R^2 + B^2R^2 + C^2R^2 - D^2. \qquad (13.17)$$
Agar bu ifoda $\Delta < 0$ va $\Delta > 0$ bo‘lsa, izlanayotgan kesim aylanadan iborat bo‘ladi; agarda (13.17) nolga teng bo‘lsa, bu holda $\Delta = 0$ bo‘ladi va izlangan kesim nuqtaga aylanadi.

Agarda tekislik sferani kessa, aylana hosil bo‘ladi, yoki tekislik bilan sferaning umumiy nuqtasi bo‘lmaydi, yoki ikkalasining umumiy nuqtasi bo‘ladi.

Tekislik bilan sferaning bir umumiy nuqtasi bo‘lganda, ya’ni tekislik sferaga urunma bo‘lganda
$$A^2R^2 + B^2R^2 + C^2R^2 - D^2 = 0 \qquad (13.18)$$
bo‘ladi. Bu munosabatga asoslanib, sferaga urinma bo‘lgan tekislikning tenglamasini tuzish mumkin. Haqiqatdan ham (13.18) ni ushbu ko‘rinishda yozish mumkin:

$$\left(-\frac{R^2A}{D}\right)^2 : R^2 + \left(-\frac{R^2B}{D}\right)^2 : R^2 + \left(-\frac{R^2C}{D}\right)^2 : R^2 = 1,$$

ya'ni koordinatalari

$$x_1 = -\frac{R^2A}{D}, \quad y_1 = -\frac{R^2B}{D}, \quad z_1 = -\frac{R^2C}{D} \tag{13.19}$$

bo'lgan nuqta sfera tenglamasini qanoatlantiradi. Ikkinchi tomondan (13.18) ta'minlanganda $(x_1; y_1; z_1)$ nuqtaning koordinatalari (13.13) tekislikning tenglamasini ham qanoatlantiradi. Demak, $(x_1; y_1; z_1)$ nuqta (13.13) tenglamaning sferaga urinish nuqtasi bo'ladi. Sferaga urinma bo'lgan (13.13) tekislikning koeffitsiyentlari (13.19) dan aniqlanadi:

$$A = -\frac{Dx_1}{R^2}, \quad B = -\frac{Dy_1}{R^2}, \quad C = -\frac{Dz_1}{R^2},$$

natijada, sferaning $(x_1; y_1; z_1)$ nuqtasidan o'tgan urinma tekislikning tenglamasi quyidagicha bo'ladi:

$$xx_1 + yy_1 + zz_1 = R^2. \tag{13.20}$$

### Mustaqil yechish uchun topshiriqlar.

**13.1.1**. $\frac{x^2}{p} + \frac{y^2}{q} = 2z \; (p > 0,\; q > 0)$ sirt to'g'ri chiziqli yasovchilarining $Oxz$ tekisligidagi proyeksiyalari $x^2 = 2pz$ parabolaga urinishini isbotlang.

**13.1.2**. $x^2 - y^2 = 2z$ giperbolik paraboloid ikkita o'zaro perpendikulyar yasovchilari kesishish nuqtalarining geometrik o'rni topilsin.

**13.1.3** Quyidagi sfera markazining koordinatalari va radiusi aniqlansin.

1) $x^2 + y^2 + z^2 - 12x + 4y - 6z = 0$;

2) $x^2 + y^2 + z^2 + 8x = 0$;

3) $x^2 + y^2 + z^2 - 2x + 4y - 6z - 22 = 0$;

4) $x^2 + y^2 + z^2 - 6z - 7 = 0$.

**13.1.4**. $A(3; 0; 4)$, $B(3; 5; 0)$, $C(3; 4; 4)$, $D(5; 4; 6)$ nuqtalarning $(x-1)^2 + (y+2)^2 + (z-1)^2 = 49$ sferaga nisbatan vaziyati aniqlansin.

**13.1.5**. $(x-a)^2+(y-b)^2+(z-c)^2=R^2$ sferaning ushbu $x=x_0+lt, \quad y=y_0+mt, \quad z=z_0+nt$ to‘g‘ri chiziqqa qo‘shma bo‘lgan diametrial tekisligining tenglamasi tuzilsin.

**13.1.6**. Ushbu $(x-1)^2+(y-4)^2+(z+1)^2=25$ sferaning $M(3;5;1)$ nuqtada teng ikkiga bo‘linadigan vatarlarining geometrik o‘rni topilsin.

**13.1.7**. $x^2+y^2+z^2-R^2=0$ sferaning $N(x_0;y_0;z_0)$ nuqtadan o‘tuvchi vatarlari o‘rtalarining geometrik o‘rni topilsin.

**13.1.8**. $x^2+y^2+z^2-R^2=0$ sferaning $M(-R;0;0)$ nuqtadan o‘tuvchi vatarlari o‘rtalarining geometrik o‘rni topilsin.

**13.1.9**. $(x-a)^2+(y-b)^2+(z-c)^2=R^2$ sferaning $K(x_0;y_0;z_0)$ nuqtadan o‘tuvchi vatarlari o‘rtalarining geometrik o‘rni topilsin.

**13.1.10**. $x^2+y^2=9,\ z=0$ va $z=2$ aylanalardan o‘tuvchi sfera tenglamasi tuzilsin.

**13.1.11**. Koordinatalar boshidan va $(x+1)^2+(y-2)^2+(z+2)^2= =49,\ 2x+2y-z+4=0$ aylanadan o‘tadigan sfera tenglamasi tuzilsin.

**13.1.12**. $A(1;-2;0)$ nuqtadan va $(x+1)^2+(y-2)^2+(z-2)^2= =49,\ 2x+2y-z+4=0$ aylanadan o‘tuvchi sfera tenglamasi tuzilsin.

**13.1.13**. Quyidagi tekisliklarning ushbu $(x-1)^2+(y-2)^2+ +(z-4)^2=25$ sferaga nisbatan vaziyati aniqlansin.

1) $2x+2y+z+2=0$;
2) $2x+2y+z+5=0$;
3) $2x+2y+z+11=0$.

**13.1.14**. $(x-1)^2+(y+3)^2+(z-2)^2=49$ sferaga $N(7;-1;5)$ nuqtada o‘tkazilgan urinma tekislik tenglamasi tuzilsin.

**13.1.15**. $(x+2)^2+(y-1)^2+z^2=169$ sferaga $K(-5;-3;12)$ nuqtada o‘tkazilgan urinma tekislik tenglamasi tuzilsin.

**13.1.16**. $(x-a)^2+(y-b)^2+(z-c)^2=R^2$ sferaga $N_0(x_0;y_0;z_0)$ nuqtada o‘tkazilgan urinma tekislik tenglamasi tuzilsin.

**13.1.17**. $x^2 + y^2 + z^2 = R^2$ sferaga $M_0(x_0; y_0; z_0)$ nuqtada o‘tkazilgan urinma tekislik tenglamasi tuzilsin.
**13.1.18**. Qanday zaruriy va yetarli shart bajarilganda $Ax + By + Cz + +D = 0$ tekislik $x^2 + y^2 + z^2 = R^2$ sferaga urinadi? Bu shart bajarilgan deb urinish nuqtasining koordinatalari topilsin.
**13.1.19**. $x^2 + y^2 + z^2 - 3x + 6y + 2z - 5 = 0$, $x - 2y - 2z + 1 = = 0$ aylanadan o‘tuvchi va $2x + 2y + z - 7 = 0$ tekislikka urinadigan sfera tenglamasi tuzilsin.
**13.1.20**. $x^2 + y^2 - 11 = 0$, $z = 0$ aylanadan o‘tuvchi va $x + y + z - -5 = 0$ tekislikka urinadigan sfera tenglamasi tuzilsin.
**13.1.21.** Koordinatalar boshi bilan $N(1; 1; 1)$ nuqtadan o‘tuvchi to‘g‘ri chiziqda $(x - 2)^2 + (y - 5)^2 + z^2 = 1$, $(x - 4)^2 + (y - 3)^2 + +(z - 6)^2 = 2$ sferalarga o‘tkazilgan urinma uzunliklari bir-biriga teng bo‘lgan nuqta topilsin.
**13.1.22.** $x = x_0 + lt$, $y = y_0 + mt$, $z = z_0 + nt$ to‘g‘ri chiziq $(x - a)^2 + (y - b)^2 + (z - c)^2 = R^2$ sferaga urinishi uchun qanday shartlarning bajarilishi zarur va yetarli?
**13.1.23.** Ikkita ayqash
$x = x_1 + lt,\ y = y_1 + mt,\ z = z_1 + nt;$
$x = x_2 + lt,\ y = y_2 + mt,\ z = z_2 + nt.$
to‘g‘ri chiziqqa urinuvchi sferalar markazlarining geometrik o‘rni topilsin, bu yerda ${l_1}^2 + {m_1}^2 + {n_1}^2 = 1$ va ${l_2}^2 + {n_2}^2 + {n_2}^2 = 1$ deb faraz qilinadi.
**13.1.24**. Quyida ko‘rsatilgan hollarning har birida:
1) berilgan nuqtadan o‘tuvchi;
2) berilgan ikki nuqtadan o‘tuvchi;
3) berilgan uch nuqtadan o‘tuvchi;
4) berilgan to‘g‘ri chiziqqa urinuvchi;
5) berilgan tekislikka urinuvchi;
6) berilgan tekislikka urinuvchi va belgili radiusga ega;
7) berilgan tekislikdagi markazga ega bo‘lgan;
8) berilgan aylanadagi markazga ega bo‘lgan;

9) berilgan aylana orqali o‘tuvchi
sferalar to‘plami nechta parametrga bog‘liq?

**13.1.25**. $x = x_1 + lt,\ y = y_1 + mt,\ z = z_1 + nt$ to‘g‘ri chiziq orqali ushbu $(x-a)^2 + (y-b)^2 + (z-c)^2 = R^2$ sferaga urinma tekisliklar o‘tkazilsin.

**13.1.26**. Qanday zaruriy va yetarli shartlarda $x = x_1 + lt,\ y = y_1 + mt,\ z = z_1 + nt$ to‘g‘ri chiziq bilan $(x-a)^2 + (y-b)^2 + (z-c)^2 = R^2$ sfera

1) kesishmaydi?

2) kesishadi?

**13.1.27.** $(x-a)^2 + (y-b)^2 + (z-c)^2 = R^2$ sferaning $Ax + By + Cz + D = 0$ tekislikka parallel bo‘lgan urinma tekisliklari tenglamalari tuzilsin.

**13.1.28**. $\frac{x^2}{27} + \frac{y^2}{12} + \frac{z^2}{75} = 1$ ellipsoidning $N(3; 2; 5)$ nuqtasidagi urinma tekisligi tenglamasi tuzilsin.

**13.1.29**. $\frac{x^2}{49} + \frac{y^2}{18} + \frac{z^2}{50} = 1$ ellipsoidning $A(0; -3; 5)$ nuqtasidagi urinma tekisligi tenglamasi tuzilsin.

**13.1.30**. $Ax + By + Cz + D = 0$ tekislikning $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ ellipsoidga urinishi uchun zaruriy va yetarli shart topilsin.

**13.1.31**. $Ax + By + Cz + D = 0$ tekislikning $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ ellipsoid bilan kesishishi uchun qanday shartning bajarilishi zarur va yetarli?

**13.1.32**. $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ ellipsoidning markazidan uning urinma tekisligiga tushurilgan perpendikulyarlar asoslarining geometrik o‘rni topilsin.

**13.1.33**. $2x + 3y - z + 1 = 0$ tenglama bilan berilgan tekislik va $\frac{x^2}{25} + \frac{y^2}{16} + \frac{z^2}{9} = 1$ tenglama bilan berilgan ellipsoidning kesishishidan hosil bo‘lgan chiziqni aniqlang.

**13.1.34.** $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ ellipsoidning $Ax+By+Cz+D=0$ tekislik bilan kesishish chizig‘ining markazi topilsin.

**13.1.35.** $\frac{x^2}{25}+\frac{y^2}{16}+\frac{z^2}{9}=1$ ellipsoidning markazidan barcha nuqtalarida unga o‘tkazilgan urinma tekisliklargacha bo‘lgan masofalar $d$ ga teng bo‘ladigan nuqtalarning geometrik o‘rni topilsin.

**13.1.36.** $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ ellipsoidning markazidan barcha nuqtalarida unga o‘tkazilgan urinma tekisliklargacha bo‘lgan masofalar $d$ ga teng bo‘ladigan nuqtalarning geometrik o‘rni topilsin.

**13.1.37.** $Oxy$ tekislikka va $x^2+y^2+z^2=a^2$ sferaga urinadigan sferalar markazlarining geometrik o‘rni topilsin.

**13.1.38.** O‘qlari koordinata o‘qlari bilan ustma - ust tushuvchi, $Oxz$ va $Oyz$ tekisliklarni mos ravishda $y=0,\ \frac{x^2}{25}+\frac{z^2}{16}=1,\ \ x=0,\ \frac{y^2}{9}+\frac{z^2}{16}=1$ chiziqlar bo‘ylab kesib o‘tuvchi ellipsoid tenglamasi tuzilsin.

**13.1.39.** O‘qlari koordinata o‘qlaridan iborat, $z=0, \frac{x^2}{9}+\frac{y^2}{16}=1$ ellips va $N\left(1;2;\sqrt{23}\right)$ nuqta orqali o‘tuvchi ellipsoid tenglamasi tuzilsin.

**13.1.40.** O‘qlari koordinata o‘qlaridan iborat bo‘lgan va $x^2+y^2+$ $+z^2=9,\ \ x=z$ aylanadan hamda $N(3;1;1)$ nuqtadan o‘tgan ellipsoid tenglamasi tuzilsin.

# 14-MAVZU: IKKINCHI TARTIBLI SIRTLAR UMUMIY TENGLAMALARINI SODDALASHTIRISH. MARKAZIY VA NOMARKAZIY SIRT TENGLAMASINI KANONIK KO‘RINISHGA KELTIRISH.

**Reja:**

**1. Ikkinchi tartibli sirtlar umumiy tenglamalarini soddalashtirish.**

**2. Markaziy sirtning tenglamasini kanonik ko‘rinishga keltirish.**

**3. Nomarkaziy sirt tenglamasini kanonik ko‘rinishga keltirish.**

**Tayanch iboralar**: markaziy sirt, nomarkaziy sirt, invariant, parallel ko‘chirish, xarakteristik tenglama, mavhum parallel tekislik.

## 14.1. Ikkinchi tartibli sirtlar umumiy tenglamalarini soddalashtirish.

Ikkinchi tartibli sirtning tenglamasi

$$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{23}yz + 3a_{31}zx +$$
$$+2a_1x + 2a_2y + 2a_3z + a = 0 \qquad (14.1)$$

ko‘rinishga ega. Bu tenglama to‘g‘ri burchakli koordinatalar sistemasiga nisbatan berilgan bo‘lsa, quyidagi ifodalar to‘g‘ri burchakli dekart koordinatalari sistemasini parallel ko‘chirish va burishga nisbatan invariantlari hisoblanadi:

$$I_1 = a_{11} + a_{22} + a_{33}, \quad I_2 = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix},$$

$$I_3 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}, \quad K_4 = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_1 \\ a_{21} & a_{22} & a_{23} & a_2 \\ a_{31} & a_{32} & a_{33} & a_3 \\ a_1 & a_2 & a_3 & a \end{vmatrix}.$$

Yariminvariant nomini olgan quyidagi ikki ifoda, to‘g‘ri burchakli dekart koordinatalar sistemasini burishga nisbatan invariantlardir.

$$K_3 = \begin{vmatrix} a_{11} & a_{12} & a_1 \\ a_{21} & a_{22} & a_2 \\ a_1 & a_2 & a \end{vmatrix} + \begin{vmatrix} a_{11} & a_{13} & a_1 \\ a_{31} & a_{33} & a_3 \\ a_1 & a_3 & a \end{vmatrix} + \begin{vmatrix} a_{22} & a_{23} & a_2 \\ a_{32} & a_{33} & a_3 \\ a_2 & a_3 & a \end{vmatrix},$$

$$K_2 = \begin{vmatrix} a_{11} & a_1 \\ a_1 & a \end{vmatrix} + \begin{vmatrix} a_{22} & a_2 \\ a_2 & a \end{vmatrix} + \begin{vmatrix} a_{33} & a_3 \\ a_3 & a \end{vmatrix}$$

$I_3 = 0$, $K_4 = 0$ holda $K_3$ yariminvariant ayni vaqtda burishga nisbatan ham invariant bo‘ladi, $I_3 = 0$, $K_4 = 0$, $I_2 = 0$, $K_3 = 0$ holda esa $K_2$ yariminvariant parallel ko‘chirishga nisbatan invariant bo‘ladi.

**I.** $I_3 \neq 1$ holda ikkinchi tartibli sirt tenglamasini to‘g‘ri burchakli koordinatalar sistemasini parallel ko‘chirish va burish natijasida quyidagi ko‘rinishga keltirish mumkin:

$$\lambda_1 X^2 + \lambda_2 Y^2 + \lambda_3 Z^2 + \frac{K_4}{I_3} = 0 \qquad (14.2)$$

bu yerda, $\lambda_1, \lambda_2, \lambda_3 -$ quyidagi xarakteristik tenglamaning ildizlaridir:

$$\begin{vmatrix} a_{11} - \lambda & a_{12} & a_{13} \\ a_{21} & a_{22} - \lambda & a_{23} \\ a_{31} & a_{32} & a_{33} - \lambda \end{vmatrix} = 0 \qquad (14.3)$$

yoki

$$\lambda^3 - I_1\lambda^2 + I_2\lambda - I_3 = 0.$$

$1^0$. Agar $\lambda_1$, $\lambda_2$, $\lambda_3$ bir xil ishorali, $\frac{K_4}{I_3}$ esa ularga teskari ishorada bo‘lsa, u holda (14.2) tenglama ellipsoidni aniqlaydi. $|\lambda_1| \leq |\lambda_2| \leq |\lambda_3|$ deb hisoblab, (14.2) tenglamani

$$\frac{x^2}{-\frac{K_4}{\lambda_1 I_3}} + \frac{y^2}{-\frac{K_4}{\lambda_2 I_3}} + \frac{z^2}{-\frac{K_4}{\lambda_3 I_3}} = 1$$

ko‘rinishda yozib olamiz. Bunda ellipsoidni yarim o‘qlarini

$$a = \sqrt{-\frac{K_4}{\lambda_1 I_3}}, \quad b = \sqrt{-\frac{K_4}{\lambda_2 I_3}}, \quad c = \sqrt{-\frac{K_4}{\lambda_3 I_3}}$$

ko‘rinishda yoza olamiz va $|\lambda_1| \leq |\lambda_2| \leq |\lambda_3|$ qilingan farazga ko‘ra $a \geq b \geq c$ munosabatlar o‘rinli bo‘ladi.

$2^0$. $\lambda_1, \lambda_2, \lambda_3$, $\frac{K_4}{I_3}$ bir xil ishorali bo‘lsa, u holda (14.2) tenglama mavhum ellipsoidni aniqlaydi: $|\lambda_1| \leq |\lambda_2| \leq |\lambda_3|$ deb hisoblagan holda uni $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} == -1$ ko‘rinishga keltiramiz, bunda: $a = \sqrt{\frac{K_4}{\lambda_1 I_3}}$,

$b = \sqrt{\frac{K_4}{\lambda_2 I_3}}$, $c = \sqrt{\frac{K_4}{\lambda_3 I_3}}$ qilingan $|\lambda_1| \leq |\lambda_2| \leq |\lambda_3|$ farazga ko'ra $a \geq b \geq c$ ekanligiga ishonch hosil qilamiz.

$3^0$. $\lambda_1, \lambda_2, \lambda_3$ sonlar bir xil ishorali, va $K_4 = 0$ bo'lsa, u holda (14.2) tenglama mavhum konusni aniqlaydi. $|\lambda_1| \leq |\lambda_2| \leq |\lambda_3|$ deb hisoblagan holda uni $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0$ ko'rinishga keltiramiz, bunda:

$$a = \sqrt{\frac{1}{|\lambda_1|}}, \quad b = \sqrt{\frac{1}{|\lambda_2|}}, \quad c = \sqrt{\frac{1}{|\lambda_3|}}$$

va shu bilan birga $a \geq b \geq c$.

$4^0$. Agar (14.3) xarakteristik tenglama ildizlarining ikkitasi bir xil ishorali, uchinchi ildizi bilan $\frac{K_4}{I_3}$ ularga teskari ishorali bo'lsa, (14.2) tenglama bir pallali giperboloidni aniqlaydi. Bu holda xarakteristik tenglamaning bir xil ishorali ildizlarini $\lambda_1$ va $\lambda_2$ deb belgilab va $|\lambda_1| < |\lambda_2|$ deb faraz qilib (14.2) tenglamani yoki

$$\frac{x^2}{-\frac{K_4}{\lambda_1 I_3}} + \frac{y^2}{-\frac{K_4}{\lambda_2 I_3}} + \frac{z^2}{-\frac{K_4}{\lambda_3 I_3}} = 1$$

ko'rinishda yozib olamiz.

Bu yerda: $a = \sqrt{-\frac{K_4}{\lambda_1 I_3}}$, $b = \sqrt{-\frac{K_4}{\lambda_2 I_3}}$, $c = \sqrt{-\frac{K_4}{\lambda_3 I_3}}$, $a \geq b$.

$5^0$. Xarakteristik tenglamaning ikki ildizi va $\frac{K_4}{I_3}$ ozod hadi bir xil ishorali, xarakteristik tenglamaning uchinchi ildizi esa ularga teskari ishorali bo'lsa, (14.2) tenglama ikki pallali giperboloidni aniqlaydi. Bu holda xarakteristik tenglamaning bir xil ishorali ildizlari $\lambda_1$ va $\lambda_2$ ni olib $|\lambda_1| \leq |\lambda_2|$ deb hisoblasak, (14.3) tenglamani

$$\frac{x^2}{\frac{K_4}{\lambda_1 I_3}} + \frac{y^2}{\frac{K_4}{\lambda_2 I_3}} - \frac{z^2}{-\frac{K_4}{\lambda_3 I_3}} = -1 \quad \text{yoki} \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$$

ko'rinishida yozamiz, bunda: $a \geq b$.

$6^0$. Xarakteristik tenglamaning ikkita ildizi bir xil ishorali, uchinchi ildizi ularga teskari va $K_4 = 0$ bo'lsa, u holda (14.2) tenglama konusni aniqlaydi. $\lambda_1$ va $\lambda_2$ sonlar bir xil ishorali ildizlar va $|\lambda_1| \leq |\lambda_2|$ deb hisoblanganda (14.2) tenglamani

$$\frac{x^2}{\frac{1}{|\lambda_1|}} + \frac{y^2}{\frac{1}{|\lambda_2|}} - \frac{z^2}{\frac{1}{|\lambda_3|}} = 0 \quad \text{yoki} \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$

ko‘rinishga keltiramiz. Bu yerda $a \geq b$ bo‘lib:

$$a = \sqrt{\frac{1}{|\lambda_1|}}, \quad b = \sqrt{\frac{1}{|\lambda_2|}}, \quad c = \sqrt{\frac{1}{|\lambda_3|}}$$

Xarakteristik tenglamadagi musbat ildizlar soni uning koeffitsiyentlari orasidagi ishoralar almashuvlari soniga teng bo‘ladi (Dekart qoidasi).

**II.** $I_3 = 0,\ K_4 \neq 0$ bo‘lsa, u holda to‘g‘ri burchakli koordinatalar sistemasini parallel ko‘chirish va burish natijasida ikkinchi tartibli sirt tenglamasini

$$\lambda_1 x^2 + \lambda_2 y^2 \pm 2\sqrt{-\frac{K_4}{I_2}}\, z = 0 \qquad (14.4)$$

ko‘rinishga keltirish mumkin. Bu tenglamada $\lambda_1$ va $\lambda_2$ xarakteristik tenglamaning noldan farqli bo‘lgan ildizlari.

$7^0$. $\lambda_1, \lambda_2$ sonlar bir xil ishorali bo‘lsa, u holda (14.4) tenglama elliptik paraboloidni aniqlaydi. $|\lambda_1| \leq |\lambda_2|$ deb hisoblab (14.4) tenglamani

$$\frac{x^2}{\pm\frac{1}{\lambda_1}\sqrt{-\frac{K_4}{I_2}}} + \frac{y^2}{\pm\frac{1}{\lambda_2}\sqrt{-\frac{K_4}{I_2}}} = 2z$$

ko‘rinishda yoza olamiz.

$$\pm\frac{1}{\lambda_1}\sqrt{-\frac{K_4}{I_2}} = p, \quad \pm\frac{1}{\lambda_2}\sqrt{-\frac{K_4}{I_2}} = q$$

deb faraz qilib, ushbu tenglamani hosil qilamiz:

$$\frac{x^2}{p} + \frac{y^2}{q} = 2z$$

bunda: $p \geq q \geq 0$.

$8^0$. $\lambda_1, \lambda_2$ sonlar har xil ishorali bo‘lsa, (14.4) tenglama giperbolik paraboloidni aniqlaydi. $\lambda_1$ musbat, $\lambda_2$ manfiy ildiz deb olib, $\sqrt{-\frac{K_4}{I_2}}$ radikal oldidagi ishoradan minusini olib, (14.4) tenglamani

$$\frac{x^2}{\frac{1}{\lambda_1}\sqrt{-\frac{K_4}{I_2}}} - \frac{y^2}{-\frac{1}{\lambda_2}\sqrt{-\frac{K_4}{I_2}}} = 2z \quad \text{yoki} \quad \frac{x^2}{p} - \frac{y^2}{q} = 2z$$

ko‘rinishda yozamiz, bu yerda:

$$p = \frac{1}{\lambda_1}\sqrt{\frac{K_4}{I_2}}, \qquad q = -\frac{1}{\lambda_2}\sqrt{-\frac{K_4}{I_2}}$$

**III.** $I_3 = 0,\ K_4 = 0,\ I_2 \neq 0$ bo‘lsa, to‘g‘ri burchakli koordinatalar sistemasini burish va parallel ko‘chirish natijasida ikkinchi tartibli sirt tenglamasini

$$\lambda_1 x^2 + \lambda_2 y^2 + \frac{K_3}{I_2} = 0 \qquad (14.5)$$

ko‘rinishga keltirish mumkin. Bu yerda: $\lambda_1$, $\lambda_2$ sonlar xarakteristik tenglamaning noldan farqli ildizlari.

$9^0$. $\lambda_1$, $\lambda_2$ sonlar bir xil ishorali, $\frac{K_3}{I_2}$ esa ularga teskari ishorali bo‘lsa, (14.5) tenglama elliptik silindrni aniqlaydi. $|\lambda_1| \leq |\lambda_2|$ deb hisoblab, (14.5) tenglamani

$$\frac{x^2}{-\frac{K_3}{\lambda_1 I_2}} + \frac{y^2}{-\frac{K_3}{\lambda_2 I_2}} = 1 \quad \text{yoki} \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

ko‘rinishda yozib olamiz, bu yerda: $a \geq b$ bo‘lib,

$$a = \sqrt{-\frac{K_3}{\lambda_1 I_2}}, \qquad b = \sqrt{-\frac{K_3}{\lambda_2 I_2}}$$

$10^0$. $\lambda_1$, $\lambda_2$, $\frac{K_3}{I_2}$ sonlar bir xil ishorali bo‘lsa, (14.5) tenglama mavhum elliptik silindrni aniqlaydi. $|\lambda_1| \leq |\lambda_2|$ deb hisoblab, (14.5) tenglamani

$$\frac{x^2}{\frac{K_3}{\lambda_1 I_2}} + \frac{y^2}{\frac{K_3}{\lambda_2 I_2}} = -1 \quad \text{yoki} \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = -1$$

ko‘rinishda yozamiz, bunda: $a \geq b$.

$11^0$. $\lambda_1, \lambda_2$ sonlar bir xil ishorali va $K_3 = 0$ bo‘lsa, u holda (14.5) tenglama kesishadigan ikkita mavhum tekisliklarni aniqlaydi. Bu holda (14.5) tenglamani

$$\frac{x^2}{\frac{1}{\lambda_1}} + \frac{y^2}{\frac{1}{\lambda_2}} = 0 \quad \text{yoki} \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 0$$

ko‘rinishda yozib olamiz, bunda $a \geq b$ bo‘lib,

$$a = \sqrt{\frac{1}{|\lambda_1|}}, \quad b = \sqrt{\frac{1}{|\lambda_2|}}.$$

$12^0$. $\lambda_1$, $\lambda_2$ sonlar har xil ishorali va $K_3 \neq 0$ bo‘lsa, (14.5) tenglama giperbolik silindrni aniqlaydi. $\lambda_1$ deb xarakteristik tenglamaning $\frac{K_3}{I_2}$ ning ishorasiga teskari ishorali ildizni olib, (14.5) tenglamani

$$\frac{x^2}{-\frac{K_3}{\lambda_1 I_2}} - \frac{y^2}{-\frac{K_3}{\lambda_2 I_2}} = 1 \quad \text{yoki} \quad \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

ko‘rinishda yozib olamiz, bu yerda:

$$a = \sqrt{-\frac{K_3}{\lambda_1 I_2}}, \qquad b = \sqrt{\frac{K_3}{\lambda_2 I_2}}.$$

$13^0$. $\lambda_1$, $\lambda_2$ sonlar har xil ishorali va $K_3 = 0$ bo‘lsa, (14.5) tenglama kesishadigan ikkita tekislikni aniqlaydi. Xarakteristik tenglamaning musbat ildizini $\lambda_1$ deb olib, (14.5) tenglamani

$$\frac{x^2}{\frac{1}{\lambda_1}} - \frac{y^2}{\frac{1}{\lambda_2}} = 0 \quad \text{yoki} \quad \frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$$

ko‘rinishda yozamiz, bunda:

$$a = \sqrt{\frac{1}{\lambda_1}}, \qquad b = \sqrt{-\frac{1}{\lambda_2}}.$$

**IV.** $I_3 = 0,\ K_4 = 0,\ I_2 = 0,\ K_3 \neq 0$ hol yuz bersa, to‘g‘ri burchakli koordinatalar sistemasini burish va parallel ko‘chirish natijasida ikkinchi tartibli sirt ternglamasini

$$\lambda_1 x^2 \pm 2\sqrt{-\frac{K_3}{I_1}}y = 0 \tag{14.6}$$

ko‘rinishga keltirish mumkin, bu yerda: $\lambda_1 = I_1$ son xarakteristik tenglamaning noldan farqli bo lgan ildizi.

$14^0$. (14.6) tenglamani ushbu $x^2 = 2\sqrt{-\frac{K_3}{I_1}}y$ ko‘rinishda yozish ham mumkin. Bu tenglama parabolik silindrni aniqlaydi. Bu silindrni yasovchilariga perpendikulyar bo‘lgan tekislik bilan kesishish natijasida hosil bo‘lgan parabolaning parametrini ushbu

$$p = \sqrt{-\frac{K_3}{{I_1}^3}}$$

formuladan aniqlanadi.

**V.** $I_3 = 0,\ K_4 = 0,\ I_2 = 0,\ K_3 = 0$ holda, to‘g‘ri burchakli koordinatalar sistemasini burish natijasida ikkinchi tartibli sirt tenglamasini

$$\lambda_1 x^2 + \frac{K_2}{I_1} = 0 \quad \text{yoki} \quad I_1 x^2 + \frac{K_2}{I_1} = 0 \quad \text{yoki}$$

$$x^2 + \frac{K_2}{{I_1}^2} = 0 \tag{14.7}$$

ko‘rinishga keltirish mumkin.

$15^0$. $K_2 < 0$ bo‘lsa, (14.7) tenglama ikkita parallel tekislikni aniqlaydi. Bu holda $\frac{K_2}{{I_1}^2} = -a^2$ deb tenglamani $x^2 - a^2 = 0$ ko‘rinishda yozib olamiz.

$16^0$. $K_2 > 0$ bo‘lsa, (14.7) tenglama ikkita mavhum parallel tekislikni aniqlaydi. $\frac{K_2}{{I_1}^2} = a^2$ deb uni $x^2 + a^2 = 0$ ko‘rinishda yozamiz.

$17^0$. Nihoyat, $K_2 = 0$ bo‘lsa, (14.7) tenglama ikkita ustma-ust tushuvchi tekislikni aniqlaydi. $x^2 = 0$.

Ikkinchi tartibli sirt aylanma sirt bo‘lishi uchun uning xarakteristik tenglamasi karrali ildizga ega bo‘lishi zarur va yetarlidir.

Kanonik tenglamasi ma’lum bo‘lgan sirt vaziyatini aniqlash uchun, kanonik sistemaning yangi koordinatalar boshi $O'$ ni va shu bilan birga bu sistemaning yo‘naltiruvchi vektorlari koordinatalarini bilish kerak.

Kanonik koordinatalar sistemasi o‘qlari yo‘naltiruvchi vektorlari koordinatalari

$$\begin{cases} (a_{11} - \lambda)l + a_{12}m + a_{13}n = 0 \\ a_{21}l + (a_{22} - \lambda)m + a_{23}n = 0 \\ a_{31}l + a_{32}m + (a_{33} - \lambda)n = 0 \end{cases} \tag{14.8}$$

tenglamalar sistemasidan aniqlandi, bunda $\lambda -$ xarakteristik tenglamaning ildizi. Aylanma sirtning joylashishini aniqlash uchun kanonik koordinatalar sistemasida yangi koordinata boshi $O'$ ni va aylanish o‘qi yo‘naltiruvchi vektorining koordinatalarini bilish lozim. Yo‘naltiruvchi vektorining koordinatalari (14.8) sistemasidan aniqlanadi, bunda $\lambda -$ xarakteristik tenglamaning oddiy ildizi.

Sirt markazga ega bo‘lsa (yagona bo‘lishi shart emas), u holda kanonik sistemasining yangi koordinata boshi $O'$ deb sirt markazi olinadi. Sirt markazining koordinatalari

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z + a_1 = 0 \\ a_{21}x + a_{22}y + a_{23}z + a_2 = 0 \\ a_{31}x + a_{32}y + a_{33}z + a_3 = 0 \end{cases} \qquad (14.9)$$

tenglamalar sistemasidan topiladi.

$1^0$. Uch o‘qli ellipsoid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \quad a > b > c.$

Bu ellipsoid markazining koordinatalari (14.9) sistemadan topiladi. Katta o‘qi $(O'X)$ ning yo‘naltiruchi vektorining koordinatalari (14.8) tenglamadan topiladi, undagi son xarakteristik tenglamalarning modul jihatdan kichik bo‘lgan ildizi; o‘rta o‘q $(O'Y)$ning yo‘naltiruvchi vektorining koordinatalari (14.8) sistemadan topiladi, $\lambda -$ son xarakteristik tenglamaning modul jihatdan o‘rta bo‘lgan ildiz; kichik o‘q $(O'z)$ ning yo‘naltiruvchi vektorning koordinatalari ham (14.8) sistemadan topiladi, bunda $\lambda -$ xarakteristik tenglamaning modul jihatidan katta bo‘lgan ildizi.

$2^0$. Agar (14.1) tenglama nuqtani aniqlasa (mavhum konus), u holda bu nuqtaning koordinatalari (14.9) sistemadan topiladi.

$3^0$. Bir pallali giperboloidning kanonik tenglamasi:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1, \quad a > b.$$

Bir pallali giperboloid markazining koordinatalari (14.9) sistemadan aniqlanadi.

$\lambda_1$, $\lambda_2$ sonlar xarakteristik tenglmaning bir xil ishorali ildizlari bo‘lib, bunda $|\lambda_1| < |\lambda_2|$ va $\lambda_3$ esa ishorasi $\lambda_1$ va $\lambda_2$ ildizlarning ishorasiga teskari ildiz bo‘lsin. Giperboloid $(O'Z)$ o‘qining yo‘naltiruvchi vektori koordinatalari (14.8) sistemadan aniqlanadi, bunda $\lambda = \lambda_3^i$ bir pallali giperboloid bo‘g‘iz kesimning $(O'x)$ katta o‘qi yo‘naltiruvchi vektorining koordinatalari (14.8) sistemadan aniqlanadi, bunda $\lambda = \lambda_1$; bir pallali $\lambda = \lambda_1$ bo‘g‘iz kesimining $(O'y)$ kichik o‘qi yo‘naltiruvchi vektorining koordinatalari (14.8) sistemadan topiladi, bunda $\lambda = \lambda_2$.

$4^0$. Ikki pallali giperboloid:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1, \quad a>b.$$

Ikki pallali giperboloid markazining koordinatalari (14.9) sistemadan topiladi. $\lambda_1, \lambda_2$ sonlar xarakteristik tenglamaning bir xil ishorali ildizlari va $|\lambda_1| < |\lambda_2|$ bo‘lsin, $\lambda_3 -$ esa xarakteristik tenglamaning $\lambda_1$, $\lambda_2$ ildizlari ishorasiga teskari ishoraga ega bo‘lgan uchinchi ildizi bo‘lsin.

U holda giperboloid $(O'Z)$ o‘qining yo‘naltiruvchi vektorining koordinatalari (14.8) sistemadan aniqlanadi, bunda $\lambda = \lambda_3$; $O'X$ o‘qini (giperboloid o‘qiga perpendikulyar bo‘lgan o‘q bilan kesishi natijasida hosil bo‘lgan ellipsning katta o‘qi) yo‘naltiruvchi vektorining koordinatalari (14.8) sistemadan topiladi. Bunda $\lambda = \lambda_1$; $O'Y$ o‘qi yo‘naltiruvchi vektorining koordinatalari (14.8) sistemadan topiladi, bunda $\lambda = \lambda_2$.

$5^0$. Konus: $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0, \quad a>b$

Konus uchlarining koordinatalari (14.9) sistemadan aniqlanadi. $\lambda_1$, $\lambda_2$ sonlar xarakteristik tenglamaning bir xil ishorali ildizlari va $|\lambda_1| < |\lambda_2|$; $\lambda_3 -$ esa xarakteristik tenglamaning ishorasi $\lambda_1, \lambda_2$ ildizlar ishorasiga teskari ishorali ildizi bo‘lsin. U holda konusning $(O'z)$ o‘qi yo‘naltiruvchi vektorning koordinatalari (14.8) sistemadan aniqlanadi, bunda $\lambda = \lambda_3$. $O'x$ o‘qi yo‘naltiruvchi vektorining koordinatalarini (14.8) sistemadan aniqlanadi, bunda $\lambda = \lambda_1$. $O'x$ o‘qi (ya‘ni konusning o‘qiga perpendikulyar bo‘lgan kesimda hosil qilingan ellipsning katta o‘qi) yo‘naltiruchi vektorning koordinatalari (14.8) sistemadan aniqlaydi, bunda $\lambda = \lambda_1$; $O'y$ o‘qi yo‘naltiruvchi vektorining koordinatalarini (14.8) sistemadan aniqlaymiz, bunda $\lambda = \lambda_2$.

II. $6^0$. Elliptik parabolid: $\frac{x^2}{p}+\frac{y^2}{q}=2z$ kanonik sistemasining boshi, bu holda paraboloid uchidan iborat. Elliptik paraboloidning sirt botiqligi tomon yo‘nalgan o‘qining vektori ushbu munosabatdan aniqlanadi: $P\{I_1A_1, I_1A_2, I_1A_3\}$ bu yerda

$$A_1=-\begin{vmatrix} a_{12} & a_{13} & a_1 \\ a_{22} & a_{23} & a_2 \\ a_{32} & a_{33} & a_3 \end{vmatrix}; \quad A_2=\begin{vmatrix} a_{11} & a_{13} & a_1 \\ a_{21} & a_{23} & a_2 \\ a_{31} & a_{33} & a_3 \end{vmatrix};$$

$$A_3=-\begin{vmatrix} a_{11} & a_{12} & a_1 \\ a_{21} & a_{22} & a_2 \\ a_{31} & a_{32} & a_3 \end{vmatrix}.$$

Bu yerdagi $A_1$, $A_2$, $A_3$ sonlar $K_4$ determinantdagi $a_1$, $a_2$, $a_3$ elementlarining algebraik to‘ldiruvchilarini bildiradi.

$\lambda_1$, $\lambda_2$ xarakteristik tenglamaning noldan farqli ildizlari va $|\lambda_1|<|\lambda_2|$ bo‘lsin, bu holda $O'X$ o‘qining (ya’ni elliptik paraboloidni o‘ziga perpendikulyar bo‘lgan tekislik bilan kesishishdan hosil bo‘lgan ellips katta o‘qi) yo‘naltiruvchi vektorining koordinatalari $\lambda=\lambda_1$ holda (14.8) sistemadan aniqlanadi, $O'Y$ o‘qining yo‘naltiruvchi vektorini koordinatalari esa $\lambda=\lambda_2$ holda (14.8) sistemadan aniqlanadi. Elliptik paraboloidni uchi ushbu

$$\begin{cases} \frac{a_{11}x+a_{12}y+a_{13}z+a_1}{A_1}=\frac{a_{21}x+a_{22}y+a_{23}z+a_2}{A_2}=\frac{a_{31}x+a_{32}y+a_{33}z+a_3}{A_3} \\ a_{11}x^2+a_{22}y^2+a_{33}z^2+2a_{12}xy+2a_{23}yz+2a_{31}zx+2a_1x+2a_2y+2a_3z+a=0 \end{cases} \tag{14.10}$$

tenglamalar sistemasidan topiladi.

$7^0$. Giperbolik paraboloid: $\frac{x^2}{p}-\frac{y^2}{q}=2z$.

Bu holda kanonik sistemaning boshi paraboloid uchidan iborat. Giperbolik paraboloidning $(O'XZ)$ tekislik bilan kesishishi natijasida hosil bo‘lgan katta parametrli bosh kesimning botiqlik tomonga yo‘nalgan parabola o‘qining yo‘naltiruvchi vektori ushbu koordinatalarga ega bo‘ladi:

$$\{I_1A_1, I_1A_2, I_1A_3\}$$

bu yerda $A_1$, $A_2$, $A_3$ sonlar $K_4$ determinantning $a_1$, $a_2$, $a_3$ elementlarining algebraik to‘ldiruvchilaridir, $\lambda_1$, $\lambda_2$ sonlar xarakteristik tenglamaning ildizlari bo‘lib, $|\lambda_1| < |\lambda_2|$. U holda $O'X$ o‘qining yo‘naltiruvchi vektorining koordinatalari (ya’ni paraboloid uchidan o‘tuvchi to‘g‘ri chiziqli yasovchilar orasidagi o‘tkir burchak bissektrisalari) (14.8) sistemadan $\lambda = \lambda_1$ deb aniqlanadi: $O'Y$ o‘qni yo‘naltiruvchi vektorining koordinatalari (14.8) sistemadan $\lambda = \lambda_2$ deb aniqlanadi. Giperbolik paraboloidning uchi (14.10) sistemadan aniqlanadi.

Agar giperbolik paraboloid uchun $\lambda = -\lambda_2$ tenglik o‘rinli bo‘lsa, tegishli tenglama ushbu $x^2 - y^2 = 2pZ$ ko‘rinishni qabul qiladi.
Bu holda paraboloidning $O'XZ$, $O'YZ$ tekisliklar bilan kesimida hosil qilingan parabolalar bir xil parametrga ega. Bunda parabola o‘qining yo‘nalishi $\{A_1, A_2, A_3\}$ vektor orqali aniqlanadi.

III. $8^0$. Elliptik silindr. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a \neq b$ bo‘lganda, elliptik silindrning joylashishini aniqlash uchun uning o‘qini va silindr o‘qiga perpendikulyar kesimidagi katta va kichik o‘qlarining yo‘naltiruvchi vektorlarini bilish kerak.

Silindr o‘qi (14.9) tenglamalar yordamida topiladi (ulardan chiziqli erklilarini olish kerak). $\lambda_1, \lambda_2$ sonlar xarakteristik tenglamaning noldan farqli ildizlari va $|\lambda_1| < |\lambda_2|$ bo‘lsin.

U holda $O'X$ o‘qi (silindr o‘qiga perpendikulyar kesimida hosil bo‘lgan katta o‘qi) yo‘naltiruvchi vektorining koordinatalari (14.8) sistemadan topiladi, bunda $\lambda = \lambda_1$; $O'Y$ o‘qi yo‘naltiruvchi vektorining koordinatalari (14.8) sistemadan aniqlanib, bunda $\lambda = \lambda_2$ farazda $\lambda_1 = \lambda_2$

$$x^2 + y^2 = a^2$$

silindr hosil qilinadi va uning joylashishini aniqlash uchun o‘qini bilish yetarli.

$9^0$. Giperbolik silindr. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Giperbolik silindrning joylashishini bilish uchun uning o‘qini va o‘qiga perpendikulyar kesimining haqiqiy va mavhum o‘qlarining yo‘naltiruvchi vektorlarini bilish kerak. $\lambda_1$, $\lambda_2$ sonlar xarakteristik tenglamaning noldan farqli ildizlari, va $\lambda_1$ deb ishorasi $\frac{K_3}{I_2}$ ishorasiga teskari bo‘lgan ildiz belgilangan. U holda $O'X$ o‘qni yo‘naltiruvchi vektorlarini koordinatalari (silindrni o‘qqa perpendikulyar kesimini haqiqiy o‘qi) (14.8) tenglamalardan ($\lambda = \lambda_1$ holda) topiladi. $O'Y$ o‘qni (mavhum o‘qni) yo‘naltiruvchi vektorini koordinatalari esa $\lambda = \lambda_2$ holda (14.8) tenglamalardan topiladi.

**1-misol**. Koordinatalarning to‘g‘ri burchakli sistemasiga nisbatan

$$x^2 + 5y^2 + z^2 + 2xy + 6xz + 2yz - 2x + 6y + 2z = 0$$

tenglama bilan berilgan sirt ko‘rinishi va uning joylashishi aniqlansin.

**Yechish**.

$I_3 = \begin{vmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{vmatrix} = -36 \neq 0,$ sirt yagona simmetriya markazga ega. So‘ngra

$$K_4 = \begin{vmatrix} 1 & 1 & 3 & -1 \\ 1 & 5 & 1 & 3 \\ 3 & 1 & 1 & 1 \\ -1 & 3 & 1 & 0 \end{vmatrix} = 36 > 0: \quad I_1 = 1 + 5 + 1 = 7; \ I_1 I_3 < 0$$

ekanidan, berilgan sirt bir pallali giperboloidligi kelib chiqadi. $I_2 -$ ni topamiz:

$$I_2 = \begin{vmatrix} 1 & 1 \\ 1 & 5 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} + \begin{vmatrix} 5 & 1 \\ 1 & 1 \end{vmatrix} = 0.$$

Xarakteristik tenglamani tuzamiz va yechamiz:

$$\lambda^3 - 7\lambda^2 + 36 = 0; \quad \lambda_1 = 3, \ \lambda_2 = 6, \quad \lambda_3 = -2.$$

Sodda tenglamasi

$3x^2 + 6y^2 - 2z^2 + \frac{36}{-36} = 0$ yoki $3x^2 + 6y^2 - 2z^2 - 1 = 0$ yoki

$$\frac{x^2}{\left(\frac{1}{\sqrt{3}}\right)^2} + \frac{y^2}{\left(\frac{1}{\sqrt{6}}\right)^2} - \frac{z^2}{\left(\frac{1}{\sqrt{2}}\right)^2} = 1$$

ko‘rinishga ega ekan, bu yerda $a = \frac{1}{\sqrt{3}}, \ b = \frac{1}{\sqrt{6}}, \ c = \frac{1}{\sqrt{2}}$.

Sirt markazini

$$\begin{cases} x + y + 3z - 1 = 0 \\ x + 5y + z + 3 = 0 \\ 3x + y + z + 1 = 0 \end{cases}$$

sistemani yechib topamiz, bundan $C\left(-\frac{1}{3}; -\frac{2}{3}; \frac{2}{3}\right)$.

**2-misol**. To‘g‘ri burchakli koordinatalar tenglamalar sistemasiga nisbatan

$$5x^2 + 2y^2 + 5z^2 - 4xy - 2xz - 4yz + 10x - 4y - 2z + 4 = 0$$

tenglama bilan berilgan sirtning ko‘rinishi va joylashishi aniqlansin.

**Yechish**.

$$I_3 = \begin{vmatrix} 5 & -2 & -1 \\ -2 & 2 & -2 \\ -1 & -2 & 5 \end{vmatrix} = 0, \quad K_4 = \begin{vmatrix} 5 & -2 & -1 & 5 \\ -2 & 2 & -2 & -2 \\ -1 & -2 & 5 & -1 \\ 5 & -2 & -1 & 4 \end{vmatrix} = 0,$$

$$I_2 = \begin{vmatrix} 5 & -2 \\ -2 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -1 \\ -1 & 5 \end{vmatrix} + \begin{vmatrix} 2 & -2 \\ -2 & 5 \end{vmatrix} = 36,$$

$$K_3 = \begin{vmatrix} 5 & -2 & 5 \\ -2 & 2 & -2 \\ 5 & -2 & 4 \end{vmatrix} + \begin{vmatrix} 5 & -1 & 5 \\ -1 & 5 & -1 \\ 5 & -1 & 4 \end{vmatrix} + \begin{vmatrix} 2 & -2 & -2 \\ -2 & 5 & -1 \\ -2 & -1 & 4 \end{vmatrix} = 36,$$

$$I_1 = 5 + 2 + 5 = 12. \quad I_3 = K_4 = 0, \quad I_2 > 0, \quad I_1 K_3 < 0$$

bo‘lgani uchun berilgan tenglama elliptik silindrni aniqlaydi. Xarakteristik $\lambda^3 - 12\lambda^2 + 36\lambda = 0$ tenglama ildizlari: $\lambda_1 = \lambda_2 = 6$, $\lambda_3 = 0$. Sodda tenglamasi $6x^2 + 6y^2 - \frac{36}{36} = 0$ yoki $x^2 + y^2 = \frac{1}{6}$ ko‘rinishga ega. Bu tenglama radiusi $\frac{1}{\sqrt{6}}$ ga teng aylanma silindrni aniqlaydi. Silindrning o‘qi ushbu

$$\begin{cases} 5x - 2y - z + 5 = 0 \\ -2x + 2y - 2z - 2 = 0 \\ -x - 2y + 5z - 1 = 0 \end{cases}$$

tenglamalar sistemasidan topiladi, ammo bu sistemadagi ikkita tenglamani olish kifoya.

## Mustaqil yechish uchun topshiriqlar.

**14.1.1 - 14.1.2** mashqlarni Lagranj usulidan foydalanib, quyidagi tenglamalar ikkita tekislikka ajraluvchi sirtni aniqlashini isbotlang, va bu tekisliklarni toping.

**14.1.1**. 1) $y^2 + 2xy + 4xz + 2yz - 4x - 2y = 0$;

2) $x^2 + 4y^2 + 9z^2 - 4xy + 6xz - 12yz - x + 2y - 3z - 6 = 0$;

3) $3x^2 - 4y^2 + 3z^2 + 4xy + 10xz - 4yz + 6x - 20y - 14z -$
$-24 = 0$.

**14.1.2**. 1) $5x^2 + 4y^2 + 3z^2 + 9xy + 8xz + 7yz + 7x + 6y + 5z +$
$+2 = 0$;

2) $4x^2 + 49y^2 + z^2 - 28xy + 4xz - 14yz + 8x - 28y + 4z + 3 =$
$= 0$;

3) $16x^2 + 9y^2 + 100z^2 + 24xy + 80xz + 60yz + 56x + 42y +$
$+140z + 49 = 0$.

**14.1.3 - 14.1.6** mashqlarni Lagranj usulidan foydalanib, tenglamalarni kvadratlar yig‘indisi shakliga keltirib, quyidagi sirtlarning ko‘rinishi aniqlansin:

**14.1.3**. 1) $4x^2 + 6y^2 + 4z^2 + 4xz - 8y - 4z + 3 = 0$;

2) $x^2 + 5y^2 + z^2 + 2xy + 6xz + 2yz - 2x + 6y - 10z = 0$;

3) $x^2 + y^2 - 3z^2 - 2xy - 6xz - 6yz + 2x + 2y + 4z = 0$.

**14.1.4**. 1) $x^2 - 2y^2 + z^2 + 4xy - 8xz - 4yz - 14x - 4y + 14z +$
$+16 = 0$;

2) $2x^2 + y^2 + 2z^2 - 2xy - 2yz + x - 4y - 3z + 2 = 0$;

3) $x^2 - 2y^2 + z^2 + 4xy - 10xz + 4yz + x + y - z = 0$.

**14.1.5**. 1) $2x^2 + y^2 + 2z^2 - 2xy - 2yz + 4x - 2y = 0$;

2) $x^2 - 2y^2 + z^2 + 4xy - 10xz + 4yz + 2x + 4y - 10z - 1 = 0$;

3) $x^2 + y^2 + 4z^2 + 2xy + 4xz + 4yz - 6z + 1 = 0$.

**14.1.6**. 1) $4xy + 2x + 4y - 6z - 3 = 0$;

2) $xy + xz + yz + 2x + 2y - 2z = 0$.

**14.1.7 - 14.1.12** mashqlarni parallel ko‘chirish va burish almashtirishlari yoki hadlarni gruppalash yordamida quyidagi sirtlarning ko‘rinishi va joylashishi aniqlansin.

**14.1.7**. 1) $z = 2x^2 - 4y^2 - 6x + 8y + 1$;
2) $z = x^2 + 3y^2 - 6y + 1$;
3) $x^2 + 2y^2 - 3z^2 + 2x + 4y - 6z = 0$.
**14.1.8**. 1) $x^2 + 2xy + y^2 - z^2 = 0$;
2) $z^2 = 3x + 4y + 5$;
3) $z = x^2 + 2xy + y^2 + 1$.
**14.1.9**. 1) $z^2 = x^2 + 2xy + y^2 + 1$;
2) $x^2 + 4y^2 + 9z^2 - 6x + 8y - 18z - 14 = 0$;
3) $2xy + z^2 - 2z + 1 = 0$.
**14.1.10**. 1) $x^2 + y^2 - z^2 - 2xy + 2z - 1 = 0$;
2) $x^2 + 4y^2 - z^2 - 10x - 16y + 6z + 16 = 0$;
3) $2xy + 2x + 2y + 2z - 1 = 0$.
**14.1.11**. 1) $3x^2 + 6x - 8y + 6z - 7 = 0$;
2) $x^2 + y^2 + 2z^2 + 2xy + 4z = 0$;
3) $3x^2 + 3y^2 + 3z^2 - 6x + 4y - 1 = 0$.
**14.1.12**. 1) $3x^2 + 3y^2 - 6x + 4y - 1 = 0$;
2) $3x^2 + 3y^2 - 3z^2 - 6x + 4y + 4z + 3 = 0$;
3) $4x^2 - y^2 - 4x + 4y - 3 = 0$.
**14.1.13** - **14.1.24** mashqlardagi sirtlarning kanonik tenglamasi va joylashishini aniqlansin.
**14.1.13.** $x^2 + 5y^2 + z^2 + 2xy + 6xz + 2yz - 2x + 6y + 2z = 0$.
**14.1.14.** $2x^2 + y^2 + 2z^2 - 2xy + 2yz + 4x - 2y = 0$.
**14.1.15.** $x^2 + y^2 + 4z^2 + 2xy + 4xz + 4yz - 6z + 1 = 0$.
**14.1.16.** $4x^2 + 9y^2 + z^2 - 12xy - 6yz + 4zx + 4x - 6y + 2z -$
$-5 = 0$.
**14.1.17.** $7x^2 + 6y^2 + 5z^2 - 4xy - 4yz - 6x - 24y + 18z + 30 =$
$= 0$.
**14.1.18.** $2x^2 + 2y^2 - 5z^2 + 2xy - 2x - 4y - 4z + 2 = 0$.
**14.1.19.** $x^2 - 2y^2 + z^2 + 4xy - 8xz - 4yz - 14x - 4y + 14z +$
$+16 = 0$.
**14.1.20.** $2x^2 + 2y^2 + 3z^2 + 4xy + 2xz + 2yz - 4x + 6y - 2z +$
$+3 = 0$.

**14.1.21.** $2x^2 + 5y^2 + 2z^2 - 2xy + 2yz - 4xz + 2x - 10y - 2z -$
$-1 = 0.$
**14.1.22.** $x^2 + 5y^2 + z^2 + 2xy + 6xz + 2yz - 2x + 6y + 2z = 0.$
**14.1.23**. $5x^2 + 2y^2 + 5z^2 - 4xy - 2xz - 4yz + 10x - 4y - 2z +$
$+4 = 0.$
**14.1.24**. $x^2 - 2y^2 + z^2 + 4xy - 10xz + 4yz + 2x + 4y - 10z -$
$-1 = 0.$
**14.1.25.** $5x^2 - y^2 + z^2 + 4xy + 6xz + 2x + 4y + 6z - 8 = 0.$
**14.1.26.** $2x^2 + 10y^2 - 2z^2 + 12xy + 8yz + 12x + 4y + 8z - 1 =$
$= 0.$
**14.1.27**. Ikkinchi tartibli sirtning tenglamasi elliptik silindrni aniqlaydi. Uning
1) ozod hadini o‘zgartirsak;
2) birinchi darajali koordinatalari oldidagi koeffitsiyentlarini o‘zgartirsak sirtda qanday o‘zgarish bo‘ladi?
**14.1.28**. Yuqoridagi masala savollarini ikkinchi sirtning umumiy tenglamasi parabolik silindrni aniqlashini bilgan holda yeching.
**14.1.29**. $\lambda$ va $\mu$ parametrni qanday qiymatlarida

$$x^2 - y^2 + 3z^2 + (\lambda x + \mu y)^2 - 1 = 0$$

tenglama doiraviy silindrni aniqlaydi?
**14.1.30.** $a(x^2 + 2yz) + b(y^2 + 2xz) + c(z^2 + 2xy) = 1$ tenglama bilan berilgan sirt aylanma sirt bo‘lishi uchun qanday shart bajarilishi kerak?

# 15-MAVZU: AFFIN VA ORTOGONAL ALMASHTIRISHLAR, XOSSALARI. IZOMETRIK ALMASHTIRISHLAR. HARAKAT.

**Reja:**

**1. $n$ o‘lchamli vektorli yevklid fazosi.**

**2. Affin almashtirishlar.**

**3. Harakat.**

**Tayanch iboralar**: invariant, parallel ko‘chirish, ortogonal, ekvivalentlik, izomorfizm, ortonormallangan bazis, reper, ikkinchi tur harakat, harakatlar gruppasi, kongruent.

## 15.1. 1. $n$ o‘lchamli vektorli yevklid fazosi.

***Ta’rif.*** *$V_n$ vektor fazoning ixtiyoriy ikki $\vec{a},\vec{b}$ vektoriga ularning skalyar ko‘paytmasi deb atalgan haqiqiy son mos qo‘yilgan bo‘lib (vektor ko‘paytmani $\vec{a}\cdot\vec{b}$ bilan belgilaymiz), quyidagi to‘rtta aksioma bajarilsa, bunday fazo $n$ o‘lchamli vektorli yevklid fazosi deb ataladi va $V_E$ kabi belgilanadi:*

$G_1$) $\forall\vec{a},\ \vec{b}\in V_n$ uchun $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$,

$G_2$) $\forall\vec{a},\ \vec{b},\ \vec{c}\in V_n$ uchun $(\vec{a}+\vec{b})\cdot\vec{c}=\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{c}$,

$G_3$) $\forall\vec{a},\ \vec{b}\in V_n$ va $\forall k\in\mathbb{R}$ uchun $k\vec{a}\cdot\vec{b}=k(\vec{a}\vec{b})$,

$G_4$) $\forall\vec{a}\neq\bar{0}\in V_n$ uchun $\vec{a}\cdot\vec{a}>0$.

Bu aksiomalar odatda vektorning ***skalyar ko‘paytirish aksiomalari*** deb yuritiladi.

Yuqorida berilgan aksiomalardan kelib chiqadigan ba’zi natijalarini ko‘ramiz.

*1-natija.* $G_2$ aksiomadagi komutativlik, assotsiativlik qonuni ikki qo‘shiluvchi vektor uchun o‘rinli bo‘lsa, u istalgan $m\in\mathbb{N}$ sondagi qo‘shiluvchlar uchun o‘rinlidir, $(\vec{a}_1+\vec{a}_2+.\dots+\vec{a}_m)\cdot\vec{b}=\vec{a}_1\vec{b}+$ $+\vec{a}_2\vec{b}+\dots+\vec{a}_m\vec{b}$ (ifodadagi barcha vektorlar $V_E$ ga tegishli).

Haqiqatdan ham $\vec{a}_2+\vec{a}_3+.\dots+\vec{a}_m=\vec{b}_1$ desak, $G_2$ ga asosan $(\vec{a}_1+\vec{b}_1)\vec{b}=\vec{a}_1\vec{b}+\vec{b}_1\vec{b},$ bu ifodaning ikkinchisi qo‘shiluvchisidagi

$\vec{b}_1$ ni $\vec{a}_2 + +\vec{b}_2$ deb olsak, bunda $\vec{b}_2 = \vec{a}_3 + \vec{a}_1 +. ... + \vec{a}_m$, u holda $G_2$ ni yana tadbiq qilsak, $\vec{a}_1\vec{b} + \vec{b}_1\vec{b} = \vec{a}_1\vec{b} + (\vec{a}_2 + \vec{b}_2)\vec{b} = \vec{a}_1\vec{b} + \vec{a}_2\vec{b} + +\vec{b}_2\vec{b};$ endi shu ishni uchinchi qo'shiluvchi uchun takrorlaymiz va h.k. $\vec{a}_1 + \vec{a}_2 +. ... + \vec{a}_m$ ning soni chekli bo'lgani uchun ma'lum qadamdan so'ng izlangan tenglik hosil bo'ladi.

*2- natija.* $\vec{0}$ vektorning har qanday vektor bilan skalyar ko'paytmasi nolga tengdir, chunki $G_3$ ga asosan

$$(\vec{0} \cdot \vec{b}) = (0\vec{b} \cdot \vec{a}) = 0(\vec{b}\,\vec{a}) = 0.$$

*3- natija.* $\vec{a} \cdot \vec{a}$ skalyar ko'paytma faqat $\vec{a} = 0$ bo'lgandagina nolga tengdir, bu bevosita $G_4$ aksioma va 2 – natijadan kelib chiqadi.

***Ta'rif.*** *$V_E$ dagi $\vec{e}_1, \vec{e}_2, \dots, \vec{e}_n$ bazis vektorlarining har biri birlik vektor bo'lib, ularning istalgan ikkitasi o'zaro ortogonal bo'lsa, bunday vektorlar sistemasi ortonormallangan bazis (yoki dekart bazisi) deb ataladi, uni ham odatdagidek $\mathcal{B} = (\vec{e}_1, \vec{e}_2, \dots, \vec{e}_n)$ deb belgilaymiz.*

Demak, ortonormallangan bazis uchun

$$\vec{e}_i\vec{e}_j = \begin{cases} 1, & agar\ i = j\ bo'lsa, \\ 0, & agar\ i \neq j\ bo'lsa, \end{cases} \tag{15.1}$$

bunda $i,\ j = 1,2, \dots, n$.

Endi ortonormal bazisda koordinatalari bilan berilgan ikki vektorning skalyar ko'paytmasi, vektorning uzunligi, ikki vektor orasidagi burchakni hisoblash formulalarini topamiz.

Faraz qilaylik, dekart bazisida

$$\vec{a}\,(x_1, x_2, \dots, x_n) = x_1\vec{e}_1 + x_2\vec{e}_2 +. ... + x_n\vec{e}_n,$$
$$\vec{b}\,(y_1, y_2, \dots, y_n) = y_1\vec{e}_1 + y_2\vec{e}_2 +. ... + y_n\vec{e}_n.$$

bo'lsin. U holda skalyar ko'paytmasining xossalarini va (15.1) tenglikdan foydalanib,

$$\vec{a} \cdot \vec{b} = (x_1\vec{e}_1 + x_2\vec{e}_2 +. ... + x_n\vec{e}_n)(y_1\vec{e}_1 + y_2\vec{e}_2 +. ... + y_n\vec{e}_n) =$$
$$= x_1y_1(\vec{e}_1\vec{e}_1) + x_1y_2(\vec{e}_1\vec{e}_2) +. ... + x_1y_n(\vec{e}_1\vec{e}_n) + x_2y_1(\vec{e}_2\vec{e}_1) +$$
$$+x_2y_2(\vec{e}_2\vec{e}_2) +. ... + x_2y_n(\vec{e}_2\vec{e}_n) +. ... + x_ny_1(\vec{e}_n\vec{e}_1) + x_ny_2(\vec{e}_n\vec{e}_2) +$$
$$+\ldots+x_ny_n(\vec{e}_n\vec{e}_n) = x_1y_1 + x_2y_2 + \ldots + x_ny_n,$$

ya'ni

$$\vec{a}\cdot\vec{b} = x_1y_1 + x_2y_2 + \ldots + x_ny_n \qquad (15.2)$$

tenglikni hosil qilamiz.

Demak, $V_E$ da ikki vektorning skalyar ko‘paytmasi shu vektorlar mos koordinatalari ko‘paytmalarining yig‘indisiga teng.

$$\vec{a} = \vec{b} \Rightarrow x_1 = y_1, x_2 = y_2, \ldots, x_n = y_n \Rightarrow$$
$$\vec{a}\,\vec{a} = x_1^2 + x_2^2 + \ldots + x_n^2$$

yoki

$$\sqrt{\vec{a}\,\vec{a}} = \sqrt{x_1^2 + x_2^2 + \ldots + x_n^2},$$

vektor uzunligining ta’rifiga ko‘ra

$$|\vec{a}| = \sqrt{x_1^2 + x_2^2 + \ldots + x_n^2}. \qquad (15.3)$$

Demak, vektorning uzunligi va uning koordinatalari yig‘indisidan olingan arifmetik kvadrat ildizga teng.

(15.2) va (15.3) tengliklardan foydalanib, ikki vektor orasidagi burchakni hisoblash formulasini topamiz:

$$cos\varphi = \frac{x_1y_1 + x_2y_2 + \ldots + x_ny_n}{\sqrt{x_1^2 + x_2^2 + \ldots + x_n^2}\sqrt{y_1^2 + y_2^2 + \ldots + y_n^2}} \qquad (15.4)$$

***Ta’rif.*** *$\vec{a}_1, \vec{a}_2, \ldots, \vec{a}_n$ vektorlar sistemasida ixtiyoriy ikki vektor o‘zaro ortogonal bo‘lsa, bungay vektorlar sistemasi* ***ortogonal sistema*** *deb ataladi.*

**1-misol.** $\vec{a}(1;3;2;-1)$, $\vec{b}(5;1;-4;0)$, $\vec{c}(0;4;1;14)$ vektorlarning ortogonal sistemani hosil qilishini isbotlang.

**Yechish.** Yuqorida berilgan ta’rifdan foydalanib, $\vec{a}\vec{b}$, $\vec{a}\vec{c}$ va $\vec{b}\vec{c}$ larning skalyar ko‘paytmalarini hisoblaymiz:

$$\vec{a}\,\vec{b} = 1\cdot5 + 3\cdot1 + 2\cdot(-4) + (-1)\cdot0 = 8 - 8 = 0,$$
$$\vec{a}\,\vec{c} = 1\cdot0 + 3\cdot4 + 2\cdot1 + (-1)\cdot14 = 14 - 14 = 0,$$
$$\vec{b}\,\vec{c} = 5\cdot0 + 1\cdot4 + (-4)\cdot1 + 0\cdot14 = 4 - 4 = 0.$$

Demak, $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlar ortogonal sistema hosil qiladi.

**15.2. Affin almashtirishlar.**

Endi $n$ o‘lchamli affin fazodagi almashtirishlar bilan tanishamiz. $A_n$ vektor fazoda ikki $\mathcal{B} = (O, \vec{e}_1, \vec{e}_2, \dots, \vec{e}_n)$ va $\mathcal{B}' = \left(O', \overrightarrow{e_1'}, \overrightarrow{e_2'}, \dots, \overrightarrow{e_n'}\right)$ reper berilgan bo‘lsin. Bu reperlar yordamida $A_n$ ning nuqtalari orasida shunday $f$ moslik o‘rnatamizki, ixtiyoriy $M \in A_n$ nuqta $\mathcal{B}$ reperda qanday koordinatalarga ega bo‘lsa, uning obrazi $M' = f(M)$ nuqta $\mathcal{B}'$ reperda xuddi shunday koordinatalarga ega bo‘lsin, ravshanki, bu moslik o‘zaro bir qiymatli bo‘lib, $A_n$ ni o‘zini – o‘ziga o‘tkazadi. Demak, $f$ biror almashtirishdir.

***Ta’rif.*** *Yuqoridagicha aniqlangan $f$ almashtirish $A_n$ ni affin almashtirish deb ataladi.*

Bu ta’rifdan ko‘rinadiki, affin almashtirish bir juft affin reperlarning berilishi bilan to‘la aniqlanadi.

Endi affin almashtirishning ba’zi xossalari bilan tanishamiz.

$1^0$. $f$ affin almashtirishda $\vec{a} \in A_n$ vektor shu fazoning biror $f(\vec{a}) = \overrightarrow{a'}$ yoki $\vec{b}$ vektorga teng vektoriga almashadi, $\vec{a} = \overrightarrow{MN}$ desak, $M, N$ nuqtalarning obrazlari $f(M) = M'$, $f(N) = N'$ bo‘lib, bu nuqtalar ham $A_n$ ga tegishli bo‘lgani uchun ularga mos kelgan $\overrightarrow{a'}$ vektor $f(\vec{a})$ bo‘ladi. Xususiy holda nol vektor yana nol vektorga almashadi.

$2^0$. $f$ affin almashtirishda $\vec{a}$ vektorning koordinatalari $\mathcal{B}$ da qanday bo‘lsa, unga mos kelgan $\overrightarrow{a'}$ vektorning ham koordinatalari $\mathcal{B}'$ da xuddi shu sonlardan iborat bo‘ladi. Bu xossa $f$ ning ta’rifi va $1^0$ dan bevosita kelib chiqadi.

$3^0$. $f$ affin almashtirishda ikki vektorning yig‘indisiga mos kelgan vektor qo‘shiluvchi vektorlarga mos kelgan vektorlar yig‘indisidan iborat, ya’ni $\vec{a} + \vec{b} = \vec{c} \Rightarrow f(\vec{c}) = f(\vec{a}) + f(\vec{b})$.

Bu xossaning isboti koordinatalari bilan berilgan vektorlarni qo‘shish qoidasi va $f$ ning ta’rifidan kelib chiqadi.

$4^0$. $k\vec{a}$ vektorga mos kelgan vektor $kf(\vec{a}) = k\overrightarrow{a'}$ vektordir.

Bu ikki $3^0, 4^0 -$ xossadan $f$ almashtirishda

$\lambda_1\vec{a}_1+\lambda_2\vec{a}_2+.\dots+\lambda_k\vec{a}_k$ vektorga $\lambda_1\overrightarrow{a_1'}+\lambda_2\overrightarrow{a_2'}+\dots+\lambda_k\overrightarrow{a_k'}$ vektorning mos kelishi kelib chiqadi, ya'ni $f$ almashtirish natijasida vektorlarning chiziqli kombinatsiyasi saqlanadi. Demak, chiziqli erkli vektorga yana chiziqli erkli vektorlar mos keladi. Bu xossalarni va ikki affin fazoning izomorfligi ta'rifini e'tiborga olsak, affin almashtirishning quyidagi ikkinchi ta'rifi kelib chiqadi.

***Ta'rif.*** *$A_n$ fazoni o'zini – o'ziga izomorf akslantiruvchi $f$* almashtirish *$A_n$ dagi affin almashtirish deb ataladi.*

***Ta'rif.*** *$P$ nuqta $MN$ kesmani $\lambda$ nisbatda bo'lsa (ya'ni $\overrightarrow{MP}=$ $=\lambda\overrightarrow{PM}$ bo'lsa), u holda $\lambda$ son $M,P,N$ nuqtalarning oddiy nisbati deb atalib, uni odatdagidek $\lambda=(MN,P)$ ko'rinishda belgilanadi.*

$5^0$. $f$ almashtirishda $k$ o'lchovli $\Pi_k$ tekislik yana $k$ o'lchovli $\Pi_k$ tekislikka almashadi, ya'ni tekislikning o'lchovi $f$ uchun invariantdir.

$6^0$. $f$ affin almashtirishda parallel tekisliklar yana parallel tekisliklarga o'tadi.

Bu xossa affin almashtirishning o'zaro bir qiymatli ekanligidan kelib chiqadi. Endi affin almashtirishning koordinatalaridagi ifodasini ko'ramiz.

$A_n$ da $\mathcal{B}=(O,\vec{e}_1,\vec{e}_2,\dots,\vec{e}_n)$ reper berilgan bo'lsin. $\forall M\in E_n$ ni olib, uning shu reperdagi koordinatalarini $x_1,x_2,\dots,x_n$, $f(M)=M'$ nuqtaning ham shu $\mathcal{B}$ reperga nisbatan koordinatalarini $x_1',x_2',\dots,x_n'$ ni bog'lovchi munosabatlarni topamiz.

Faraz qilaylik, $f(O)=O'$, $f(\vec{e}_1)=\overrightarrow{e_1'},\dots,f(\vec{e}_n)=\overrightarrow{e_n'}$ bo'lib, bularning $\mathcal{B}$ ga nisbatan koordinatalari $O'(c_1,c_2,\dots,c_n)$, $\overrightarrow{e_1'}(c_{11},c_{12},\dots,c_{1n})$, $\overrightarrow{e_2'}(c_{21},c_{22},\dots,c_{2n})$, $\overrightarrow{e_n'}(c_{n1},c_{n2},\dots,c_{nn})$ bo'lsin. U holda $\mathcal{B}'=\left(O',\overrightarrow{e_1'},\overrightarrow{e_2'},\dots,\overrightarrow{e_n'}\right)$ affin reper hosil bo'lib, $f(\mathcal{B})=\mathcal{B}'$ bo'ladi. Xususiy holda $f\left(\overrightarrow{OM}\right)=\overrightarrow{O'M'}$. U holda $M$ nuqtaning koordinatalari affin almashtirishning ta'rifiga ko'ra $\mathcal{B}'$ reperga nisbatan $x_1,x_2,\dots,x_n$ bo'lib,

$$x_1'=c_{11}x_1+c_{21}x_2+\dots+c_{n1}x_n+c_1,$$
$$x_2'=c_{12}x_1+c_{22}x_2+\dots+c_{n2}x_n+c_2, \qquad (15.5)$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$x_n' = c_{1n}x_1 + c_{2n}x_2 + \dots + c_{nn}x_n + c_n$$
o‘rinli bo‘ladi. Bu (15.5) formulalar affin almashtirishning koordinatalardagi ifodasidir.

**2-Misol.** $M(1; 4; -5; 3; 2)$ nuqtadan $\Pi_4 = 3x_1 + x_2 + +2x_3 - x_4 + x_5 - 3 = 0$ gipertekislikkacha bo‘lgan masofani toping.

**Yechish.** $\rho(M_0\Pi_{n-1}) = \frac{|a_1x_1^0 + a_2x_2^0 + \dots + a_nx_n^0 + a_0|}{\sqrt{a_1^2 + a_2^2 + \dots + a_n^2}}$ formulaga asosan

$$\rho(M_0\Pi_4) = \frac{|3 \cdot 1 - 1 \cdot 4 + 2 \cdot (-5) + (-1) \cdot 3 + 2 \cdot 1 - 3|}{\sqrt{3^2 + 1^2 + 2^2 + (-1)^2 + 1^2}} =$$
$$= \frac{|-15|}{\sqrt{16}} = \frac{15}{4} = 3\frac{3}{4}.$$

### 15.3. Harakat.

$E_n$ da ikkita $\mathcal{B} = (O, \vec{e}_1, \vec{e}_2, \dots, \vec{e}_n)$, $\mathcal{B}' = \left(O', \overrightarrow{e_1'}, \overrightarrow{e_2'}, \dots, \overrightarrow{e_n'}\right)$ dekart reperi berilgan bo‘lsin. $E_n$ ning har bir $M$ nuqtasini shu fazoning shunday $M'$ nuqtasiga akslantiramizki, $\mathcal{B}$ reperda $M$ nuqta qanday koordinatalarga ega bo‘lsa, $\mathcal{B}'$ reperda $M'$ nuqta shunday koordinatalarga ega bo‘lsin. Bu yerda $E_n$ nuqtalari yana shu fazo nuqtalariga mos qo‘yilib, bunday moslik o‘zaro bir qiymatlidir. Demak, $E_n$ da almashtirish hosil qilindi, u $E_n$ ning harakati deb ataladi. $E_n$ dagi harakat ikkita dekart reperining berilishi bilan to‘liq aniqlanadi. Bu ta’rifni affin almashtirishning ta’rifi bilan taqqoslasak, harakat affin almashtirishning xususiy holi ekanligi ayon bo‘ladi. Shu sababli figuraning barcha affin xossalari harakatda saqlanib qoladi.

Harakat quyidagi xossalarga ega. Harakatda ikki nuqta orasidagi masofa saqlanadi. Haqiqatan, $\mathcal{B}$ reperdagi $M(x_1, x_2, \dots, x_n)$, $N(y_1, y_2, \dots, y_n)$ nuqtalarga harakat natijasida $\mathcal{B}'$ reperda mos kelgan $M', N'$ nuqtalar ta’rifga asosan xuddi shunday koordinatalarga ega, ya’ni $M'(x_1, x_2, \dots, x_n)$, $N'(y_1, y_2, \dots, y_n)$ u holda
$$\rho(M, N) = \rho(M', N').$$

Masofa harakatning asosiy invariant hisoblanib, ba’zan harakat shu invariant orqali ta’riflanadi.

***Teorema.*** *$E_n$ ning biror $f$ almashtirishida ikki nuqtasi orasidagi masofa saqlansa, bu almashtirish harakatdir.*

$E_n$ da ixtiyoriy uchta $O, A, B$ nuqtani olsak, yuqoridagi berilgan aksiomalarga asosan

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} \tag{15.6}$$

yoki

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}.$$

Bu tenglikning chap va o‘ng tomonida turgan vektorlarni o‘zini – o‘ziga skalyar ko‘paytiraylik:

$$\overrightarrow{AB} \cdot \overrightarrow{AB} = (\overrightarrow{OB} - \overrightarrow{OA})(\overrightarrow{OB} - \overrightarrow{OA}) \Rightarrow$$

$$\overrightarrow{AB^2} = \overrightarrow{OB^2} - 2\left(\overrightarrow{OB} \cdot \overrightarrow{OA}\right) + \overrightarrow{OA^2} \Rightarrow$$

$$2\left(\overrightarrow{OA} \cdot \overrightarrow{OB}\right) = \overrightarrow{OA^2} + \overrightarrow{OB^2} - \overrightarrow{AB^2}. \tag{15.7}$$

$f(O) = O',\ f(A) = A',\ f(B) = B'$ bo‘lsin, u holda $O', A', B'$ nuqtalar uchun ham aksiomani tadbiq qilib va (9.6) singari tenglik yozib, tegishlicha ixchamlasak,

$$2\left(\overrightarrow{OB'} \cdot \overrightarrow{OA'}\right) = \overrightarrow{O'A'}^2 + \overrightarrow{O'B'}^2 - \overrightarrow{A'B'}^2 \tag{15.8}$$

Lekin teorema shartiga ko‘ra $\rho(O, A) = \rho(O', A'),\ \rho(O, B) = = \rho(O', B'), \rho(A, B) = \rho(A', B')$ bo‘lgani uchun (15.7) bilan (15.8) ning o‘ng tomonlarini taqqoslasak, ular o‘zaro tengdir, demak, chap tomonlari ham teng bo‘ladi:

$$\left(\overrightarrow{O'A'} \cdot \overrightarrow{O'B'}\right) = \left(\overrightarrow{OA} \cdot \overrightarrow{OB}\right). \tag{15.9}$$

$E_n$ da biror $\mathcal{B} = (O, \vec{e}_1, \vec{e}_2, \dots, \vec{e}_n)$ dekart reperini olaylik, u holda $\overrightarrow{OA_1} = \overrightarrow{e_1},\ \overrightarrow{OA_2} = \overrightarrow{e_2}, \dots,\ \overrightarrow{OA_n} = \overrightarrow{e_n}$ desak, $\mathcal{B}$ reperni quyidagicha yozish mumkin: $\mathcal{B} = (O, A_1, A_2, \dots, A_n)$. Shu reperni $f$ bo‘yicha almashtirsak, $f(O) = O',\ f(A_1) = A_1', \dots,\ (A_n) = A_n'$ bo‘lgani uchun bu nuqtalar sistemasi ham biror $\mathcal{B}' = (O', A_1',, \dots, A_n')$ reperni aniqlaydi. Bu reper ham dekart reperidan iboratdir, chunki:

1) almashtirishga asosan $\rho(O, A_i) = \rho(O', A_i'),\quad (i = 1,2, \dots, n)$ ya'ni birlik vektor obrazi yana birlik vektordir;

2) (15.9) shartga asosan o'zaro perpendikulyar vektorlar yana perpendikulyar vektorga o'tadi.

$E_n$ dagi ixtiyoriy $M$ nuqtani olaylik, uning $\mathcal{B}$ dekart reperidagi koordinatalari $x_1, x_2, \dots, x_n$ bo'lsin. $M$ nuqtaga $f$ almashtirishda mos kelgan $M'$ nuqtaning shu reperdagi koordinatalari $y_1, y_2, \dots, y_n$ bo'lsin. U holda

$$\overrightarrow{OM} \cdot \overrightarrow{OA_1} = \left|\overrightarrow{OM}\right| \cdot \left|\overrightarrow{OA_1}\right| cos\varphi = \left|\overrightarrow{OM}\right| \cdot \left|\overrightarrow{e_1}\right| cos\varphi =$$
$$= \left|\overrightarrow{OM}\right| cos\varphi = \left|\overrightarrow{OM_1}\right| = x_1$$

(bunda $\varphi = \left(\overrightarrow{OM}, ^\wedge \overrightarrow{OA_1}\right)$, $\overrightarrow{OM_1}$ vektor $\overrightarrow{OM}$ ning $Ox$ o'qdagi proyeksiyasi) bo'lgani uchun

$$x_1 = \overrightarrow{OM} \cdot \overrightarrow{OA_1} \tag{15.10}$$

shunga o'xshash

$$\begin{cases} x_2 = \overrightarrow{OM} \cdot \overrightarrow{OA_2}, \\ x_3 = \overrightarrow{OM} \cdot \overrightarrow{OA_3}, \\ \dots \dots \dots \dots \dots \dots \dots \\ x_n = \overrightarrow{OM} \cdot \overrightarrow{OA_n}, \\ y_1 = \overrightarrow{O'M'} \cdot \overrightarrow{O'A_1{}'}, \\ y_2 = \overrightarrow{O'M'} \cdot \overrightarrow{O'A_2{}'}, \\ \dots \dots \dots \dots \dots \dots \dots \\ y_n = \overrightarrow{O'M'} \cdot \overrightarrow{O'A_n{}'} \end{cases} \tag{15.11}$$

$$(15.9) - (15.11) \Rightarrow \begin{cases} x_1 = y_1, \\ x_2 = y_2, \\ \dots \dots \dots \\ x_n = y_n \end{cases} \Rightarrow f \text{ harakatdan iborat.}$$

Ortogonal matritsa deyiladi qachonki, uning determinant $\pm 1$ ga teng, ya'ni (15.5) ning determinanti $\Delta = \pm 1$. Agar harakatning analitik ifodasida $\Delta = \pm 1$ bo'lsa, bunday harakat, ***birinchi tur harakat***, deb ataladi, bu tur harakatda ikkita mos reper bir xil oreyentatsiyali bo'ladi. $\Delta = -1$ holda bunday harakat ***ikkinchi tur harakat*** deyilib, undagi mos reperlar har xil oriyentatsiyali.

$E_n$ ning barcha harakatlari to‘plamini $E$ bilan belgilaylik hamda $\forall f, g \in E$ ni olaylik; harakatda ikki nuqta orasidagi masofa o‘zgarmaganligi uchun ketma – ket bajarilgan ikki $f, g$ harakat natijasida ham ikki nuqta orasidagi masofa o‘zgarmaydi, demak, $g \cdot f$ “ko‘paytma” harakat bo‘lib, $E$ ga tegishlidir. $f$ da ikki nuqta orasidagi masofa o‘zgarmagani uchun unga teskari $f^{-1}$ da ham masofa o‘zgarmaydi, demak, $f^{-1} \in E$. Xullas, $E_n$ ning barcha harakatlari to‘plami $E$ gruppa hosil qiladi, u $E_n$ ning ***harakatlar gruppasi*** deb ataladi. Harakat affin almashtirishning xususiy holi ekanligidan harakatlar gruppasi affin gruppaning qism gruppasi bo‘ladi. Demak, $A$ ning barcha invariantlari $E$ uchun ham invariant bo‘ladi, lekin buning teskarisi doimo to‘g‘ri bo‘lavermaydi; masalan, $E$ ning invariantlaridan biri ikki nuqta orasidagi masofadir, bu esa $A$ da invariant emas, shu nuqtai nazardan $E_n$ dagi figura geometrik xossalar nuqtai nazardan $A_n$ dagi figuraga nisbatan boyroqdir.

Endi Yevklid geometriyasiga quyidagicha ta’rif berish mumkin. Yevklid geometriyasi geometriyaning harakat natijasida figuraning o‘zgarmay qoladigan xossalarini o‘rganadigan bir bo‘limidir. O‘rta maktab geometriya kursida ikki va uch o‘lchovli $(E_2, E_3)$ yevklid fazolari geometriyasi o‘rganiladi.

$n$ o‘lchovli $(n > 3)$ yevklid geometriyasida ham o‘rta maktab geometriya kursida qaraladigan ba’zi tushunchalarni umumlashtirish mumkin. Masalan, kongruentlik tushunchasi $E_n$ da quyidagicha kiritiladi: $F, F'$ figuralardan birini ikkinchisiga o‘tkazuvchi harakat mavjud bo‘lsa, bu figuralar ***kongruent*** deb ataladi, yoki oddiy sferani umumlashtirib, $E_n$ da gipersfera kiritiladi: $E_n$ ning markaz deb atalgan $C$ nuqtadan berilgan $r$ masofada yotgan barcha nuqtalari to‘plami ***gipersfera*** deb ataladi.

Endi harakatlar gruppasining ba’zi qism gruppalari bilan tanishaylik.

1. I turdagi barcha harakatlar to‘plamini $E_1$ deb belgilasak, bu to‘plam gruppani hosil qiladi, chunki 1) $E_1$ ning har bir almashtirishida

reper oriyentatsiyasi (demak, fazo oriyentatsiyasi) o‘zgarmaganligi uchun unga tegishli ikki harakatning kompozitsiyasi natijasida ham oriyentatsiya o‘zgarmaydi; 2) $E_1$ ning har bir harakatiga teskari harakat ham oriyentatsiyani o‘zgartirmaydi, demak, $E_1$ ham $E$ ning qism gruppasidir.

2. $E_1$ dagi barcha parallel ko‘chirishlar to‘plamini olaylik. Avvalo parallel ko‘chirishning harakat ekanligini isbotlaylik. $M, N$ nuqtalar $M', N'$ nuqtalarni $\vec{u}$ vektor bo‘yicha parallel ko‘chirishdan $\left(\overrightarrow{MM'} = \vec{u}, \overrightarrow{NN'} = \vec{u}\right)$ hosil qilingan bo‘lsa, $\left|\overrightarrow{MN}\right| = \left|\overrightarrow{M'N'}\right| \Longrightarrow \rho(M, N) = \rho(M', N')$. Demak, parallel ko‘chirish harakatdir. U holda bunday harakatlarning to‘plami ham $E$ ning qismidir.

3. $E$ ning shunday harakatlari to‘plamini qaraymizki, bu harakatlar natijasida $E$ ning biror $O$ nuqtasi o‘z – o‘ziga o‘tsin, bunday xossaga ega bo‘lgan harakatlarni $E_n$ ni $O$ nuqta atrofida ***burish*** deyiladi, bu to‘plamni $E_0$ deb belgilasak, $E_0$ ning gruppa hosil qilishini ko‘rsatish osondir (buni ko‘rsatishni o‘quvchiga havola qilamiz); demak, $E_0$ ham $E$ ning qism gruppasidir.

**3-misol.** $E = R^3$ Yevklid fazosida $\overrightarrow{b_1}(1; 0; 0)$, $\overrightarrow{b_2}(1; 1; 0)$, $\overrightarrow{b_3}(1; 1; 1)$ vektorlar sistemasiga ortogonallashtirish jarayonini qo‘llang.

**Yechish.** Ma’lumki, $R^n$ fazoda $n$ ta vektordan iborat sistemaning chiziqli erkli bo‘lishi uchun bu vektorlarning koordinatalaridan tuzilgan determinantning noldan farqli bo‘lishi zarur va yetarlidir. Berilgan vektorlar uchun bu determinant

$$\begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} = 1 \neq 0$$

bo‘lganligi sababli, ular chiziqli erklidir. Endi bu elementlarga ortogonallashtirish jarayonini qo‘llaymiz. $\overrightarrow{c_1} = \overrightarrow{b_1} = (1; 0; 0)$ deb olsak, $|\overrightarrow{c_1}| = \sqrt{1^2 + 0^2 + 0^2} = 1$ bo‘ladi. $\overrightarrow{c_2}$ elementni
$\overrightarrow{c_2} = \overrightarrow{b_2} - a_{21}\overrightarrow{c_1}$ ko‘rinishda olib, $a_{21}$ koeffitsiyentni $(\overrightarrow{c_2}, \overrightarrow{c_1}) = 0$ ortogonallik shartini qanoatlantiradigan qilib tanlaymiz:

$$0 = (\overrightarrow{c_2}, \overrightarrow{c_1}) = \left(\overrightarrow{b_2}, \overrightarrow{c_1}\right) - a_{21}(\overrightarrow{c_2}, \overrightarrow{c_1}) \quad \text{yoki}$$

$$a_{21} = \frac{\left(\overrightarrow{b_2}, \overrightarrow{c_1}\right)}{|\overrightarrow{c_1}|^2} = \frac{1}{1} = 1.$$

U holda

$$c_2 = (1;1;0) - (1;0;0) = (0;1;0), \quad |\overrightarrow{c_2}| = 1,$$

bo‘ladi. $\overrightarrow{c_3}$ vektorni quyidagi ko‘rinishda izlaymiz:

$$\overrightarrow{c_3} = \overrightarrow{b_3} - a_{31}\overrightarrow{c_1} - a_{32}\overrightarrow{c_2}. \tag{15.12}$$

Bunda $a_{31}, a_{32}$ koeffitsiyentlar, ortogonallik shartlaridan, ya’ni

$$(\overrightarrow{c_3}, \overrightarrow{c_1}) = (\overrightarrow{c_3}, \overrightarrow{c_2}) = 0 \tag{15.13}$$

shartlardan topiladi. Buning uchun (15.12) ni $\overrightarrow{c_1}$ va $\overrightarrow{c_2}$ ga skalyar ko‘paytirib, (15.13) shartlardan foydalansak, $a_{31}, a_{32}$ koeffitsiyentlarga nisbatan chiziqli tenglamalar sistemasi hosil bo‘ladi. Bu tenglamaning yechimi:

$$a_{31} = \frac{\left(\overrightarrow{b_3}, \overrightarrow{c_1}\right)}{|\overrightarrow{c_1}|^2} = \frac{1 \cdot 1 + 1 \cdot 0 + 0 \cdot 1}{1} = 1,$$

$$a_{32} = \frac{\left(\overrightarrow{b_3}, \overrightarrow{c_2}\right)}{|\overrightarrow{c_2}|^2} = \frac{1}{1} = 1.$$

Demak, $\overrightarrow{c_3} = (1;1;1) - (1;0;0) - (0;1;0) = (0;0;1), \quad |\overrightarrow{c_3}| = 1.$

Hosil bo‘lgan $\overrightarrow{c_1}$, $\overrightarrow{c_2}$, $\overrightarrow{c_3}$ vektorlar sistemasi ortonormaldir.

## Mustaqil yechish uchun topshiriqlar.

**15.1.1 – 15.1.9** - misollarda keltirilgan vektorlarning chiziqli erkliligini tekshiring va ortogonallashtirish jarayonini qo‘llab, ortonormal sistema hosil qiling.

**15.1.1.** $E = \mathbb{R}^2, \ \vec{x}(0;1), \ \ \vec{y}(1;0).$

**15.1.2.** $E = \mathbb{R}^2, \ \vec{x}(1;0), \ \ \vec{y}(0;1).$

**15.1.3.** $E = \mathbb{R}^3, \ \vec{x}(0;0;1), \ \vec{y}(0;1;1), \ \vec{z}(1;1;1).$

**15.1.4.** $E = \mathbb{R}^3, \ \vec{x}(1;1;1), \ \vec{y}(0;1;1), \ \vec{z}(0;0;1).$

**15.1.5.** $E = \mathbb{R}^3, \ \vec{x}(1;1;0), \ \vec{y}(2;0;-1), \ \vec{z}(0;-1;1).$

**15.1.6.** $E = \mathbb{R}^3, \ \vec{x}(0;2;-1), \ \vec{y}(1;-1;1), \ \vec{z}(1;0;0).$

**15.1.7.** $E = \mathbb{R}^3, \ \vec{x}(-1;0;0), \ \vec{y}(0;-1;1), \ \vec{z}(2;0;-1).$

**15.1.8.** $E=\mathbb{R}^4$, $\vec{x}(0;1;-1;1)$, $\vec{y}(1;-1;1;0)$, $\vec{z}(1;0;0;1)$.
**15.1.9.** $E=\mathbb{R}^4$, $\vec{x}(-1;0;0;1)$, $\vec{y}(0;-1;1;0)$, $\vec{z}(2;0;-1;1)$.
**15.1.10.** Chiziqli fazoda mos ravishda $\vec{a}_1(1;1;0;0)$, $\vec{a}_2(0;1;1;0)$, $\vec{a}_3(0;0;1;1)$ va $\vec{b}_1(1;0;1;0)$, $\vec{b}_2(0;2;1;1)$, $\vec{b}_3(1;2;1;2)$ bazislarga ega $V_1$ va $V_2$ qism fazolar yig‘indisi va kesishmasining bazisini toping.
**15.1.11.** Chiziqli fazoda mos ravishda $\vec{a}_1(1;2;0;1)$, $\vec{a}_2(1;1;1;0)$ va $\vec{b}_1(1;0;1;0)$, $\vec{b}_2(1;3;0;1)$ bazislarga ega $V_1$ va $V_2$ qism fazolar yig‘indisi va kesishmasining bazisini toping.
**15.1.12.** To‘g‘ri chiziq va gipertekislik mos ravishda $x_1=8\mathrm{t}$, $x_2=4\mathrm{t}$, $x_3=3\mathrm{t}$, $x_4=-3\mathrm{t}$ va $2x_1+2x_2-x_3+x_4=0$ tenglamalar bilan berilgan. Berilgan $\vec{x}(1;2;3;4)$ vektorni to‘g‘ri chiziqqa tegishli $\vec{y}$ vektor va gipertekislikka tegishli $\vec{z}$ vektorlarning yig‘indisi ко‘rinishida ifodalang.
**15.1.13.** Tekislikning $(-1;1;0;1;5)$, $(2;-1;3;4;0)$, $(1;2;7;6;1)$ nuqtalardan o‘tishi ma’lum bo‘lsa, uning parametrik va umumiy tenglamalari tuzilsin.
**15.1.14.** Umumiy tenglamasi bilan berilgan
$$\begin{cases}5x_1+6x_2-2x_3+7x_4+4x_5-3=0\\ 2x_1+3x_2-x_3+4x_4+2x_5-6=0\end{cases}$$
tekislikning parametrik tenglamasini yozing.
**15.1.15.** Birinchi to‘g‘ri chiziq $(1;0;-2;1)$ nuqta va $\vec{a}(1;2;-1;-3)$ vektor bilan, ikkinchi tekislik esa $(0;1;1;-1)$ nuqta va $\vec{b}(2;3;-2;-4)$ vektor bilan aniqlangan bo‘lsa, ularni o‘z ichiga oluvchi eng kichik o‘lchamli tekislik tenglamasini yozing.
**15.1.16.** Ikkita $x_1=1+\mathrm{t}$, $x_2=2+\mathrm{t}$, $x_3=3+\mathrm{t}$, $x_4=4+\mathrm{t}$, $x_1=0$, $x_4-3=0$, $x_2-x_3+1=0$ to‘g‘ri chiziq o‘zichiga oluvchi eng kichik o‘lchamli tekislik tenglamasini yozing.
**15.1.17.** To‘rt o‘lchamli fazoda
$$\begin{cases}x_1+2x_2+3x_3+x_4-3=0\\ x_1+4x_2+5x_3+2x_4-2=0\\ 2x_1+9x_2+8x_3+3x_4-7=0\end{cases}$$

sistema bilan berilgan tekislik va $5x_1 + 7x_2 + 9x_3 + 2x_4 - 20 = 0$ to‘g‘ri chiziqning o‘zaro vaziyatini aniqlang.

**15.1.18.** To‘rt o‘lchamli fazoda $5x_1 + 9x_3 + 2x_4 - 20 = 0,\ \ x_2 = 0$ tekislik va

$$\begin{cases} x_1 + 2x_2 + 3x_3 + x_4 - 3 = 0 \\ x_1 + 4x_2 + 5x_3 + 2x_4 - 2 = 0 \\ 2x_1 + 9x_2 + 8x_3 + 3x_4 - 7 = 0 \end{cases}$$

to‘g‘ri chiziq berilgan. Ularning o‘zaro vaziyatini aniqlang.

**15.1.19**. To‘g‘ri chiziq va tekislik mos ravishda $x_1 = 1 + \mathrm{t},\ \ x_2 = 2 + 2\mathrm{t},\ \ x_3 = 3 + 3\mathrm{t},\ x_4 = 4 + 4\mathrm{t}$ va $x_1 + x_2 + 1 = 0,\ x_3 - x_4 = 0$ tenglamalar bilan berilgan. Ularning kesishmasligini ko‘rsating va to‘g‘ri chiziqqa parallel bo‘lib berilgan tekislikdan o‘tuvchi eng kichik o‘lchamli tekislik tenglamasini yozing.

**15.1.20.** Ortonormal bazisga nisbatan uchta $\vec{a}(1;2;2;1)$, $\vec{b}(1;1;-5;3)$, $\vec{c}(3;2;8;-7)$ vektorlar berilgan. Berilgan vektorlarga tortilgan qism fazoning bazisini toping va uni fazoning bazisigacha to‘ldiring.

**15.1.21.** Besh o‘lchamli fazoda ortonormal bazisga nisbatan $x_1 - x_2 - 2x_3 + 4 = 0$ gipertekislik berilgan. Birinchi to‘rttasi berilgan gipertekislikda yotuvchi yangi bazisni toping.

**15.1.22.** Gipertekislik $2x_1 - 2x_2 - x_3 + x_4 = 0$ va $\vec{x}(2;0;4;6)$ vektor berilgan. $\vec{x}$ vektorni berilgan gipertekislikka tegishli $\vec{y}$ vektor va shu gipertekislikka orthogonal $\vec{z}$ vektorlarning yig‘indisi ko‘rinishida ifodalang.

**15.1.23.** Yevklid fazosi $V$ da $\overrightarrow{x_1}, \overrightarrow{x_2}$ vektorlar, $V$ ning qism fazosi $V'$ da $\overrightarrow{y_1}, \overrightarrow{y_2}$ vektorlar va $V'$ ga ortogonal bo‘lgan $\overrightarrow{z_1}, \overrightarrow{z_2}$ vektorlar berilgan. Agar, $\overrightarrow{x_2} - \overrightarrow{x_1}\ \ V'$ fazoga tegishli bo‘lsa, $\overrightarrow{z_1} = \overrightarrow{z_2}$ munosabat o‘rinli bo‘lishini isbotlang.

**15.1.24.** O‘zining $M(x_0, y_0)$ bazislari bilan berilgan qism fazoga $\vec{a}(4;-1;3;4)$ vektorning ortogonal proeksiyasini toping.

**15.1.25.** Tenglamalar sistemasi

$$\begin{cases} 2x_1 + x_2 + x_3 + 3x_4 = 0 \\ 3x_1 + 2x_2 + 2x_3 + x_4 = 0 \end{cases}$$

bilan berilgan qism fazoga $\vec{a}(7;-4;-1;2)$ vektorning ortogonal proyeksiyasini toping.

**15.1.26.** To‘rt o‘lchamli fazoda $x_1 = x_2,\ x_3 = x_4,\ x_2 = 2x_3$ to‘g‘ri chiziq va $3x_1 - 2x_2 + x_4 = 0,\ x_2 + x_3 = 0$ tekislik orasidagi burchak topilsin.

**15.1.27.** To‘rt o‘lchamli yevklid fazosida $\vec{a}(1;1;1;1)$, $\vec{b}(1;-1;1;-1)$ vektorlarga hamda $\vec{c}(2;2;1;0)$, $\vec{d}(1;-2;2;0)$ vektorlarga qurilgan qism fazolar orasidagi burchak topilsin.

**15.1.28.** Berilgan $M(5;1;0;8)$ nuqtadan $A(1;2;3;4)$, $B(2;3;4;5)$, $C(2;2;3;7)$ nuqtalardan o‘tuvchi tekislikka tushirilgan perpendikulyarning uzunligi va asosi topilsin.

**15.1.29.** Berilgan $M(4;2;-5;1)$ nuqtadan

$$\begin{cases} 2x_1 - 2x_2 + x_3 + 2x_4 = 9 \\ 2x_1 - 4x_2 + 2x_3 + 3x_4 = 12 \end{cases}$$

tekislikka tushirilgan perpendikulyarning uzunligi va asosi topilsin.

**15.1.30.** Berilgan $A(1;1;1;1)$, $B(2;2;0;0)$, $C(1;2;0;1)$ nuqtalardan o‘tuvchi tekislik va $D(1;1;1;2)$, $E(1;1;2;1)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqning o‘zaro vaziyatini aniqlang, ularning umumiy perpendikulyarining tenglamasini yozing va uzunligini toping.

# SINOV TESTI

1. $A(x;0;0)$ nuqta $B(1;2;3)$ va $C(-1;3;4)$ nuqtalardan teng uzoqlikdaligi ma'lum bo'lsa, $x$ ni toping.

A) $-1$ B) $-2$ C) $-3$ D) $3$

2. $M(1;-2)$ va $N(-2;-6)$ nuqtalar orasidagi masofaning yarmini toping.

A) $3{,}5$ B) $5$ C) $4{,}5$ D) $2{,}5$

3.Agar $A(1;0)$, $B(1;3)$ va $C(4;3)$ bo'lsa $ABC$ uchburchakning turi qanday bo'ladi?

A) teng yonli

B) to'g'ri burchakli

C) teng yonli to'g'ri burchakli

D) teng tomonli

4. Uchlari $A(3;2)$ va $B(-4;1)$ nuqtalarda bo'lgan $AB$ kesma o'rtasining koordinatalarini toping.

A) $(-0{,}5;1{,}5)$ B) $(1{,}5;-0{,}5)$ C) $(1{,}5;0{,}5)$ D) $(0{,}5;-1{,}5)$

5. $ABCD$ parallelogram $C(5;8)$ uchining koordinatalari, $O(4;5)$ esa parallelogram diagonallarining kesishish nuqtasi. Parallelogramm $A$ uchining koordinatalarini toping.

A) $(2;3)$ B) $(3;2)$ C) $(1;4)$ D) $(4;1)$

6. Uchburchakning koordinatalari $A(1;2)$, $B(3;4)$ va $C(5;-1)$ nuqtalarda joylashgan. Shu uchburchak medianalarining kesishgan nuqtasi koordinatalarini toping.

A) $(2;3)$ B) $(3;2)$ C) $(3;3)$ D) $(3;5/3)$

7. Uchlari $A(4;5;1)$, $B(2;3;0)$ va $C(2;1;-1)$ nuqtalarda joylashgan uchburchakning $BD$ medianasi uzunligini toping.

A) $1$ B) $2$ C) $10$ D) $3$

8. $\vec{a}(2;-5)$ vektorning $\vec{\imath}$ va $\vec{\jmath}$ ortlar bo'yicha yoyilmasi to'g'ri ko'rsatilgan javobni toping.

A) $\vec{a} = 2\vec{\imath} - 5\vec{\jmath}$

B) $\vec{a} = -5\vec{\imath} + 2\vec{\jmath}$

C) $\vec{a} = -2\vec{i} + 5\vec{j}$

D) $\vec{a} = 5\vec{i} - 2\vec{j}$

9. $\vec{a}(0; -4)$ va $\vec{b}(-2; 2)$ vektorlar berilgan. Agar $\vec{b} = 3\vec{a} - \vec{c}$ bo‘lsa, $\vec{c}$ vektorning koordinatalarini toping.

A) $(2; -14)$ B) $(3; -6)$ C) $(-2; 10)$ D) $(-2; -10)$

10. $\vec{a}(1; -2; 3)$ vektorning oxiri $B(2; 0; 4)$ nuqta bo‘lsa, bu vektorning boshini toping.

A) $(1; 2; 1)$

B) $(-1; 2; 1)$

C) $(1; -2; 1)$

D) $(1; 2; -1)$

11. $\vec{a} = 2\vec{i} + 3\vec{j}$ va $\vec{b} = 2\vec{j}$ bo‘lsa, $\vec{p} = 2\vec{a} - 3\vec{b}$ vektorlarning koordinatalarini ko‘rsating.

A) $(-4; 12)$ B) $(-4; 0)$ C) $(4; 0)$ D) $(2; -6)$

12. To‘rtburchakning uchi $M(2; 4)$, $N(-4; 0)$ va $P(2; -2)$ uchlari berilgan. Agar $\overrightarrow{MN} = 4\overrightarrow{QP}$ bo‘lsa, $Q$ uchining koordinatalarini toping.

A) $(-7; -2)$ B) $(3{,}5; -1)$ C) $(7; -1)$ D) $(3{,}5; 2)$

13. $\vec{m}(-1; 2)$, $\vec{p}(4; -2)$, va $\vec{n}(2; -3)$ vektorlar berilgan. $\vec{a} = \vec{m} + 2\vec{n}$ vektorni $\vec{m}$ va $\vec{p}$ vektorlar orqali ifodalang.

A) $-\frac{5}{3}\vec{m} + \frac{1}{3}\vec{p}$

B) $-\vec{m} + 2\vec{p}$

C) $3\vec{m} - 4\vec{p}$

D) $2\vec{m} + \vec{p}$

14. Agar $A(-5; 2; 8)$ nuqta va $\overrightarrow{AB}(-3; 4; 1)$, $\overrightarrow{BD}(-2; 4; 1)$ vektorlar berilgan bo‘lsa, $ABCD$ parallelogram $C$ uchining koordinatalari yig‘indisini toping.

A) 8 B) 10 C) 11 D) 12

15. $\vec{a}(1; 4/3)$ vektor berilgan. $3\vec{a}$ vektorning modulini toping.

A) 4,5 B) 3,5 C) 5 D) 5,5

16. $\vec{a}(1; 2; 3)$ va $\vec{b}(4; -2; 9)$ bo‘lsa, $\vec{c} = \vec{a} + \vec{b}$ vektorning uzunligini toping.

A) 5,5    B) 4    C) 13    D) 8

17. $\vec{a}(-2;6;3)$ vektorga yo‘nalishdosh bo‘lgan birlik vektorning koordinatalarini toping.

A) $\left(\frac{2}{7};\frac{6}{7};\frac{3}{7}\right)$

B) $(-1;-3;-1)$

C) $\left(-\frac{1}{3};1;\frac{1}{2}\right)$

D) $\left(-\frac{2}{7};\frac{6}{7};\frac{3}{7}\right)$

18. $\vec{a}(3;1)$ va $\vec{b}(1;3)$ vektorlarga qurilgan parallelogram diagonallarining uzunliklari yig‘indisini toping.

A) $2\sqrt{2}$    B) 6    C) $6\sqrt{2}$    D) 8

19. $|\vec{a}| = \sqrt{137}$, $|\vec{a}+\vec{b}| = 20$ va $|\vec{a}-\vec{b}| = 18$ bo‘lsa, $|\vec{b}|$ ni toping.

A) $11\sqrt{6}$    B) 15    C) 12    D) 8

20. $\vec{a}(4;-12;z)$ vektorning moduli 13 ga teng bo‘lsa, *z* ning qiymatini toping.

A) 3    B) 4    C) $-3$    D) $\pm 3$

21.Absissa o‘qiga nisbatan $M(-3;5)$ nuqtaga simmetrik bo‘lgan nuqtani toping.

A)$(-3;-5)$    B)$(3;5)$    C)$(3;-5)$    D)$(-3;5)$

22. Uchlari $A(2;4)$, $B(-3;-2)$, $C(-3;4)$ va $D(2;-2)$ nuqtalarda bo‘lgan to‘g‘ri to‘rtburchakni perimetrini toping.

A) 10    B) 23    C) 40    D) 22

23.Ordinata o‘qiga nisbatan $M(-4;-9)$ nuqtaga simmetrik bo‘lgan nuqtani toping.

A)$(4;9)$    B)$(4;-9)$    C)$(-4;-9)$    D)$(-4;9)$

24. Uchlari $A(1;1)$, $B(-2;1)$ va $C(1;7)$ nuqtalarda bo‘lgan uchburchakning yuzini toping.

A) 9    B) 18    C) 8    D) 5

25. $ABCD$ parallelogrammda $A(1;3)$ va $C(-5;7)$ bo‘lsa, uning digonallar kesishgan nuqtasi koordinatasini toping.

A)$(5;2)$    B)$(-4;3)$    C)$(-2;5)$    D)$(2;1)$

26. $ABCD$ rombda $B(4;3)$ uchi va $O(2;1)$ diagonallar kesishgan nuqtasi koordinatasi bo‘lsa, uning $D(x;y)$ uchi koordinatasini toping.
A)$(1;2)$ B)$(0;-1)$ C)$(2;2)$ D)$(1;3)$
27. Agar $|\vec{a}|=4$, $|\vec{b}|=5$ va $\varphi=30^0$ bo‘lsa, $|\vec{a}\vec{b}|=$?
A) $20$ B) $10$ C) $10\sqrt{3}$ D) $41$
28. Koordinatalari bilan berilgan $\vec{a}(2;-3;1), \vec{b}(1;0;4)$ va $\vec{c}(5;-2;0)$ vektorlarning aralash ko‘paytmasi $\vec{a}\vec{b}\vec{c}$ hisoblansin.
A) $0$ B) $23$ C) $-46$ D) $-23$
29. $m$ parametrning qanday qiymatlarida $\vec{a}(2;0;1)$, $\vec{a}(1;1;m)$, va $\vec{c}(-1;3m;1)$ vektorlar komplanar bo‘ladi?
A) $1$ va $-0{,}5$
B) $1$ va $-1$
C) $0{,}5$ va $1$
D) $0{,}5$ va $-1$
30. $\vec{a}(2;4;1)$ va $\vec{b}(-1;1;3)$ vektorlarning vektor ko‘paytmasini toping.
A)$(11;-7;6)$
B)$(11;3;8)$
C)$(13;7;6)$
D)$(14;7;1)$
31. $\vec{a}(-2;1;3)$ va $\vec{b}(0;1;2)$ vektorlarning vektor ko‘paytmasini toping.
A)$(-3;7;-6)$
B)$(11;-3;8)$
C)$(-1;4;-2)$
D)$(14;-7;-1)$
32. $\vec{a}(1;2;1)$ va $\vec{b}(1;-1;3)$ vektorlarning vektor ko‘paytmasini toping.
A)$(-1;-7;6)$
B)$(7;-2;-3)$
C)$(-3;7;-6)$

D)$(-4;-7;1)$

33. $\vec{a}(1;0;4)$ va $\vec{b}(3;-2;4)$ vektorlarning skalyar ko‘paytmasini toping.

A) $19$ B) $15$ C)$13$ D)$14$

34. $\vec{a}(3;-1;2)$ va $\vec{b}(3;1;0)$ vektorlarning skalyar ko‘paytmasini toping.

A) $19$ B) $8$ C) $5$ D) $6$

35. $\begin{cases}2x+3y=5\\ x-y=0\end{cases}$ tenglamalar sistemasini yeching.

A)$(3;2)$ B)$(1;-1)$ C)$(1;1)$ D)$(3;3)$

36.$\begin{cases}2x+y=4\\ x-2y=7\end{cases}$ tenglamalar sistemasini yeching.

A)$(3;-2)$ B)$(7;-1)$ C)$(1;1)$ D)$(3;3)$

37. $\vec{a}=8\vec{\imath}+\vec{\jmath}-4\vec{k}$ va $\vec{b}=-\vec{\imath}-2\vec{k}$ vektorlar orasidagi burchakni aniqlang.

A) $90^0$ B) $30^0$ C) $0^0$ D) $60^0$

38. $\vec{a}(1;2)$ va $\vec{b}(2;1)$ vektorlar orasidagi burchak sinusini toping.

A) $1/5$ B) $3/5$ C) $4/7$ D) $2/3$

39. $\vec{a}(1;6;-4)$, $\vec{b}(-3;2;7)$ va $\vec{c}(-5;-6;2)$vektorlar berilgan bo‘lsa $[\vec{a}\,\vec{b}]\vec{c}$ aralash ko‘paytmani toping.

A) $-240$ B) $240$ C) $244$ D) $144$

40. $\vec{a}(1;6;-4)$, $\vec{b}(-3;2;7)$ va $\vec{c}(-5;-6;2)$ vektorlar berilgan bo‘lsa $[\vec{a}\,\vec{c}]\vec{b}$ aralash ko’paytmani toping.

A) $156$ B) $240$ C) $-240$ D) $144$

41. $\vec{a}$ va $\vec{b}$ vektorlar o‘zaro $\varphi=\frac{2\pi}{3}$ burchak tashkil qiladi. $|\vec{a}|=3$ va $|\vec{b}|=4$ bo‘lsa, $\vec{a}\vec{b}$ ni toping.

A) $4$ B) $6$ C)$-6$ D)$-9$

42. $\vec{a}(5;2)$, $\vec{b}(7;-3)$ vektorlar berilgan. Bir vaqtning o‘zida ikkita $\vec{a}\vec{x}=38$, $\vec{b}\vec{x}=30$ tenglamani qanoatlantiradigan $\vec{x}$ vektor topilsin.

A)$(-6;4)$ B)$(6;4)$ C)$(-4;6)$ D)$(6;-4)$

43. Tomonlari birga teng bo‘lgan teng tomonli $ABC$ uchburchak berilgan. $\overrightarrow{BC} = \vec{a}$, $\overrightarrow{CA} = \vec{b}$, $\overrightarrow{AB} = \vec{c}$ deb $\vec{a}\,\vec{b} + \vec{b}\,\vec{c} + \vec{a}\,\vec{c}$ ifoda hisoblansin.

A) $-\frac{1}{3}$ B) $\frac{2}{3}$ C) $\frac{3}{2}$ D) $-\frac{3}{2}$

44. $\vec{a} = \alpha\vec{\imath} - 3\vec{\jmath} + 2\vec{k}$ va $\vec{b} = \vec{\imath} + 2\vec{\jmath} - \alpha\vec{k}$ vektorlar $\alpha$ ning qanday qiymatida o‘zaro perpendikulyar bo‘ladi?

A) $-6$ B) $3$ C) $-3$ D) $2$

45. $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlar bir-birlari bilan $60^0$ ga teng bo‘lgan burchak tashkil qilsa, hamda $|\vec{a}| = 4$, $|\vec{b}| = 2$ va $|\vec{c}| = 6$ berilgan bo‘lsa, $\vec{p} = \vec{a} + \vec{b} + \vec{c}$ vektorning modulini aniqlang.

A) $12$ B) $10$ C) $9$ D) $11$

46. Determinantni hisoblang. $\begin{vmatrix} \sqrt[4]{5} & \sqrt{2} \\ \sqrt{2} & \sqrt[4]{125} \end{vmatrix}$

A) $3$ B) $4$ C) $-3$ D) $2$

47. Determinantni hisoblang. $\begin{vmatrix} 3 & x+1 \\ -4 & -21 \end{vmatrix} = 1$

A) $17$ B) $14$ C) $15$ D) $-17$

48. Determinantni hisoblang. $\begin{vmatrix} 4 & -3 & 5 \\ 3 & -2 & 8 \\ 1 & -7 & -5 \end{vmatrix}$

A) $92$ B) $100$ C) $-87$ D) $102$

49. Determinantni hisoblang. $\begin{vmatrix} 4 & -3 & 5 & 1 \\ 3 & -2 & 8 & 3 \\ 8 & -6 & 10 & 2 \\ 1 & -5 & 4 & 3 \end{vmatrix}$

A) $51$ B) $207$ C) $-43$ D) $0$

50. Quyidagi chiziqli tenglamalar sistemasini Kramer usulida yeching.

$$\begin{cases} 3y - x = -17 \\ 5x + 3y = -5 \end{cases}$$

A) $(5;-2)$ B) $(2;-5)$ C) $(-2;5)$ D) $(-5;2)$

51. Quyidagi chiziqli tenglamalar sistemasini Kramer usulida yeching.

$$\begin{cases} x+y+4z=1 \\ 2x+y+6z=2 \\ 3x+3y+13z=2 \end{cases}$$

A)$(-3;2;1)$ B)$(2;3;-1)$ C)$(1;2;3)$ D)$(3;2;-1)$

52. $\vec{a}$ va $\vec{b}$ vektorlar o'zaro $\varphi=\frac{2\pi}{3}$ burchak hosil qiladi. $|\vec{a}|=1$, $|\vec{b}|=2$ ni bilgan holda, quyidagini hisoblang: $\left[\vec{a}\,\vec{b}\right]^2$.

A) 3 B) 4 C) 9 D) 2

53. $\vec{a}(3;-1;-2)$ va $\vec{b}(1;2;-1)$ vektorlar berilgan. Vektor ko'paytmalar koordinatasini toping: $\left[\vec{a}\,\vec{b}\right]$.

A)$(5;-1;7)$ B)$(-3;1;-7)$ C)$(5;1;-4)$ D)$(5;1;7)$

54. $\vec{a}(3;-1;-2)$ va $\vec{b}(1;2;-1)$ vektorlar berilgan. Vektor ko'paytmaning koordinatasini toping: $\left[(2\vec{a}+\vec{b})\vec{b}\right]$.

A)$(5;1;6)$ B)$(10;2;14)$ C)$(2;5;3)$ D)$(4;1;5)$

55. $A(2;-1;2)$, $B(1;2;-1)$ va $C(3;2;1)$ nuqtalar berilgan. Vektor ko'paytmaning koordinatasini toping: $\left[\overrightarrow{AB}\,\overrightarrow{BC}\right]$.

A)$(8;2;-4)$ B)$(5;3;12)$ C)$(6;-4;-6)$ D)$(3;-2;5)$

56. $A(2;-1;2)$, $B(1;2;-1)$ va $C(3;2;1)$ nuqtalar berilgan. Vektor ko'paytmalar koordinatalarini toping: $\left[(\overrightarrow{BC}-2\overrightarrow{CA})\overrightarrow{CB}\right]$.

A)$(12;-8;12)$ B)$(10;4;-6)$ C)$(-11;6;4)$ D)$(-12;8;12)$

57. $\vec{a}(2;-2;1)$ va $\vec{b}(2;3;6)$ vektorlar orasidagi burchak sinusini hisoblang.

A) $sin\alpha=\frac{5\sqrt{17}}{21}$ B) $sin\alpha=-\frac{3\sqrt{17}}{21}$

C) $sin\alpha=-\frac{5\sqrt{17}}{21}$ D) $sin\alpha=\frac{4\sqrt{17}}{21}$

58. $A(3;-2;5)$, $B(1;4;-3)$ va $C(-6;2;4)$ nuqtalar berilgan bo'lsa, $\left[\overrightarrow{BC}\,\overrightarrow{AC}\right]\overrightarrow{AB}$ aralash ko'paytmasini toping.

A) $-13$ B) $-28$ C) 0 D) 28

59. $C(-2;4;3)$, $D(1;-5;6)$ va $E(3;7;-4)$ nuqtalar berilgan bo'lsa, $\left(2\overrightarrow{CD}-3\overrightarrow{DE}\right)\left(\overrightarrow{DC}+3\overrightarrow{CE}\right)\left(2\overrightarrow{CD}-\overrightarrow{ED}\right)$ aralash ko'paytmasini toping.

A) $-3$ B) $0$ C) $3$ D)$-2$

60. $\vec{a}=\vec{\imath}+6\vec{\jmath}-4\vec{k}$, $\vec{b}=-3\vec{\imath}+2\vec{\jmath}+7\vec{k}$ va $\vec{c}=-5\vec{\imath}-6\vec{\jmath}+2\vec{k}$ vektorlar berilgan bo‘lsa, $[\vec{a}\ \vec{c}]\vec{b}$ aralash ko‘paytmasini toping.

A) $240$ B) $244$ C) $-240$ D)$120$

61. $\vec{a}=\vec{\imath}+6\vec{\jmath}-4\vec{k}$, $\vec{b}=-3\vec{\imath}+2\vec{\jmath}+7\vec{k}$ va $\vec{c}=-5\vec{\imath}-6\vec{\jmath}+2\vec{k}$ vektorlar berilgan bo‘lsa, $(\vec{b}+2\vec{a})(\vec{c}+3\vec{b})(2\vec{a}-\vec{c})$ aralash ko‘paytmasini toping.

A) $-920$ B) $940$ C) $960$ D)$-930$

62. $\vec{a}(6;-4;8)$ va $\vec{b}(-2;4;0)$ vektorlar berilgan bo‘lsa:
$\left[\frac{\vec{a}+\vec{b}}{2}(\vec{b}-\frac{\vec{a}}{2})\right]$ topilsin.

A)$(24;12;-12)$ B)$(-14;13;12)$
C)$(12;24;-12)$ D)$(-24;-12;12)$

63. $\vec{a}(8;4;1)$ va $\vec{b}(2;-2;1)$ vektorlardan yasalgan parallelogramm yuzi hisoblansin.

A) $8\sqrt{3}$ B) $18\sqrt{2}$ C)$18\sqrt{3}$ D)$9\sqrt{2}$

64. Berilganlarga ko‘ra $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlarning aralash ko‘paytmasini toping. $\vec{a}=\vec{k}$, $\vec{b}=\vec{\imath}$, $\vec{c}=\vec{\jmath}$.

A) $1$ B) $-1$ C) $0$ D)$-2$

65. $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlarning aralash ko‘paytmasini toping.
$\vec{a}=\vec{\imath}+\vec{\jmath}$, $\vec{b}=\vec{\imath}-\vec{\jmath}$, $\vec{c}=\vec{k}$.

A) $1$ B) $-1$ C) $0$ D)$-2$

66. $\vec{a}$ va $\vec{b}$ vektorlar o‘zaro $\varphi=\frac{\pi}{6}$ burchak tashkil qiladi va $\vec{c}$ vektor bilan perpendikulyar. $|\vec{a}|=6$, $|\vec{b}|=3$ va $|\vec{c}|=4$ berilgan bo‘lsa, $\vec{a}\vec{b}\vec{c}$ ni toping.

A) $20$ B) $36$ C)$-12$ D) $24$

67. $\vec{a}=2\vec{\imath}+3\vec{\jmath}+4\vec{k}$, $\vec{b}=3\vec{\imath}+2\vec{\jmath}+\vec{k}$ va $\vec{c}=\vec{\jmath}-\vec{k}$ vektorlar berilgan bo‘lsa, $([\vec{a}\vec{c}]\vec{b})$ ni toping.

A) $-15$ B) $12$ C) $15$ D) $10$

68. $\vec{a} = 2\vec{i} + 3\vec{j} + 4\vec{k}$, $\vec{b} = 3\vec{i} + 2\vec{j} + \vec{k}$ va $\vec{c} = \vec{j} - \vec{k}$ vektorlar berilgan bo‘lsa, $\left[(\vec{a} - 2\vec{c})\left(3\vec{b} - 2\vec{a}\right)\right]$ ni toping.

A)$(5; -40; 5)$ B)$(40; 15; -5)$
C)$(-5; 40; -5)$ D)$(-15; 45; 5)$

69. $\vec{a} = -\vec{i} + 3\vec{j} + \vec{k}$, $\vec{b} = \vec{j} + 2\vec{k}$ va $\vec{c} = 2\vec{a} - 3\vec{b}$ vektorlar berilgan bo‘lsa, $\left[\vec{a}\left[\vec{b}\vec{c}\right]\right]\vec{c}$ ni toping.

A)$-146$ B) $-156$ C) $180$ D) $-180$

70. $\vec{a} = -\vec{i} + 3\vec{j} + \vec{k}$, $\vec{b} = \vec{j} + 2\vec{k}$ va $\vec{c} = 2\vec{a} - 3\vec{b}$ vektorlar berilgan bo‘lsa, $[\vec{a}[\vec{a}\vec{c}]]\left[\vec{b}[\vec{a}\vec{c}]\right]$ ni toping.

A)$1926$ B) $1350$ C) $2120$ D)$2020$

71. Quyida berilgan aylana tenglamasidan aylana markazi va radiusi topilsin.
$x^2 + y^2 - 6y = 0$

A)$R = 3$, $(0; 3)$; C) $R = 9$, $(0; 3)$;
B) $R = 3$, $(0; -3)$; D) $R = 9$, $(0; -3)$.

72. Ellipsning yarim o‘qlarini toping: $\frac{x^2}{9} + \frac{y^2}{25} = 1$.

A) $\pm 3$ va $\pm 5$ C) 3 va 5
B) $\pm 5$ va $\pm 3$ D) 5 va 3

73. Quyidagilardan qaysi biri ellips tenglamasini ifodalaydi?

A) $x^2 + 25y^2 = 4$ C) $x^2 - 16y^2 = 16$
B) $x^2 + 9y^2 = 0$ D) $x^2 - y^2 = 1$

74. Radiusi $R = 5$, markazi $(2; -4)$ nuqtada bo‘lgan aylana tenglamasini toping.

A) $x^2 + y^2 + 4x + 8y + 5 = 0$
C) $x^2 + y^2 - 4x - 8y + 5 = 0$
B) $x^2 + y^2 + 4x - 8y - 5 = 0$
D) $x^2 + y^2 - 4x + 8y - 5 = 0$

75. Ellips fokuslarining koordinatalarini toping: $\frac{x^2}{64} + \frac{y^2}{100} = 1$.

A) $(\pm 6; 0)$ C) $(6; 0)$ B) $(0; \pm 6)$ D) $(0; -6)$

76. Quyidagi ellips tenglamasining ekssentrisitetini aniqlang:
$x^2 + 4y^2 = 1$

A) $\varepsilon = \sqrt{3}$ C) $\varepsilon = \frac{\sqrt{3}}{2}$

B) $\varepsilon = \pm\sqrt{3}$ D) $\varepsilon = \pm\frac{\sqrt{3}}{2}$

77. Direktrisalari $x = \pm\frac{7}{2}$ ekssentrisiteti $\varepsilon = \frac{2}{\sqrt{7}}$ bo‘lgan ellips tenglamasi topilsin.

A) $7x^2 + 3y^2 = 21$ C) $3x^2 + 7y^2 = 1$

B) $7x^2 + 3y^2 = 1$ D) $3x^2 + 7y^2 = 21$

78. Quyidagi ellips tenglamasining fokuslari orasidagi masofani aniqlang: $\frac{x^2}{25} + \frac{y^2}{169} = 1$

A) 8 B) 12 C)18 D) 24

79. Yarim o‘qlari 2 va 5 bo‘lgan ellips tenglamasini ko‘rsating.

A) $\frac{x^2}{4} + \frac{y^2}{25} = 1$ C) $\frac{x^2}{2} + \frac{y^2}{5} = 1$

B) $\frac{x^2}{5} + \frac{y^2}{2} = 1$ D) $\frac{x^2}{25} + \frac{y^2}{4} = 1$

80. Tekislikda berilgan nuqtadan bir xil uzoqlikdagi nuqtalarning geometrik o‘rniga …… . . deyiladi.

A) ellips C) shar

B) aylana D) giperbola

81. Parabola tenglamasining umumiy ko‘rinishini ko‘rsating.

A) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ C) $y^2 = 2px$

B) $x^2 + y^2 = R^2$ D) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

82. $y^2 = 8x$ parabolaning direktrisasini toping.

A) $y = 2$ C) $x = 2$

B) $y = -2$ D) $x = -2$

83. Direktrisasi $y = -6$ bo‘lgan parabola tenglamasini aniqlang.

A) $y^2 = 24x$ C) $y^2 = -24x$

B) $x^2 = 24y$ D) $x^2 = -24y$

84. $y^2 = 12x$ parabola tenglamasining fokusi nimaga teng?

A) $F(0;3)$ C) $F(3;0)$

B) $F(0;-3)$ D) $F(-3;0)$

85. Agar $F(-5;0)$ fokus va direktrisa 0tenglamasi $x = 5$ bo‘lsa, parabola tenglamasini tuzing.

A) $y^2 = -20x$ C) $y^2 = -10x$

B) $y^2 = 20x$ D) $y^2 = 10x$

86. Quyidagi nuqtalardan qaysilari $y^2 = 18x$ parabolaga tegishli?

A) $A(2;6)$ C) $C(1;18)$

B) $B(2;36)$ D) $D(-1;18)$

87. Ushbu nuqtalar tegishli bo‘lgan parabola tenglamasi toping?

$A(-7;7), B(-1;\sqrt{7})$.

A) $y^2 = -6x + 7$ C) $y^2 = -2x + 5$

B) $y^2 = -7x$ D) $y^2 = -\sqrt{7}x$

88. Parabolaning fokusidan direktrisasigacha bo‘lgan masofa 4 ga teng. Uning kanonik tenglamasini tuzing.

A) $y^2 = 16x$ C) $y^2 = -8x$

B) $y^2 = -16x$ D) $y^2 = 8x$

89. $y^2 = 20x$ parabola tenglamasi berilgan. Fokal radiusi 10 ga teng bo‘ladigan $M$ nuqtani toping.

A) $(8;-11), (8;11)$ C) $(11;-8), (11;8)$

B) $(-11;8), (11;8)$ D) $(-8;11), (8;11)$

90. $x^2 = 10y$ parabola $(5;7)$ nuqtadan o‘tganda ushbu nuqtada fokal radius topilsin.

A) $\sqrt{41}$ C) $\sqrt{26}$

B) $\sqrt{53}$ D) $\sqrt{50}$

91. Giperbola tenglamasi uchun qaysi shart bajarilganda teng yonli giperbola deyiladi?

A) $a \neq b$ C) $a > b$

B) $a < b$ D) $a = b$

92. Haqiqiy o‘qi 10, mavhum o‘qi 8 ga teng bo‘lgan giperbola tenglamasi topilsin.

A) $\frac{x^2}{64} - \frac{y^2}{100} = 1$ C) $\frac{x^2}{10} - \frac{y^2}{8} = 1$

B) $\frac{x^2}{100} - \frac{y^2}{64} = 1$ D) $\frac{x^2}{8} - \frac{y^2}{10} = 1$

93.Fokuslari orasidagi masofa $2c = 8$, ekssentrisiteti $\varepsilon = \frac{4}{3}$ bo‘lgan giperbolaning kanonik tenglamasini aniqlang.

A) $\frac{x^2}{9} - \frac{y^2}{16} = 1$ C) $\frac{x^2}{7} - \frac{y^2}{9} = 1$

B) $\frac{x^2}{9} - \frac{y^2}{7} = 1$ D) $\frac{x^2}{9} + \frac{y^2}{7} = 1$

94. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ giperbolaning ixtiyoriy nuqtasidan uning ikki asimptotasigacha bo‘lgan masofalar ko‘paytmasi har doim …….. ga teng bo‘ladi.

A) $\frac{ab}{a^2+b^2}$ C) $\frac{a^2b^2}{a^2+b^2}$

B) $\frac{a^2b^2}{a+b}$ D) $\frac{ab}{a+b}$

95. $\frac{x^2}{9} - \frac{y^2}{64} = 1$ giperbola tenglamasi berilgan bo‘lsa, uning yarim o‘qlari topilsin.

A) 3 va 8 C) -8 va -3

B) 8 va 3 D) -3 va -8

96. Yarim o‘qlari $a = 6$, $b = 4$ bo‘lgan giperbolaning ekssentrisiteti nimaga teng?

A) $\frac{\sqrt{13}}{2}$ B)$-\frac{\sqrt{13}}{2}$ C) $-\frac{\sqrt{13}}{3}$ D) $\frac{\sqrt{13}}{3}$

97. $\frac{x^2}{7} - \frac{y^2}{15} = 1$ giperbolaning asimptotalarini aniqlang.

A) $y = \pm\sqrt{\frac{7}{15}}x$ C) $y = \pm\sqrt{\frac{15}{7}}x$

B) $x = \sqrt{\frac{7}{15}}y$ D) $x = \pm\sqrt{\frac{15}{7}}y$

98. Asimptotalari $y = \pm\frac{4}{3}x$ , fokuslari orasidagi masofa 20 bo‘lgan giperbolaning ekssentrisitetini toping.

A) $\varepsilon = \frac{5}{4}$ B) $\varepsilon = \frac{5}{3}$ C) $\varepsilon = \frac{4}{5}$ D) $\varepsilon = \frac{3}{5}$

99. Quyidagi nuqtalardan qaysi biri ushbu giperbolani qanoatlantiradi: $\frac{x^2}{20} - \frac{y^2}{14} = 1$

A) $(2\sqrt{10}\,;\ \sqrt{14})$ C) $(\sqrt{10}\,;\ \sqrt{14})$

B) $(2\sqrt{10}\,;\ 2\sqrt{14})$ D) $(\sqrt{10}\,;\ 2\sqrt{14})$

100. Teng tomonli giperbola $x^2 - y^2 = 18$ berilgan. Unga fokusdosh bo‘lib, $M(10; 8)$ nuqtadan o‘tuvchi giperbolaning tenglamasi topilsin.

A) $\frac{x^2}{16} - \frac{y^2}{20} = 1$ C) $\frac{x^2}{20} - \frac{y^2}{16} = 1$

B) $\frac{x^2}{30} - \frac{y^2}{6} = 1$ D) $\frac{x^2}{6} - \frac{y^2}{30} = 1$

101. $A(4; \frac{2\pi}{3})$ nuqtaga qutb o‘qiga nisbatan simmetrik bo‘lgan $B$ nuqtani toping.

A) $B(-4; \frac{2\pi}{3})$ B) $B(4; \frac{5\pi}{3})$ C) $B(4; \frac{\pi}{3})$ D) $B(-4; \frac{4\pi}{3})$

102. $B(2; \frac{2\pi}{3})$ nuqtaga qutb o‘qiga nisbatan simmetrik bo‘lgan $C$ nuqtani toping.

A) $C(2; \frac{\pi}{3})$ B) $C(-2; \frac{2\pi}{3})$ C) $C(2; \frac{5\pi}{3})$ D) $C(2; \frac{4\pi}{3})$

103. $A(3; \frac{\pi}{6})$ nuqtani qutb o‘qi atrofida $\frac{3\pi}{4}$ burchakka musbat yo‘nalishda burilsa bu nuqtaning koordinatalarini aniqlang.

A) $(3; \frac{17\pi}{12})$ B) $(3; \frac{2\pi}{3})$ C) $(-3; \frac{2\pi}{5})$ D) $(3; \frac{5\pi}{6})$

104. Qutb koordinatalar sistemasida $A(8; -\frac{2\pi}{3})$ va $B(6; \frac{\pi}{3})$ nuqtalar berilgan. $AB$ kesma o‘rtasining koordinatalarini toping.

A) $(3; -\frac{2\pi}{3})$ B) $(2; \frac{\pi}{3})$ C) $(1; -\frac{2\pi}{3})$ D) $(1; \frac{2\pi}{3})$

105. Dekart koordinatalar sistemasida $M(\sqrt{3}; 1)$ nuqta berilgan. Uni qutb koordinatalarini toping.

A) $(2; \frac{\pi}{6})$ B) $(1; \frac{2\pi}{3})$ C) $(2; \frac{\pi}{3})$ D) $(3; \frac{\pi}{3})$

106.Qutb koordinatalarida $a$ radiusli, markazi koordinatalar boshida bo‘lgan aylana tenglamasini toping.

A) $r = 2a$ B) $r = a$ C) $r = a^2$ D) $r = 3a$

107.Qutb koordinatalar sistemasida $M(3;\frac{5\pi}{6})$ va $N(2;\frac{\pi}{6})$ nuqtalar orasidagi masofani toping.

A) $\sqrt{19}$ B) 2 C) 3 D)$\sqrt{5}$

108.Qutb koordinatalar sistemasida $r = \frac{2}{1-\cos\varphi}$ tenglama bilan berilgan chiziqni dekart koordinatalar sistemasida tenglamasini toping.

A) $y^2 = 4(x+1)$

B) $y^2 + x^2 = 1$

C) $x = y^2$

D) $\frac{x^2}{9} + \frac{y^2}{4} = 1$

109. Qutb koordinatalar sistemasida $M(3;\frac{5\pi}{6})$ va $N(4;\frac{\pi}{3})$ nuqtalar orasidagi masofani toping.

A) 6 B) 3 C) 7 D) 5

110. $\rho = \frac{4cos\varphi}{sin^2\varphi}$ parabolaning direktrisa tenglamasini toping.

A) $x = 5$ B) $x = -3$ C) $x = -1$ D) $x = -2$

111. Quyidagi $3x^2 - 2xy + 3y^2 + 2x - 4y + 1 = 0$ egri chiziqni markazi topilsin.

A) $(-1;1)$ B) $(-\frac{1}{8};\frac{5}{8})$ C) $(\frac{1}{2};-4)$ D) $(2;-3)$

112. $4xy + 3y^2 + 16x + 12y - 36 = 0$ berilgan ikkinchi tartibli chiziqning turini aniqlang.

A)giperbola B)parabola C)parallel to‘g‘ri chiziqlar D)ellips

113. Ushbu $9x^2 - 16y^2 - 54x - 64y - 127 = 0$ ikkinchi tartibli chiziqning eksentrisitetini aniqlang.

A) $\frac{3}{4}$ B) $\frac{4}{5}$ C) $\frac{5}{4}$ D) $\frac{7}{5}$

114. Quyidagi $32x^2 + 52xy - 7y^2 + 180 = 0$ egri chiziqning asimptotalarini toping.

A) $\pm\frac{2}{3}x$ B) $\pm\frac{5}{2}x$ C) $\pm\frac{4}{3}x$ D) $\pm\frac{1}{3}x$

115. $14x^2 + 24xy + 21y^2 - 4x + 18y - 139 = 0$ ellipsning fokuslari orasidagi masofani aniqlang.

A) 5 B) 6 C) 10 D) 8

116. Ushbu $7x^2 + 60xy + 32y^2 - 14x - 60y + 7 = 0$ ikkinchi tartibli chiziqning tipini aniqlang.

A) giperbola B) parallel to‘g‘ri chiziqlar C) ellips D) parabola

117. Quyidagi $9x^2 + 24xy + 16y^2 - 230x + 110y - 475 = 0$ tenglama bilan berilgan ikkikinchi tartibli chiziqning direktrisasini aniqlang.

A) $x = -\frac{5}{3}$ B) $x = -\frac{7}{2}$ C) $x = -\frac{4}{3}$ D) $x = -\frac{5}{2}$

118. Ushbu $5x^2 + 12xy - 12x - 22y - 19 = 0$ egri chiziqning haqiqiy o‘qining burchak koeffitsiyentini aniqlang.

A) $k = \frac{1}{3}$ B) $k = \frac{3}{4}$ C) $k = \frac{2}{3}$ D) $k = \frac{1}{2}$

119. Quyidagi $5x^2 + 8xy + 5y^2 - 18x - 18y + 9 = 0$ ikkinchi tartibli chiziqning markazi qaysi nuqtada joylashgan?

A) $(1; 1)$ B) $(-2; 3)$ C) $(-3; 1)$ D) $(-1; 1)$

120. $6xy - 8y^2 + 12x - 26y - 11 = 0$ tenglama bilan berilgan ikkinchi tartibli chiziqning turini aniqlang.

A) parabola B) ellips C) parallel to‘g‘ri chiziqlar D) giperbola.

# SINOV TESTI JAVOBLARI

| № | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | | C | D | C | A | B | D | A | A | A |
| 1 | A | C | B | A | D | C | C | D | C | B |
| 2 | D | A | D | B | A | C | B | B | C | A |
| 3 | A | C | B | A | B | C | A | A | B | A |
| 4 | B | C | B | D | A | B | A | C | B | D |
| 5 | B | D | A | D | B | C | D | A | C | B |
| 6 | A | C | D | B | A | D | B | A | C | D |
| 7 | B | A | C | A | D | B | C | D | B | A |
| 8 | B | C | D | B | C | A | A | B | D | C |
| 9 | A | D | B | B | C | A | D | C | B | A |
| 10 | C | B | D | A | C | A | B | A | A | D |
| 11 | C | B | A | C | A | C | B | D | C | A |
| 12 | D | | | | | | | | | |

# JAVOBLAR

**1.2.1.** 1) $5$; 2) $\sqrt{34}$; 3) $13$; 4) $\sqrt{2}$. **1.2.2.** 1) $\sqrt{137}$; 2) $5$; 3) $11$; 4) $1$. **1.2.3.** $(2;4)$. **1.2.4.**$(3;3)$. **1.2.5.**$\left(\frac{65}{24};6\right)$. **1.2.6.** $(0;-10)$. **1.2.7.** $\left(0;\frac{13}{2}\right)$. **1.2.8.** $\left(-\frac{5}{3};0\right)$. **1.2.9.** $\left(\frac{89}{10};0\right)$. **1.2.10.** $(5;3)$. **1.2.11.** $ABC$ uchburchak to‘g‘ri burchakli. **1.2.12.** $(-7;0)$ va $(17;0)$; $\left(0;9-10\sqrt{2}\right)$, $\left(0;9+10\sqrt{2}\right)$. **1.2.13.** $\left(0;11+4\sqrt{6}\right)$, $\left(0;11-4\sqrt{6}\right)$. **1.2.14.** $5$. **1.2.15.** $(2;2)$; $(12;-12)$; $(6;-6)$; $(-4;4)$. **1.2.16.** $M(-5;4)$. **1.2.17.** Markazi $(-1;-2)$ nuqtada, radiusi $r=5$ ga teng. **1.2.18.** $B(2;5)$; $D(16;3)$. **1.2.19.** $M(2;10)$. **1.2.20.** $M_1(1;-1)$, $r_1=1$; $M_2(-5;-5)$, $r_2=5$. **1.2.21.** $M_1\left(4+\sqrt{6};4+\sqrt{6}\right)$, $r_1=4+\sqrt{6}$; $M_2\left(4-\sqrt{6};4—\sqrt{6}\right)$, $r_2=4-\sqrt{6}$. **1.2.22.** $5$. **1.2.23.** $\sqrt{29}$. **1.2.24.** $5+2\sqrt{10}+5\sqrt{5}$. **1.2.25.** $(-5;2)$. **1.2.27.** $(3;5)$; $(4;2)$; $(5;-1)$. **1.2.28.** $(4;-4)$; $(2;5)$. **1.2.29.** $8$. **1.2.30.** $13$. **1.3.1.** $(0;2)$. **1.3.2.** $(1;-1)$. **1.3.3.** $(3;-3)$. **1.3.4.** $(-2;2)$. **1.3.5.** $(1;3)$. **1.3.6.**$(-1;4)$; $(0;0)$; $\left(\frac{1}{2};\frac{1}{2}\right)$. **1.3.7.** $\left(\frac{11}{5};0\right)$ va $(0;-11)$. **1.3.8.** $x=\frac{1}{3}(x_1+x_2+x_3)$; $y=\frac{1}{3}(y_1+y_2+y_3)$. **1.3.9.** $(-3;3)$; $(7;5)$; $(-3;-3)$. **1.3.10.** $(4;1)$; $(1;4)$; $(4;4)$. **1.3.11.** $B(0;-7)$. **1.3.12.** $N(6;9)$. **1.3.13.** $B(12;-4)$. **1.3.14.** $C(10;9)$; $D(4;-4)$. **1.3.15.** $4$. **1.3.16.** $M(12;-11)$ **1.3.17.** $D(8;-18)$. **1.3.18.** $C(0;-1)$; $D(4;-4)$. **1.3.19.** $\left(0;\frac{19}{3}\right)$; $\left(-3;\frac{26}{3}\right)$. **1.3.20.** $A(3;-1)$; $B(0;8)$. **1.3.21.** $A(-5;3)$; $B(4;3)$. **1.3.22.** $B\left(-5;\frac{16}{3}\right)$. **1.3.23.** $C(1;-4)$. **1.3.24.** $C(-9;7)$. **1.3.25.** $A(160;-131)$; $B(-225;184)$. **1.3.26.** $\frac{\sqrt{157}}{2}$. **1.3.27.** $\left(-5;-\frac{19}{4}\right)$; $\left(-3;-\frac{13}{4}\right)$. **1.3.28.** $\left(\frac{7}{13};-\frac{4}{13}\right)$; $\left(\frac{13}{33};\frac{292}{165}\right)$. **1.3.29.**$\frac{10\sqrt{2}}{3}$. **1.3.30.** $D(11;7)$. **2.1.1.** $\overrightarrow{AB}(-7;-3)$. **2.1.2.** $\overrightarrow{CD}(8;-10;5)$ va $\overrightarrow{DC}(-8;10;-5)$. **2.1.3.** $\overrightarrow{AB}(-12;-2;7)$; $\overrightarrow{BA}(12;2;-7)$. **2.1.4.** $B(14;-15)$. **2.1.5.** $B(-5;8;-1)$. **2.1.6.** $B(7;-1;4)$. **2.1.7.** $A(9;-8)$. **2.1.8.** $A(-8;6;1)$. **2.1.9.** $A(0;-10;7)$. **2.1.10.** $\left(-\frac{4}{5};\frac{3}{5}\right)$. **2.1.11.** $\left(-\frac{4}{5};-\frac{3}{5}\right)$. **2.1.12.** $\left(-\frac{9}{16};\frac{12}{16}\right)$. **2.1.13.**

$\vec{a}^0\left(\frac{6}{7};-\frac{2}{7};-\frac{3}{7}\right)$. **2.1.14.** $\left(-\frac{4}{13};\frac{3}{13};\frac{12}{13}\right)$. **2.1.15.** $\left(\frac{2}{11};-\frac{6}{11};-\frac{9}{11}\right)$. **2.1.16.** $\left(-\frac{3}{13};-\frac{4}{13};\frac{12}{13}\right)$. **2.1.17.** $\left(\frac{1}{17};-\frac{12}{17};\frac{12}{17}\right)$. **2.1.18.** $\left(-\frac{2}{15};\frac{10}{15};\frac{11}{15}\right)$. **2.1.19.** $\{0{,}6;-0{,}8\};\{-0{,}6;0{,}8\}$. **2.1.20.** $\left(-\frac{9}{\sqrt{82}};\frac{1}{\sqrt{82}}\right)$; $\left(\frac{9}{\sqrt{82}};\frac{1}{\sqrt{82}}\right)$. **2.1.21.** $\left(\frac{11}{\sqrt{179}};-\frac{7}{\sqrt{179}};\frac{3}{\sqrt{179}}\right)$; $\left(-\frac{11}{\sqrt{179}};\frac{7}{\sqrt{179}};-\frac{3}{\sqrt{179}}\right)$. **2.1.22.** $\left(\frac{8}{9};\frac{4}{9};\frac{1}{9}\right)$. **2.1.23.** $\left(\frac{9}{11};-\frac{2}{11};\frac{6}{11}\right)$. **2.1.24.** $\left(-\frac{10}{15};-\frac{2}{15};\frac{11}{15}\right)$. **2.1.25.** $\left(\frac{9}{\sqrt{34}};-\frac{15}{\sqrt{34}}\right)$. **2.1.26.** $\left(-\frac{10}{\sqrt{29}};\frac{20}{\sqrt{29}};-\frac{15}{\sqrt{29}}\right)$. **2.1.27.** $\left(\frac{24}{\sqrt{37}};-\frac{4}{\sqrt{37}}\right)$. **2.1.28.** $\left(\frac{36}{\sqrt{46}};-\frac{6}{\sqrt{46}};\frac{18}{\sqrt{46}}\right)$. **2.1.29*.** $\left(\frac{3}{\sqrt{130}};\frac{11}{\sqrt{130}}\right)$. **2.1.30*.** $\left(-\frac{2}{\sqrt{6}};-\frac{1}{\sqrt{6}};-\frac{1}{\sqrt{6}}\right)$.
**2.2.3.** 1) $\{21;0\}$; 2) $\{18;-15\}$; 3) $\{-36;9\}$; 4) $\{1;-2\}$; 5) $\{39;6\}$; 6) $\{10;-13\}$. **2.2.4.** 1) $\{2;-7\}$; 2) $\{-10;9\}$; 3) $\{-8;2\}$; 4) $\{-3;4\}$; 5) $\{10;-22\}$; 6) $\left\{-7;8\frac{1}{4}\right\}$. **2.2.5.** 1) $\{42;-6\}$; 2) $\{-10;-10\}$; 3) $\{-16;8\}$; 4) $\{-3;-1\}$; 5) $\{-42;6\}$; 6) $\{22;-5\}$. **2.2.6.** 1) $\{2;-7;16\}$; 2) $\{-2;9;-22\}$; 3) $\{8;-12;16\}$; 4) $\left\{0;\frac{4}{3};-4\right\}$; 5) $\{-8;10;-10\}$; 6) $\left\{-1;\frac{15}{2};-20\right\}$. **2.2.7.** 1) $\{-11;33;-12\}$; 2) $\{7;-13;-12\}$; 3) $\{2;-10;12\}$; 4) $\left\{\frac{4}{3};-3;-1\right\}$; 5) $\{11;-33;12\}$; 6) $\left(-8\frac{1}{4};19\frac{1}{4};\frac{9}{2}\right)$.
**2.2.8.** 1) $\{7;1;9\}$; 2) $\{7;-9;17\}$; 3) $\{-12;4;-20\}$; 4) $\{-1;2;-3\}$; 5) $\{1;13;-9\}$; 6) $\left\{-1;3\frac{2}{3};-4\frac{1}{3}\right\}$. **2.2.9.** 1) $\{7;-5;18\}$; 2) $\{5;-3;6\}$; 3) $\{6;-4;12\}$; 4) $\left\{1;-\frac{1}{2};0\right\}$; 5) $\{0;-1;12\}$; 6) $\left\{3;-\frac{5}{3};2\right\}$. **2.2.10.** 1) $\{-30;21\}$; 2) $\{114;-38\}$. **2.2.11.** 1) $\{21;2\}$; 2) $\{45;-10\}$. **2.2.12.** 1) $\{-58;-39\}$; 2) $\{19;27\}$. **2.2.13.** 1) $\{3;22;-3\}$; 2) $\{19;39;30\}$. **2.2.14.** 1) $\{-9;6;17\}$; 2) $\{-14;5;15\}$. **2.2.15.** 1) $\{-5;-3;-25\}$; 2) $\{25;12;13\}$. **2.2.16.** $\vec{a}+\vec{b}=\{4;3\}$; $\vec{a}-\vec{b}=\{0;5\}$ **2.2.17.** $\vec{a}+\vec{b}=\{-6;8\}$; $\vec{a}-\vec{b}=\{2;-2\}$. **2.2.18.** $\vec{a}+\vec{b}=\{12;9\}$; $\vec{a}-\vec{b}=\{4;3\}$. **2.2.19.** $\vec{a}+\vec{b}=\{2;-4;4\}$; $\vec{a}-\vec{b}=\{4;-6;12\}$.
**2.2.20.** $\vec{a}+\vec{b}=\{-2;6;9\}$; $\vec{a}-\vec{b}=\{-4;-8;-1\}$. **2.2.21.** $2\vec{a}-5\vec{b}=$ $=\{-1;-9;-16\}$; **2.2.22.** $\overrightarrow{AM}(3;4;-3)$; $\overrightarrow{BN}(0;-5;3)$; $\overrightarrow{CP}(-3;1;0)$.

**2.2.23.** $\overrightarrow{AD} = \frac{\overrightarrow{AB}+\overrightarrow{AC}}{2}$. **2.2.24.** 0. **2.2.25.** $\overrightarrow{BC} = p + q$; $\overrightarrow{CD} = -q$; $\overrightarrow{DE} = -p$; $\overrightarrow{EF} = -p - q$. **2.2.26.** **2.3.1.** $\vec{d} = -48\vec{i} + 45\vec{j} - 36\vec{k}$. **2.3.2.** $\vec{a} = 2\vec{p} + 5\vec{q}$. **2.3.3.** $\vec{a} = -2\vec{p} + \vec{q}$. **2.3.4.** $\vec{a} = 3\vec{p} - 2\vec{q}$. **2.3.5.** $\vec{a} = 2\vec{b} + \vec{c}$; $\vec{c} == \vec{a} - 2\vec{b}$. **2.3.6.** $\vec{c} = 2\vec{p} - 3\vec{q} + \vec{r}$. **2.3.7.** $\vec{p} = \frac{\vec{c}+3\vec{q}-\vec{r}}{2}$. **2.3.8.** $\vec{q} = \frac{2}{3}\vec{p} - \frac{1}{3}\vec{c} + \frac{5}{3}\vec{r}$. **2.3.9.** $\vec{r} = -2\vec{p} + 3\vec{q} + \vec{c}$. **2.3.10.** $\vec{c} = \frac{41}{7}\vec{p} + + \frac{5}{14}\vec{q} + \frac{13}{14}\vec{r}$. **2.3.11.** $\vec{p} = 2\vec{a} - 3\vec{b}$. **2.3.12.** $\vec{d} = 2\vec{a} - 3\vec{b} + \vec{c}$; $\vec{c} = -2\vec{a} + 3\vec{b} + \vec{d}$; $\vec{b} = \frac{2\vec{a}+\vec{c}-\vec{d}}{3}$; $\vec{a} = \frac{3\vec{b}-\vec{c}+\vec{d}}{2}$. **2.3.13.** Qarama-qarshi yo‘nalgan, 3 marta uzun. **2.3.14.** $\alpha = 4$; $\beta = = -1$. **2.3.15.** $\alpha = -3$; $\beta = -\frac{2}{3}$. **2.3.16.** $\vec{a}$ va $\vec{b}$; $\vec{c}$ va $\vec{d}$. **2.3.17.** $\lambda = \frac{(n+1)(n-3)}{n(n-1)}$; $\mu = \frac{(n-2)(n-1)}{n(n+1)}$. **2.3.18.** $\frac{-2(2n^2-n+1)}{n^2+n+1}$. **2.3.19.** 1) $\vec{c} = \vec{a} - \vec{b}$;2) $\vec{c} = 2\vec{a} - 3\vec{b}$; 3) $\vec{c} = -\frac{3}{2}\vec{a}$.**2.3.20.** 1) $\vec{a} = 2\vec{b} + +\vec{c}$;2) $\vec{a} = \vec{b} - -3\vec{c}$; 3) $\vec{a} = \frac{1}{2}\vec{b}$. **2.3.21.** 1) $\vec{b} = \vec{a} + 2\vec{c}$;2) $\vec{b} = \vec{c} - 3\vec{c}$; 3) $\vec{b} = \vec{a} - \vec{c}$. **2.3.22.** 1) $\vec{d} = \vec{a} + \vec{b} - \vec{c}$;2) $\vec{d} = 5\vec{a} + 4\vec{b}$; 3) $\vec{d} = 4\vec{a} - -\vec{c}$. **2.3.23.** 1) $\vec{a}, \vec{b}, \vec{c}$ vektorlar chiziqli bog‘liq emas; 2) $\vec{a}, \vec{b}, \vec{c}$ vektorlar chiziqli bog‘liq $\vec{c} = \frac{1}{2}\vec{a} + \frac{2}{3}\vec{b}$ 3) $\vec{a}, \vec{b}, \vec{c}$ vektorlar chiziqli bog‘liq, lekin $\vec{c}$ vektorni $\vec{a}$ va $\vec{b}$ vektorlarning chiziqli kombinatsiyalari ko‘rinishida tasvirlab bo‘lmaydi. **2.3.25.** $m = \pm 2$. **2.3.26.** $\frac{-2(2n^2-n+1)}{n^2+n+1}$. **2.3.27.** $\lambda = \frac{(n+1)(1-n)}{n(2n-5)}$. **2.3.28.** $x = \frac{n+5}{2}\vec{e}_1 + \frac{n-5}{2}\vec{e}_2 + \frac{n-3}{2}\vec{e}_3$. **2.3.29.** $-3\vec{a} + +4\vec{b} + \vec{c} = 0$.

**3.1.1.** $|\vec{b}| = 10$. **3.1.2.** $|\vec{d}| = 7$. **3.1.3.** $|\vec{a}| = 11$. **3.1.4.** $|\vec{c}| = 13$. **3.1.5.** $|\vec{d}| = 17$. **3.1.6.** $\cos\alpha = \frac{12}{15}$; $\cos\beta = -\frac{9}{15}$. **3.1.7.** $\cos\alpha = -\frac{10}{15}$; $\cos\beta = \frac{2}{15}$; $\cos\gamma = \frac{11}{15}$. **3.1.8.** $\cos\alpha = \frac{12}{25}$; $\cos\beta = -\frac{15}{25}$; $\cos\gamma = \frac{16}{25}$. **3.1.9.** $\cos\alpha = \frac{1}{17}$; $\cos\beta = -\frac{12}{17}$; $\cos\gamma = \frac{12}{17}$. **3.1.10.** $\cos\alpha = \frac{3}{7}$; $\cos\beta = -\frac{6}{7}$;$\cos\gamma = \frac{2}{7}$. **3.1.11.** $\cos\alpha = \frac{12}{25}$; $\cos\beta = -\frac{15}{25}$; $\cos\gamma = -\frac{16}{25}$.

**3.1.12.** $\cos\alpha = \frac{8}{10}$; $\cos\beta = -\frac{6}{10}$ ; $|\overrightarrow{AB}|$=10. **3.1.13.** $\cos\alpha = -\frac{12}{13}$; $\cos\beta = \frac{12}{13}$; $\cos\gamma = -\frac{1}{13}$; $\left|\overrightarrow{CD}\right| = 13$. **3.1.14.** $\cos\alpha = \frac{2}{\sqrt{33}}$; $\cos\beta = = -\frac{2}{\sqrt{33}}$; $\cos\gamma = \frac{5}{\sqrt{33}}$; $|\overrightarrow{MN}|$=$\sqrt{33}$. **3.1.15.** $K(2;3;18)$. **3.1.16.** $D(-10;3;1)$. **3.1.17.** $\vec{a}(-\sqrt{3},-\sqrt{3},-\sqrt{3})$; $\vec{a}(\sqrt{3},\sqrt{3},\sqrt{3})$. **3.1.18.** $\vec{a}\left(-\frac{17}{\sqrt{3}},-\frac{17}{\sqrt{3}},-\frac{17}{\sqrt{3}}\right)$; $\vec{a}\left(\frac{17}{\sqrt{3}},\frac{17}{\sqrt{3}},\frac{17}{\sqrt{3}}\right)$. **3.1.19.** $\left(\frac{3}{\sqrt{130}};\frac{11}{\sqrt{130}}\right)$. **3.1.20.** $60^0$, $120^0$. **3.1.21.** $120^0$, $60^0$. **3.1.22.** $z = \pm 3$. **3.1.23.** $y = \pm 12$. **3.1.24.** $B(9;5;11)$, $B(9;5;-1)$. **3.1.25.** $A(7;6;-6)$ yoki $A(-17;6;-6)$. **3.1.26.** $B(8;1;9)$ yoki $B(8;-5;9)$. **3.1.27.** 1) bo‘lmaydi, 2) bo‘ladi, 3) bo‘lmaydi. **3.1.28.** 1) bo‘ladi, 2) bo‘lmaydi, 3) bo‘ladi**.** **3.1.29.** $\overrightarrow{AB}(n+3;-4;3n-5)$. **3.1.30.** $\overrightarrow{CD}(3;-6;-1)$.

**3.2.1.** $5\vec{a} - 2\vec{b} = 22\vec{i} - 13\vec{j} + 20\vec{k}$. **3.2.2.** $\vec{c} = -6\vec{i} + 8\vec{j} + 8\vec{k}$. **3.2.3.** $\vec{d} = 18\vec{i} - 28\vec{j} + 23\vec{k}$. **3.2.4.** $\vec{d} = -31\vec{i} - 8\vec{j} - 36\vec{k}$. **3.2.5.** $N(4;1;1)$. **3.2.6.** $M(-1;2;3)$. **3.2.7.** $60^0$; $120^0$. **3.2.8.** $pr_{\vec{x}}\vec{a} = \sqrt{2}$, $pr_{\vec{y}}\vec{a} = 1$, $pr_{\vec{z}}\vec{a} = -1$. **3.2.9.** $\vec{a}(1;-1)$. **3.2.11.** $D(1;-2)$. **3.2.12.** $D(1;-1)$. **3.2.13.** $B(-5;-2)$. **3.2.14** $D = A + C - B$. **3.2.15.** $n\vec{a}(n^2-2n;\ n^2+3n;\ n^2-n)$, $\vec{a}+\vec{b} = \{2n-2;2n-1;2n+1\}$, $\vec{a}-\vec{b} = \{-2;7;-3\}$, $3\vec{a}+n\vec{b} = \{n^2+3n-6;\ n^2-n-9;n^2+ +5n-3\}$. **3.2.16.** $(9;4;8)$. **3.2.17.** $(4;2;5)$. **3.2.18.** $(2;-2;2)$. **3.2.19.** $(1;-2;3)$. **3.2.20.** $(2{,}5;6;-10)$. **3.2.21.** $m = 2, n = 4$. **3.2.22.** $k = 12$, $l = -4$. **3.2.23.** $\left|5\vec{a} - 3\vec{b}\right| > |2\vec{a} + 3\vec{c}|$. **3.2.24.** $\left|4\vec{b} - \vec{c}\right| > |2\vec{a} + 3\vec{c}|$. **3.2.25.**$\left|6\vec{b} - 3\vec{c}\right| > \left|2\vec{a} + 5\vec{b}\right|$. **3.2.26.** $\left|3\vec{a} + 2\vec{b} - 4\vec{c}\right| < \left|2\vec{b} - -3\vec{c} - \vec{a}\right|$. **3.2.27.** $\left|3\vec{a} + 5\vec{b} - 4\vec{c}\right| < \left|2\vec{b} - 3\vec{c} - \vec{a}\right|$. **3.2.28.** $C\left(\frac{2n^2-5}{2n+1};\frac{2n^2+n+9}{2n+1};\frac{2n^2+2n-1}{2n+1}\right)$. **3.2.29.** $\left(3;\frac{2}{3};2\right)$. **3.2.30.** $\left(\frac{11}{7};4;-4\right)$.
**4.1.1.** 1) $0$; 2) $-48$; 3) $-9$. **4.1.2.** 1) $31$; 2) $-14$; 3) $-28$; 4) $63$; 5) $76$. **4.1.3.** 1) $90^0$; 2) $135^0$; 3) $180^0$. **4.1.4.** 1) $\varphi = arccos\frac{1}{3}$; 2) $90^0$. **4.1.5.** $\vec{a}\vec{b} = 20$. **4.1.6.** $\vec{c}\vec{d} = -\frac{\sqrt{2}}{2}$. **4.1.7.** $\vec{a}\vec{b} = 18$. **4.1.8.** $\vec{c}\vec{d} = -21$. **4.1.9.** 1)$-6$; 2)$9$; 3) $16$; 4) $13$; 5) $37$; 6) $73$; 7) $252$; 8)$-61$. **4.1.10.** 1)$-62$;

2) $162$; 3) $-107$; 4) $-30$; 5) $-445$. **4.1.11.** 1) $716$; 2) $-353$; 3) $-721$. **4.1.12.** 1) $(21;42;21)$; 2) $280$; 3) $(115;242;137)$. **4.1.13.** 1) $13$; 2) $3$; 3) $6\sqrt{3}$; 4) $-188$; 5) $452$. **4.1.14.** $\frac{5}{21}$. **4.1.15.** $\vec{x}(6;4)$. **4.1.16.** $\vec{x}(2;-1)$. **4.1.17.** $\vec{x}(2;7;3)$. **4.1.18.** $\vec{x}\left(1;\frac{1}{2};-\frac{1}{2}\right)$. **4.1.19.** $(4;6;12)$. **4.1.20.** $\vec{x}(2;-2;3)$. **4.1.21.** $\vec{a}\vec{b}+\vec{b}\vec{c}+\vec{a}\vec{c}=-\frac{3}{2}$. **4.1.22.** $\alpha=-6$. **4.1.23.** 1) $19$; 2) $19$; 3) $30$; 4) $30$; 5) $6$. **4.1.24.** $\vec{a}\vec{b}+\vec{b}\vec{c}+\vec{a}\vec{c}=-13$. **4.1.25.** $10$. **4.1.26.** $\pm\frac{3}{5}$. **4.1.27.** $|\vec{a}|=\left|\vec{b}\right|$. **4.1.28**. $arccos\frac{2}{\sqrt{7}}$. **4.1.29.** $45^0$. **4.1.30**. $arccos\frac{4}{9}$. **4.2.1.** 1) $26$; 2) $0$; 3) $3$; 4) $15$; 5) $1$; 6) $3$. **4.2.2.** 1) $-3$; 2) $100$; 3) $-4$; 4) $0$ va $1$; 5) $-3$ va $2$. **4.2.3.** 1) $(2;-5)$; 2) $(-2;-3)$; 3) $(\cos\alpha;\sin\alpha)$. **4.2.4.** 1) $(3;2;-1)$; 2) $(1;-2;3)$; 3) $(2;4;1)$. **4.2.5.** $15$. **4.2.6.** $16$. **4.2.7.** $\vec{a}\vec{b}=-30$. **4.2.8.** 1) $24$; 2) $60$. **4.2.9.** 1) $3$; 2) $32$; 3) $300$. **4.2.11.** 1) $(5;1;7)$; 2) $(10;2;14)$; 3) $(20;4;28)$. **4.2.12.** 1) $(6;-4;-6)$; 2) $(-12;8;12)$. **4.2.13.** $14$. **4.2.14.** $5$. **4.2.15.** $\sin\alpha=\frac{5\sqrt{17}}{21}$. **4.2.16.** 1) $0$; 2) $0$; 3) $-28$. **4.2.17.** 1) $0$; 2) $0$; 3) $0$. **4.2.18.** 1) $-240$; 2) $240$; 3) $-240$; 4) $372$; 5) $-7200$. **4.2.19.** 1) $(-7;14;-7)$; 2) $(-27;-23;-7)$. **4.2.20.** 1) $(64;32;-32)$; 2) $(-32;-16;16)$; 3) $(-24;-12;12)$. **4.2.21.** 1) $(6;-3;-3)$; 2) $(-12;-26;8)$; 3) $0$. **4.2.22.** $18\sqrt{2}$. **4.2.23.** 1) $-7$; 2) $(-44;16;12)$; 3) $(-7;7;7)$. **4.2.24.** 1) $1$; 2) $-1$; 3) $-1$; 4) $1$; 5) $0$; 6) $-2$. **4.2.25.** $24$. **4.2.26.** $36$. **4.2.27.** $-1$. **4.2.28.** 1) $(-5;10;-3)$; 2) $(-3;3;3)$; 3) $(-7;2;2)$; 4) $(-2;13;-20)$; 5) $(-5;-5;-5)$; 6) $(-3;-18;15)$; 7) $15$; 8) $-15$; 9) $15$; 10) $(-35;145;-35)$; 11) $(-5;40;-5)$; 12) $48$; 13) $-285$; 14) $(0;16;-16)$; 15) $(2;3;4)$. **4.2.29.** 1) $25$; 2) $25$; 3) $(-10;-8;34)$; 4) $(-10;-30;15)$; 5) $-180$; 6) $1350$.

**5.1.1.** 1) $y=-\frac{3}{2}x+\frac{9}{2}$; 2) $y=\frac{5}{7}x+\frac{8}{7}$. **5.1.3.** $y=-5x+13$. **5.1.5.** 1) $5$; 2) $-7$; 3) $\frac{21}{20}$; 4) $\frac{56}{33}$. **5.1.6.** $x-3=0$, $y+2=0$. **5.1.7.** $x+y-$ $-1=0$. **5.1.8.** $3x+4y-16=0$; $5x+3y-1=0$; $2x-y-7=0$. **5.1.9.** $91x-26y-2=0$. **5.1.11.** $\left(\frac{29}{18};\frac{47}{54}\right)$. **5.1.12.** $45^0$ va $135^0$.

**5.1.13.** $5x+y-16=0$; $5x-y-14=0$. **5.1.14.** $72x-y=0$; $12x+71y=0$. **5.1.15.** $M_1(4;0)$, $M_1(-1;5)$. **5.1.16.** 1) $m=-4$, $n\neq 2$ yoki $m=4$, $n\neq -2$; 2) $m=-4$, $n=2$ yoki $m=4$, $n=-2$; 3)$m=0$, $n$ ixtiyoriy qiymatida. **5.1.17.** $x-10=0$, $x+$ $+4=0$. **5.1.18.** $135^0$. **5.1.19.** $arctg\frac{1}{2}$. **5.1.20.** $x=-\sqrt{3}t$, $y=t$. **5.1.21.** $5x-2y-33=0$, $x+4y-11=0$, $7x+6y+33=0$. **5.1.22.** $A_1(1;1)$, $B_1(2;4)$, $A_2(5;-3)$, $B_2(4;-6)$.-izlangan to‘g‘ri chiziq vektorining koordinatalari topilsin. **5.1.26.** $(\sqrt{2}+1)x+(\sqrt{2}-$ $-1)y-10=0, (\sqrt{2}-1)x+(\sqrt{2}+1)y+10=0, x-y-10=0;$ 2) $3x-2y-12=0$; $3x-8y+24=0$; 3) $x+3y-30=0, 3x+$ $+4y-60=0$, $3x-y-30=0$, $x-12y+60=0$. **5.1.27.** $2x+$ $+5y-20=0$. **5.1.28.** $(2\sqrt{34}+5\sqrt{5})x+(\sqrt{34}-3\sqrt{5})y-7\sqrt{34}-$ $-\sqrt{5}=0$. **5.1.29.** $3x+4y-2=0$, $4x+3y-5=0$. **5.1.30.** $-7$; 2; $\frac{1}{3}$. **5.2.1.** $K_1$, $K_3$ va $K_4$ nuqtalar bir to‘g‘ri chiziqda yotadi. Qolgan nuqtalar bir to‘g‘ri chiziqda yotmaydi. **5.2.2.** 3;$-3$; 0; $-6$ va 12. **5.2.3.** 1; $-2$; 4; $-5$ va 7. **5.2.4.** 1) $-\frac{5}{3}$. 2) $\frac{3}{5}$. **5.2.5.** 1) $3x-7y-27=0$; 2) $2x+9y+23=0$; 3) $2x-3y-13=0$; 4) $x-2=0$; 5) $y+3=0$. **5.2.6.** $2x+3y-26=0$. **5.2.7.** Berilgan nuqta berilgan to‘g‘ri chiziqda yotganligi uchun bunday to‘g‘ri chiziq mavjud emas.**5.2.8.** $x-y=0$; $x-3y+13=0$. **5.2.9.** $x+y-7=0$. **5.2.10.** $A(2;-1)$, $B(-1;3)$, $C(2;4)$. **5.2.12.** $45^0$, $90^0$, $0^0$, $arc\ tg\frac{16}{11}$. **5.2.13.**$x-5y+$ $+3=0$ yoki $5x+y-11=0$. **5.2.15.** $\frac{9}{8}$. **5.2.17.** 1) $M_1M_2 \parallel l$; 2) $M_1M_2 \parallel l$; 3) $M_1$ va $M_2$ nuqtalar $l$ to‘g‘ri chiziqdan har xil tomonda; 4) $l$ to‘g‘ri chiziq $M_1M_2$ kesmaning davomini $M_1$ nuqtadan keyin kesib o‘tadi; 5) $l$ to‘g‘ri chiziq $M_1M_2$ kesmaning davomini $M_2$ nuqtadan keyin kesib o‘tadi. **5.2.18.** Berilgan to‘g‘ri chiziq $CB$ va $BA$ tomonlarni kesib o‘tadi, shu bilan birga $CA$ tomonni $A$ nuqtadan keyin kesib o‘tadi. **5.2.19.** $5x-2y=0$. **5.2.20.** $25x+29y-21=0$. **5.2.21.** $38x-19y+30=0$. **5.2.22.** $32x-9=0$, $32y-19=0$. **5.2.23.**

$x+y-6=0$. **5.2.24.** $x-49y+20=0$. **5.2.25.** $91x-26y-2=$ $=0$. **5.2.26.** $3x+8y-9=0$. **5.2.27.** $5x+3y-15=0$. **5.2.28.** $x+$ $+y-6=0$. **5.2.29.** $x-2y-4=0$. **5.2.30.** $S=9$. **5.3.1.** 1) $\frac{5}{13}x+$ $+\frac{12}{13}y-\frac{7}{13}=0$; 2) $\frac{4}{5}x-\frac{3}{5}y+\frac{9}{5}=0$. **5.3.3.** $\frac{13}{5}$; 2; $\frac{11}{5}$; $\frac{12}{5}$; 0. **5.3.4.** $\frac{7}{\sqrt{10}}$; $\frac{1}{\sqrt{10}}$. **5.3.7.** $5x+12y+64=0$; $5x+12y-66=0$. **5.3.8.** $7x-$ $-2y+57=0$; $7x-2y-49=0$. **5.3.9.** $\frac{1}{\sqrt{58}}$. **5.3.10.** $\left(-\frac{3}{10};0\right)$, $\left(0;\frac{9}{2}\right)$. **5.3.11.** $(5;5)$, $(-3;11)$, $(3;19)$ va $(11;13)$. **5.3.12.** $(-12;5)$. **5.3.13.** $x+4=0$, $3x-4y+20=0$. **5.3.14.** $x+2y\pm 5=0$. **5.3.15.** $(3\pm\sqrt{3})x+4y=0$. **5.3.16.** $x+4y+1=0$; $13x+16y-23=0$. **5.3.17.** $x=3-4t$, $y=-5+2t$. **5.3.18.** $x=-6+7t$, $y=-4-$ $-3t$. **5.3.19.** $x=3+3t$, $y=5t$. **5.3.20.** $(2;0)$ va $(-1;7)$. **5.3.21.** $M_1(10;-5)$.**5.3.22.** $3x-4y+12=0$. **5.3.23.** $(2;-7)$. **5.3.24.** $M'(2;3)$. **5.3.25.** $2x-y+3=0$ to‘g‘ri chiziq $M_1M_3$ tomonga parallel va $M_1M_2$, $M_3M_2$ tomonlarning davomoni $M_2$ nuqtadan keyin kesib o‘tadi. **5.3.26.** $(8;1)$ va $\left(\frac{888}{49};\frac{465}{49}\right)$. **5.3.30.** $4x-3y-1=0$; $6x-$ $-8y+9=0$. **5.4.1.** $S=17$ kv birlik. **5.4.2.** $C_1(-1;4)$ yoki $C_2\left(\frac{25}{7};-\frac{36}{7}\right)$. **5.4.3.** $C_1(1;-1)$ yoki $C_2(-2;-10)$. **5.4.4.** $C(2;4)$. **5.4.5.** $2x+7y+22=0$; $7x+2y-13=0$; $x-y+2=0$. **5.4.6.** $M_1$ va $M_8$ nuqtalar bir to‘g‘ri chiziqda yotadi. Qolgan nuqtalar bir to‘g‘ri chiziqda yotmaydi. $M_2$, $M_4$, $M_5$, $M_6$ nuqtalar to‘g‘ri chiziqdan bir tomonda, $M_3$, $M_7$ nuqtalar esa ikkinchi tomonda yotadi. **5.4.7.** $(1;-3)$, $(-2;5)$, $(5;-9)$, va $(8;-17)$. **5.4.8.** $3x+2y=0$; $2x-3y-13=0$. **5.4.9.** $(2;1)$, $(4;2)$, $(-1;7)$. **5.4.10.** $2x-5y+3=0$; $2x-5y-$ $-26=0$; $7x-3y-33=0$. **5.4.11.** $P(2;5)$. **5.4.12.** $4x+3y-11=$ $=0$; $x+y+2=0$; $3x+2y-13=0$. **5.4.13.** $(3;4)$. **5.4.14.** $m=\frac{7}{12}$. **5.4.17.** $3x-4y+15=0$; $4x+3y-30=0$; $3x-4y-$ $-10=0$; $4x+3y-5=0$. **5.4.18.** $A$, $B$ va $C$ nuqtalar parallel to‘g‘ri chiziqlar orasidagi sohaga; $D$ va $F$ nuqtalar parallel to‘g‘ri chiziqlar hosil qilgan bir tashqi sohaga, $E$ nuqta boshqa tashqi sohaga tegishli. **5.4.19.** $A$ nuqta 2- tomonning 3-uchdan keyingi davomida yotadi. $B$ nuqta 1 - tomon, 2 - va 3 - tomonlarning mos ravishda 3 - va 2 - uchlaridan keyingi davomlari bilan chegaralangan sohada yotadi. $C$ nuqta 3-tomon, 1- va 2- tomonlarning mos ravishda 2- va 1- uchlaridan keyingi davomlari bilan chegaralangan sohada yotadi. $D$ nuqta 1- va 2- tomonlarni 3- uchdan keyingi davomlari bilan chegaralangan sohada yotadi. Bu natijalarni yasash usuli bilan tekshirib ko‘rish tavsiya etiladi. **5.4.20.** 1) $6x+1=0$; $2y-9=0$; 2) $64x+8y+11=0$; $14x-$ $-112y+41=0$; 3) $x=0, y=0$; 4) $(3+\sqrt{5})x+2(2+\sqrt{5})y=0$, $(3-\sqrt{5})x+2(2-\sqrt{5})y=0$. **5.4.21.** $(0;6)$, $\left(-1;\frac{13}{2}\right)$. **5.4.22.** $4x+$ $+3y+3=0$, $y+1=0$. **5.4.23.** $3x-y+9=0$; $3x-y-3=0$; $x+3y+7=0$. **5.4.24.** $\left(\frac{5}{12},-\frac{5}{12}\right)$. **5.4.25.** $7x+y+18=0$. **5.4.26.** $y=0$; $y=2\sqrt{3}$; $y=\sqrt{3}x+5\sqrt{3}$; $y=-\sqrt{3}x+5\sqrt{3}$. **5.4.27.** 0,5. **5.4.28.** $\left(\frac{11}{7},2\right),\left(-\frac{1}{7},0\right)$. **5.4.29.** $(19;0)$, $(21;5)$. **5.4.30.** $17x-7y+49=$ $=0$; $7x-3y+23=0$; $2x-y+7=0$ yoki $11x-7y+49=0$; $5x-3y+19=0$; $2x-y+7=0$.

**6.1.1.** 1) $x+24y+14z-60=0$; 2) $5x-2y-9z+5=0$; 3) $x+$ $+24y-14z+29=0$; 4) $14x+31y-13z-21=0$; 5) $x+24y-$ $-14z+44=0$; 6) $5x-2y-9z+37=0$. **6.1.2.** 1) $4x+2y+$ $+7z-7=0$; 2) $2x+2y+4z-6=0$; 3) $x-18y+11z+63=0$; 4) $4x+23y-13z-71=0$; 5) $x-14y+9z+49=0$; 6) $2x+$ $+17y-12z-52=0$. **6.1.3.** 1) $14x+9y+8z-89=0$; 2) $42x-$ $-3y-12z-129=0$; 3) $16x+5y-12z-107=0$; $28x-6y+$ $+4z-118=0$; 5) $52x-4y-14z-162=0$; 6) $32x-14y-$ $-24z-132=0$. **6.1.4.** 1) $z-1=0$; 2) $x+3=0$; 3) $y-2=0$; 4) $z-3=0$; 5) $x-2=0$; 6) $y+4=0$. **6.1.5.** 1) $x-4=0$; 2) $z+$ $+1=0$; 3) $x+2=0$; 4) $y-4=0$; 5) $y+2=0$; 6) $z-2=0$. **6.1.6.** 1) $x-1=0$; 2) $y+2=0$; 3) $z-3=0$; 4) $x-2=0$; 5) $y-$ $-4=0$; 6) $y-4=0$. **6.1.7.** 1) $-6y+z+13=0$; 2) $2y-7z+$

$+29=0$; 3) $3x+z-8=0$; 4) $4x+z-11=0$; 5) $x+2y-7=0$; 6) $7x+y-10=0$. **6.1.8.** 1) $x+2y+z-9=0$, 2) $x+y-2=0$. **6.1.9.** $14x-10y+33z-70=0$. **6.1.10.** $x+y+z-1=0$. **6.1.11.** $7x+7y-6z-50=0$. **6.1.12.** $35x+21y-15z-105=0$. **6.1.13.** $a=4,\ \ b=-4,\ \ c=\frac{4}{7}$. **6.1.14.** $27x+11y+z-65=0$. **6.1.15.** 1) $x-4y-z+16=0$; 2) $x+5y-z+5=0$. **6.1.16.** 1) kesishadi; 2) kesishadi; 3) parallel; 4) kesishadi; 5) ustma –ust tushadi. **6.1.17.** **6.1.18**. $(ABC)=4/39$. **6.1.19.** $(12;0;0)$; $(0;-8;0)$; $(0;0;-6)$. **6.1.20.** $a==-4$; $b=3$; $c=1/2$. **6.1.21.** $S=240$ kv birlik. **6.1.22.** $V=8$ kub birlik. **6.1.23.** $\frac{x}{-3}+\frac{y}{-4}+\frac{z}{2}=1$. **6.1.24.** 1 va 4 berilgan tekislik tenglamalari normal hisoblanadi. **6.1.25.** 1) $\frac{2}{3}x-\frac{2}{3}y+\frac{1}{3}z-6=0$; 2) $-\frac{3}{7}x+\frac{6}{7}y-\frac{2}{7}z-3=0$; 3) $\frac{2}{7}x-\frac{3}{7}y-\frac{6}{7}z-\frac{11}{14}=0$; 4) $\frac{2}{3}x++\frac{2}{3}y-\frac{1}{3}z-\frac{1}{6}=0$; 5) $-\frac{5}{13}y+\frac{12}{13}z-2=0$; 6) $\frac{3}{5}x-\frac{4}{5}y+\frac{1}{5}=0$. **6.1.26.** $d=4$. **6.1.27.** 1) $d=2$; 2) $d=3{,}5$; 3) $d=6{,}5$. **6.1.28.** 1) $4x--y-2z-4=0$; 2) $3x+2y-z+1=0$; 3) $20x-12y+4z++13=0$. **6.1.29.** $2x-2y-z-18=0$; $2x-2y-z+12=0$. **6.1.30.** 1)$23x-2y+21z-33=0$; 2) $y+z-18=0$; 3) $x+z--3=0$; 4) $x-y+15=0$. **6.2.1.** 1. a) $\frac{x-2}{4}=\frac{y-3}{1}=\frac{z-1}{2}$; b)$\begin{cases}x=4t+2\\ y=t+3\\ z=2t+1\end{cases}$; 2. a)$\frac{x-3}{4}=\frac{y-1}{1}=\frac{z-4}{2}$; b) $\begin{cases}x=4t+3\\ y=t+1\\ z=2t+4\end{cases}$; 3. a) $\frac{x-2}{-2}=\frac{y-3}{3}=\frac{z-1}{5}$; b) $\begin{cases}x=-2t+2\\ y=3t+3\\ z=5t+1\end{cases}$; 4. a) $\frac{x-3}{-2}=\frac{y-1}{3}=\frac{z-4}{5}$; b) $\begin{cases}x=-2t+3\\ y=3t+1\\ z=5t+4\end{cases}$; 5. a) $\frac{x-2}{5}=\frac{y-3}{-1}=\frac{z-1}{3}$ b) $\begin{cases}x=5t+2\\ y=-t+3\\ z=3t+1\end{cases}$;6. a)$\frac{x-3}{5}=\frac{y-1}{-1}=\frac{z-4}{3}$; b) $\begin{cases}x=5t+3\\ y=-t+1\\ z=3t+4\end{cases}$. **6.2.2.** 1. a) $\frac{x-2}{1}=\frac{y-3}{-2}=\frac{z-1}{3}$; b) $\begin{cases}x=t+2\\ y=-2t+3\\ z=3t+1\end{cases}$; 2. a)

$\frac{x-2}{0}=\frac{y-3}{-2}=\frac{z-1}{4}$; b) $\begin{cases} x=2 \\ y=-2t+3; \\ z=4t+1 \end{cases}$ 3. a) $\frac{x-2}{-2}=\frac{y-3}{-1}=\frac{z-1}{-2}$;

b) $\begin{cases} x=-2t+2 \\ y=-t+3 \\ z=-2t+1 \end{cases}$; 4. a) $\frac{x-3}{-1}=\frac{y-1}{0}=\frac{z-4}{1}$; b) $\begin{cases} x=-t+3 \\ y=1 \\ z=t+4 \end{cases}$;

5. a) $\frac{x-3}{-3}=\frac{y-1}{1}=\frac{z-4}{-5}$;

b) $\begin{cases} x=-3t+2 \\ y=t+1 \\ z=-5t+4 \end{cases}$; 6. a) $\frac{x-2}{-2}=\frac{y-1}{1}=\frac{z-5}{-6}$; b) $\begin{cases} x=-2t+2 \\ y=t+1 \\ z=-6t+5 \end{cases}$. **6.2.3.** 1) $\frac{13x+12}{-17}=\frac{13y+44}{7}=\frac{z}{1}$; 2) $\frac{4x+7}{-4}=\frac{8y-15}{12}=\frac{z}{1}$; 3) $\frac{18x-35}{16}=\frac{18y-73}{47}=\frac{z}{1}$; 4) $\frac{11x-3}{5}=\frac{11y+21}{-2}=\frac{z}{1}$. **6.2.4.** 1) $z-1=0$; 2) $x-2=0$; 3) $y-1=0$; 4) $x-2=0$; 5) $z-5=0$; 6) $y-1=0$. **6.2.5.** 1) $x+y-3=0$; 2) $x+z+4=0$; 3) $y+z-1=0$; 4) $x+y+6=0$; 5) $x+z+$ $+7=0$; 6) $y+z-7=0$. **6.2.6.** 1) Bir to‘g‘ri chiziqda yotadi; 2) uchburchak hosil qiladi; 3) bir to‘g‘ri chiziqda yotadi. **6.2.7.** $A$, $B$, $D$ to‘g‘ri chiziqda yotadi; $C$ va $E$ yotmaydi. **6.2.8.** 1) $x=-2t$; $y=7t$; $z=4t$.2) $x=t$; $y=-8-4t$; $z=-3-3t$. **6.2.9.** 1) $x=3+4t$; $y=5-3t$; $z=1$. 2) $x+2y+10=0$; $z-4=0$. **6.2.10.** 1) $11x-$ $-4y+6=0$; $z=0$; 2) $6x+5y-38=0$; $z=0$. **6.2.11.** 1) $(-1;7{,}5;0)$, $(2;0;3)$, $(0;5;1)$; 2) $(6;-2;0)$, **6.2.12.** $\left(-\frac{x_2z_1}{z_2-z_1},\frac{y_1z_2}{z_2-z_1},0\right)$.

**6.2.13.** $(2;-1;0)$; $(1\frac{1}{3};0;-\frac{1}{3})$; $(0;2;-1)$. **6.2.14.** $5x+5z-8=0$. **6.2.15.** $\alpha(5x-y-z-3)+\beta(x+3y-2z+5)=0$. **6.2.16.** $x-$ $-9y+5z+20=0$, $x-2y-5z+9=0$. **6.2.17.** 1) To‘g‘ri chiziq bilan tekislik $(0;0;-2)$ nuqtada kesishdi; 2) to‘g‘ri chiziq tekislikka parallel; 3) to‘g‘ri chiziq tekislikda yotadi; 4) to‘g‘ri chiziq bilan tekislik $(2;3;1)$ nuqtada kesishadi. **6.2.18.** 1) $\cos\alpha=\frac{4}{13}$, $\cos\beta=-\frac{3}{13}$, $\cos\gamma=\frac{12}{13}$; 2) $\cos\alpha=\frac{12}{25}$, $\cos\beta=\frac{9}{25}$, $\cos\gamma=\frac{20}{25}$. **6.2.19.** $\cos\varphi=\pm\frac{72}{77}$. **6.2.20.** $\cos\varphi=\pm\frac{98}{195}$. **6.2.21.** 1) $\pm\frac{7}{2\sqrt{91}}$; 2) $\pm\frac{9}{\sqrt{2}\sqrt{66}}$. **6.2.23.** $\arcsin\frac{33}{\sqrt{46}\sqrt{62}}$. **6.2.24.**

$\arcsin\frac{1}{10\sqrt{19}}$. **6.2.25** $x - z + 4 = 0,\ \ y = 0$. **6.2.26.** $(0; -3; 5)$ va $\left(\frac{98}{23}, -\frac{363}{23}, -\frac{375}{23}\right)$. **6.2.27.** 1) $9x + 10y - 7z - 58 = 0$ **6.2.30.** $4x + {}$ $+3z = 0,\ y + 2z + 9 = 0$. **6.3.1.** 1)$\pm\frac{20}{\sqrt{14}\sqrt{53}}$; 2) tekisliklar o‘zaro perpendikulyar.**6.3.2.** $(-2; 1; 4)$. **6.3.3.** $3x + 2y + 4z - 38 = 0$.**6.3.4.** 1) Uch tekislik $(3; 5; 7)$ nuqtada kesishadi; 2) uch tekislik juft- jufti bilan parallel; 3) uch tekislik bitta to‘g‘ri chiziqdan o‘tadi; 4) tekisliklar juft – jufti bilan kesishadi va ikkata tekislikninfg kesishish chizig‘i uchinchi tekislikka parallel. **6.3.5.** $6x + 9y - 22z = 0$. **6.3.6.** $20x + 19y - 5z + 41 = 0$. **6.3.7.** $2x - 2y - 2z - 1 = 0$. **6.3.8.** $3x + 5y - {}$ $-4z + 25 = 0$. **6.3.9.** 1) $10x - 7z = 0$; 2) $6y - 7 = 0$;3) $39x - 29y - 7z = 0$.**6.3.10.** $5x - 13y - 12z + 20 = 0$; $2x - 2y + +3z - 5 = 0$. **6.3.11.** $\frac{x-3}{5} = \frac{y+2}{3} = \frac{z-4}{-7}$.**6.3.12.**$(7; 1; 0)$. **6.3.13**. $4x + +5y - 2z = 0$. **6.3.14.** $(2; 9; 6)$. **6.3.15.** $y - 2z = 0$; $x = 3$. **6.3.16.** $y + 2z - 8 = 0$; $x + 2y - z + 5 = 0$. **6.3.17.** 1)Bir tomonda; 2)Bir tomonda;3)Har xil tomanda; 4)Bir tomonda. **6.3.20.** $11x - 2y - {}$ $-15z - 3 = 0$. **6.3.21.** $\alpha(5x - y - z - 3) + \beta(3x - 2y - 5z + 2) = {}$ $= 0$. **6.3.22.** $9x + 7y + 8z + 7 = 0$. **6.3.23.** $\arcsin\frac{33}{\sqrt{46}\sqrt{62}}$. **6.3.24.** $\arcsin\frac{1}{10\sqrt{19}}$. **6.3.25.** $x = \frac{3}{7},\ z = \frac{18}{7}$. **6.3.26.** Bunday to‘g ‘ri chiziq mavjud emas. **6.3.27.** $7x + y - 3z = 0$.**6.3.28.** $x = x_0 + At$; $y = y_0 + Bt$; $z = z_0 + Ct$. **6.3.29.** 1) $\alpha = 60^0$, $\beta = 45^0$, $\gamma = 60^0$, $p = 5$; 3) $\alpha = 45^0$, $\beta = 90^0$, $\gamma = 45^0$, $p = 3\sqrt{2}$; 4) $\alpha = 90^0, \beta = 135^0$, $\gamma = 45^0, p = \sqrt{2}$. **6.3.30.** 1)matritsaning rangi 3 ga teng;

**7.1.2.** 1) $S(3; 0), r = 3$; 2) $S(-3; 4), r = 5$; 3) $S(5; -12), r = 15$; 4) $S\left(-1; \frac{2}{3}\right),\ r = \frac{4}{3}$; 5) $(x-1)^2 + (y+2)^2 - 5 = 0$; 6) $(x - \frac{1}{3})^2 + {}$ $+(y + \frac{7}{6})^2 - \frac{41}{36} = 0$. **7.1.3.** $A, C, D$ nuqtalar aylana tashqarisida, $B$ nuqta aylanada yotadi. **7.1.4.** 1) Izlangan nuqtalar markazi $S(1; 3)$ nuqtada va radiusi 5 ga teng aylanada, yoki uning tashqarisida yotadi;

2) nuqtalar markazi $(1;-3)$ nuqtada va radiuslari 4 va 5 ga teng konsentrik aylanalarda, yoki bu aylanalar orasida yotadi; 3) markazlari $S(1;2), S(4;6)$ nuqtalarda bo‘lgan va radiuslari mos ravishda 5 va 3 ga teng bo‘lgan doiralarning umumiy qismiga va chegaralariga tegishli; $x^2+y^2-4y=0x=\pm 1$ **7.1.5.** $(x-6)^2+(y-5)^2=25$.**7.1.6.** $(x-2)^2+(y-3)^2-\frac{9}{5}=0$. **7.1.8.** $(x—3)^2+(y-2)^2--26=0$, $(x+3)^2+(y-6)^2-26=0$.**7.1.9.** 1)$x^2+y^2=9$; 2)$(x-2)^2+(y+3)^2=49$; 3) $(x-6)^2+(y+8)^2=100$; 4) $(x+1)^2+(y-2)^2=25$; 5) $(x-1)^2+(y-4)^2=8$; 6) $x^2++y^2=16$; 7) $(x-1)^2+(y+1)^2=4$; 8) $(x-2)^2+(y-4)^2=10$ 9) $(x-1)^2+y^2=1$; 10) $(x-2)^2+(y-1)^2=25$.**7.1.10.** $(x+2)^2+(y+1)^2=20$. **7.1.11.** 1) $x+5y-3=0$; 2) $x+2=0$; 3) $3x-y-9=0$; 4) $y+1=0$. **7.1.12.** $x^2+y^2+6x-9y-17==0$. **7.1.13.** $7x-4y=0$. **7.1.14.** 1)$\frac{x^2}{25}+\frac{y^2}{16}=1$;. 2)$\frac{x^2}{25}+\frac{y^2}{9}=1$;. 3)$\frac{x^2}{169}+\frac{y^2}{25}=1$ **7.1.15.** $(\pm 3;0)$. **7.1.16.** $(0;\pm 12)$. **7.1.17.** 1) ichki; 2) ichki; 3) tashqi; 4) tashqi; 5)elepsga tegishli. **7.1.18.** $3x^2+5y^2=32$. **7.1.19.** $3x^2++2xy+3y^2-4x-4y=0$. **7.1.20.** $\frac{x^2}{16}+\frac{y^2}{7}=1$. **7.1.21.** $x=\pm 9$. **7.1.22.** $\frac{x^2}{32}+\frac{y^2}{16}=1$. **7.1.23.** $\left(-\frac{15}{2};\pm\frac{3\sqrt{7}}{2}\right)$.**7.1.24.** 1) $\frac{x^2}{25}+\frac{y^2}{4}=1$; 2) $\frac{x^2}{25}++\frac{y^2}{9}=1$; 3) $\frac{x^2}{169}+\frac{y^2}{144}=1$; 4) $\frac{x^2}{25}+\frac{y^2}{16}=1$; 5)$\frac{x^2}{100}+\frac{y^2}{64}=1$; 6) $\frac{x^2}{169}++\frac{y^2}{25}=1$; 7) $\frac{x^2}{5}+5y^2=1$; 8) $\frac{x^2}{16}+\frac{y^2}{12}=1$; 9) $\frac{x^2}{13}+\frac{y^2}{9}=1$ yoki $\frac{x^2}{\frac{117}{4}}+\frac{y^2}{9}=1$; 10) $\frac{x^2}{64}+\frac{y^2}{48}=1$.**7.1.25.** 1) $\frac{x^2}{4}+\frac{y^2}{9}=1$; 2) $\frac{x^2}{9}+\frac{y^2}{25}=1$; 3) $\frac{x^2}{25}+\frac{y^2}{169}=1$; 4) $\frac{x^2}{64}+\frac{y^2}{100}=1$; 5) $\frac{x^2}{16}+\frac{y^2}{25}=1$; 6) $\frac{x^2}{7}+\frac{y^2}{16}=1$. **7.1.26.** 1) 4 va 3; 2) 2 va 1; 3) 5 va 1; 4) $\sqrt{15}$ va $\sqrt{3}$; 5) $\frac{5}{2}$ va $\frac{5}{3}$; 6) $\frac{1}{3}$ va $\frac{1}{5}$; 7) 1 va $\frac{1}{2}$; 8) 1 va 4; 9) $\frac{1}{5}$ va $\frac{1}{3}$; 10) $\frac{1}{3}$ va 1. **7.1.27.** 1) 5 va 3; 2) $F_1(-4;0)$, $F_2(4;0)$; 3) $\varepsilon=\frac{4}{5}$; 4) $x=\pm\frac{25}{4}$. **7.1.28.** 1) $\sqrt{5}$ va 3; 2)$F_1(0;-2)$, $F_2(0;-2)$; 3)$\varepsilon=$

$\frac{2}{3}$; 4) $y = \pm\frac{9}{2}$. **7.1.30.** 1) $\frac{x^2}{36}+\frac{y^2}{9}=1$; 2) $\frac{x^2}{16}+\frac{y^2}{\frac{16}{3}}=1$; 3) $\frac{x^2}{20}+\frac{y^2}{15}=1$; 4) $\frac{x^2}{20}+\frac{y^2}{4}=1$; 5) $\frac{x^2}{9}+\frac{y^2}{5}=1$; 6) $\frac{x^2}{256}+\frac{y^2}{192}=1$; 7) $\frac{x^2}{15}+\frac{y^2}{6}=1$.

**7.2.1.** A – ichki, B- tashqi, C- giperbola nuqtasi. **7.2.2.** 1) $\frac{x^2}{25}-\frac{y^2}{9}=1$; $\frac{x^2}{16}-\frac{y^2}{9}=1$. **7.2.3.** 1) $\frac{x^2}{576}-\frac{y^2}{100}=1$; $\frac{x^2}{64}-\frac{y^2}{36}=1$. **7.2.4.** $\sqrt{2}$. **7.2.5.** $\frac{x^2}{432}-\frac{y^2}{75}=1$. **7.2.6.** $F_1(-13;0)$, $F_2(13;0)$. **7.2.7.** $F_1(0;17)$, $F_2(0;-17)$. **7.2.8.** 1) $\frac{x^2}{16}-\frac{y^2}{9}=1$; 2) $\frac{x^2}{9}-\frac{y^2}{3}=1$. **7.2.9.** $\frac{x^2}{16}-\frac{y^2}{9}=1$. **7.2.10.** 1) $F_1(5;0)$, $F_2(-5;0)$; $e=\frac{5}{3}$; $y=\pm\frac{4}{3}x$, $x=\pm\frac{9}{5}$; $\frac{y^2}{16}-\frac{x^2}{9}=1$, $e=\frac{5}{4}$. **7.2.11.** 1) $a=2\sqrt{3}$, $b=2$; 2) $a=b=6$;3) $a=\sqrt{5}$, $b=2\sqrt{5}$; 4) $a=\frac{3\sqrt{19}}{5}$, $b=\sqrt{19}$**7.2.12.** $\frac{x^2}{10}-\frac{y^2}{6}=1$. **7.2.13.** 1) $\frac{x^2}{25}-\frac{y^2}{16}=1$; 2) $\frac{x^2}{9}-\frac{y^2}{16}=1$ 3) $\frac{x^2}{4}-\frac{y^2}{5}=1$; 4) $\frac{x^2}{64}-\frac{y^2}{36}=1$; 5) $\frac{x^2}{36}-\frac{y^2}{64}=1$; 6) $\frac{x^2}{144}-\frac{y^2}{25}=1$; 7) $\frac{x^2}{16}-\frac{y^2}{9}=1$; 8) $\frac{x^2}{4}-\frac{y^2}{5}=1$; 9) $\frac{x^2}{64}-\frac{y^2}{36}=1$.

**7.2.14.** 1) $\frac{x^2}{36}-\frac{y^2}{324}=-1$; 2) $\frac{x^2}{16}-\frac{y^2}{9}=-1$ 3) $\frac{x^2}{100}-\frac{y^2}{576}=-1$; 4) $\frac{x^2}{24}-\frac{y^2}{25}=-1$; 5) $\frac{x^2}{9}-\frac{y^2}{16}=-1$. **7.2.15.** 1) $a=3$, $b=2$; 2) $a=4$, $b=1$; 3) $a=4$, $b=2$; 4) $a=1$, $b=1$; 5) $a=\frac{5}{2}$, $b=\frac{5}{3}$; 6) $a=\frac{1}{5}$, $b=\frac{1}{4}$; 7) $a=\frac{1}{3}$, $b=\frac{1}{8}$.

**7.2.16.** 1) $a=3$, $b=4$; 2) $F_1(-5;0)$, $F_2(5;0)$; 3) $\varepsilon=\frac{5}{3}$; 4) $y=\pm\frac{4}{3}x$; 5)$x=\pm\frac{9}{5}$. **7.2.17.** 1) $a=3$, $b=4$; 2) $F_1(0;-5)$, $F_2(0;5)$; 3) $\varepsilon=\frac{5}{4}$; 4) $y=\pm\frac{4}{3}x$; 5) $y=\pm\frac{16}{5}$. **7.2.18.** 12. **7.2.20.** $x-4\sqrt{5}y+10=0$ yoki $x-10=0$. **7.2.21.** $r_1=2\frac{1}{4}$; $r_2=10\frac{1}{4}$. **7.2.22.** $\left(10;\frac{9}{2}\right)$ va $\left(10;-\frac{9}{2}\right)$. **7.2.23.** $(-6;4\sqrt{3})$ va $(-6;-4\sqrt{3})$. **7.2.24.** 1) $\frac{x^2}{32}-\frac{y^2}{8}=1$; 2) $x^2-y^2=16$; 3) $\frac{x^2}{18}-\frac{y^2}{8}=1$; 4) $\frac{x^2}{4}-\frac{y^2}{5}=1$ yoki $\frac{x^2}{\frac{61}{9}}-\frac{y^2}{\frac{305}{16}}=1$; 5)

$\frac{x^2}{16}-\frac{y^2}{9}=1$. **7.2.25.** $\varepsilon=\sqrt{2}$. **7.2.26.** $\frac{x^2}{4}-\frac{y^2}{12}=1$. **7.2.27.** $\frac{x^2}{60}-\frac{y^2}{40}=1$. **7.2.30.** 1) $\frac{(x-x_0)^2}{a^2}-\frac{(y-y_0)^2}{b^2}=1$; 2) $\frac{(x-x_0)^2}{a^2}-\frac{(y-y_0)^2}{b^2}=-1$. **7.3.4.** $(1;0)$. **7.3.5.** $(0;1)$. **7.3.6.** $(-2;0)$. **7.3.7.** $x=-\frac{3}{2}$. **7.3.8.** $y^2=12x$. **7.3.9.** $y^2=4x$. **7.3.10.** $y^2=8x-8$. **7.3.11.** $y^2=\pm12x$. **7.3.12.** $y^2=10x-25$. **7.3.13.** $y^2=16x$. **7.3.14.** $x^2=8y$. **7.3.15.** $x^2=-18y$. **7.3.16.** $(18;12)$, $(18;-12)$. **7.3.17.** $y^2=4x$. **7.3.18.** $y^2=-9x$. **7.3.19.** $x^2=y$. **7.3.20.** $x^2=-2y$. **7.3.21.** $x^2=-12y$. **7.3.22.** $F(6;0)$, $x+6=0$. **7.3.25.** 12. **7.3.26.** 6. **7.3.27.** $(9;12)$, $(9;-12)$. **7.3.30.** $y^2=-28x$.
**8.1.2.** 1), 2), 3) markazlari qutbda va radiuslari mos ravishda 1,5 va $a$ ga teng bo‘lgan aylanalarda. 4), 5), 6), 7) qutbdan chiquvchi va qutb o‘qi bilan $30^0$, $60^0$, $90^0$ va $\varphi$ burchaklar tashkil etuvchi nurlarda joylashgan. **8.1.3.** 1) $\left(1;\frac{5\pi}{4}\right)$, $\left(3;\frac{5\pi}{3}\right)$, $\left(\frac{2}{3};\frac{5\pi}{6}\right)$, 2) $(\rho;\ \varphi+\pi)$; 3) $\left(1;\frac{7\pi}{4}\right)$, $\left(3;\frac{4\pi}{3}\right)$, $\left(\frac{2}{3};\frac{\pi}{6}\right)$, 4) $(\rho;\ 2\pi-\varphi)$. **8.1.4.** $(a;0)$, $\left(a\sqrt{3};\frac{\pi}{6}\right)$, $\left(2a;\frac{\pi}{3}\right)$, $\left(a\sqrt{3};\frac{\pi}{2}\right)$, $\left(a;\frac{2\pi}{3}\right)$. $\left(0;\frac{0}{0}\right)$. Izoh. Qutb nolga teng radius – vektorga va noma’lum amplitudaga ega. **8.1.5.** Jadvalga qaralsin.

| $\varphi$ | $0^0$ | $15^0$ | $30^0$ | $45^0$ | $60^0$ | $75^0$ | $90^0$ |
|---|---|---|---|---|---|---|---|
| $2\varphi$ | $0^0$ | $30^0$ | $60^0$ | $90^0$ | $120^0$ | $150^0$ | $180^0$ |
| $\rho=a\cdot sin2\varphi$ | 0 | $\frac{a}{2}$ | $\frac{a\sqrt{3}}{2}$ | $a$ | $\frac{a\sqrt{3}}{2}$ | $\frac{a}{2}$ | 0 |

**8.1.6.** $AB=\sqrt{3}, CD=10, EF=5$. **8.1.7.** $AB=BC=CA=7$. **8.1.8.** $M_1(1;0)$ va $M_2(7;0)$. **8.1.9.** $S=\frac{1}{2}\rho_1\rho_2\cdot\sin(\varphi_2-\varphi_1)$. Ko‘rsatma. Uchburchakning yuzi uchun trigonometrik $S=\frac{a\cdot b\cdot sinC}{2}$ formuladan foydalanamiz. **8.1.10.** $S=1$ kv birlik. **8.1.11.** $S=6\left(5\sqrt{3}-3\right)$ kv birlik. Ko‘rsatma. Shaklni yasang va izlanayotgan yuzni, bir uchi qutbda bo‘lgan $OAB$, $OBC$ va $OAC$ uchburchakning yuzlari orqali hisoblang. **8.1.12.** 1) $\rho=a$; 2) $\rho=2a\cdot cos2\varphi$;

3)$\rho^2 — 2\rho_1\rho\cos(\varphi - \varphi_1) = a^2 - \rho_1^2$. **8.1.13.** $\rho^2 = \frac{b^2}{1-e^2cos^2\varphi}$. **8.1.14.** $\varphi = \arccos\left(\pm\frac{4}{5}\right)$. **8.1.15.** 1) $\rho = \frac{P}{1-ecos\varphi}$; 2) $\rho = \frac{P}{1+ecos\varphi}$, bunda 1) $P = \frac{b^2}{a}$ miqdor ellipsning parametri deyiladi. **8.1.16.** $a = 2\sqrt{2}$; $b = \sqrt{6}$; $2c = 2\sqrt{2}$. **8.1.17.** $\rho^2 = \frac{-b^2}{1-e^2cos^2\varphi}$. **8.1.18.** Ichiga giperbola joylashgan burchaklardan biri $\theta$ bilan belgilansa, $\theta = \frac{2\pi}{3}$. **8.1.19.** $\rho = \frac{P}{1-ecos\varphi}$, bunda $P = \frac{b^2}{a}$. **8.1.20.** Asimptotalarning tenglamalari: $\rho = -\frac{2}{\sin\left(\varphi - \frac{\pi}{4}\right)}$ va $\rho = -\frac{2}{\sin\left(\varphi - \frac{3\pi}{4}\right)}$; direktrisalar tenglamalari: $\rho = -\frac{\sqrt{2}}{\cos\varphi}$ va $\rho = -\frac{3\sqrt{2}}{\cos\varphi}$. **8.1.21.** $\rho = \frac{2Pcos\varphi}{sin^2\varphi}$. **8.1.22.** $M\left(3; arccos\frac{1}{3}\right)$b o‘qiga nisbatan simmetrik bo‘lgan ikkita nuqta. **8.1.23.** $\rho = \frac{P}{1-\cos\varphi}$. **8.1.24.** 1) $\left(\frac{P}{2}; \pi\right)$ – parabolaning uchi; 2) ikkita nuqta: $\left(P; \frac{\pi}{2}\right)$ va $\left(P; \frac{3\pi}{2}\right)$. **8.1.25.**$|\delta_1\delta_2| = P^2$. **8.1.26.** 1)$\frac{x^2}{169} + \frac{y^2}{25} = 1$; 2)$y^2 = \frac{2}{3}x$; 3)$\frac{x^2}{16} - \frac{y^2}{9} = 1$; 4) $\frac{x^2}{5} + \frac{y^2}{4} = 1$. **8.1.27.** $x + 3y + 2 = 0$. **8.1.28.**1)$x + 3y + 2 = 0$; 2) $x - 4 = 0$. **8.1.29.**1)$x - 6y + 1 = 0$; 2) $2x + 3y - 3 = 0$. **8.1.30.** $(5; 1)$. **9.1.1.** $5x^2 + 16xy + 5y^2 - 5x -$ $-5y = 0$. **9.1.2.** 1) $O'(1; 1)$; 2) $O'(-1; 2)$; 3) $4x + 2y - 5 = 0$. **9.1.3.** 1) Qarama- qarshi tomonlarining o‘rtalarining birlashtiruvchi to‘g‘ri chiziqlar egri chiziq diametrlari; 2) Qarama- qarshi tomonlarining urinish nuqtalarini tutashtiruvchi egri chiziqlar to‘g‘ri chiziq diametri. **9.1.6.** $3x^2 + 2xy + 2y^2 + 3x - 4 = 0$. Ikkinchi tartibli egri chiziqning umumiy tenglamasini olamiz: $a_{11}x^2 +$ $+2a_{12}xy + a_{22}y^2 + 2a_{13}x + 2a_{23}y + a_{33} = 0$. Agar bu tenglama izlangan egri chiziqni tasvirlasa, berilgan nuqtalarning koordinatalari tenglamani qanoatlantirishi kerak. Berilgan har bir nuqtaning koordinatalarini umumiy tenglamaga qo‘yib, $a_{ik}$ koeffitsiyentlarni bog‘lovchi 5 ta shart hosil qilamiz, bu munosabatlardan beshta koeffitsiyentning 6 - koeffitsiyentga nisbatlarini aniqlaymiz va ularni,oldin oltinchi koeffitsiyentga bo‘lingan umumiy tenglamaga qo‘yamiz. **9.1.7.**$x^2+4xy+4y^2-6x-12y=0$. **9.1.8.** Masalaning shartini ikkita parabola tipidagi egri chiziq qanoatlantiradi: $x^2-8x-$ $-y+15=0$ va $9x^2+6xy+y^2-72x-24y+135=0$. **9.1.9.** $x^2-4xy+3y^2-4y+3=0$. **9.1.10.** $xy+15=0$. **9.1.11.** $x^2-$ $-8y=0$. **9.1.12.** 1) $(7;5)$; 2) $(-1;-1)$; 3) $(0;1)$; 4) markazi bo‘lmagan, parabola tipidagi egri chiziq; 5) egri chiziq $x+y+1=0$ markazlar chizig‘iga ega; 6) $\left(\frac{10}{3};\frac{4}{3}\right)$. **9.1.13.** Agar $a\neq 9$ bo‘lsa, tenglama markaziy chiziqni ifodalaydi. Agar $a=9$ va $b\neq 9$ bo‘lsa, tenglama parabola tipidagi egri chiziqni ifodalaydi.Agar $a=9$; $b\neq 9$ bo‘lsa, egri chiziq $2x+6y+3=0$ markazlar chizig‘iga ega bo‘ladi. *Ko‘rsatma.* Masala markazining koordinatalari aniqlanadigan ikkita tenglama $\begin{cases}2x+6y+3=0\\6x+2ax+b=0\end{cases}$ sistemasini tekshirishga olib keladi. Javob esa bu sistemaning aniq, birgalikda bo‘la olmaydi, yoki aniqmas bo‘lishligiga bog‘liqdir. **9.1.14.** a), b), c)egri chiziqlar koordinatalar boshida markazga ega; d) egri chiziq $3x-2y=0$ markazlar chizig‘iga ega; agarda $\delta\neq 0$ bo‘lsa. e) egri chiziqning markazi koordinatalar boshida bo‘ladi, agarda $\delta=0$ bo‘lsa, egri chiziq $a_{11}x+a_{12}y=0$ ko‘rinishdagi markazlar chizig‘iga ega. **9.1.15.** $2x^2-6xy+5y^2-$ $-11=0$. Ko‘rsatma. Egri chizqning markazi koordinatalar boshi deb olinganda uning tenglamasi $a_{11}x^2+2a_{12}xy++a_{22}y^2+\frac{\Delta}{\delta}=0$ bo‘ladi. Berilgan $(2x^2-6xy+5y^2)$ tenglamaning yuqori hadlarini o‘zgartirmasdan birinchi darajali hadlarini tanlab, yangi tenglamaning ozod hadini topish uchun ikkita $\Delta=-11$ va $\delta=1$ diskriminantni hisoblashimiz kerak. **9.1.16.** a) $7x^2+4xy+4y^2-83=0$; b) $x^2-$ $-2xy+4=0$; c) $6x^2-4xy+9y^2-40=0$. **9.1.17.** $a_{11}(x-x_0)^2+$ $+2a_{12}(x-x_0)(y-y_0)+a_{22}(y—y^0)2+a_{33}=0$. *Ko‘rsatma.* Egri chiziqning markaziga nisbatan yozilgan tenglamasini olib, qaytadan oldingi koordinatalar sistemasiga o‘tish kerak. **9.1.18.** $5x^2-5xy+$ $+2y^2-5x-2y=0$. **9.1.19.** $3x+y=0$ to‘g‘ri chiziq. Markazning koordinatalarini topish uchun tenglamalar tuzib, ularda $a$ parametrni yo‘qotib, izlanayotgan geometrik o‘rinning tenglamasini hosil qilamiz. **9.1.20.** $4x^2-8xy-2y^2+9y-4=0$. Berilgan to‘rtta nuqta orqali cheksiz ko‘p to‘g‘ri chiziqlar o‘tadi; ularning hammasi $2x^2-4\lambda xy+$ $+(4\lambda+1)y^2-4x-(4\lambda+1)y=0$ tenglama bilan ifodalanadi, bunda $\lambda$ –o‘zgaruvchi parametr.**10.1.1.** $17x-4y-4=0$. **10.1.2.** $2x+y+6=0$. **10.1.3.** $4x-6y+1=0$. **10.1.4.** $y=x-1$.**10.1.5.** $7x-y-3=0$.**10.1.6.** $20x-9y-91=0$. **10.1.8.** $\left(\pm\frac{ab}{\sqrt{b^2-a^2}};\pm\frac{ab}{\sqrt{b^2-a^2}}\right)$; masala $b>a$ holdagina o‘rinli. **10.1.9.** $\frac{2b^2}{a}$. **10.1.10.** $\frac{2ab}{\sqrt{a^2+b^2}}$. **10.1.11.** $8x+25y=0$. **10.1.12.** $32x+25y-$ $-89=0$. **10.1.13.** $2p$. **10.1.14.** $y=2x-5$. **10.1.15.** $(2;1)$, $(-6;9)$. **10.1.16.** $(-4;6)$. **10.1.17.** Kesishmaydi. **10.1.18.** $(6;12)$ va $(6;-12)$. **10.1.19.** $(10;\sqrt{30})$, $(10;-\sqrt{30})$, $(2;\sqrt{6})$, $(2;-\sqrt{6})$. **10.1.20.** $(2;1)$, $(-1;4)$, $\left(\frac{3+\sqrt{13}}{2};\frac{7+\sqrt{13}}{2}\right)$ va $\left(\frac{3-\sqrt{13}}{2};\frac{7-\sqrt{13}}{2}\right)$. **10.1.21.** $\left(4;\frac{3}{2}\right)$; $(3;2)$. **10.1.22.** $\left(3;\frac{8}{5}\right)$. **10.1.23.** To‘g‘ri chiziq ellips bilan kesishmaydi. **10.1.24.** 1) To‘g‘ri chiziq ellips bilan kesishadi; 2)To‘g‘ri chiziq ellips bilan kesishmaydi; 3) To‘g‘ri chiziq ellips bilan urinadi. **10.1.25.** 1) $|m|<5$ bo‘lganda to‘g‘ri chiziq ellipsni kesib o‘tadi; 2) $m=\pm 5$ to‘g‘ri chiziq ellipsga urinadi; 3) $|m|>5$ bo‘lganda to‘g‘ri chiziq ellipsni kesib o‘tmaydi. **10.1.26.** $(6;2)$ va $\left(\frac{14}{3};-\frac{2}{3}\right)$. **10.1.27.** $\left(\frac{25}{4};3\right)$ nuqtada urinadi. **10.1.28.** To‘g‘ri chiziqni giperbola bilan kesishmaydi. **10.1.29.** 1) $|m|>4{,}5$ bo‘lganda to‘g‘ri chiziq giperbolani kesib o‘tadi; 2) $m=\pm 4{,}5$ bo‘lganda to‘g‘ri chiziq giperbolaga urinadi; **10.1.30.** $k^2a^2-b^2=m^2$. **10.2.2.** 1) $3x-y+3=0$; $2y+3=0$; 2)$3x-y=$ $=0$; $4y-9=0$.**10.2.3.**$2x+3y-5=0$; $5x+3y-8=0$. **10.2.4.** 1) $7x-35y+22=0$; $7x+14y+20=0$; 2) $6x-2y+19=0$; $2x+2y-1=0$; 3) $3x+4y+14=0$; $x+y-3=0$; 4) $25x-$ $-5y+13=0$; $5y+3=0$. **10.2.5.** 1) $x-4y-2=0$, 2) $x+4y-$ $-3=0$. **10.2.6**. $x+y-1=0$; $3x+3y+13=0$. **10.2.7.** $7x+$

$+1=0$. **10.2.8.** $7x-2y-13=0$; $x-3=0$.**10.2.9.** $\begin{cases} x'=\alpha x+\beta y+\gamma \\ y'=Ax+By+C \end{cases}$ almashtirishda chiziq tenglamasi $x'^2+2y'=0$. ko‘rinishga ega. **10.2.10.** urunma tenglamasi $(a_1+a_{11}x_0+a_{12}y_0)x+(a_2+a_{12}x_0+a_{22}y_0)y++(a_1x_0+a_2y_0+a)=0$ ; bu yerda $a_{11}=\alpha^2$, $a_{12}=\alpha\beta$, $a_{22}=\beta^2$. **10.2.12.** $x+y-1=0$. **10.2.13.** $3x+y-8==0$. **10.2.14.** 1) $x=1$; 2) $5x-2y+3=0$. **10.2.15.** 1) $3x-y\pm\pm3\sqrt{5}=0$; 2) $5x-2y\pm 9=0$. **10.2.16.** $x^2-\frac{y^2}{4}=1$. **10.2.17.** $3x-4y-10=0$; $3x-4y+10=0$. **10.2.18.** $10x-3y-32=0$; $10x-3y+32=0$. **10.2.19.** $x+2y-4=0$; $x+2y+4=0$; $d=\frac{8\sqrt{5}}{5}$. **10.2.20.** $M_1(-6;3)$; $d=\frac{11\sqrt{13}}{11}$. **10.2.21.** $5x-3y-16=0$, $13x+5y+48=0$. **10.2.22.** $2x+5y-16=0$. **10.2.23.** $24x++25y=0$.**10.2.24.** $3x+4y-24=0$. **10.2.25.** 1) $y=4$; 2) $16x--15y-100=0$. **10.2.26.** $x+y\pm 5=0$. **10.2.27.** $\pm3x\pm4y++15=0$. **10.2.28.** $x\pm y\pm 3=0$. **10.2.29.** $arcctg\frac{c^2}{2ab}\le\varphi\le\frac{\pi}{2}$. **10.2.30.** $\frac{\sqrt{a^2+b^2}}{\sqrt{2}}$. **10.2.31.** Yig‘indi $\frac{2(a^2+b^2)}{a}$ ga teng. **10.2.32.**1) $C(2;-3)$, $a=3$, $b=4$, $\varepsilon=\frac{5}{3}$, direktrisa tenglama $5x-1=0$; $5x-19=0$, assimptota tenglamasi: $4x-3y-17=0$; $4x+3y+1=0$; 2) $C(-5;1)$, $a=8$, $b=6$, $\varepsilon=1{,}25$, direktrisa tenglama $x=-11{,}4$ va $x=1{,}4$, assimptota tenglamasi: $3x+4y+11=0$; $3x-4y++19=0$; 3) $C(2;-1)$, $a=3$, $b=4$, $\varepsilon=1{,}25$, direktrisa tenglama $y=-4{,}2$, $y=2{,}2$ asimptota tenglamasi: $4x+3y-5=0$; $4x-3y-11=0$. **10.2.34.** $x-3y+9=0$. **10.2.35.** $y^2=4x$. **10.2.36.** 1) $k<\frac{1}{2}$; 2) $k=\frac{1}{2}$; 3) $k>\frac{1}{2}$. **10.2.37.** $y_1y=p(x+x_1)$. **10.2.38.** $x+y+2=0$. **10.2.39.** $2x-y16=0$. **10.2.40.** $y-p=0$. **11.1.1.** $\frac{(x-1)^2}{5^2}-\frac{(y+2)^2}{3^2}=1$. **11.1.2.** 1) ellips $(\Delta\neq 0;\ \delta>0)$; 2) giperbola $(\Delta\neq 0;\ \delta<0)$; 3) parabola $(\Delta\neq 0;\ \delta=0)$; 4) haqiqiy $(7;5)$ nuqtada kesishadigan mavhum to‘g‘ri chiziqlar $(\Delta=0;\ \delta>0)$;

5)ikkita kesishuvchi haqiqiy to‘g‘ri chiziq ($\Delta = 0$; $\delta < 0$). **11.1.3.** 1) Giperbola ($\Delta = 16$; $\delta = -8$); 2) ellips ($\Delta = -64$; $\delta = 8$); 3) ikkita haqiqiy kesishuvchi to‘g‘ri chiziq ($\Delta = 0$; $\delta = -1$); 4) ikkita haqiqiy kesishuvchi to‘g‘ri chiziq $\left(\Delta = 0;\ \delta = -\frac{81}{4}\right)$; 5) giperbola $\left(\Delta = -\frac{1}{4};\ \delta = -\frac{5}{4}\right)$. **11.1.4.** 1) Koordinata o‘qlariga parallel bo‘lgan ikkita to‘g‘ri chiziq: $x - a = 0$ va $y - b = 0$; 2) ordinatalar o‘qi $x = 0$ va $x - 2y + 5 = 0$ to‘g‘ri chiziq; 3) ikki marta olingan $x - 2y = 0$ to‘g‘ri chiziq; 4) ikki marta olingan $3x + 5y = 0$ to‘g‘ri chiziq; 5) ikkita parallel to‘g‘ri chiziq $2x - 3y + 5 = 0$ va $2x - 3y - 5 = 0$. **11.1.5.** $y + 5 = 0$ va $y = x - 2$. **11.1.6.** 1) $3x - 2y = 0$ va $7x + 5y = = 0$; 2) $x + y + 1 + \sqrt{5} = 0$ va $x + y + 1 - \sqrt{5} = 0$; 3) $y - 5x = 0$ va $x + y - 1 = 0$; 4) $2x - y + 3 = 0$ to‘g‘ri chiziqlar ustma – ust tushadi. **11.1.7.** 1) $x - y = 0$ va $2x + 5y = 0$ qo‘sh to‘g‘ri chiziq; 2) ikki marta olingan $x + 2y = 0$ to‘g‘ri chiziq; 3) $5x - y = 0$ va $2x - -y = 0$ qo‘sh to‘g‘ri chiziq; 4) koordinatalar boshida kesishuvchi qo‘sh mavhum to‘g‘ri chiziq. **11.1.8.** 1) $x^2 - y^2 = 11\sqrt{2}$; $I_1 = 0$; $I_2 = -2$; $I_3 = 44$; 2) $\frac{x^2}{16} - \frac{y^2}{9} = 1$; $I_1 = 7$; $I_2 = -144$; $I_3 = -144^2$; 3)$\frac{x^2}{9} + y^2 = 1$; $I_1 = 10$; $I_2 = 9$; $I_3 = -81$; 4)$\frac{x^2}{4} - \frac{y^2}{9} = 1$; $I_1 = 5$; $I_2 = -36$; $I_3 = 36^2$; 5) $x^2 - \frac{y^2}{9} = 1$; $I_1 = 8$; $I_2 = -9$; $I_3 = 81$. **11.1.9.** 1) $y^2 = 4\sqrt{2}x$; 2) $y = \frac{6}{\sqrt{5}}x$; 3) $y^2 = \frac{3}{2\sqrt{2}}x$; 4) $y^2 = \frac{1}{5\sqrt{5}}x$; **11.1.10.** 1) $15x^2 - y^2 + 3 = 0$; 2) $\frac{x^2}{3} + y^2 = 1$; 3)$y^2 = \sqrt{3}x$. **11.1.11.**$xy = \frac{5}{2}$. **11.1.12.** 1)$xy = 1{,}2$; 2) $xy = \frac{29}{25}$; 3) $xy = \frac{3\sqrt{5}}{2}$. **11.1.13.** $9x^2 - 4y^2 \pm 36x = 0$. *Ko‘rsatma.* Oxirgi had oldidagi ishora koordinatalar boshi giperbolaning qaysi bir uchiga ko‘chirilganligiga bog‘liq. **11.1.14.** 1) $C(3; -1)$, yarim o‘qi 3 va $\sqrt{5}$, $\varepsilon = \frac{2}{3}$, direktrisa tenglamalari $2x - 15 = 0$; $2x + 3 = 0$; 2) $C(-1; 2)$, yarim o‘qi 5 va 4, $\varepsilon = \frac{3}{5}$, direktrisa tenglamalari $3x - 22 = 0$; $3x + 28 = 0$; 3) $C(1; -2)$, yarim o‘qi $2\sqrt{3}$ va 4, $\varepsilon = \frac{1}{2}$, direktrisa tenglamalari $y - 6 = 0$; $y + +10 = 0$. **11.1.15.** 1) $C(2; -3)$, $a = 3; b = 4; \varepsilon = \frac{5}{3}$, direktrisa tenglamalari $5x - 1 = 0$; $5x - 19 = 0$, asimptotasi $4x - 3y - 17 = = 0; 4x + 3y + 1 = 0$; 2) $C(-5; 1), a = 8, b = 6, \varepsilon = 1{,}25$, direktrisa tenglamalari $x = -11{,}4$ va $x = 1{,}4$; asimptotasi $3x + 4y + 11 = 0$; $3x - 4y + 19 = 0$; 3) $C(2; -1)$, $a = 3$, $b = 4$, $\varepsilon = 1{,}25$, direktrisa tenglamalari $y = -4{,}2$, $y = 2{,}2$ asimptotasi $4x + 3y - 5 = 0$; $4x - -3y - 11 = 0$. **11.1.16.** 1)1, 2, 5 va 8 – yagona markazga; 2) 4 va 6- cheksiz ko‘p markazlarga; 3) 3 va 7 markazga ega emas. **11.1.17.** 1) $(3; -2)$; 2) $(0; -5)$; 3) $(0; 0)$; 4) $(-1; 3)$. **11.1.18.** 1) $x - 3y - 6 = 0$; 2) $2x + y - 2 = 0$; 3) $5x - y + 4 = 0$. **11.1.19.** 1) $9x^2 - 18y + +6y^2 + 2 = 0$; 2) $6x^2 + 4xy + y^2 - 7 = 0$; 3) $4x^2 + 6xy + y^2 - -5 = 0$; 4) $4x^2 + 2xy + 6y^2 + 1 = 0$. **11.1.20.** 1) $m \neq 4$, $n -$ har qanday qiymatida; 2) $m = 4$, $n \neq 6$; 3) $m = 4$, $n = 6$. **11.1.21.** 1) Elliptik tenglama; $\frac{x'^2}{9} + \frac{y'^2}{4} = 1$ ellipsni ifodalaydi; $O'(5; -2) -$ yangi koordinatalar sistemasi; 2) Giperbolik tenglama; $\frac{x'^2}{16} - \frac{y'^2}{9} = 1$ giperbolani ifodalaydi; $O'(3; -2) -$ yangi koordinatalar sistemasi; 3) $\frac{x'^2}{4} + \frac{y'^2}{9} = -1$ elliptik tenglamasi, hech qanday geometrik shaklni ifodalamaydi; $O'(5; -2) -$ yangi koordinatalar sistemasi. **11.1.22.** 1) Giperbolik tenglama; $\frac{x'^2}{9} - \frac{y'^2}{4} = 1$ giperbolani ifodalaydi; $tg\alpha = -2$, $cos\alpha = \frac{1}{\sqrt{5}}$, $sin\alpha = -\frac{2}{\sqrt{5}}$; 2) Elliptik tenglama; $\frac{x'^2}{16} + \frac{y'^2}{4} = 1$ellipsni ifodalaydi; $\alpha = 45^0$. **11.1.23.** 1) $\frac{x^2}{30} + \frac{y^2}{30} = 1$ ellipsni ifodalaydi; 2) $9x^2 - 16y^2 = 5$ giperbolani ifodalaydi. **11.1.24.** 1) 3 va 1; 2) 3 va 2; 3) 1 va $\frac{1}{2}$; 4) 3 va 7. **11.1.25.** 1) $x = 2$, $y = 3$; 2) $x = 3$, $y = -3$; 3) $x = 1$, $y = -1$; 4) $x = -2$, $y = 4$. **11.1.26.** 1) 2 va 1; 2) 5 va 1; 3) 4 va 2; 4) 1 va $\frac{1}{2}$. **11.1.27.** 1) $x + y - 1 = 0$, $3x + y + 1 = 0$; 2) $x - -4y - 2 = 0$, $x - 2y + 2 = 0$; 3) $x - y = 0$; $x - 3y = 0$; 4) $x + y -$

$-3=0;\ x+3y+3=0$. **11.1.28.** 1) $y^2=6x$ – parabola. **11.1.29.** 1) 3; 2) 3; 3) $\sqrt{2}$; 4) $\frac{1}{2}\sqrt{10}$. **11.1.30.** 1) $2x+y-5=0,\ 2x+y-1=0$; 2) $2x-3y-1=0,\ \ 2x-3y+11=0$; 3) $5x-y-3=0,\ \ 5x-$ $-y+5=0$.

**13.1.2.** 1) $x-y=0, z=0$; 2) $x+y=0, z=0$. **13.1.3.** 1) $(6;-2;3)$, $r=7$; 2) $(-4;0;0)$, $r=4$; 3) $(1;-2;3)$, $r=6$; 4) $(0;0;3)$, $r=4$. **13.1.4.** **13.1.5.** $l(x-a)m(y-b)+n(z-c)=0$. **13.1.6.** $2x+y+$ $+2z-13=0$. **13.1.7.** $x(x-x_0)+y(y-y_0)+z(z-z_0)=0$ (sfera). **13.1.8.** $x^2+y^2+z^2+R, x=0$ (sfera). **13.1.9.** $(x-a)(x-$ $-x_0)+(y-b)(y-y_0)+(z-c)(z-z_0)=0$. **13.1.10.** $x^2+y^2+$ $+z^2—10z-9=0$. **13.1.11.** $x^2+y^2+z^2+22x+16y-6z=0$. **13.1.12.** $x^2+y^2+z^2+27x+21y-\frac{33}{2}z+10=0$. **13.1.13.** 1) kesib o‘tadi; 2) $\left(-\frac{7}{3};-\frac{4}{3};\frac{7}{3}\right)$ nuqtada urinadi; 3) kesib o‘tmaydi. **13.1.14.** $6x+2y+3z-55=0$. **13.1.16.** $(x_0-a)(x-x_0)+$ $+(y_0-b)(y-y_0)+(z_0-c)(z-z_0)=0$. **13.1.17.** $xx_0+yy_0+$ $+zz_0-R^2=0$. **13.1.18.** $R^2(A^2+B^2+C^2)-D^2=0$. $\left(-\frac{AR^2}{D};-\frac{BR^2}{D};-\frac{CR^2}{D}\right)$ urinish nuqtasi. **13.1.19.** $x^2+y^2+z^2-2x+$ $+4y-4=0$ va $x^2+y^2+z^2-\frac{58}{65}x+\frac{116}{65}y-\frac{114}{65}z-\frac{188}{65}=0$. **13.1.20.** $x^2+y^2+(z+1)^2=12$ va $x^2+y^2+(z+4)^2=27$. **13.1.21.** $\left(\frac{31}{12};\frac{31}{12};\frac{31}{12}\right)$. **13.1.22.** $(l^2+m^2+n^2)R^2=$

$$\begin{vmatrix} a-x_0 & b-y_0 \\ l & m \end{vmatrix}^2+\begin{vmatrix} b-y_0 & c-z_0 \\ m & n \end{vmatrix}^2+\begin{vmatrix} c-z_0 & a-x_0 \\ n & l \end{vmatrix}^2.$$

**13.1.23.**

$$\begin{vmatrix} x-x_1 & y-y_1 \\ l_1 & m_1 \end{vmatrix}^2+\begin{vmatrix} y-y_1 & z-z_1 \\ m_1 & n_1 \end{vmatrix}^2+\begin{vmatrix} z-z_1 & x-x_1 \\ n_1 & l_1 \end{vmatrix}^2=\begin{vmatrix} x-x_2 & y-y_2 \\ l_2 & m_2 \end{vmatrix}^2+$$
$$+\begin{vmatrix} y-y_2 & z-z_2 \\ m_2 & n_2 \end{vmatrix}^2+\begin{vmatrix} z-z_2 & x-x_2 \\ n_2 & l_2 \end{vmatrix}^2.$$

**13.1.24.** 1) 3; 2) 2; 3) 1; 4) 3; 5) 3; 6) 2; 7) 3; 8) 2; 9) 1. **13.1.25.**

$$\begin{vmatrix} a-x_1 & b-y_1 & c-z_1 \\ x-x_1 & y-y_1 & z-z_1 \\ \boldsymbol{l} & \boldsymbol{m} & \boldsymbol{n} \end{vmatrix}=R^2\left[\begin{vmatrix} y-y_1 & z-z_1 \\ \boldsymbol{m} & \boldsymbol{n} \end{vmatrix}^2+\begin{vmatrix} z-z_1 & x-x_1 \\ \boldsymbol{n} & \boldsymbol{l} \end{vmatrix}^2+\begin{vmatrix} x-x_1 & y-y_1 \\ \boldsymbol{l} & \boldsymbol{m} \end{vmatrix}^2\right]$$

**13.1.26.** $\begin{vmatrix} b-y_1 & c-z_1 \\ \boldsymbol{m} & \boldsymbol{n} \end{vmatrix}^2 + \begin{vmatrix} c-z_1 & a-x_1 \\ \boldsymbol{n} & \boldsymbol{l} \end{vmatrix}^2 + \begin{vmatrix} a-x_1 & b-y_1 \\ \boldsymbol{l} & \boldsymbol{m} \end{vmatrix}^2 > R^2(\boldsymbol{l}^2+\boldsymbol{m}^2+\boldsymbol{n}^2)$ .

**13.1.27.** $A(x-a)+B(y-b)+C(z-c)+R\sqrt{A^2+B^2+C^2}=0$.
**13.1.28.** $10x+15y+6z-90=0$. **13.1.30.** $a^2A^2+b^2B^2+c^2C^2=$ $=D^2$. **13.1.31.** $a^2A^2+b^2B^2+c^2C^2>D^2$. **13.1.32.** $(x^2+y^2+$ $+z^2)^2=a^2x^2+b^2y^2+c^2z^2$. **13.1.34.** $\left(-\frac{a^2AD}{\Delta};-\frac{b^2BD}{\Delta};-\frac{c^2CD}{\Delta}\right)$.

**13.1.36.** Berilgan ellipsoidning $\frac{x^2}{a^4}+\frac{y^2}{b^4}+\frac{z^2}{c^4}=\frac{1}{d^4}$ ellipsoid bilan kesishish chizig‘i. **13.1.37.** $x^2+y^2=a^2\pm 2az-$ ikkita aylanma paraboloid. **13.1.38.** $\frac{x^2}{25}+\frac{y^2}{9}+\frac{z^2}{16}=1$. **13.1.39.** $\frac{x^2}{9}+\frac{y^2}{16}+\frac{z^2}{36}=1$.
**13.1.40.** $\frac{x^2}{12}+\frac{y^2}{9}+\frac{z^2}{7{,}2}=1$.

**14.1.1.** 1) Ikkita tekislik: $2x+y=0$; $y+2z-2=0$; 2) ikkita tekislik: $x-2y+3z+2=0$; $x-2y+3z-3=0$3) ikkita tekislik:$x+2y+3z+4=0$; $3x-2y+z-6=0$**14.1.2**. 1) ikkita tekislik: $x+y+z+1=0$; $5x+4y+3z+2=0$;2) ikkita tekislik: $2x-7y+z+1=0$; $2x-7y+z+3=0$;) ustma – ust tushgan qo‘sh tekislik: $(4x+3y+10z+7)^2=0$. **14.1.3.** 1) Ellipsoid; 2) bir pallali giperboloid; 3) ikki pallali giperboloid. **14.1.4**. 1)konus; 2) elliptik paraboloid; 3) giperbolik paraboloid. **14.1.5**. 1) elliptik silindr; 2) giperbolik silindr; 3) parabolik silindr. **14.1.6**. 1) giperbolik paraboloid; 2) bir pallali giperboloid. **14.1.7.** 1) Uchi $\left(\frac{3}{2};1;\frac{1}{2}\right)$ da bo‘lgan $Z=2X^2-4Y^2$giperbolik paraboloid ; 3)uchi $(-1;-1;-1)$ nuqtada bo‘lgan konus $X^2+2Y^2-3Z^2=0$.**14.1.8**. 1) bir juft tekislik: $x+y\pm z=0$ 2) Parabolik silindr: $Z=5X^2$ 3) $Z=2X^2$ parabolik silindr. **14.1.9**. 1) $z^2-2x^2=1$2) $(3;-1;1)$ $\frac{X^2}{36}+\frac{Y^2}{9}+\frac{Z^2}{4}=1$ ellipsoid; 3) $x^2-y^2+z^2=0$konus. **14.1.10**. 1) ikkita tekislik: $x-y\pm(z-1)^2=0$; 2) markazi $(5;2;3)$ nuqtada bo‘lgan bir pallali giperboloid $\frac{X^2}{16}+\frac{Y^2}{4}-\frac{Z^2}{16}=1$; 3) giperbolik paraboloid: $x^2-y^2=-2z$.

**14.1.11**. 1) parabolik silindr: $x^2-10y=0$. 2) $x^2+z^2=1$ doiraviy silindr; 3) $(x-1)^2+\left(y+\frac{2}{3}\right)^2+z^2=\frac{16}{9}$ sfera. **14.1.12**. 1) $(x-1)^2+\left(y+\frac{2}{3}\right)^2=\frac{16}{9}$ doiraviy silindr; 2) $X^2+Y^2-Z^2=0$ doiraviy konus. 3) $(2x-1)\pm(y-2)==0$ ikkita tekislik. **14.1.13.** Markazi $\left(-\frac{1}{3},-\frac{2}{3};\frac{2}{3}\right)$nuqtadagi bir pallali giperboloid $\frac{X^2}{\left(\frac{1}{\sqrt{3}}\right)^2}+\frac{Y^2}{\left(\frac{1}{\sqrt{6}}\right)^2}-\frac{Z^2}{\left(\frac{1}{\sqrt{2}}\right)^2}=1$ yangi sistema birlik vektorlarining koordinatalari: $e_1'=\left\{\frac{1}{\sqrt{3}};-\frac{1}{\sqrt{3}};\frac{1}{\sqrt{3}}\right\}$, $e_2'=\left\{\frac{1}{\sqrt{6}};\frac{2}{\sqrt{6}};\frac{1}{\sqrt{6}}\right\}$, $e_3'=\left\{\frac{1}{\sqrt{2}};0;\frac{1}{\sqrt{2}}\right\}$. **14.1.14.** $\frac{X^2}{\left(\sqrt{\frac{2}{3}}\right)^2}+\frac{Y^2}{1}=1$ elliptik silindr, simmetriya o‘qi tenglamalari $x=t,\ y=2+2t,\ z=-1-t,\ O'X$ o‘qining vektori $e_1'=\left\{-\frac{1}{\sqrt{3}};\frac{1}{\sqrt{3}};\frac{1}{\sqrt{3}}\right\}$, $O'Y$ o‘qining vektori $e_2'=\left\{\frac{1}{\sqrt{2}};0;\frac{1}{\sqrt{2}}\right\}$. **14.1.15.** Parabolik silindr: $6x^2--2\sqrt{3}y=0$. **14.1.16.** Ikkita parallel tekislik: $2x-3y+z=-1\pm\pm\sqrt{6}$. **14.1.17.** Markazi $(1;2;-1)$ nuqtada bo‘lgan ellipsoid: $\frac{X^2}{2}+\frac{Y^2}{1}+\frac{Z^2}{\frac{2}{3}}=1$, $e_1'=\left\{\frac{1}{3};\frac{2}{3};\frac{2}{3}\right\}$, $e_2'=\left\{\frac{2}{3};\frac{1}{3};-\frac{2}{3}\right\}$, $e_3'=\left\{\frac{2}{3};-\frac{2}{3};\frac{1}{3}\right\}$. **14.1.18.** Markazi $\left(0,1,-\frac{2}{5}\right)$ nuqtada bo‘lgan ikki pallali giperboloid: $\frac{X^2}{\frac{4}{5}}+\frac{Y^2}{\frac{4}{15}}-\frac{Z^2}{\frac{4}{25}}=-1$, $e_1'=\left\{\frac{1}{\sqrt{2}};-\frac{1}{\sqrt{2}};0\right\}$, $e_2'=\left\{\frac{1}{\sqrt{2}};\frac{1}{\sqrt{2}};0\right\}$, $e_3'=\{0;0;1\}$. **14.1.19.** Uchi $(1;1;-1)$ nuqta bo‘lgan $x^2+y^2-2z^2=0$ aylanma konus, $(2;1;-2)$ konus o‘qiga parallel vektor. **14.1.20.** $\frac{X^2}{\frac{5}{2\sqrt{2}}}+\frac{Y^2}{\frac{1}{\sqrt{2}}}=2Z$ elliptik paraboloid. Paraboloid tomoniga yo‘nalgan o‘qining birlik vektori $\left\{\frac{1}{\sqrt{2}},\frac{-1}{\sqrt{2}},0\right\}$, $(1;1;-2)$, $(1;1;1)$ paraboloid o‘qlariga perpendikulyar kesimlarning bosh o‘qlariga parallel

vektorlar, paraboloid uchi $\left(-\frac{1}{40},-\frac{19}{40},\frac{1}{2}\right)$. **14.1.21.** $\frac{X^2}{2}+\frac{Y^2}{1}=1$ elliptik silindr, $(0;1;0)$ silindr o‘qidagi nuqta: $\{1;0;1\}$, silindr o‘qidagi parallel vektor: $\{1;1;-1\}$
va $\{-1;2;1\}$ silindr o‘qiga perpendikulyar kesimlarining bosh o‘qlariga parallel vektorlar. **14.1.22.** Markazi $0'\left(-\frac{1}{3},-\frac{2}{3},\frac{2}{3}\right)$ nuqtada bo‘lgan $\frac{X^2}{\frac{1}{3}}+\frac{Y^2}{\frac{1}{6}}-\frac{Z^2}{\frac{1}{2}}=1$ bir pallali giperboloid, o‘qlarning birlik vektorlari $e_1'=\left\{-\frac{1}{\sqrt{3}};\frac{1}{\sqrt{3}};-\frac{1}{\sqrt{3}}\right\}$, $e_2'=\left\{\frac{1}{\sqrt{6}};\frac{2}{\sqrt{6}};\frac{1}{\sqrt{6}}\right\}$, $e_3'=\left\{\frac{1}{\sqrt{2}};0;-\frac{1}{\sqrt{2}}\right\}$.
**14.1.23**. $x^2+y^2=\frac{1}{6}$ doiraviy silindr, o‘qining tenglamalari: $5x-$ $-2y-z+5=0;$ $x-y+z+1=0.$ **14.1.24**. $x^2-y^2=\frac{1}{3}$ giperbolik silindr, markazlar o‘qining tenglamalari: $x+2y-5z+$ $+1=0;$ $x-y+z+1=0$. Bosh kesim haqiqiy o‘qining yo‘nalishi, $e_1'=\left\{\frac{1}{\sqrt{2}};0;-\frac{1}{\sqrt{2}}\right\}$, mavhum o‘qining yo‘nalishi: $e_2'=\left\{\frac{1}{\sqrt{3}};-\frac{1}{\sqrt{3}};\frac{1}{\sqrt{3}}\right\}$. **14.1.25.** $\frac{X^2}{\frac{4}{7\sqrt{14}}}-\frac{Y^2}{\frac{2}{\sqrt{14}}}=2Z$ giperbolik paraboloid. $O'XZ$ tekisligi bilan sirt kesishidan hosil bo‘lgan parabola o‘qining musbat yo‘nalishini $\{1;2;-3\}$vektor aniqlaydi. $O'X$o‘qining musbat yo‘nalishi $\{4;1;2\}$vektor bilan, $O'Y$ o‘qining musbat yo‘nalishi $\{-1;2;1\}$vektor bilan aniqlanadi. $O'\left(-\frac{617}{392},-\frac{113}{196},\frac{1011}{392}\right)$nuqtada. **14.1.26.** Uchi $\left(-\frac{183}{784},-\frac{499}{784},\frac{509}{392}\right)$ nuqtada bo‘lgan giperbolik paraboloid: $7X^2-2Y^2-\frac{8Z}{\sqrt{14}}=0$. $O'X$ o‘qining yo‘nalishi $\{2;4;1\}$, $O'X$ o‘qining yo‘nalishini $\{1;-1;2\}$vektor aniqlaydi. $\{-3;1;2\}$vektor paraboloid o‘qi bo‘ylab kichik parametrli bosh kesim o‘qi tomonga yo‘nalgan ($O'XZ$tekislik). **14.1.27**. 1) Simmetriya o‘qlari aniqlanadi, hosil bo‘lgan silindrlar avvalgilarga gomotetik bo‘ladi; 2) simmetriya o‘qi o‘ziga

parallel siljiydi, hosil bo‘lgan silindr avvalgiga o‘xshash bo‘ladi. **14.1.28**.1) Parametri, botiqlik yo‘nalishi va yasovchilar yo‘nalishi o‘zgarmagan holda silindrning ko‘chishi ro‘y beradi; 2) yasovchilar yo‘nalishi o‘zgaradi, parameter o‘zgaradi. **14.1.29**. $\lambda = \pm 1$; $\mu = \pm\sqrt{2}$ parametrlar $I_3 = 0$, $K_4 = 0$, ${I_1}^2 = 4I_2$ shartlardan aniqlanadi. **14.1.30**. $ab + bc + ca = 0$.

## Foydalanilgan adabiyotlar ro‘yxati

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# ANALITIK GEOMETRIYADAN MISOL VA MASALALAR
**o‘quv qo‘llanma**

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