1-MAVZU: To'plamlar nazariyasining asosiy elementlari

Adabiyotlar:

1. Valentin Deaconu. Don Pfaff **. A bridge course to higher mathematics.** Department of Mathematics, University of Nevada, Reno NV 89557-0084, USA E-mail address: vdeaconu@unr.edu, don@unr.edu, **Elementary theory of sets. pp. 21-35 (75%)**

2. Herbert Gintis , Mathematical Literacy for Humanists, p.p11-12,14-15

To‘plam tushunchasi matematikaning boshlang‘ich tushunchalark bo‘lib, u ta’rifsiz qabul qilinadi. To‘plamni tashkil qiluvchi obyektlar uning elementlari deyiladi. To‘plamlarni *A*, *a*, a, A yoki A harflari bilan belgilaymiz. To’plam bir qancha elementlardan iborat bo’lishi mumkin, quyidagi yozuv:

$$a \in A \tag{1}$$

a elementni *A* to’plamga tegishliligini bildiradi.

$$a \notin A \tag{2}$$

a elementni *A* to’plamga tegishli emasligini bildiradi, yoki mantiq belgisidan foydalangan holda $\neg(a \in A)$ ko’rinishda yozishimiz mumkin. Agar $a \in A$ bo’lsa, u holda *a* element *A* to’plamga tegishli deyiladi.

The central primitive concepts in set theory are those of a *set* and set *membership*. We often denote a set by a letter, such as A, a, $\mathbf{a}$, $\mathbb{A}$, or $\mathbf{A}$. A set is simply a collection of things, and we call an element of such a collection a *member* of the set. We allow sets to be members of other sets. We write

$$a \in A \tag{3.1}$$

to mean that a is a member of set A, and

$$a \notin A \tag{3.2}$$

to mean that a is not a member of set A, or $\neg(a \in A)$, using the logical notation of the previous chapter. If $a \in A$, we also say that a is an *element* of A. The concept of set membership $\in$ is obviously primitive—we can interpret it clearly, but we do define it in terms of more elementary concepts.

To'plamlar nazariyasi aksiomalariga 3.15 paragrafda to'xtalib o'tamiz. Hajmlilik Aksiomasiga ko'ra to'plam elementlarini quyidagicha belgilashimiz ham mumkin,

$$A = \{1, a, t, x\}, \qquad (3)$$

bunda, A to'plam tarkibida 1 soni va *a,t,x* harfiy belgilar kiradi.

To'liqlik Aksiomasiga ko'ra to'plam elementlari soni uning tarkibiga kiruvchi elementlar bilan aniqlanib ularning qanday tartiblanganiga bog'liq emas.

(3) A to'plam $\{a, x, 1, t\}$ to'plam bilan xam va $\{x, t, a, 1, 1, 1, t, a, t, x\}$ to'plam bilan xam bir xildir.

We develop the axioms of set theory in section 3.15. According to one of these, the **Axiom of Extensionality**, we can always denote a set unambiguously by its elements, as for instance in

$$A = \{1, a, t, x\}, \qquad (3.3)$$

meaning the set A consists of the number 1, and whatever the symbols a, t, and x represent.

By the Axiom of Extensionality, a set is completely determined by its members, and not how they are ordered. Thus the set A in (3.3) is the same as the set

$$\{a, x, 1, t\}$$

and even the same as the set

$$\{x, t, a, 1, 1, 1, t, a, t, x\}.$$

To’plamlar ustida amallar

Agar A va B to’plamlar bir xil elementlardan tashkil topgan bo’lsa bu to’plamlar teng deyiladi. U holda to’liqlik aksiomasiga ko’ra agar ikkita to’plam bir xil elemantlar jamlanmasidan tuzilgan bo’lsa ular teng bo’ladi. Masalan

$$\{1,2,3\} = \{2,1,3\} = \{1,1,2,3\}.$$

Agar A to’plamning xar bir elementi B to’plamning ham elementi bo’lsa, A to’plam B to’plamning to’plamostisi deyiladi va $A \subset B$ yoki $A \subseteq B$ orqali belgilanadi.

3.3 Operations on Sets

We say set A *equals* set B if they have the same members. Then, by the Axiom of Extensionality, two sets are equal if and only if they are the same set. For instance

$$\{1,2,3\} = \{2,1,3\} = \{1,1,2,3\}.$$

If every member of set A is also a member of set B, we say A is a *subset* of B, and we write

$$A \subset B \qquad \text{or} \qquad A \subseteq B.$$

Bu belgilshlardan birinchisi A to'plam B to'plamning qismi va $A \neq B$ ekanligini ikkinchisi esa A to'plam B to'plamning qismi bo'lib ular teng bo'lishiyam va teng bo'lmasligiyam mumkinligini bildiradi. Masalan $\{x,t\} \subset \{x,t,1\}$. Ihtiyoriy A to'plam uchun $A \subseteq A$ munosabat o'rinli bo'ladi.

Yuqoridagilarni matematik tilda quyidagicha yozish mumkin:

$$A \subseteq B \equiv (\forall x \in A)(x \in B)$$
$$A \subsetneq B \equiv (\forall x \in A)(x \in B) \wedge (A \neq B)$$

Bu yozuvda $\wedge$ yozuvi "va" ma'nosini bildiradi. Ba'zida ayrimlar $\subset$ belgisi o'rniga $\subseteq$belgisini ayrimlar esa $\subsetneq$ belgisini ishlatadi. A $\subsetneq$B bo'lganda A to'plam B to'plamning xos to'plam ostisi deyiladi.

The first of these expressions says that A is a subset of B and $A \neq B$, while the second says that A is a subset of B but A may or may not equal B. For instance $\{x, t\} \subset \{x, t, 1\}$. For any set A, we then have $A \subseteq A$.

In formal mathematical notation, we define

$$A \subseteq B \equiv (\forall x \in A)(x \in B)$$
$$A \subsetneq B \equiv (\forall x \in A)(x \in B) \wedge (A \neq B)$$

In the second equation, the symbol $\wedge$ is logical notation for "and." Sometimes writers treat $\subset$ as meaning $\subseteq$ and others treat $\subset$ as meaning $\subsetneq$. When $A \subsetneq B$, we say A is a *proper subset* of B.

Ixtiyoriy A to'plam uchun $\varnothing \subseteq A$, agar $A \neq \varnothing$ u holda $\varnothing \subsetneq A$.

A va *B* <u>to'plamlarning ayirmasi</u> deb, *A* to'plamning *B* to'plamga kirmagan barcha elementlardan tashkil topgan to'plamga aytiladi va *A \ B* yoki *A-B*

Ko'rinishlarda belgilanadi. *A* va *B* to'plamlarning ayirmasini mantiq qoidalariga ko'ra bunday yozamiz:

$$A - B = A \backslash B = \{x | x \in A \wedge x \notin B\}$$

A va *B* to'plamlarning kamida biriga tegishli bo'lgan barcha elementlardan tashkil topgan $A \cup B$ to'plam *A* va *B* to'plamlarning birlashmasi yoki yig'indisi deyiladi. Buni matematik tilda quyidagicha yozamiz

$$A \cup B = \{x | x \in A \vee x \in B\}.$$

Masalan: $\{1, x, a\} \cup \{2, 7, x\} = \{1, x, a, 2, 7\}$

For any set A we have $\emptyset \subseteq A$, and if $A \neq \emptyset$, then $\emptyset \subsetneq A$. Can you see why? Hint: use (3.6) and show that there is no member of $\emptyset$ that is not a member of A.

Another commonly used notation is $A \backslash B$ or $A - B$, where A and B are sets, to mean the subset of A consisting of elements not in B. We call $A \backslash B$ the *difference* between A and B. Using logical notation,

$$A - B = A \backslash B = \{x | x \in A \wedge x \notin B\}. \qquad (3.11)$$

We know that $A \backslash B$ is a set because it is the subset of A defined by the property $P(x) = x \notin B$.

The *union* sets A and B, which we write $A \cup B$, is the set of things that are members of either A or B. In mathematical language, we can write this as

$$A \cup B = \{x | x \in A \vee x \in B\}.$$

Here, the symbol $\vee$ means "or." For instance, $\{1, x, a\} \cup \{2, 7, x\} = \{1, x, a, 2, 7\}$.

A va B to'plamlarning kesishmasi yoki ko'paytmasi deb, A va B to'plamlarning barcha umumiy, ya'ni A ga ham, B ga ham tegishli elementlardan tashkil topgan $A \cap B$ to'plamga aytiladi. A va B to'plamlarning I kesishmasi mantiq qoidalariga ko'ra bunday yozamiz:

$$A \cap B = \{x | x \in A \wedge x \in B\}$$

The *intersection* of two sets A and B, which we write $A \cap B$, is the set of things that are members of both A and B. We can write this as

$$A \cap B = \{x | x \in A \wedge x \in B\}.$$

Matematikaning ba'zi sohalarida faqatgina birorta to'plam va uning barcha to'plamostilari bilan ish ko'rishga to'g'ri keladi. Masalan, planimetriya tekislik va uning barcha to'plamostilari bilan, stereometriya esa fazo va uning barcha to'plamostilari bilan ish ko'radi.

Agar biror E to'plam va faqat uning to'plamostilari bilan ish ko'rsak, bunday E to'plamni universal to'plam deb ataymiz. Universal to'plamning barcha to'plamostilari to'plamini β (E) orqali belgilaymiz.

To'plamlar ustida bajariladigan algebraik amallar quyidagi xossalarga ega.

1^0. $A \cap A = A$ kesishmaning idempotentligi;

2^0. $A \cup A = A$ birlashmaning idempotentligi;

3^0. $A \cap B = B \cap A$, $A \cup B = B \cup A$ kesishma va birlashmaning kommutativligi;

4^0. 2°. $(A \cap B) \cap C = A \cap (B \cap C)$, $(A \cup B) \cup C = A \cup (B \cup C)$ kesishma va birlashmaning assosiativligi

5^0. Kesishmaning birlashmaga nisbatan distributivligi:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C);$$

6^0. Birlashmaning kesishmaga nisbatan distributivligi:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C);$$

7^0. $(A \setminus B) \cap C = (A \cap C) \setminus B = (A \cap C) \setminus (B \cap C);$

$A_1 \cup A_2 \cup ... A_n \cup ...$ birlashmani $\bigcup_{i=1}^{\infty} A_i$, $A_1 \cap A_2 \cap ... A_n \cap ...$ kesishmani $\bigcap_{i=1}^{\infty} A_i$ deb belgilab olsak, yana quyidagi xossalarga ega bo'lamiz. $A_i, i = 1...$ to'plamlar birorta X to'plamning to'plamostilari bo'lsin, u holda

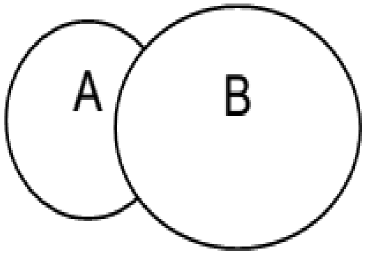
8°. $X \setminus \bigcup_{i=1}^{\infty} A_i = \bigcap_{i=1}^{\infty} (X \setminus A_i)$;

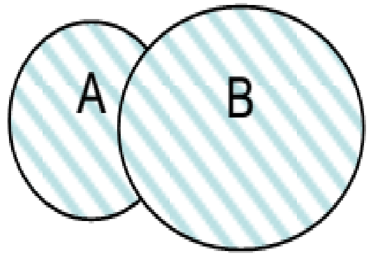
9°. $X \setminus \bigcap_{i=1}^{\infty} A_i = \bigcup_{i=1}^{\infty} (X \setminus A_i)$.

Bu tengliklarni isbotlash uchun, tengliklarning chap tomonidagi to'plamga tegishli ixtiyoriy element, tenglikning o'ng tomonidagi to'plamga tegishli va to'plamning chap tomonidagi to'plamga tegishli ixtiyoriy element chap tomonidagi to'plamga ham tegishli bo'lishini ko'rsatish etarli.

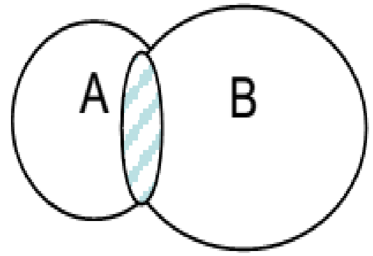
To'mlamlar ustida amallarni Eyler-Venn diagrammalari deb ataladigan quyidagi shakllar yordamida ifoda qilish, amallarning xossalarini isbot qilishni ancha engillashtiradi.

Universal to'plam to'g'ri to'rt burchak shaklida, uning to'plamostilarini to'g'ri to'rtburchak ichidagi doiralar orqali ifoda qilinadi. U xolda, ikki to'plam birlashmasi, kesishmasi, ayirmasi, to'lduruvchi to'plamlar, ikki to'plamning simmetrik ayirmasi mos ravishda quyidagicha ifodalanadi:

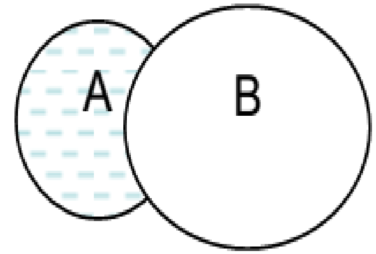




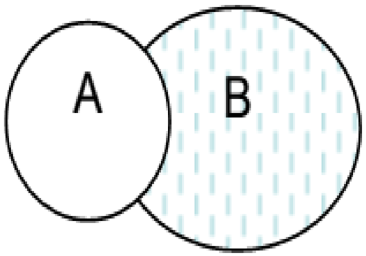
$A \cup B$



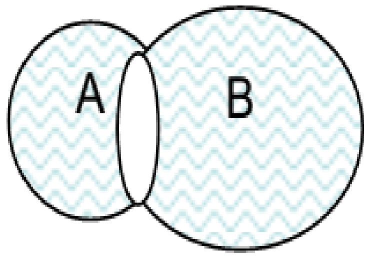
$A \cap B$



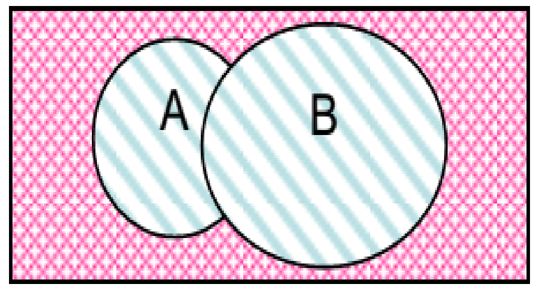
$A \setminus B$



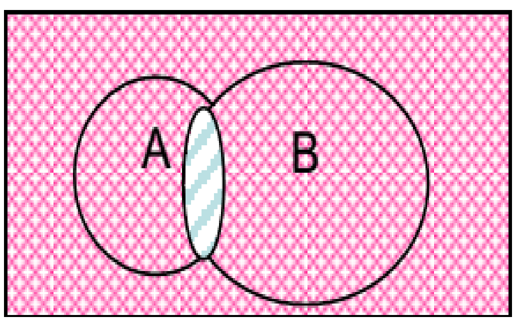
$B \setminus A$



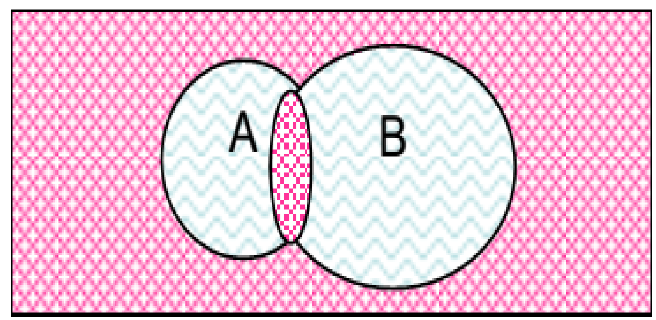
$A \Delta B$



$(A \cup B)'$



$(A \cap B)'$



$(A \triangle B)'$