O‘ZBEKISTON RESPUBLIKASI
OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

GULISTON DAVLAT UNIVERSITETI

OLIY MATEMATIKA FANIDAN

O’QUV USLUBIY MAJMUA

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Ushbu o’quv majmua Oily o‘quv yurtlari biologiya, oziq ovqat texnologiyalari, biotexnologiyalari ta’lim yo’nalishlari talabalari uchun mo’ljallangan. Guliston davlat universiteti ilmiy kengashi tasdiqlagan fan dasturi asosida tayyorlangan.

Tuzuvchi: matematika kafedrasi katta o’qituvchisi G’oyibnazarov H R.

Taqrizchilar: fizika-matematika fanlari nomzodi, dotsent X. Norjigitov

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Oliy matematika fanini oqitishda ilg’or xorij talim tajribalaridan foydalanib matematik tushunchalar qanday o’rganilishi, hayotga tadbiq qilinishi, matematik muammolarning yechimlari qay yo’sinda hal qilinishini aniqlash.

Bu ishlar bo`lg`usi biologiya o‘qituvchilarini tayyorlash ishini takomillashtirishda, talabalarni o‘qituvchilik kasbiga tayyorlashda muhim bo‘lgan bilim, ko‘nikma va malakalarni hosil qilishda zarur bo‘lgan o‘quv-tarbiya ishlarini tashkil qilishda katta yordam beradi.

Bundan chiqdiki biologiya darslarida matematik bilimlardan foydalanish dolzarbdir.

Respublikamiz ta’lim tizimi yosh avlodlarning o’qish faoliyati mazmunini, maqsadini, vazifasini, vositalarini, metodlarini va shakllarini ilmiy pedagogik asoslarga tayangan xolda takomillashtirishni taqoza etmoqda. Buning mohiyati O’zbekiston Respublikasining prezidenti Shavkat Mirziyoyevning **“Maktabda o’qitish metodikasi o’zgarmasa, ta’lim sifati ham, mazmuni ham, muhit ham o’zgarmaydi”** degan dasturiy fikrlaridan ham bilib olish mumkin. Ushbu vazifaning samaradorligi masalalari ularning bilim egallashdagi faolligini, mustaqil bilish faoliyatini shakllantirishga taqaladi. Bunda talabalarning matematik tayyorgarligi jarayonini shakllantirishni dolzarbligi birinchi kurs talabalarida yaqqol namoyon bo’ladi.

Matematika o’qitishda qiziqarlilikni keng ma’noda tushunmoq kerak. Og’zaki masalalar mazmunining qiziqarli bo’lishi dars matriallariga oid qiziq bosh qotirmalar kiritish o’quvchilarning ba’zi matematik o’yinlar bilan tanishtirish talabalar ongini o’stiradigan o’ylatadigan qiziqarli misollar yechish.

Talabalarning mustaqil bilish faoliyatining mohiyati va uni tashkil etishda yozma adabiyotlardan foydalanib borish natijasida ko’p yutuqlarga erishishimiz ma’lum shuning uchun mazkur qollanmaning yaratilishi maqsadga muvofiqdir.

I bob. Chiziqli algebra elementlari.

Matritsalar va ular ustida amallar.

1. Matritsa haqida tushuncha

$a_{i\kappa}$ haqiqiy sonlar n ta satr va m ta ustunda joylashgan quyidagi to`g`ri to`rtburchak

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{pmatrix} = (a_{ik})$$

shaklidagi jadvalga n x m o`lchamli **matritsa** deyiladi.

$a_{i\kappa}$ haqiqiy sonlar matritsa elementlari deb ataladi.

1 x m o`lchamli matritsaga **satr matritsa**, n x 1 o`lchamli matritsaga **ustun matritsa** deyiladi. **Nol matritsa** deb, har bir elementi nolga teng bo`lgan matritsaga aytiladi.

n x m o`lchamli A = ($a_{i\kappa}$) va B = ($b_{i\kappa}$) matritsalar berilgan bo`lsin. Agar matritsalarning barcha mos elementlari o`zaro teng bo`lsa, matritsalar o`zaro teng deyiladi va A = B ko`rinishda yoziladi.

$$(1 \quad 3 \quad 2 \quad 0)\ , \qquad \begin{pmatrix} 2 \\ 1 \\ 0 \\ 3 \end{pmatrix}\ , \qquad \begin{pmatrix} 5 & 2 & 0 & 1 & 2 \\ 3 & 1 & 4 & 0 & 3 \\ 1 & 3 & 2 & 4 & 4 \\ 4 & 1 & 0 & 1 & 2 \end{pmatrix}\ ,$$

$$\begin{pmatrix} 2 & 1 & 4 \\ 0 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 4 \\ 0 & 3 & 1 \end{pmatrix}$$

2. Matritsalar ustida amallar.

O`lchamlari aynan teng A va B matritsalarni qo`shganda, ularning mos elementlari qo`shiladi: A + B = ($a_{i\kappa}$) + ($b_{i\kappa}$) = ($a_{i\kappa}$ + $b_{i\kappa}$).

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \dots & \dots & \dots & \dots \\ b_{m1} & b_{m2} & \dots & b_{mn} \end{pmatrix} = \begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \dots\dots\dots & a_{1n}+b_{1n} \\ a_{21}+b_{21} & a_{22}+b_{22} & \dots\dots\dots & a_{2n}+b_{2n} \\ \dots\dots\dots & \dots\dots\dots & \dots\dots\dots & \dots\dots\dots \\ a_{m1}+b_{m1} & a_{m2}+b_{m2} & \dots\dots\dots & a_{mn}+b_{mn} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 5 \\ -3 & 5 & 0 \\ 2 & 1 & 3 \\ 1 & 0 & 4 \end{pmatrix} + \begin{pmatrix} 5 & 2 & -3 \\ 2 & 5 & 0 \\ -1 & 0 & 3 \\ 1 & -4 & -2 \end{pmatrix} = \begin{pmatrix} 6 & 4 & 2 \\ -1 & 10 & 0 \\ 1 & 1 & 6 \\ 2 & -4 & 2 \end{pmatrix}$$

Haqiqiy son matritsaga ko`paytirilganda, matritsaning har bir elementi shu songa ko`paytiriladi: $k\,(a_{ik}) = (k\,a_{ik})$.

Misol. Amallarni bajaring:

$$k \cdot \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{pmatrix} = \begin{pmatrix} ka_{11} & ka_{12} & \dots & ka_{1m} \\ ka_{21} & ka_{22} & \dots & ka_{2m} \\ \dots & \dots & \dots & \dots \\ ka_{n1} & ka_{n2} & \dots & ka_{nm} \end{pmatrix}$$

$$3\begin{pmatrix} 1 & 0 \\ -3 & -4 \\ 4 & 5 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ -9 & -12 \\ 12 & 15 \end{pmatrix} \qquad 2\begin{pmatrix} 2 & 1 \\ 3 & -2 \\ 2 & 3 \end{pmatrix} - 3\begin{pmatrix} 5 & 3 \\ 7 & -1 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} -11 & -7 \\ -15 & -1 \\ 4 & 0 \end{pmatrix}$$

Matritsalarni qo`shish va songa ko`paytirish amallari quyidagi xossalarga bo`ysinadi:

1) A + B = B + A; 2) A + (B + C) = (A + B) + C; 3) k(A + B) = kA + kB; 4) k(nA) = (kn)A ; 5) (k + n)A = kA + nA.

Agar A matritsaning ustunlari soni B matritsaning satrlari soniga teng bo`lsa, A va B matritsalar **o`zaro zanjirlangan matritsalar** deyiladi. O`zaro zanjirlangan matritsalarni ko`paytirish mumkin.

n x m o`lchamli $A = (a_{ik})$ matritsani m x p o`lchamli $B = (b_{ik})$ matritsaga ko`paytmasi n x p o`lchamli $C = (c_{ik})$ matritsaga teng bo`lib, uning c_{ik} elementlari quyidagicha aniqlanadi

$$c_{ik} = \sum_{j=1}^{n} a_{ij} b_{jk} ,$$

ya`ni $c_{ik}$ element A matritsa i-satri elementlarining B matritsa k-ustuni mos elementlariga ko`paytmalarining yig`indisiga teng.

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} \\ a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22} \\ a_{31}b_{11}+a_{32}b_{21} & a_{31}b_{12}+a_{32}b_{22} \end{pmatrix}$$

Masalan: $$\begin{pmatrix} 1 & 3 \\ 2 & 4 \\ 5 & 0 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 1\cdot 2+3\cdot 1 & 1\cdot 3+3\cdot 4 \\ 2\cdot 2+4\cdot 1 & 2\cdot 3+4\cdot 4 \\ 5\cdot 2+0\cdot 1 & 5\cdot 3+0\cdot 4 \end{pmatrix} = \begin{pmatrix} 5 & 15 \\ 8 & 22 \\ 10 & 15 \end{pmatrix}$$

Matritsalarni ko\`paytirish quyidagi xossalarga bo\`ysinadi:

1. (kA)B = k(AB); 2. (A + B)C = AC + BC;

3. A(B + C) = AB + AC; 4. A(BC) = (AB)C.

Matritsalarning ko\`paytmasi ko\`paytuvchi matritsalar nolmas bo\`lishiga qaramasdan, nol matritsani berishi ham mumkin.

A va B matritsalarning ko\`paytmasi har doim o\`rin almashtirish qonuniga bo\`ysinavermaydi, ya\`ni umuman olganda AB $\neq$ BA. AB = BA tenglikni qanoatlantiruvchi A va B matritsalarga **o\`rin almashinuvchi** matritsalar deyiladi.

Berilgan **n x m** o\`lchamli A matritsaning har bir satri mos ustunlari bilan almashtirilsa, hosil bo\`lgan **m x n** o\`lchamli matritsaga A matritsaning **transponirlangan matritsa**si deyiladi va $\mathbf{A^T}$ ko\`rinishda belgilanadi.

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{pmatrix} \qquad A^T = \begin{pmatrix} a_{11} & a_{21} & \dots & a_{n1} \\ a_{12} & a_{22} & \dots & a_{n2} \\ \dots & \dots & \dots & \dots \\ a_{1m} & a_{2m} & \dots & a_{nm} \end{pmatrix}$$

Misol. $$A = \begin{pmatrix} 1 & 2 & 4 & 1 \\ 3 & 2 & 0 & 3 \\ 2 & 1 & 3 & 0 \\ 5 & 1 & 2 & 4 \end{pmatrix} \qquad A^T = \begin{pmatrix} 1 & 3 & 2 & 5 \\ 2 & 2 & 1 & 1 \\ 4 & 0 & 3 & 2 \\ 1 & 3 & 0 & 4 \end{pmatrix}$$

Matritsalar ko\`paytmasi transponirlangani uchun quyidagi formula o\`rinli:

$$(AB)^T = B^T A^T.$$

Satrlari soni **n** ustunlari soni **n** ga teng bo\`lgan matritsaga **n**–tartibli **kvadratik matritsa** deyiladi. Kvadratik matritsaning quyidagi xususiy ko\`rinishlari bir-biridan farqlaniladi:

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ 0 & a_{22} & \dots & a_{2n} \\ 0 & \dots & \dots & \dots \\ 0 & 0 & \dots & a_{nn} \end{pmatrix}$$ – **yuqori uchburchakli matritsa;**

$$\begin{pmatrix} a_{11} & 0 & \dots & 0 \\ a_{21} & a_{22} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$$ – **quyi uchburchakli matritsa;**

$$\begin{pmatrix} a_{11} & 0 & \dots & 0 \\ 0 & a_{22} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & a_{nn} \end{pmatrix}$$ – **diagonal matritsa;**

$$\begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{pmatrix} = E$$ - **birlik matritsa.**

Mustaqil yechish uchun misollar:

Berilgan matritsalarni qo'shing, ko'paytiring va λ songa ko'paytiring.

a) $A = \begin{pmatrix} 1 & -3 \\ 2 & -4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 2 \\ -2 & 1 \end{pmatrix}$, $\lambda = 3$

b) $A = \begin{pmatrix} 0 & 1 \\ 3 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ 4 & -3 \end{pmatrix}$, $\lambda = 4$

c) $A = \begin{pmatrix} 5 & -3 \\ -4 & 1 \end{pmatrix}$, $B = \begin{pmatrix} -3 & 4 \\ -2 & 0 \end{pmatrix}$, $\lambda = -5$

d) $A = \begin{pmatrix} -2 & 1 & -1 \\ 3 & 2 & 0 \\ -2 & -3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 2 & 1 \\ 1 & 3 & -2 \\ -1 & -3 & 1 \end{pmatrix}$, $\lambda = -2$

e) $A = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 3 & 0 \\ -1 & 2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 & 1 \\ 2 & 3 & -3 \\ -1 & 0 & 1 \end{pmatrix}$, $\lambda = -3$

f) $A=\begin{pmatrix} 0 & 1 & -2 \\ -1 & 3 & 0 \\ 0 & 2 & 1 \end{pmatrix}$, $B=\begin{pmatrix} 3 & 0 & 1 \\ 1 & 4 & 5 \\ -2 & -3 & 1 \end{pmatrix}$, $\lambda=-1$

Ikkinchi va uchinchi tartibli determinantlar. Determinantlarni hisoblash. Determinantlarning asosiy xossalari.

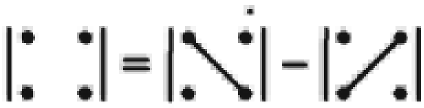
1.Determinantlarni hisoblash.

To'rtta sondan iborat $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

kvadrat jadval ikkinchi tartibli kvadrat matritsa deyiladi.

Ikkinchi tartibli kvadrat matritsaga mos keluvchi ikkinchi tartibli determinant deb quyidagi belgi va tenglik bilan aniqlanuvchi songa aytiladi:

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}\cdot a_{22} - a_{12}\cdot a_{21}$$


- chizmasi

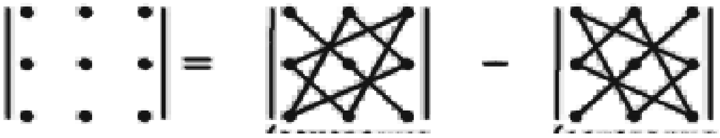
Misol: $\begin{vmatrix} 2 & -3 \\ -1 & 8 \end{vmatrix} = 2\cdot 8-(-3)\cdot(-1)=16-3=13$

$$\begin{vmatrix} -1 & -4 \\ -2 & 5 \end{vmatrix} = -1\cdot 5-(-4)\cdot(-2) = -5-8=-13$$

Shunga o'xshash ushbu

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}$$

ifoda uchinchi tartibli determinant deyiladi. Bu ifodaga musbat ishora bilan kiradigan har bir ko'paytma, hamda manfiy ishorali ko'paytmalar ko'paytuvchilarini alohida-alohida punktir chiziqlar yordamida tutashtirib, uchinchi tartibli determinantlarni hisoblash uchun xotirada oson saqlanadigan "uchburchaklar qoidasi"ga ega bo'lamiz (1-shakl).



Misol:

$$\begin{vmatrix} 1 & -3 & 5 \\ 4 & 2 & -1 \\ -3 & 0 & 4 \end{vmatrix} = 1\cdot 2\cdot 4 + (-3)\cdot(-1)\cdot(-3) + 5\cdot 4\cdot 0 - 5\cdot 2\cdot(-3) - 1\cdot(-1)\cdot 0 - (-3)\cdot 4\cdot 4 =$$
$$= 8 - 9 + 0 + 30 - 0 + 48 = 77$$

$$\begin{vmatrix} -1 & -4 & 0 \\ -3 & -2 & -5 \\ -2 & -3 & -4 \end{vmatrix} = -1\cdot(-2)\cdot(-4) + (-3)\cdot 0\cdot(-3) + (-5)\cdot(-4)\cdot(-2) -$$
$$- 0\cdot(-2)\cdot(-2) - (-1)\cdot(-3)\cdot(-5) - (-3)\cdot(-4)\cdot(-4) = 15$$

Determinant a_{ik} elementining M_{ik}minori deb, shu determinantdan bu element turgan qator va ustunni o'chirish natijasida hosil bo'lgan determinantga aytiladi. Determinant a_{ik} elementining A_{ik} algebraik to'ldiruvchisi $A_{ik} = (-1)^{i+k} M_{ik}$ munosabat biln aniqlanadi.

Misol: $\begin{vmatrix} 2 & 4 & 1 \\ 1 & 0 & 3 \\ 3 & 2 & 5 \end{vmatrix}$ ni A_{23} algebraic to'ldiruvchisini toping.

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 2 & 4 \\ 3 & 2 \end{vmatrix} = -(2\cdot 2 - 4\cdot 3) = -(4-12) = -(-8) = 8$$

2.Determinantlarning xossalari:

a) agar determinantning barcha satrlari mos ustunlari biln almashtirilsa, uning qiymati o'zgarmaydi;

Keyingi xossalarni ta'riflashda satrlar va ustunlarni bir so'z bilan qator deb ataymiz.

b) agar determinant nollardan iborat qatorga ega bo'lsa, uning qiymati nolga teng bo'ladi.

c) agar determinant ikkita bir xil parallel qatorga ega bo'lsa, uning qiymati nolg teng bo'ladi;

d) agar determinant ikkita parallel qatorining mos elementlari mutanosib (proportsional) bo'lsa, uning qiymati nolga teng bo'ladi;

e) biror qator elementlarining umumiy ko’paytuvchisini determinant belgisidan tashqariga chiqarish mumkin;

f) agar determinant ikkita parallel qatorining o’rinlari almashtirilsa, determinanbt ishorasini qarama-qarshisiga o’zgartiradi;

g) determinantning qiymati biror qator elementlari bilan shu elementlarga tegishli algebraik to’ldiruvchilari ko’paytmalari yig’indisiga teng.
Bu xossa determinantni qator elementlari bo’yicha yoyish deyiladi. Undan determinantlarni hisoblashda foydalaniladi.

h) biror qator elementlari bilan parallel qator mos elementlari algebraik to’ldiruvchilari ko’paytmalarining yig’indisi nolga teng.

i) agar determinant biror qatorining har bir elementi ikki qo’shiluvchining yig’indisidan iborat bo’lsa, u holda determinant ikki determinant yig’indisiga teng bo’lib, ularning biri tegishli qator birinchi qo’shiluvchilardan, ikkinchisi esa ikkinchi qo’shiluvchilardan iborat bo’ladi.

Masalan:

$$\begin{vmatrix} a_{11} & a_{12}+b_1 & a_{13} \\ a_{21} & a_{22}+b_2 & a_{23} \\ a_{31} & a_{32}+b_3 & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}$$

j) agar determinantlarning biror qatori elementlariga parallel qatorning mos elementlarini biror o’zgarmas songa ko’paytirib qo’shilsa, determinantning qiymati o’zgarmaydi.

Masalan:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31}+\lambda a_{11} & a_{32}+\lambda a_{12} & a_{33}+\lambda a_{13} \end{vmatrix}$$

Mustaqil yechish uchun misollar:

Determinantni hisoblang.

a) $\begin{vmatrix} 5 & 2 \\ -6 & 1 \end{vmatrix}$, b) $\begin{vmatrix} -2 & 1 \\ -3 & 4 \end{vmatrix}$, c) $\begin{vmatrix} 0 & 2 \\ -5 & 4 \end{vmatrix}$,

d) $\begin{vmatrix} 4 & -3 \\ -2 & 5 \end{vmatrix}$, e) $\begin{vmatrix} 1 & 2 \\ 0 & 7 \end{vmatrix}$, f) $\begin{vmatrix} 5 & 3 \\ -4 & -2 \end{vmatrix}$

j) $\begin{vmatrix} 1 & -1 & 5 \\ 3 & 2 & 0 \\ -2 & -4 & 1 \end{vmatrix}$, k) $\begin{vmatrix} 2 & 3 & -1 \\ -2 & 5 & 1 \\ 0 & -4 & -3 \end{vmatrix}$, l) $\begin{vmatrix} 0 & -2 & 3 \\ -2 & 1 & -1 \\ 4 & 5 & 2 \end{vmatrix}$,

m) $\begin{vmatrix} 1 & -2 & 4 \\ 1 & 2 & 3 \\ -1 & -3 & 1 \end{vmatrix}$, n) $\begin{vmatrix} 0 & -2 & 5 \\ -1 & 4 & -1 \\ 2 & -3 & 1 \end{vmatrix}$ o) $\begin{vmatrix} 1 & -3 & 2 \\ -1 & -1 & -4 \\ 3 & -5 & 3 \end{vmatrix}$

p) $\begin{vmatrix} 3 & 0 & 1 \\ 2 & 1 & -2 \\ -2 & -4 & 2 \end{vmatrix}$ q) $\begin{vmatrix} 1 & -2 & 3 \\ -1 & 4 & 2 \\ -4 & -3 & 5 \end{vmatrix}$ r) $\begin{vmatrix} 0 & -1 & 2 \\ -2 & 3 & 2 \\ 3 & -4 & 3 \end{vmatrix}$

Teskari matritsa haqida tushuncha

n – tartibli kvadratik **A = (a_{ik})** matritsa berilgan bo`lsin. Agar A matritsa determinanti noldan farq qilib, uning rangi tartibi n ga teng bo`lsa, matritsaga maxsusmas matritsa deyiladi. Agarda det(A) = 0 bo`lib, rangi n dan kichik bo`lsa, A matritsaga maxsus matritsa deyiladi.

Teorema. Ikki teng tartibli kvadrat matritsalarning ko`paytmasi, ko`paytuvchi matritsalarning har biri maxsusmas bo`lgandagina, maxsusmas matritsadan iborat bo`ladi.

To`g`ridan-to`g`ri ko`paytirish yo`li bilan n - tartibli birlik E va n –tartibli har qanday A matritsalarning o`zaro o`rin almashinuvchi ekanligini, ko`paytma A matritsani berishini, ya`ni $A\cdot E = E\cdot A = A$ tengliklar o`rinli bo`lishini misollarda tekshirib ko`rish qiyin emas.

Berilgan A kvadratik matritsaning teskari matritsasi deb, tartibi A matritsaning tartibiga teng va A matritsaga chapdan yoki o`ngdan ko`paytmasi birlik E matritsaga teng bo`lgan A^{-1} matritsaga aytiladi: $A^{-1}\cdot A = A\cdot A^{-1} = E$.

Yuqoridagi teoremaga asosan E birlik matritsaning maxsusmas ekanligini e`tiborga olsak, maxsus matritsaning teskari matritsaga ega emasligini xulosa qilamiz. Har qanday maxsusmas kvadrat matritsaning yagona teskari matritsasi mavjudligi quyidagi teoremadan kelib chiqadi.

Teorema. Teskari matritsa mavjud bo`lishi uchun det(A) $\neq$ 0 bo`lib, A matritsaning maxsusmas bo`lishi zarur va yetarli.

Misol. $A = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$ maxsusmas matritsaning teskari matritsasini klassik usulda quring.

Klassik usulda ikkinchi tartibli maxsusmas matritsa teskarisi

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}^{-1} = \frac{1}{\det A}\begin{pmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{pmatrix}$$

formula asosida quriladi. Formulani qo`llab,

$$A^{-1} = \frac{1}{7}\begin{pmatrix} 2 & 1 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} 2/7 & 1/7 \\ -3/7 & 2/7 \end{pmatrix}$$

natijani olamiz. Teskari matritsa to`g`ri qurilganini ta`rif asosida tekshirib ko`ramiz:

$$AA^{-1} = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 2/7 & 1/7 \\ -3/7 & 2/7 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = E$$

Demak, berilgan A matritsaning teskarisi $A^{-1} = \begin{pmatrix} 2/7 & 1/7 \\ -3/7 & 2/7 \end{pmatrix}$.

Misol. $A = \begin{pmatrix} 0 & 1 & -3 \\ 1 & -5 & 4 \\ 2 & 3 & 2 \end{pmatrix}$ maxsusmas matritsaning teskari matritsasini klassik usulda quring.

Matritsaning teskarisi $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}^{-1} = \frac{1}{\det A}\begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}$

formula asosida quriladi.

Formulani qo`llab,

$$A = \begin{vmatrix} 0 & 1 & -3 \\ 1 & -5 & 4 \\ 2 & 3 & 2 \end{vmatrix} = 0 + 8 - 9 - 30 - 0 - 2 = -33$$

$$A_{11} = \begin{vmatrix} -5 & 4 \\ 3 & 2 \end{vmatrix} = -10 - 12 = -22 \qquad A_{12} = -\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} = -(2-8) = 6$$

$$A_{13} = \begin{vmatrix} 1 & -5 \\ 2 & 3 \end{vmatrix} = 3 + 10 = 13 \qquad A_{21} = -\begin{vmatrix} 1 & -3 \\ 3 & 2 \end{vmatrix} = -(2+9) = -11$$

$$A_{22} = \begin{vmatrix} 0 & -3 \\ 2 & 2 \end{vmatrix} = 0 + 6 = 6 \qquad A_{23} = -\begin{vmatrix} 0 & 1 \\ 2 & 3 \end{vmatrix} = -(0-2) = 2$$

$$A_{31}=\begin{vmatrix}1 & -3\\ -5 & 4\end{vmatrix}=4-15=-11 \qquad A_{32}=-\begin{vmatrix}0 & -3\\ 1 & 4\end{vmatrix}=-(0+3)=-3$$

$$A_{33}=\begin{vmatrix}0 & 1\\ 1 & -5\end{vmatrix}=0-1=-1 \qquad A_{33}=\begin{vmatrix}2 & 2\\ -3 & -4\end{vmatrix}=-8+6=-2$$

$$A^{-1}=-\frac{1}{33}\begin{pmatrix}-22 & -11 & -11\\ 6 & 6 & -3\\ 13 & 2 & -1\end{pmatrix}=\begin{pmatrix}\frac{2}{3} & \frac{1}{3} & \frac{1}{3}\\ -\frac{2}{11} & -\frac{2}{11} & \frac{1}{11}\\ -\frac{13}{33} & -\frac{2}{33} & \frac{1}{33}\end{pmatrix}$$

$$AA^{-1}=\begin{pmatrix}0 & 1 & -3\\ 1 & -5 & 4\\ 2 & 3 & 2\end{pmatrix}\cdot\begin{pmatrix}\frac{2}{3} & \frac{1}{3} & \frac{1}{3}\\ -\frac{2}{11} & -\frac{2}{11} & \frac{1}{11}\\ -\frac{13}{33} & -\frac{2}{33} & \frac{1}{33}\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

Teskari matritsa o`zining quyidagi xossalariga ega:

1) $(A^{-1})^{-1} = A$; 2) $(A^{T})^{-1} = (A^{-1})^{T}$; 3) $(AB)^{-1} = B^{-1}A^{-1}$.

Mustaqil yechish uchun misollar:

Teskari matritsasini toping.

$$A=\begin{pmatrix}1 & 3\\ 2 & 4\end{pmatrix},\quad B=\begin{pmatrix}3 & 2\\ 2 & 5\end{pmatrix},\quad C=\begin{pmatrix}1 & 2\\ 4 & 3\end{pmatrix},\quad D=\begin{pmatrix}-2 & 2\\ 3 & 1\end{pmatrix},$$

$$G=\begin{pmatrix}5 & 0\\ 1 & 3\end{pmatrix},\quad F=\begin{pmatrix}-2 & -3\\ 1 & 5\end{pmatrix},\quad J=\begin{pmatrix}4 & 3\\ 1 & 2\end{pmatrix},\quad K=\begin{pmatrix}1 & 5\\ -2 & 4\end{pmatrix},$$

$$M=\begin{pmatrix}3 & 1 & 0\\ 2 & 2 & 1\\ 6 & 3 & 7\end{pmatrix},\quad N=\begin{pmatrix}4 & 2 & 1\\ 1 & 3 & 3\\ 3 & 2 & -1\end{pmatrix},\quad L=\begin{pmatrix}4 & 2 & 1\\ 1 & 3 & 3\\ 3 & 2 & -1\end{pmatrix}$$

$$P=\begin{pmatrix}1 & 2 & 0\\ 3 & 2 & 4\\ 1 & 3 & 2\end{pmatrix},\quad O=\begin{pmatrix}0 & 1 & 3\\ 2 & 4 & 1\\ 1 & 3 & 2\end{pmatrix},\quad V=\begin{pmatrix}3 & 1 & 3\\ 1 & 0 & 4\\ 2 & 2 & 3\end{pmatrix}$$

Chiziqli tenglamalar sistemasining umumiy ko'rinishi va uning yechimi.

Ikkita noma'lumli ikkita chiziqli tenglamalar sistemasi $\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}$

ning bosh determinanti $\Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0$ bo'lganda, yagona yechimga ega va u Kramer qoidasi bo'yicha quyidagi formulalar bilan hisoblanadi: $x_1 = \frac{\Delta x_1}{\Delta}$, $x_2 = \frac{\Delta x_2}{\Delta}$

bu yerda

$$\Delta x_1 = \begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix} \neq 0 \qquad \Delta x_1 = \begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix} \neq 0$$

Agar $\Delta = 0$ va shu bilan birga Δx_1 , Δx_2 lardan aqalli bittasi nolga teng bo'lmasa, sistema yechimga ega emas. Agar $\Delta = \Delta x_1 = \Delta x_2 = 0$ bo'lsa, u xolda berilgan sistema cheksiz ko'p yechimga ega bo'ladi.

Misol.

$$\begin{cases} 2x_1 + 3x_2 = 12 \\ 4x_1 - 2x_2 = 8 \end{cases} \qquad \Delta = \begin{vmatrix} 2 & 3 \\ 4 & -2 \end{vmatrix} = -4 - 12 = -16$$

$$\Delta x_1 = \begin{vmatrix} 12 & 3 \\ 8 & -2 \end{vmatrix} = -24 - 24 = -48 \qquad \Delta x_2 = \begin{vmatrix} 2 & 12 \\ 4 & 8 \end{vmatrix} = 16 - 48 = -32$$

$$x_1 = \frac{\Delta x_1}{\Delta} = \frac{-48}{-16} = 3 \qquad x_2 = \frac{\Delta x_2}{\Delta} = \frac{-32}{-16} = 2$$

Uch noma'lumli uchta chiziqli tenglamalar sistemasi $\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases}$

bosh determinanti $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \neq 0$ bo'lganda, yagona yechimga ega va u Kramer qoidasi bo'yicha quyidagi formulalar bilan hisoblanadi:

$x_1 = \frac{\Delta x_1}{\Delta}$, $x_2 = \frac{\Delta x_2}{\Delta}$, $x_3 = \frac{\Delta x_3}{\Delta}$ bu yerda

$$\Delta x_1 = \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix} \qquad \Delta x_2 = \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix} \qquad \Delta x_3 = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}$$

Agar $\Delta = 0$ va shu bilan birga Δx_1 , Δx_2 , Δx_3 lardan aqalli bittasi nolga teng bo’lmasa, sistema yechimga ega emas. Agar $\Delta = \Delta x_1 = \Delta x_2 = \Delta x_3 = 0$ bo’lsa, u xolda berilgan sistema cheksiz ko’p yechimga ega bo’ladi.

$$\begin{cases} x_1 - 2x_2 + x_3 = 1 \\ 3x_1 + x_2 - 2x_3 = 5 \\ x_1 + x_2 + x_3 = 4 \end{cases} \qquad \Delta = \begin{vmatrix} 1 & -2 & 1 \\ 3 & 1 & -2 \\ 1 & 1 & 1 \end{vmatrix} = 1+4+3-1+2+6=15$$

$$\Delta x_1 = \begin{vmatrix} 1 & -2 & 1 \\ 5 & 1 & -2 \\ 4 & 1 & 1 \end{vmatrix} = 1+16+5-4+2+10=30$$

$$\Delta x_2 = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 5 & -2 \\ 1 & 4 & 1 \end{vmatrix} = 5-2+12-5+8-3=15$$

$$\Delta x_3 = \begin{vmatrix} 1 & -2 & 1 \\ 3 & 1 & 5 \\ 1 & 1 & 4 \end{vmatrix} = 4-10+3-1-5+24=15$$

$$x_1 = \frac{\Delta x_1}{\Delta} = \frac{30}{15} = 2 \qquad x_2 = \frac{\Delta x_2}{\Delta} = \frac{15}{15} = 1 \qquad x_3 = \frac{\Delta x_3}{\Delta} = \frac{15}{15} = 1$$

Demak, $\mathbf{x_1} = 2$, $\mathbf{x_2} = 1$, $\mathbf{x_3} = 1$

n ta noma’lumli n ta chiziqli tenglamalar sistemasini **n** ning (n> 4) qiymatlarida Kramer qoidasi bilan yechish bir necha yuqori tartibli determinantlarni hisoblashni talab etadi. Shu sababli bunday sistemalarni yechishda Gauss usulidan foydalanish maqsadga muvofiq. Bu usulning mohiyati shundan iboratki, unda noma’lumlar ketma- ket yo’qotilib, sistema uchburchak shaklga keltiriladi. Agar sistema uchburchaksimon shaklga kelsa, u yagona yechimga ega bo’ladi va uning noma’lumlari oxirgi tenglamadan boshlab topib boriladi. Sistema cheksiz ko’p yechimga ega bo’lsa, noma’lumlar ketma- ket yo’qotilgach, u trapetsiya shakliga keladi

$$\begin{cases} x_1 + 2x_2 - 3x_3 + x_4 = -1 \\ 3x_1 - 2x_2 + x_3 + 2x_4 = 7 \\ 2x_1 - 3x_2 - x_3 + 4x_4 = 5 \\ 2x_1 + x_2 - 3x_3 + x_4 = -1 \end{cases} \qquad \begin{cases} x_1 + 2x_2 - 3x_3 + x_4 = -1 \\ 8x_2 - 10x_3 + x_4 = -10 \\ 7x_2 - 5x_3 - 2x_4 = -7 \\ 3x_2 - 3x_3 + x_4 = -1 \end{cases}$$

$$\begin{cases} x_1 + 2x_2 - 3x_3 + x_4 = -1 \\ 8x_2 - 10x_3 + x_4 = -10 \\ -30x_3 + 23x_4 = -14 \\ -6x_3 - 5x_4 = -22 \end{cases} \qquad \begin{cases} x_1 + 2x_2 - 3x_3 + x_4 = -1 \\ 8x_2 - 10x_3 + x_4 = -10 \\ -30x_3 + 23x_4 = -14 \\ 288x_4 = 576 \end{cases}$$

$x_4 = 2$, $-30x_3 + 23x_4 = -14$, $-30x_3 = -60$, $x_3 = 2$

$8x_2 - 10x_3 + x_4 = -10$, $8x_2 = 8$, $x_2 = 1$, $x_1 + 2x_2 - 3x_3 + x_4 = -1$, $x_1 = 1$

Shunday qilib $x_1 = 1$, $x_2 = 1$, $x_3 = 2$, $x_4 = 2$

Mustaqil yechish uchun misollar:

Tenglamalar sistemasini Kramer formulasida va Gauss usulida yeching.

1. $\begin{cases} 2x + 3y = 8 \\ 5x - 4y = -3 \end{cases}$ **2.** $\begin{cases} x - 2y = -2 \\ 3x - y = 9 \end{cases}$ **3.** $\begin{cases} 2x + 2y = -4 \\ -6x + 4y = 2 \end{cases}$

4. $\begin{cases} 5x + 6y - 2z = 12 \\ 2x + 5y - 3z = 9 \\ 4x - 3y + 2z = -15 \end{cases}$ **5.** $\begin{cases} 3x + y - 2z = 6 \\ 2y - z = 0 \\ 4x - 3y + 5z = 19 \end{cases}$

6. $\begin{cases} 3x + 2y + z = 5 \\ 2x + 3y + z = 1 \\ 2x + y + 3z = 11 \end{cases}$ **7.** $\begin{cases} 3x + 2y + z = 5 \\ 2x + 3y + z = 1 \\ 2x + y + 3z = 11 \end{cases}$

8. $\begin{cases} 2x - y - z = 4 \\ 3x + 4y - 2z = 11 \\ 3x - 2y + 4z = 11 \end{cases}$ **9.** $\begin{cases} x - 2y + 3z = 6 \\ 2x + 3y - 4z = 20 \\ 3x - 2y - 5z = 6 \end{cases}$

10. $\begin{cases} x - 9y + 2z = 5 \\ x + 3y - 8z = 3 \\ x - 3y - 2z = 8 \end{cases}$ **11.** $\begin{cases} 3x - 3y - z = -5 \\ 3x + 5y + 4z = 17 \\ 4x - 2y + 3z = 8 \end{cases}$

12. $\begin{cases} x - y - z = 4 \\ x + 3y - 7z = 3 \\ x - 5y + 5z = 7 \end{cases}$ **13.** $\begin{cases} 4x + 5y - 3z = 5 \\ 2x - y + z = 3 \\ 3x + 2y - z = 7 \end{cases}$

14. $\begin{cases} 4x + y - 2z = 3 \\ 2x - 3y - 4z = 2 \\ 3x - y - 3z = 6 \end{cases}$ **15.** $\begin{cases} 4x - 3y + 2z = 2 \\ 3x - 2y - z = 3 \\ x - y + 3z = 5 \end{cases}$

16. $\begin{cases} 4x + y - 3z = -6 \\ 8x + 3y - 6z = -15 \\ x + y - z = -4 \end{cases}$ **17.** $\begin{cases} 5x + 7y + 10z = 4 \\ x - 3y + 4z = 5 \\ 2x + 5y + 3z = 8 \end{cases}$

II bob. Analitik geometriya.

Tekislikda ikkinchi tartibli egri chiziqlar. Ellips, giperbola va parabola ta'riflari, kanonik tenglamalari va xossalari.

Bu modulda ikkinchi tartibli egri chiziqlardan aylana, ellips, giperbola va parabolalar hamda ularga doir masalalarni keltiramiz.

1. Aylana

Ma'lumki, tekislikda berilgan **A(a;b)** nuqtadan bir xil **R** masofada joylashgan nuqtalarning geometrik o'rni aylana deb ataladi. Bunda A(a;b) nuqta aylana markazi, **R** esa aylana radiusidir.

Aylana ta'rifidan foydalanib, markazi A(a;b) nuqtada va radiusi R bo'lgan aylananing

$$(x-a)^2+(y-b)^2=R^2$$

kanonik tenglamasini keltirib chiqarish mumkin.

Agar aylananing markazi koordinata boshida bo'lsa, ya'ni **a=0, b=0** bo'lsa, u holda tenglama $x^2+y^2=R^2$ ko'rinishga keladi.

Agar aylananing markazi **Ox** o'qida yotsa, u holda uning tenglamasi

$$(x-a)^2+y^2=R^2$$

ko'rinishda, agar aylananing markazi **Oy** o'qida yotsa, u holda uning tenglamasi

$$x^2+(y-b)^2=R^2$$

ko'rinishda bo'ladi.

Aylana tenglamasi **X** va **Y** larga nisbatan ikkinchi darajali tenglama bo'lib, unda x^2 va y^2 lar bir xil koeffitsientlar bilan qatnashadi. Bundan tashqari, tenglamada **x** va **y** lar ko'paytmasi qatnashmaydi.

Demak, aylana tenglamasi umumiy ko'rinishda quyidagicha yoziladi.

$$Ax^2+Ay^2+Bx+Cy+D=0$$

Bu tenglamani $(x-a)^2+(y-b)^2=R^2$ tenglama bilan solishtirib,

$$a=-\frac{B}{2A},\qquad b=-\frac{C}{2A},\qquad R=-\frac{\sqrt{B^2+C^2-4AD}}{2A}$$

ekanligini ko'rishimiz mumkin.

Bunda quyidagi xususiy hollar bo'lishi mumkin:

a) $B^2+C^2>4AD$ bo'lsa, u holda **R>0** bo'lib, $Ax^2+Ay^2+Bx+Cy+D=0$ tenglama aylana tenglamasini ifodalaydi;

b) $B^2+C^2>4AD$ bo'lsa, u holda **R=0** bo'lib, qaralayotgan tenglama bitta nuqtani anglatadi.

c) $B^2+C^2>4AD$ bo'lsa, u holda **R** mavhum son bo'lib, qaralayotgan tenglama ma'noga ega bo'lmaydi.

$(x-x_0)^2+(y-y_0)^2=a^2$ tenglama markazi $C(x_0;y_0)$ nuqtada bo'lgan ellipsni aniqlaydi.

Mavzuga doir yechimlari bilan berilgan topshiriqlardan namunalar

1. Markazi A(2;3) nuqtada va radiusi R=6 bo'lgan aylananing tenglamasini yozing.

Yechish: Masalaning shartiga asosan a=2, b=3 va R=6. Bularni aylananing kanonik tenglamasiga qo'yib $(x-2)^2+(y-3)^2=36$ ni hosil qilamiz.

2. $x^2+6x+y^2-8y-11=0$ tenglama bilan berilgan aylananing markazini va radiusini toping.

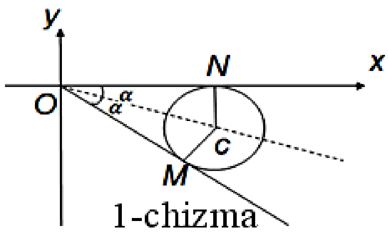
Yechish: Berilgan tenglamani kanonik ko'rinishga keltiramiz:

$x^2+6x+y^2-8y-11=(x+3)^2-9+(y-4)^2-16-3=(x+3)^2+(y-4)^2-36.$

Demak, $(x+3)^2+(y-4)^2-36=0$ yoki $(x+3)^2+(y-4)^2=6^{2.}$ Bundan esa A(-3;4) va R=6 kelib chiqadi.

3. Markazi C(5;-1) nuqtada va radiusi R=1 bo'lgan aylanaga koordinatalar boshidan o'tkazilgan urinmalar yasalsin hamda ularning tenglamalari tuzilsin.

Yechish: Aylananing markazi absissa o'qidan radiusga teng masofada joylashganligi uchun, urinmalardan biri absissa o'qidan iborat bo'ladi (1-chizma).



1-chizma

Aytaylik, $\angle CON = \alpha$ bo'lsin, u holda $tg\alpha = \frac{1}{5}$ bo'ladi, $tg\alpha$ ni hisoblaymiz:

$$tg\alpha = \frac{2tg\alpha}{1-tg^2\alpha} = \frac{2\cdot\frac{1}{5}}{1-\frac{1}{25}} = \frac{\frac{2}{5}}{\frac{24}{25}} = \frac{5}{12}$$

Demak, OM to'g'ri chiziqning burchak koeffitsienti $-\frac{5}{12}$ ga teng bo'lib, uning tenglamasi $y = -\frac{5}{12}x$ dan iborat bo'ladi.

Mustaqil yechish uchun topshiriqlar

1. Quyidagi tenglamalar bilan berilgan aylanalar yasalsin.

1) $(x-2)^2+(y-4)^2=25$; 2) $(x-2)^2+(y+4)^2=25$;

3) $x^2+(y-3)^2=9$; 4) $x^2+y^2=9$

2. A(5;3) nuqtadan o'tuvchi va markazi 5x-3y-13=0 va x+4y+2=0 to'g'ri chiziqlarning kesishish nuqtasida bo'lgan aylana tenglamasi tuzilsin.

Javob: $(x-2)^2+(y+1)^2=25$.

3. $x^2+y^2=169$ aylana bilan 5x-12y =0 to'g'ri chiziqning kesishish nuqtalari topilsin.

Javob: (12;5) va (-12;-5).

4. $x^2+4y^2=4$ ellips fokuslaridan otuvchi va markazi ellipsning yuqori uchida bo'lgan aylana tenglamasini tuzing.

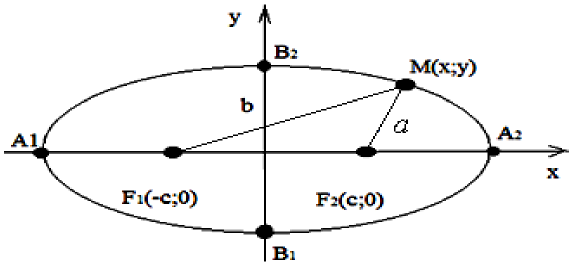
Javob: $x^2+(y-1)^2=4$

2. Ellips

Ellips deb tekislikdagi shunday nuqtalar to'plamiga aytiladiki, bu nuqtalarning har biridan shu tekislikning fokuslar deb ataluvchi $\mathbf{F_1}$ va $\mathbf{F_2}$ ikki nuqtasigacha bo'lgan masofalar yig'indisi o'zgarmas miqdordir. Bunda $\mathbf{F_1(-c;0)}$ va $\mathbf{F_2(c;0)}$ nuqtalar ellipsning fokuslari deyiladi (1-chizma).

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, \quad a > b, \quad b^2 = a^2 - c^2$$

ga ellipsning kanonik tenglamasi deyiladi.



1-chizma

$A_1A_2 = 2a$ ellipsning katta o'qi, $B_1B_2 = 2b$ ellipsning kichik o'qi deyiladi. $OA_1 = OA_2 = a$ va $OB_1 = OB_2 = b$ lar mos ravishda ellipsning katta va kichik yarim o'qlari deyiladi. $OF_1 = OF_2 = c$ ellipsning fokusi deyiladi (1-chizma).

Ellipsning fokuslari orasidagi masofani uning katta o'qi uzunligiga nisbati ellipsning eksentrisiteti deyiladi va ε bilan belgilanadi. Demak,

$$\varepsilon = \frac{2c}{2a} = \frac{c}{a}$$

$r_1 = a + \varepsilon x$ va $r_2 = a - \varepsilon x$ larni ellipsning fokal radiuslari deyiladi.

$x = \frac{a}{\varepsilon}$ va $x = -\frac{a}{\varepsilon}$ tenglamalar bilan berilgan to'g'ri chiziqlarga ellipsning direktrisalari deyiladi.

$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1$ tenglama markazi $C(x_0; y_0)$ nuqtada bo'lgan ellipsni aniqlaydi.

Mavzuga doir yechimlari bilan berilgan topshiriqlardan namunalar

1. $x^2 + 9y^2 = 81$ tenglama ellipsni ifodalashini ko'rsating va uning barcha xarakteristikalarini toping.

Yechish: Dastlab berilgan tenglamaning ikkala tomonini 9 ga bo'lamiz: $\frac{x^2}{81} + \frac{y^2}{9} = 1$

Bu yerdan berilgan tenglama yarim o'qlari $a = 9$ va $b = 3$ bo'lgan ellipsni bildirishini ko'ramiz. Unda $c^2 = a^2 - b^2 =$ 81-9=72 bo'lgani uchun qaralayotgan ellipsning fokuslari $F_1(-\sqrt{72};0)$ va $F_2(\sqrt{72};0)$ nuqtalarda joylashganini aniqlaymiz. Topilganlardan foydalanib, ellipsning eksentrisiteti va direktrisasini topamiz:

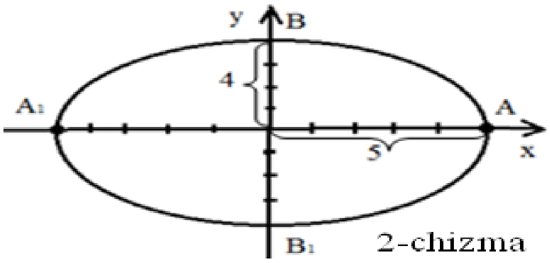
$$\varepsilon = \frac{c}{a} = \frac{\sqrt{72}}{9}, \quad x = \pm\frac{a}{\varepsilon} = \pm\frac{9}{\frac{\sqrt{72}}{9}} = \pm\frac{81}{\sqrt{72}}$$

Ellipsga tegishli M(x;y) nuqtaning fokal radiuslari $r_1 = 9 + 9x$, $r_2 = 9 - 9x$ lardan iborat.

2. $9x^2 + 16y^2 = 144$ ellips berilgan. Bu ellips o'qlarining uzunliklarini, uchlari va fokuslarining koordinatalarini hamda ekstentrisitetini toping.

Yechish: Berilgan tenglamani ikkala tomonini hadma-had 144 ga bo'lsak, $\frac{x^2}{16} + \frac{y^2}{9} = 1$ tenglama hosil bo'ladi.

Bu tenglamadan $a^2 = 16,\ b^2 = 9$ bo'lib, ulardan $a = 4,\ b = 3$ kelib chiqadi. Demak, ellipsning katta o'qi $2a = 8$, kichik o'qi esa $2b = 6$ (2-chizma).



2-chizma

Demak ellipsning uchlari A(4;0), A_1(-4;0), B(0;3), B_1(0;-3) nuqtalarda. Ellipsning fokuslari $c = \sqrt{a^2 - b^2}$ formuladan topiladi.

Demak, $c = \sqrt{16 - 9} = \sqrt{7}$. Shunday qilib, ellipsning fokuslari F_1 $(\sqrt{7};0)$ va F_2 $(-\sqrt{7};0)$ nuqtalarda bo'ladi.

Ellipsning eksentrisiteti $\varepsilon = \frac{c}{a} = \frac{\sqrt{7}}{4}$ ga teng.

3. Fokuslari orasidagi masofa 8 ga va kichik o'qi 6 ga teng bo'lgan ellipsning tenglamasi tuzilsin.

Yechish: Berilganlarga ko'ra, 2c= 8, 2b=6 bo'lib, ulardan c=4 va b=3 ekanini aniqlaymiz. a ni topish uchun $a = \sqrt{b^2 + c^2}$ dan foydalanamiz.

Demak, $a = \sqrt{9 + 16} = \sqrt{25} = 5$

a va b ning qiymatlarini ellipsning kanonik tenglamasiga qo'yilsa, izlangan tenglama hosil bo'ladi. U quyidagicha:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad \frac{x^2}{25}+\frac{y^2}{9}=1$$

4. $\frac{x^2}{25}+\frac{y^2}{16}=1$. tenglama bilan berilgan ellipsning chap fokusi va qo'yi uchidan o'tuvchi to'g'ri chiziq tenglamasini tuzing.

Yechish: Quyidagilarni topamiz:

1) qo'yi uchi koordinatalari: x = 0; y^2 =16; y = -4.
2) Chap fokusi koordinatalari: $c^2 = a^2 - b^2$ =25 – 16 = 9; c = 3; F_2(-3; 0).
3) Ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasi:

$$\frac{x-0}{-3-0}=\frac{y+4}{0+4}; \qquad \frac{x}{-3}=\frac{y+4}{4}; \qquad 4x=-3y-12; \qquad 4x+3y+12=0$$

5. Fokuslari $F_1(0;\ 0)$, $F_2(1;\ 1)$ katta yarim o'qi uzunligi 2 bo'lgan ellips tenglamasini tuzing.

Yechish:Ellips tenglamasi: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Fokuslar orasidagi masofa:

2c = $\sqrt{(1-0)^2+(1-0)^2}=\sqrt{2}$

demak, $a^2 - b^2 = c^2 = \frac{1}{2}$

Shartga ko'ra 2a = 2 bundan a = 1, $b=\sqrt{a^2-c^2}=\sqrt{1-\frac{1}{2}}=\frac{\sqrt{2}}{2}$.

Izlanayotgan ellips tenglamasi : $\frac{x^2}{1}+\frac{y^2}{\frac{1}{2}}=1$.

Mustaqil yechish uchun topshiriqlar

1. Uchlari (± 4;0) va (0; ± 2) nuqtalarda bo'lgan ellipsning kanonik tenglamasi tuzilsin hamda uning fokuslari topilsin.

Javob: $\frac{x^2}{16}+\frac{y^2}{4}=1$, $\quad F_1(-\sqrt{12};0) \quad F_2(\sqrt{12};0)$

2. Fokuslari Ox o'qida bo'lib, o'qlari 16 va 8 bo'lgan ellipsning tenglamasi tuzilsin.

Javob: $\frac{x^2}{64}+\frac{y^2}{16}=1$.

3. Fokuslari $(\pm 8;0)$ nuqtada va ekssentrisiteti $\varepsilon = 0.8$ bo’lgan ellipsning tenglamasi yozing.

Javob: $\frac{x^2}{100} + \frac{y^2}{36} = 1$.

4. Katta o’qi 6 ga teng, fokusi esa $F(\sqrt{5};0)$ nuqtada bo’lgan ellipsning tenglamasini tuzing.

Javob: $\frac{x^2}{9} + \frac{y^2}{4} = 1$

5. Ellipsning katta yarim o’qi 9 ga, ekstsentrisiteti esa $\frac{\sqrt{17}}{9}$ ga teng bo’lsa uning kanonik tenglamasini tuzing.

Javob: $\frac{x^2}{81} + \frac{y^2}{64} = 1$

6. Ellipsning kichik yarim o’qi 4 ga, focus nuqtasi esa $F(-3;0)$ ga teng bo’lsa uning kanonik tenglamasini tuzing.

Javob: $\frac{x^2}{25} + \frac{y^2}{16} = 1$

7. Ellipsning katta o’qi 6 ga, ekstsentrisiteti esa $\frac{\sqrt{5}}{3}$ ga teng bo’lsa uning kanonik tenglamasini tuzing.

Javob: $\frac{x^2}{9} + \frac{y^2}{4} = 1$

8. Ellipsning kanonik tenglamasi $\frac{x^2}{81} + \frac{y^2}{64} = 1$ berilgan bo’lsa, uning katta yarim o’qi ekstsentrisitetini toping.

Javob: $a = 9, \quad \varepsilon = \frac{\sqrt{17}}{9}$

9. Ellipsning kanonik tenglamasi $\frac{x^2}{25} + \frac{y^2}{16} = 1$ berilgan bo’lsa, kichik o’qi va fokus nuqtasini toping.

Javob**:** $2b = 8, \quad F(-3;0)$

10. Ellipsning kanonik tenglamasi $36x^2+100y^2=3600$ berilgan bo'lsa, focus nuqtalarini va ekssentrisitetini toping.

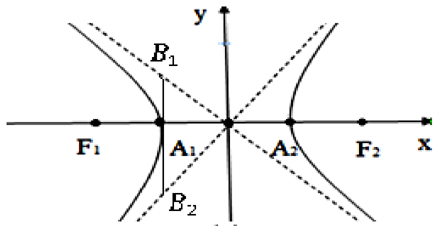
Javob: $\mathbf{F}(\pm 8;0)$, $\varepsilon=0.8$

3 Giperbola

Giperbola deb tekislikdagi shunday nuqtalar to'plamiga aytiladiki, bu nuqtalarning har biridan shu tekislikning fokuslar deb ataluvchi F_1 va F_2 ikki nuqtasigacha bo'lgan masofalar ayirmasining absolyut qiymati o'zgarmas miqdordir. Bunda $F_1(-c;0)$ va $F_2(c;0)$ nuqtalar giperbolaning fokuslari deyiladi (1-chizma).

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1, \quad a>b, \quad b^2=c^2-a^2$$

tenglamaga giperbolaning kanonik tenglamasi deyiladi.



1-chizma

Giperbola Ox o'qini ikkita $A_1(-a;0)$ va $A_2(a;0)$ nuqtalarda kesib o'tadi. Ular giprbolaning uchlari deyiladi. Ular orasidagi $|A_1A_2|=2a$ masofa giperbolaning haqiqiy o'qi deyiladi. $B_1(0;-b)$ va $B_2(0;b)$ nuqtalar giperbolaning mavhum uchlari, ular orasidagi $|B_1B_2|=2b$ masofa esa giperbolaning mavhum o'qi deb ataladi. a va b sonlariga giperbolaning haqiqiy va mavhum yarim o'qlari deb ataladi. Giperbolaning o'qlari kesishadigan nuqta uning markazi deyiladi.

$$y=\pm\frac{b}{a}x$$

tenglamalar bilan aniqlangan chiziqlarga giperbolaning asimptotalari deb ataladi.

Giperbolaning fokuslari orasidagi masofani uning haqiqiy o'qi uzunligiga nisbati giperbolaning eksentrisiteti deyiladi va u ε bilan belgilanadi. Demak,

$$\varepsilon=\frac{2c}{2a}=\frac{c}{a}>1$$

Giperbolaning $M(x;y)$ nuqtasidan uning F_1 va F_2 fokuslarigacha bo'lgan masofalar shu nuqtaning fokal radiuslari deyiladi. Ular uchun

$$r_1 = \pm(a+\varepsilon x) \qquad \text{va} \qquad r_2 = \pm(a-\varepsilon x)$$

formulalar o'rinlidir.

Tenglamalari $x = \pm\dfrac{a}{\varepsilon}$ bo'lgan ikkita ℓ_1 va ℓ_2 vertikal to'g'ri chiziqlar giperbolaning direktrisalari deb ataladi. Ularning biri O markaz bilan A_1 nuqta orasida, ikkinchisi esa O markaz bilan A_2 nuqta orasida joylashgan bo'ladi.

Agar giperbolaning kanonik tenglamasida $a=b$ bo'lsa, u holda tenglama $x^2 - y^2 = a^2$ ko'rinishga keladi va u teng tomonli giperbola deyiladi.

$\dfrac{(x-x_0)^2}{a^2} - \dfrac{(y-y_0)^2}{b^2} = 1$ tenglama markazi $C(x_0;y_0)$ nuqtada bo'lgan giperbolani aniqlaydi.

Mavzuga doir yechimlari bilan berilgan topshiriqlardan namunalar

1. $\dfrac{x^2}{25} - \dfrac{y^2}{16} = 1$ tenglama bilan berilgan giperbolaning barcha xarakteristikalarini toping. Absissasi 5 ga teng va ordinatasi musbat bo'lgan M nuqtaning fokal radiuslarini aniqlang.

Yechish: $a^2 = 25$ va $b^2 = 16$ lardan giperbolaning yarim o'qlari $a = 5$ va $b = 4$ larni topamiz. $c^2 = a^2 + b^2 = 25 + 16 = 41$ bo'lib, undan $c = \sqrt{41}$ ni topamiz. Demak, giperbolaning fokuslari $F_1(-\sqrt{41_1};0)$ va $F_2(\sqrt{41_1};0)$ nuqtalarda joylashadi.

Berilgan giperbolaning asimptotalari

$$y = \pm\frac{b}{a}x = \pm\frac{4}{5}x = \pm 0.8x$$

Eksentrisiteti $\varepsilon = \dfrac{c}{a} = \dfrac{\sqrt{41}}{5}$ ga teng va direktrisalarining tenglamalari

$x = \pm\dfrac{a}{\varepsilon} = \pm\dfrac{5}{\frac{\sqrt{41}}{5}} = \pm\dfrac{25}{\sqrt{41}}$ bo'ladi.

Giperboladagi $M(5;y)$ nuqtaning fokal radiuslari

$$r_1 = a + \varepsilon x = 5 + \frac{\sqrt{41}}{5}5 = 5 + \sqrt{41} \quad r_2 = -a + \varepsilon x = -5 + \frac{\sqrt{41}}{5}5 = -5 + \sqrt{41}.$$

2. Fokuslari orasidagi masofa 10 ga va mavhum yarim o'qi 3 ga teng bo'lgan giperbolaning kanonik tenglamasini tuzing.

Yechish: Berilganlarga ko'ra b=3, 2c=10 ga teng, bundan c=5 hamda $a = \sqrt{c^2 - b^2} = \sqrt{25-9} = \sqrt{16} = 4$ ni topamiz.

Demak, giperbolaning kanonik tenglamasi $\frac{x^2}{16} - \frac{y^2}{9} = 1$

3. $9x^2 + 16y^2 = 144$ giperbola berilgan. Bu giperbolaning yarim o'qlari uzunliklarini va fokuslarining koordinatalarini hamda ekstentrisitetini toping.

Yechish: Dastlab berilgan tenglamaning ikkala tomonini 144 ga bo'lamiz: $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Bu yerdan berilgan tenglama yarim o'qlari $a = 16$ va $b = 9$ bo'lgan giperbolani bildirishini ko'ramiz. Unda $c^2 = a^2 + b^2 = 256 + 9 = 265$ bo'lgani uchun qaralayotgan giperbolaning fokuslari $F_1(-\sqrt{265};0)$ va $F_2(\sqrt{265};0)$ nuqtalarda joylashganini aniqlaymiz. Topilganlardan foydalanib, giperbolaning eksentrisitetini topamiz:

$$\varepsilon = \frac{c}{a} = \frac{\sqrt{265}}{9},$$

4. Fokuslari abstsissalar o'qida yotuvchi giperbolaning fokuslari orasidagi masofa 6 ga va eksentrisiteti $\frac{3}{2}$ ga teng bo'lsa, uning kanonik tenglamasini tuzing.

Yechish: Berilgan shartga asosan fokuslari orasidagi masofa 2c=6, eksentrisiteti $\varepsilon = 1{,}5$ ga teng. Bundan c=3, $\varepsilon = \frac{3}{2}$ boladi. $\varepsilon = \frac{c}{a}$ ga asosan $a = \frac{c}{\varepsilon} = \frac{3}{\frac{3}{2}} = 2$ bulardan $b^2 = c^2 - a^2 = 9 - 4 = 5$. Demak: Giperbolaning kanonik tenglamasi $\frac{x^2}{4} - \frac{y^2}{5} = 1$

5. Giperbolaning mavhum yarim o'qi 7 ga, focus nuqtasining koordinatasi esa $F_1(-\sqrt{130};0)$ ga teng bo'lsa uning kanonik tenglamasini tuzing.

Yechish: Berilgan shartga asosan b=7, $c=\sqrt{130}$ ga teng. Bundan $b^2=49$, $c^2=130$ bo'ladi. $b^2=c^2-a^2$ dan $a^2=c^2-b^2=130-49=81$ ekani kelib chiqadi.

Demak: Giperbolaning kanonik tenglamasi $\frac{x^2}{81}-\frac{y^2}{49}=1$ bo'ladi.

6. Giperbolaning haqiqiy yarim o'qi 8 ga, eksentrisiteti $\varepsilon=\frac{5}{4}$ ga teng bo'lsa uning kanonik tenglamasini tuzing.

Yechish: Berilgan shartga asosan $a=8$, $\varepsilon=\frac{5}{4}$ ga teng. Bundan $a^2=64$ bo'ladi. Berilganlardan $\varepsilon=\frac{c}{a}$ ga asosan $c=\varepsilon\cdot a=\frac{5}{4}\cdot 8=10$, $c^2=100$ni topamiz. So'ng $b^2=c^2-a^2=100-64=36$ ekani kelib chiqadi.

Demak: Giperbolaning kanonik tenglamasi $\frac{x^2}{64}-\frac{y^2}{36}=1$ bo'ladi.

7.Agar giperbolaning haqiqiy o'qi 18 ga, mavhum o'qi esa 8ga teng bo'lsa, fokuslari Ox o'qda yotgan giperbolaning tenglamasini tuzing.

Yechish: Giperbolaning tenglamasini tuzish uchun a va b parametrlarni bilish zarur. Masalaning shartidan: $2a=18\Rightarrow a=9$; $2b=8\Rightarrow b=4$.

Topilgan qiymatlarni (1.4) ga qo'ysak: $\frac{x^2}{81}-\frac{y^2}{16}=1$

8. Agar giperbolaning uchlari A_1 (-2 ; 0) va A_2 (2 ; 0) nuqtalarda joylashgan, fokuslri esa F_1 (-4 ; 0) va F_2 (4 ; 0) nuqtalarda joylashgan bo'lsa, giperbola tenglamasini tuzing.

Yechish: Shartdan a=2, c=4 ekani kelib chiqadi. (1.3) formulaga ko'ra $b^2 = 4^2 - 2^2 = 12$. Bu qiymatlarni (1.4) tenglamaga qo'yib, $\frac{x^2}{4} - \frac{y^2}{12} = 1$ ni hosil qilamiz.

9. Giperbolaning tenglamasi berilgn $\frac{x^2}{64} - \frac{y^2}{57} = 1$. Uning uchlarining va fokuslarining koordinatalarini topig.

Yechish: Giperbolaning tenglamasidan: $a^2 = 64 \Rightarrow a = \pm 8$.

(1.3) formulaga ko'ra $c^2 = a^2 + b^2 = 64 + 57 = 121 \Rightarrow c = \pm 11$. Demak, giperbolaning uchlari (-8 ; 0) va (8 ; 0) nuqtalar, fokuslari esa (-11;0) va (11;0) nuqtalar ekan.

10. Giperbola asimptotalarining tenglamalari $8y + 6x = 0$ va $8y - 6x = 0$ hamda fokuslar orasidagi masofa 20. Uning kanonik tenglamasini tuzing.

Yechish Masala shartiga asosan formulaga ko'ra: $y = \pm \frac{6}{8} x$. Bundan: $b = \frac{6}{8} a$

Masala shartiga asosan: $2c = 20 \Rightarrow c = 10$; $c^2 = b^2 + a^2 \Rightarrow 100 = b^2 + a^2$

a va b larni topamiz:

$$\begin{cases} 100 = a^2 + \frac{36}{64} a^2 \\ b = \frac{6}{8} a \end{cases} \Rightarrow \begin{cases} a^2 = 64 \\ b^2 = 36 \end{cases}$$

Demak, izlanayotgan giperbola tenglamasi: $\frac{x^2}{64} - \frac{y^2}{36} = 1$.

11. Asimptotalar orasidgi burchak 150 ^{o} va fokuslari abssisssalar o'qida bo'lib, ular orasidagi masofa $2c = 8\sqrt{3}$ bo'lsa giperbola tenglamasini tuzing.

Yechish: Agar giperbola asimptotalari o'zaro 150 ^{o} li burchak tashkil etsa, ularda bittasi bilan Ox o'qning musbat yo'nalishi orasidagi burchak 30^{o} bo'ladi.

Shuning uchun: $\frac{b}{a} = tg30^0 \Rightarrow a = \sqrt{3}b$.

a va b larning qiymatlarini aniqlaymiz. Masala shartiga asosan:

$c^2 = 48$. Bundan: $\begin{cases} a = \sqrt{3}b \\ a^2 + b^2 = 48 \end{cases} \Rightarrow \begin{cases} 3b^2 + b^2 = 48 \\ a^2 = 3b^2 \end{cases} \Rightarrow \begin{cases} b^2 = 12 \\ a^2 = 36 \end{cases}$

Demak, izlanayotgan giperbola tenglamasi: $\frac{x^2}{36} - \frac{y^2}{12} = 1$

12. Quyidagi giperbolaning tenglamasini eng soda shaklga keltiring:
$16x^2 - 4y^2 + 32x + 16y - 64 = 0$.

Yechish: Bu berilgan tenglamani giperbolaning kanonik ko'rinishdagi tenglamasiga keltiramiz.

$16x^2 + 32x = 16(x^2 + 2x + 1) - 16$ va $-4y^2 + 16y = -4(y^2 - 4y + 4) + 16$ ekanliklarini e'tiborga olsak, berilgan tenglamaning ko'rinishi:

$16(x+1)^2 - 4(y-2)^2 = 64$ yoki $\frac{(x+1)^2}{4} - \frac{(y-2)^2}{16} = 1$. Bu tenglama markazi $(-1;2)$ nuqtada, haqiqiy yarim o'qi 2 ga mavhum yarim o'qi esa 4 bo'lgan giperbolaning kanonik tenglamasidir.

Mustaqil yechish uchun topshiriqlar

1. Fokuslari orasidagi masofa 10 ga va mavhum yarim o'qi 4 ga teng bo'lgan giperbolaning kanonik tenglamasini tuzing.

Javob: $\frac{x^2}{25}-\frac{y^2}{16}=1$

2. Giperbolaning quyidagi tenglamalarini kanonik ko'rinishga keltiring:

1) $9x^2-16y^2-144=0$ 2) $2x^2-y^2-6=0$

Javob: 1) $\frac{x^2}{16}-\frac{y^2}{9}=1$ 2) $\frac{x^2}{3}-\frac{y^2}{6}=1$

3. $3x^2-4y^2-12=0$ tenglama bilan berilgan giperbolani asimptotalarining tenglamalari tuzilsin.

Javob : $y=\pm\frac{b}{a}x=\pm\frac{\sqrt{3}}{2}x$

4. Giperbolaning fokuslari abstsissalar o'qida yotib, uning fokuslari orasidagi masofa 12 ga va ekstsentrisiteti 1.5 ga teng bo'lsa uning kanonik tenglamasini tuzing

Javob: $\frac{x^2}{16}-\frac{y^2}{20}=1$

5. $9x^2-4y^2-144=0$ giperbolaning fokuslarini toping va eksentrisitetini hisoblang.

Javob: $F_1(-2\sqrt{13};0)$ $F_2(2\sqrt{13};0)$ $\varepsilon=\frac{c}{a}=\frac{2\sqrt{13}}{4}=\frac{\sqrt{13}}{2}$

6.Giperbolaning fokuslari abstsissalar o'qida yotib, uning mavhum yarim o'qi 1, focus nuqtasining koordinatasi $F_1(-\sqrt{17};0)$ bo'lsa uning kanonik tenglamasini tuzing. **Javob**: $\frac{x^2}{16}-\frac{y^2}{1}=1$

7. Giperbolaning fokuslari abstsissalar o'qida yotib, uning haqiqiy yarim o'qi 3, eksentrisiteti $\varepsilon=\frac{\sqrt{13}}{3}$ bo'lsa uning kanonik tenglamasini tuzing.

Javob: $\frac{x^2}{9}-\frac{y^2}{4}=1$

8.Giperbolaning fokuslari abstsissalar o'qida yotib, uning mavhum yarim o'qi 4, focus nuqtasining koordinatasi $F_1(-4\sqrt{5};0)$ bo'lsa uning kanonik tenglamasini tuzing.

Javob: $\frac{x^2}{64}-\frac{y^2}{16}=1$

9. Giperbolaning fokuslari abstsissalar o'qida yotib, uning haqiqiy yarim o'qi 9, eksentrisiteti $\varepsilon=\frac{\sqrt{85}}{9}$ bo'lsa uning kanonik tenglamasini tuzing.

Javob: $\frac{x^2}{81}-\frac{y^2}{4}=1$

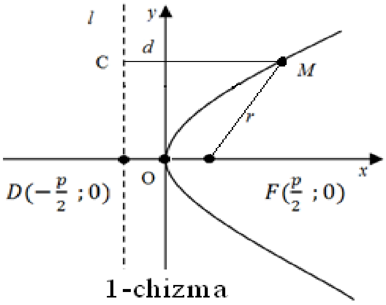
10. Giperbolaning fokuslari abstsissalar o'qida yotib, uning mavhum yarim o'qi 5, focus nuqtasining koordinatasi $F_1(-\sqrt{89};0)$ bo'lsa uning kanonik tenglamasini tuzing.

Javob: $\frac{x^2}{64}-\frac{y^2}{25}=1$

4 Parabola

Har bir nuqtasidan fokus deb ataluvchi berilgan nuqtagacha va direktrisa deb ataluvchi berilgan to'g'ri chiziqqacha bo'lgan masofa o'zaro teng bo'lgan tekislikdagi barcha nuqtalar to'plamiga parabola deb ataladi. Bunda F nuqta fokus, ℓ to'g'ri chiziq esa direktrisa deyiladi.

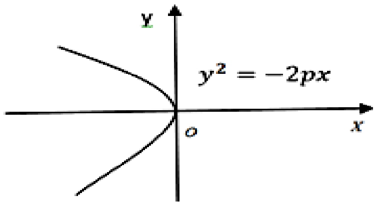
$y^2=2px$ tenglamaga parabolaning kanonik tenglamasi, p (p>0) uning parametri deyiladi.



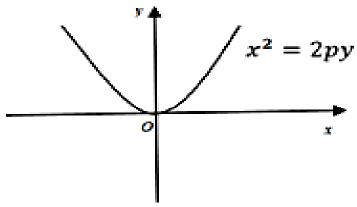
1-chizma

$r = d = x + \frac{p}{2}$ parabolaning fokal radiusi deb ataladi. Parabola uchun $\varepsilon = \frac{r}{d} = 1$ bo'ladi (1-chizma).

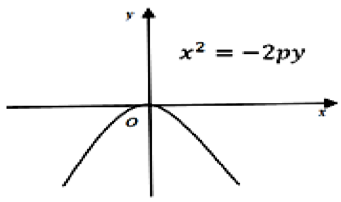
$y^2 = -2px$ $x^2 = 2py$ va $x^2 = -2py$ lar ham parabolaning tenglamalari bo'lib, ular mos ravishda Ox o'qining chap qismida, Oy o'qining yuqori qismida va Oy o'qining quyi qismida joylashgan bo'ladi (2,3,4-chizmalar).



2-chizma



3-chizma



4-chizma

$(y - y_0)^2 = 2p(x - x_0)^2$, $(x - x_0)^2 = 2p(y - y_0)^2$ tenglamalar uchi $C(x_0; y_0)$ nuqtada yotgan parabolani aniqlaydi.

Mavzuga doir yechimlari bilan berilgan topshiriqlardan namunalar

1. Ox o'qi parabolaning simmetriya o'qi bo'lib, uning uchi koordinatalar boshida yotadi. Parabola uchidan fokusigacha bo'lgan masofa 6 birlikka teng. Parabola va uning direktrisasi tenglamasini yozing.

Yechish: Dastlab, masala shartiga asosan, parabolaning p parametrini topamiz: $|OF| = 6$, $\Rightarrow \frac{p}{2} = 6$, $p = 12$. p ning bu qiymatini parabolaning kanonik tenglamasiga qo'yib $y^2 = 2px = 2 \cdot 12 \cdot x = 24x$, $y^2 = 24x$ ni topamiz. Bu parobala tenglamasidir. Direktrisa tenglamasi $x = -\frac{p}{2} = -\frac{12}{2} = -6$, yani x=-6 dan iborat.

2. Parabola Ox o'qiga nisbatan simmetrik va A (3;-6) nuqtadan o'tadi, uning tenglamasi tuzilsin.

Yechish: Parabolaning uchi koordinatalar boshida va u Ox o'qiga nisbatan simmetrik bo'lganligi uchun uning tenglamasi $y^2 = 2px$ yoki $y^2 = -2px$ dan iborat bo’ladi. Parabola musbat absissali nuqtadan o’tganligi uchun uning tenglamasi $y^2 = 2px$ ko’rinishda bo’lishi ravshan. Bu tenglamaga A nuqtaning koordinatalarini qo’yib, $36 = 2p \cdot 3$ ni hosil qilamiz. Undan esa 2p=12 kelib chiqadi. Demak, parabolaning izlangan tenglamasi: $y^2 = 12x$

3. Uchi koordinatalar boshida bo’lgan parabolaning fokusi F (0;4) nuqtada yotadi. Bu parabolaning tenglamasini yozing.

Yechish: Masalaning shartiga asosan bu parabola Oy o’qqa nisbatan simmetrik, uning tarmoqlari pastga yo’nalgan. Shuning uchun izlanayotgan tenglama $x^2 = -2py$ dan topiladi. $\frac{p}{2} = 4$ bo’lgani uchun p=8 bo’lib parabolaning tenglamasi $x^2 = -16y$ bo’ladi.

4.Agar uchi koordinatalar boshida bo’lgan parabolaning fokusi $F(4;0)$ nuqtada yotsa, bu parabola tenglamasini tuzing.

Yechish: Parabolaning fokusi Ox o’qining musbat yarim o’qida yotibdi.

Unda parabolaning tenglamasi $y^2 = 2px$ bo’ladi. $\frac{p}{2} = 4 \Rightarrow p = 8$.

Demak, $y^2 = 16x$.

5. Uchi koordinatalar boshida, Ox o’qiga nisbatan simmetrik va $A(2;-2)$ nuqtadan o’tuvchi parabolaning tenglamasi topilsin.

Yechish: Shartga ko’ra izlanayotgan parabola $(2;-2)$ nuqtadan o’tadi. Binobarin, bu nuqtaning koordinatalari parabola tenglamasini qanoatlantiradi. $(-2)^2 = 2p \cdot 2 \Rightarrow p = 1$.

Demak, parabolaning tenglamasi $y^2 = 2 \cdot 1 \cdot x = 2x$ bo’ladi.

6. Parabola tenglamasi $y^2 = 10x$ berilgan. Uning direktrisasi tenglamasini tuzing.

Yechish:. Parabola tenglamasi $y^2 = 10x$ dan $2p = 10 \Rightarrow p = 5$.

$x = -\frac{p}{2}$ bo'lgani uchun $x = -\frac{5}{2}$ yoki $2x + 5 = 0$ direktrisa tenglamasidir.

Mustaqil yechish uchun topshiriqlar

1. $y^2 = 8x$ parabola berilgan. Uning direktrisasi tenglamasi tuzilsin va fokusining koordinatalari topilsin.

Javob: $x = -2$, F (2;0);

2. (0;0) va (1;-3) nuqtalardan o'tuvchi va Oy o'qqa nisbatan simmetrik bo'lgan parabolaning tenglamasi yozilsin.

Javob: $y^2 = 9x$.

3. (0;0) va (2;4) nuqtalardan o'tuvchi va Oy o'qqa nisbatan simmetrik bo'lgan parabolaning tenglamasi yozilsin.

Javob: $y = x^2$

4. $y^2 = 6x$ parabola berilgan. Uning direktrisasi tenglamasi tuzilsin va fokusining koordinatalari topilsin.

Javob: $p = 3$, $F(-\frac{3}{2};0)$, $x = -\frac{3}{2}$

5.Direktrisasi y=-3 bo'lgan parabolaning tenglamasi tuzilsin.

Javob: $x^2 = 12y$

6. Direktrisasi y=-16 bo'lgan parabolaning tenglamasi tuzilsin.

Javob: $x^2 = 64y$

7. Ox o’qiga nisbatan simmetrik va $A(-3;6)$ nuqtadan o’tuvchi parabolaning tenglamasi topilsin.

Javob: $y^2 = -12x$

8. Direktrisasi $x = \frac{5}{8}$ bo’lgan parabolaning tenglamasi tuzing.

Javob: $y^2 = \frac{5}{2}x$

9. Ox o’qiga nisbatan simmetrik va $A(-2;8)$ nuqtadan o’tuvchi parabolaning tenglamasi topilsin.

Javob: $y^2 = -16x$

Vektorlarning skalyar, vektor va aralash ko’paytmasi.

1. Vektor tushunchasi.

Fizik, kimyoviy va boshqa hodisalarni o’rganishda uchraydigan kattaliklarni ikki sinfga bo’lish mumkin. Skalyar kattaliklar deb ataladigan kattaliklar sinfi mavjudki, ularni xarakterlash uchun bu kattaliklarning son qiymatlarini ko’rsatish etarlidir. Bular, masalan, hajm, massa, zichlik, harorat va boshqalardir. Lekin shunday kattaliklar mavjudki ular faqat son qiymatlari bilangina emas, balki yo’nalishi bilan ham harakterlanadi.

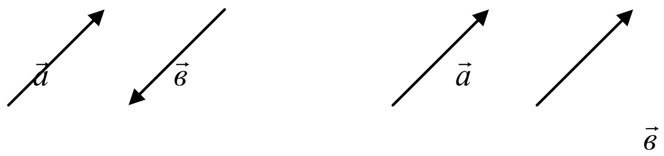
Ular yo’nalgan kattaliklar yoki vektor kattaliklar tdeb ataladi. Masalan, harakatlanayotgan nuqtaning bir vaziyatdan ikkinchi vaziyatga ko’chishda ta’sir etayotgan kuchni xarakterlash uchun kuchning ulchamalrini ko’rsatish kifoya qilishdan, balki bu kuchning yo’nalishini ham ko’rsatish zarurdir. Harakat tezligi magnit yoki elektr maydonning kuchlanganligi va boshqa kattaliklar ham shunga o’xshash xarakterlanadi. Bularning hammasi vektor kattaliklarga oid misoldir. Ularni tasvirlash uchun vektor tushinchasi kiritilgan bo’lib, u matematikaning o’zi uchun ham foydali bo’lib chiqdi.

1-ta’rif. Yo’nalishga va uzunlikka ega bo’lgan kesma vektor deb ataladi.

Vektorni $\overrightarrow{AB}$ ko’rinishda belgilaymiz, bunda birinchi xarf vektorning bosh nuqtasini 2-chi harf esa uning oxirgi nuqtasini belgilaydi. Vektorni, shuningdek, ustiga «$\rightarrow$» chizilgan bitta xarf bilan ham belgilaymiz. $\vec{a}$ vektorning uzunligini uning moduli deb ataymiz va $\left|\overrightarrow{AB}\right|$ ko’rinishda belgilaymiz. Agar vektor $\vec{a}$ bilan belgilangan bo’lsa, u holda uning moduli $\left|\vec{a}\right|$ bilan belgilanadi. Boshi oxiri bilan ustma ust tushgan vektor nol vektor deb ataladi va $\vec{0}$ bilan belgilanadi. Bunday vektor tayin yo’nalishga ega emas, uning moduli nolga teng, ya’ni $\left|\vec{0}\right| = 0$.

2-ta’rif. Bitta to’g’ri chiziqqa yoki paralel to’g’ri chiziqda yotuvchi $\vec{a}$ va $\vec{в}$ vektorlar kolliniar vektorlar deb ataladi.

Kollinear vektorlar bir xil yoki qarama-qarshi yo’nalgan bo’lishi mumkin.

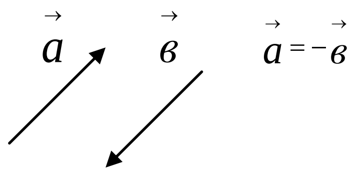


3-ta’rif. $\vec{a}$ va $\vec{в}$ vektorlar kollinear, bir xil yo’nalgan va uzunliklari teng bo’lsa, ular teng vektorlar deb ataladi. $\vec{a}$ va $\vec{в}$ vektorlar teng bo’lsa, bunday yoziladi: $\vec{a} = \vec{в}$. Agar berilgan vektorni o’z-o’ziga paralel ko’chirsak, 3-ta’rifga asosan, yana berilagn vektorga teng vektor hosil qilamiz. Shu ma’noda analitik geometriyada vektorlar erkin vektorlar deb hisoblanadi.

4-ta’rif. Bitta tekislikda yoki paralel tekislikda yotuvchi vektorlar komplanar vektorlar deb ataladi.

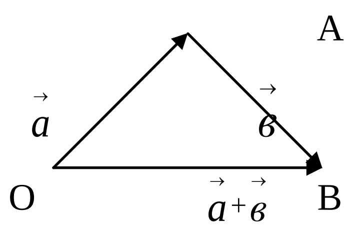
Agar komlanar vektorlarning boshlari umumiy nuqtaga ega bo’lsa, ular bitta tekslikda yotishini ko’rsatish qiyin emas.

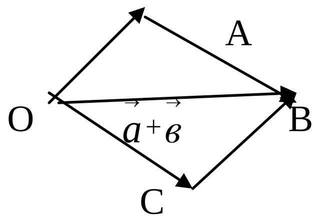
$\overrightarrow{AB}$ va $\overrightarrow{BA}$ vektorlar qarama-qarshi vektrolar deb ataladi. Agar $\overrightarrow{AB} = \vec{a}$ kabi belgilansa, u holda unga qarama-qarshi vektor $\overrightarrow{BA} = -\vec{a}$ bilan belgilanadi. (2-shakl)



5-ta’rif. Vektorlar ustida chiziqli amallar deb, vektorlarni qo’shish va ayirish hamda vektorni songa ko’paytirishga aytiladi.

a) Noldan farqli ikkita ixtiyoriy $\vec{a}$ va $\vec{в}$ berilgan bo’lsin. Ixtiyoriy O nuqtani olamiz va $\overrightarrow{OA}=\vec{a}$ vektorni yasaymiz, so’ngra A nuqtaga $\overrightarrow{AB}=\vec{в}$ vektorni qo’yamiz. Ikkita $\vec{a}$ va $\vec{в}$ vektorning yig’indisi $\vec{a}+\vec{в}$ deb birinchi qo’shiluvchi vektorning boshini ikkinchi qo’shiluvchi vektorning oxiri bilan tutashtiruvchi $\overrightarrow{OB}$ vektorga aytiladi. Vektorlarni bunday qo’shish usuli uchburchak usuli deyiladi. (3-shakl)

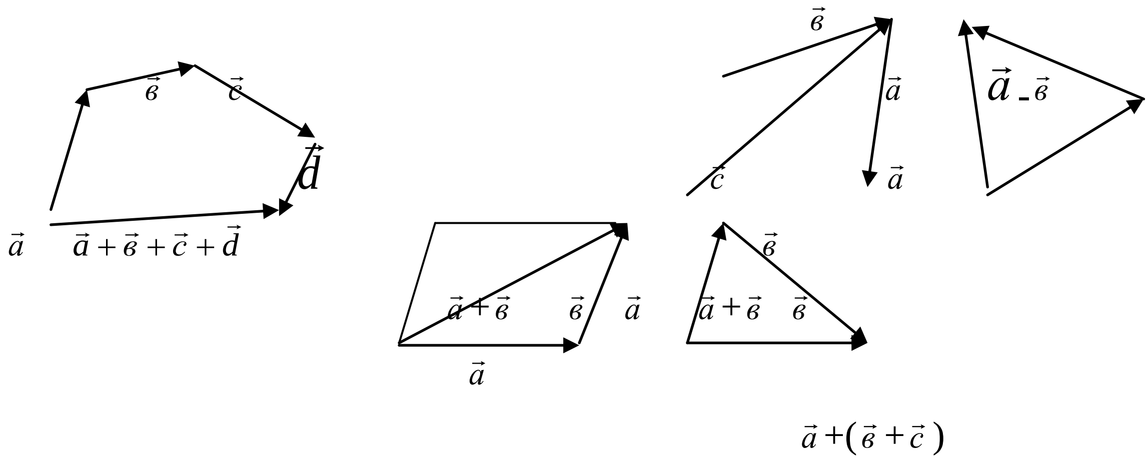




(3-shakl) 4-shakl

Vektorning yig’indisini boshqacha usul bilan ham aniqlash mumkin. Biror O nuqtadan $\overrightarrow{OA}=\vec{a}$ va $\overrightarrow{OC}=\vec{в}$ vektorlarni qo’yamiz. Bu vektorlar tomonlar sifatida olib, OABC parolelogramm yasaymiz. Parolelogrammning uchidan o’tkazilgan diogonali $\overrightarrow{OB}$ vektor, $\vec{a}$ va $\vec{в}$ vektorlar yig’indisi $\vec{a}+\vec{в}$ vektordir. Vektorlarni bunday qo’shish usuli parolelogramm qoidasi deb ataladi. (4-shakl)

Vektorni qo’shishning ikkinchi usuli siniq chiziqda ketma-ket joylashtirilgan istalgan sondagi vektorlar uchun ham o’rinlidir. Bunda yig’indi siniq chiziqni ko’pburchak yopadigan vektor bo’lib, uning boshi 1-chi vektorning oxiri bilan ustma-ust tushadi. Bir necha vektorni bunday qo’shish usuli ko’pburchak qoidasi deb ataladi (5-shakl).



5-shakl 6-shakl 7-shakl 8-shakl

Qo'shish amalining asosiy xossasini shakllarda tushintirish mumkin.

1.O'rin almashtirish xossasi (6-shakl) $\vec{a}+\vec{в}=\vec{в}+\vec{a}$

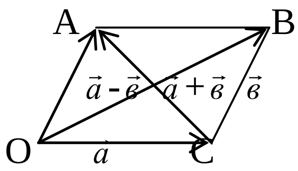
2.Gruppalash (asotsiyativlik) xossasi (7-shakl) $(\vec{a}+\vec{в})+\vec{c}=\vec{a}+(\vec{в}+\vec{c})$

3.Har qanday $\vec{a}$ vektorga nol vektor qo'shilsa $\vec{a}$ hosil bo'ladi, ya'ni $\vec{a}+\vec{0}=\vec{a}$

4.Har qanday $\vec{a}$ vektor uchun shunday $-\vec{a}$ vektor mavjudki, uning uchun: $\vec{a}+(-\vec{a})=\vec{0}$

b) Endi vektorlarni ayirish amalini qo'shishga teskari amal sifatida aniqlaymiz $\vec{a}$ va $\vec{в}$ vektorlarning ayirmasi deb, $\vec{a}-\vec{в}$ bilan belgilanadigan va $\vec{в}$ vektor bilan yig'indisi $\vec{a}$ vektorni beradigan vektorga aytiladi. Bundan vektorlarni ayirish qoidasi kelib chiqadi, agar $\vec{a}$ va $\vec{в}$ vektorlarning boshi umumiy nuqtaga qo'yilsa u holda $\vec{a}-\vec{в}$ vektor hosil bo'lgan siniq chiziqni yopadi va ayiriluvchi vektorning oxiridan kamayuvchi vektorning oxiriga yo'nalgan bo'ladi (8-shakl).

Agar umumiy O nuqtadan qo'yilgan $\vec{a}$ va $\vec{в}$ vektorlardan OABC paralelogramm yasalsa, u holda paralolelogrammning O uchidan chiquvchi dioganali bilan ustma-ust tushadigan $\overrightarrow{OB}$ vektor $\vec{a}+\vec{в}$ yig'indiga teng, ikkinchi dioganal bilan ustma-ust tushadigan $\overrightarrow{CA}$ vektor esa $\vec{a}-\vec{в}$ ayirmaga teng bo'ladi



(9-shakl)

v) vektorni songa ko’paytirish. $\vec{a}$ vetor va skalyar $\lambda = 0$ son berilgan bo’lsin. $\vec{a}$ vektorning λ songa ko’paytmasi deb quyidagi shartlarni qanoatlantiradigan $\vec{в}$ vektorga aytiladi:

1.Agar $\lambda > 0$ bo’lsa, $\vec{в}$ vektor $\vec{a}$ vektor bilan bir xil yo’nalishda ($\vec{a} \neq 0$), aks xolda $\lambda < 0$ bo’lsa, $\vec{в}$ va $\vec{a}$ vektorlar qarama-qarshi yo’nalishda bo’ladi.

2. $\vec{в}$ vektorning uzunligi(moduli) $|\vec{в}| = |\lambda| \cdot |\vec{a}|$ formula bilan hisoblanadi.

Masalan,

$\frac{1}{2}\vec{a}$ vektor $\vec{a}$ vektor bilan bir xil yo’nalgan va $\vec{a}$ vektorning uzunligidan ikki marta kichik uzunlikka ega bo’lgan vektordir.

Vektorni songa ko’paytirish qoidasidan quyidagi xulosalar kelib chiqadi:

1.Ixtiyoriy $\vec{a}$ vektor uchun: $0 \cdot \vec{a} = \vec{0}$

2.Ixtiyoriy $\lambda \in R$ son uchun: $\lambda \cdot \vec{0} = \vec{0}$

3.Ixtiyoriy $\vec{a}$ vektor uchun: $1 \cdot \vec{a} = \vec{a}; \quad (-1) \cdot \vec{a} = -\vec{a}$

4. $\vec{a}$ va $\lambda \vec{a}$ vektorlar kollinear vektorlar bo’ladi.

Biror $\vec{a} \neq 0$ vektorni o’zining uzunligiga teskari $\frac{1}{|\vec{a}|}$ songa ko’paytirilsa, shu vektor yo’nallishdagi birlik vektor (ort) hosil bo’ladi, ya’ni $\frac{1}{|\vec{a}|} \cdot \vec{a} = \vec{a}_0 \qquad (|\vec{a}_0| = 1)$

Teorema. Agar $\vec{a} \,||\, \vec{в}$ ($\vec{a} \neq 0$) bo’lsa, u holda shunday λ son mavjudki,

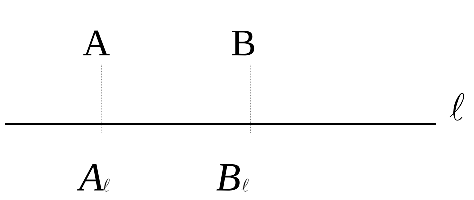
$\vec{a} = \lambda \vec{в}$ (1) tenglik o’rinli bo’ladi.

Demak, vektorni songa ko’paytirish qoidasidan va bu teoremadan quyidagi xulosani chiqarish mumkin: $\vec{a} \| \vec{в}$ berilagn bo’lsa, u holda bu vektorlar uchun $\vec{в} = \lambda \vec{a}$ tenglik o’rinli bo’ladi.

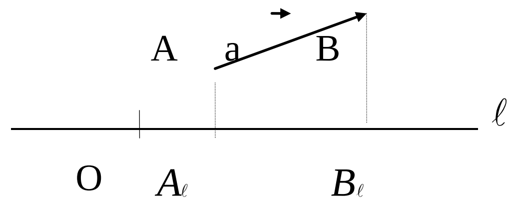
Vektorlarning o’qqa proyektsiyasi

Proektsiya so’zi lotincha «projectiv» so’zidan olingan bo’lib, «tasvir» yoki «soya» degan ma’noni bildiradi.

Biror A nuqtaning ℓ o’qdagi proektsiyasi deb, shu nuqtadan ℓ o’qqa tushirilgan perpendikulyarning ℓ o’qdagi A_ℓ asosiga aytiladi va quyidagicha yoziladi $\mathrm{Pr}_y A = A_\ell$, $\mathrm{Pr}_y B = B_\ell$ (3- rasmga qarang)



3-rasm

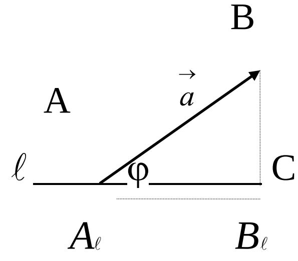


4-rasm

$\overrightarrow{AB}$ vektorning ℓ o’qdagi geometrik proektsiyasi deb, vektor boshining proektsiyasi bo’lgan A_ℓ nuqta va uchining proektsiyasi bo’lgan B_ℓ nuqta bilan chegaralangan kesma uzunligiga aytiladi.(4-rasm.) $\overline{AB}$ vektorning ℓ o’qdagi proektsiyasi $\mathrm{Pr}_y \overrightarrow{AB} = A_\ell B_\ell$ kabi belgilanadi.

Agar A_ℓ va B_ℓ nuqtalarning sonlar *o’qdagi koordinatalarini* mos ravishda x_1 va x_2 desak $\mathrm{Pr}_y \overrightarrow{AB} = x_2 - x_1$ orqali aniqlanadi.

Teorema. $\vec{a}$ vektorning ℓ o’qdagi proektsiyasi $\vec{a}$ vektor uzunligini, $\vec{a}$ vektor bilan ℓ o’q orasidagi φ burchakning kosinusiga ko’paytmasiga teng bo’ladi: $\mathrm{Pr}_y \vec{a} = |\vec{a}| \cos\varphi$



Chizmadagi ABC uchburchakdan: $|AC| = |\overrightarrow{AB}|\cos\varphi = |\vec{a}|\cos\varphi$, $|AC| = |A_\ell B_\ell|$

ekanini ko'ramiz, demak, $\Pr_y \overline{AB} = |\overline{A_\ell B_\ell}| = |\overline{AC}| = |\overline{AB}|\cos\varphi \Rightarrow \Pr_y \overline{a} = |\overline{a}|\cos\varphi$

Shuni ta'kidlash lozimki, φ burchak o'tkir bo'lsa proektsiya qiymati musbat son, φ burchak o'tmas bo'lsa, proektsiya qiymati manfiy son bo'ladi.

Tekislikdagi vektor koordinatalari deganda vektorning uchi bilan boshining mos koordinatalari ayirmalariga aytiladi va quyidagicha yoziladi $\overline{a} = \{x_2 - x_1; y_2 - y_1\}$

Tekislikdagi vektor koordinatalari kvadratlarining yig'indisidan olingan kvadrat ildizga *vektor uzunligi* deyiladi: $|\vec{a}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Mavzuga doir yechimlari bilan berilgan topshiriqlardan namunalar

1. Ikkita $\vec{a} = \{2; 1; -3\}$ va $\vec{b} = \{5; 0; -2\}$ vektor berilgan. Quyidagi vektorlarning koordinata o'qlaridagi proeksiyalarini toping:

$a)$ $2\vec{a} + 3\vec{b}$, b) $\vec{a} - 2\vec{b}$, c) $3\vec{a} - \vec{b}$, d) $3\vec{a} - 2\vec{b}$, e) $\vec{a} - \vec{b}$

Yechish:

$a)$ $2\vec{a} + 3\vec{b} = \{2\cdot 2 + 3\cdot 5;\ 2\cdot 1 + 3\cdot 0;\ 2\cdot(-3) + 3\cdot(-2)\} = \{19;\ 2;\ -12\}$

b) $\vec{a} - 2\vec{b} = \{2 - 2\cdot 5; 1 - 2\cdot 0;\ -3 - 2\cdot(-2)\} = \{-8; 1; 1\}$

c) $3\vec{a} - \vec{b} = \{3\cdot 2 - 5;\ 3\cdot 1 - 0;\ 3\cdot(-3) - (-2)\} = \{1;\ 3;\ -7\}$

d) $3\vec{a} - 2\vec{b} = \{3\cdot 2 - 2\cdot 5;\ 3\cdot 1 - 2\cdot 0;\ 3\cdot(-3) - 2\cdot(-2)\} = \{-4;\ 3;\ -5\}$

e) $\vec{a} - \vec{b} = \{2 - 5;\ 1 - 0;\ -3 - (-2)\} = \{-3;\ 1;\ -1\}$

2. Ikkita $\vec{a} = \{2; 2; -1\}$ va $\vec{b} = \{4; 0; -3\}$ vektor berilgan. Quyidagilar hosil qilgan burchak bisektrissasini uzunligini toping.

$a)$ $2\vec{a}$ va $4\vec{b}$, b) $-3\vec{a}$ va $-2\vec{b}$, c) $\vec{a}$ va $-\vec{b}$, d) $3\vec{a}$ va $4\vec{b}$

Yechish

a) $2\vec{a}+4\vec{b}=\{2\cdot 2+4\cdot 4;\ 2\cdot 2+4\cdot 0;\ 2\cdot(-1)+4\cdot(-3)\}=\{20;\ 4;\ -14\}$

$$\left|2\vec{a}+4\vec{b}\right|=\sqrt{20^2+4^2+(-14)^2}=\sqrt{612}$$

b) $-3\vec{a}+(-2)\vec{b}=\{-3\cdot 2+(-2)\cdot 4;\ -3\cdot 2+(-2)\cdot 0;\ -3\cdot(-1)+(-2)\cdot(-3)\}=\{-14;\ -6;\ 9\}$

$$\left|-3\vec{a}+(-2)\vec{b}\right|=\sqrt{(-14)^2+(-6)^2+9^2}=\sqrt{313}$$

c) $\vec{a}+(-\vec{b})=\{2+(-4);\ 2+0;\ -1+(-(-3))\}=\{-2;\ 2;\ 2\}$

$$\left|\vec{a}+(-\vec{b})\right|=\sqrt{(-2)^2+2^2+2^2}=\sqrt{12}$$

d) $3\vec{a}+4\vec{b}=\{3\cdot 2+4\cdot 4;\ 3\cdot 2+4\cdot 0;\ 3\cdot(-1)+4\cdot(-3)\}=\{22;\ 6;\ -15\}$

$$\left|3\vec{a}+4\vec{b}\right|=\sqrt{22^2+6^2+(-15)^2}=\sqrt{745}$$

Mustaqil yechish uchun topshiriqlar

1. Ikkita $\vec{a}=\{1;\ 1;-2\}$ va $\vec{b}=\{2;\ 0;-3\}$ vektor berilgan. Quyidagi vektorlarning koordinata o'qlaridagi proeksiyalarini toping:

a) $3\vec{a}+3\vec{b}$, b) $\vec{a}+2\vec{b}$, c) $3\vec{a}+2\vec{b}$, d) $-2\vec{a}-3\vec{b}$, e) $\vec{a}-3\vec{b}$

Javob: $a)\ \{9;\ 3;-15\}$, $b)\ \{5;\ 1;-8\}$, $c)\ \{7;\ 3;-12\}$, $d)\ \{-8;\ -2;13\}$, $e)\ \{-5;\ 1;7\}$

2. Ikkita $\vec{a}=\{2;\ 2;-1\}$ va $\vec{b}=\{4;\ 0;-3\}$ vektor berilgan. Quyidagilar hosil qilgan burchak bisektrissasini uzunligini toping.

a) $\vec{a}$ va $2\vec{b}$, b) $-2\vec{a}$ va $-3\vec{b}$, c) $2\vec{a}$ va $-\vec{b}$, d) $2\vec{a}$ va $-3\vec{b}$

Javob: $a)\ \sqrt{153}$, $b)\ \sqrt{393}$, $c)\ \sqrt{17}$, $d)\ \sqrt{129}$

Chiziqli bog'liq va chiziqli bog'liqsiz vektorlar.

<u>1-ta'rif.</u> $\vec{a}_1, \vec{a}_2, \dots, \vec{a}_n$ vektorlar *chiziqli bog'liq* vektorlar deyiladi agar

$$\lambda_1\vec{a}_1+\lambda_2\vec{a}_2+\dots+\lambda_n\vec{a}_n=0 \quad (1)$$

munosabat λ_1, λ_2,..., λ_n haqiqiy sonlarning kamida bittasi noldan farqli holda o'rinli bo'lsa.

2-ta'rif. Agar (1) tenglik $\lambda_1 = \lambda_2 =...= \lambda_n = 0$ bo'lgandagina o'rinli bo'lsa $\vec{a}_1, \vec{a}_2, ..., \vec{a}_n$ vektorlar *chiziqli bog'liqsiz* vektorlar deyiladi..

Ushbu ma'lumotlar yordamida qo'yidagi xulosalarga kelamiz:

- Tekislikdagi har qanday ikkita vektorning chiziqli bog'liqli bo'lishi uchun ularning kollinear vektorlar bo'lishi zarur va yetarli.
- Fazodagi har qanday uchta vektorning chiziqli bog'liqli bo'lishi uchun , ularning komplanar vektorlar bo'lishi shart.

Vektorlarni bazislar bo'yicha yoyish.

1-ta'rif. Tekislikdagi ikkita o'zaro chiziqli bog'liqsiz , ya'ni kollinear bo'lmagan $\vec{a_1}$ va $\vec{a_2}$ vektorlarga *bazis vektorlar* deb aytiladi.

1-teorema. Tekislikdagi biror $\vec{a}$ vektorning $\vec{a_1}$ va $\vec{a_2}$ bazislar orqali yoyilmasi yagona $\vec{a} = \lambda_1 \vec{a}_1 + \lambda_2 \vec{a}_2$ ko'rinishda bo'ladi.

2-ta'rif. Fazodagi har qanday uchta o'zaro chiziqli bog'liqsiz, ya'ni komplanar bo'lmagan $\vec{a_1}$, $\vec{a_2}$, $\vec{a_3}$ vektorlarga *bazis vektorlar* deb aytiladi.

2-teorema. Fazodagi biror $\vec{a}$ vektorning $\vec{a_1}$, $\vec{a_2}$, $\vec{a_3}$ bazislar orqali yoyilmasi yagona $\vec{a} = \lambda_1 \vec{a}_1 + \lambda_2 \vec{a}_2 + \lambda_3 \vec{a}_3$ ko'rinishda bo'ladi.

Endi dekart koordinata sistemasidagi *bazis vektorlarni* va ular bo'yicha har qanday vektorni yoyishni ko'raylik. Dekart koordinata sistemasida Ox, Oy, Oz sonlar o'qlari yo'nalishida mos ravishda uzunliklari birga teng bo'lgan $\vec{i}$, $\vec{j}$, $\vec{k}$ vektorlarni ($|i| = |j| = |k| = 1$) olaylik. Ushbu vektorlarga birlik vektorlari yoki *oorta vektorlar* deyiladi. Bu vektorlar komplanar bo'lmagani va o'zaro perpendikulyar bo'lgani uchun (ya'ni chiziqli bog'liqsiz vektorlar bo'lgani uchun) bazis tashkil qiladi. Shuning uchun ularni dekart ortogonal bazislari deyiladi.

Vektorlarning yo'naltiruvchi kosinuslari.

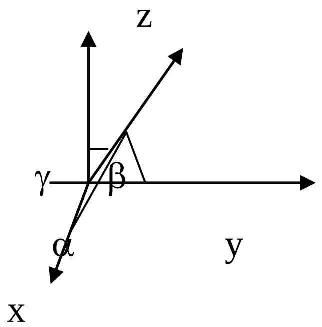
Bizga berilgan $\vec{a} = \{x, y, z\}$ vektor Ox, Oy va Oz koordinata o'qlari bilan mos ravishda α, β, γ burchaklar tashkil qilsin.

Ta'rif. $\vec{a}$ vektorning koordinata o'qlari bilan hosil qilgan burchaklari kosinuslari, ya'ni cosα ,cosβ, cosγ larga $\bar{a}$ *vektorning yo'naltiruvchi kosinuslari* deyiladi.

Proektsiyalash qoidalaridan foydalansak

$$\cos\alpha = \frac{x}{|\vec{a}|} = \frac{x}{\sqrt{x^2 + y^2 + z^2}}$$

$$\cos\beta = \frac{y}{|\vec{a}|} = \frac{y}{\sqrt{x^2 + y^2 + z^2}}$$



$$\cos\gamma = \frac{z}{|\vec{a}|} = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

Mavzuga doir yechimlari bilan berilgan topshiriqlardan namunalar

1. $\vec{a} = \{1;\ 3; 5\}$ vektorning yo'naltiruvchi kosinuslarini toping

Yechish: $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$ ga asosan $|\vec{a}| = \sqrt{1^2 + 3^2 + 5^2} = \sqrt{35}$

2.A(1,2,3) va B(2,4,5) nuqtalar bo'lsa, $\vec{a} = \overrightarrow{AB}$ vektorning yo'naltiruvchi kosinuslarini toping.

Yechish. $\overrightarrow{AB}(x_2 - x_1; y_2 - y_1; z_2 - z_1)$ ga asosan $\vec{a} = \overrightarrow{AB}$ ={1;2;2} ,

$|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$ ga asosan $|\vec{a} = \overrightarrow{AB}|$=3 ,

$\cos\alpha = \frac{x}{|\vec{a}|} = \frac{x}{\sqrt{x^2 + y^2 + z^2}}$ ga asosan cosα=1/3 ;

$\cos\beta = \frac{y}{\left|\vec{a}\right|} = \frac{y}{\sqrt{x^2+y^2+z^2}}$ ga asosan cosβ=2/3 ;

$\cos\gamma = \frac{z}{\left|\vec{a}\right|} = \frac{z}{\sqrt{x^2+y^2+z^2}}$ ga asosan cosγ=2/3.

3. Ikkita $\vec{a} = \{-5;\ 3; 8\}$ va $\vec{b} = \{1;\ -1; -4\}$ vektor berilgan bo'lsa, ularga qurilgan parallelogram diognallarini uzunligini toping.

Yechish: $\vec{a}+\vec{b} = \{-5+1;\ 3+(-1);\ 8+(-4)\} = \{-4;\ 2;\ 4\}$

$|\vec{a}+\vec{b}| = \sqrt{x^2+y^2+z^2} = \sqrt{(-4)^2+2^2+4^2} = \sqrt{36} = 6$

$\vec{a}-\vec{b} = \{-5-1;\ 3-(-1);\ 8-(-4)\} = \{-6;\ 4;\ 12\}$

$|\vec{a}-\vec{b}| = \sqrt{x^2+y^2+z^2} = \sqrt{(-6)^2+4^2+12^2} = \sqrt{196} = 14$

Mustaqil yechish uchun topshiriqlar

1. $\vec{a} = \{3;\ 3; 2\}$ vektorning yo'naltiruvchi kosinuslarini toping

Javob: $\cos\alpha = \frac{3}{\sqrt{22}}, \quad \cos\beta = \frac{3}{\sqrt{22}}, \quad \cos\gamma = \frac{2}{\sqrt{22}}$

2. $\vec{a} = \{-1;\ -2; 4\}$ vektorning yo'naltiruvchi kosinuslarini toping

Javob: $\cos\alpha = -\frac{1}{\sqrt{21}}, \quad \cos\beta = -\frac{2}{\sqrt{21}}, \quad \cos\gamma = \frac{4}{\sqrt{21}}$

3. A(2,-1,2) va B(3,4,2) nuqtalar bo'lsa, $\vec{a} = \overrightarrow{AB}$ vektorning yo'naltiruvchi kosinuslarini toping.

Javob: $\cos\alpha = \frac{1}{\sqrt{26}}, \quad \cos\beta = \frac{5}{\sqrt{26}}, \quad \cos\gamma = 0$

4. A(3,-4,1) va B(-3,2,1) nuqtalar bo'lsa, $\vec{a} = \overrightarrow{AB}$ vektorning yo'naltiruvchi kosinuslarini toping.

Javob: $\cos\alpha = -\frac{3}{5}, \quad \cos\beta = \frac{4}{5}, \quad \cos\gamma = 0$

5. Ikkita $\vec{a} = \{-1;\ 2; 4\}$ va $\vec{b} = \{2;\ 2; -3\}$ vektor berilgan bo'lsa, ularga qurilgan parallelogram diognallarini uzunligini toping.

Javob: $\sqrt{18};\ \sqrt{58}$

6. Ikkita $\vec{a}=\{-5;\ 2;6\}$ va $\vec{b}=\{1;\ -2;-1\}$ vektor berilgan bo'lsa, ularga qurilgan parallelogram diognallarini uzunligini toping.

Javob: $\sqrt{41};\ \sqrt{101}$

Kesmani berilgan nisbatda bo'lish.

$A(x_1, y_1, z_1)$ $\qquad$ $N(x,y,z)$ $\qquad$ $B(x_2, y_2, z_2)$

x=?; y= ?; z =?

$\frac{\overrightarrow{AN}}{\overrightarrow{NB}}=\lambda$ ekanidan $\overrightarrow{AN}=\lambda\,\overrightarrow{NB}$

$\overrightarrow{AN}$ *ва* $\overrightarrow{NB}$ vektorlarning kollinearlik shartidan

$$\overrightarrow{AN}=\lambda\,\overrightarrow{NB}\ \Rightarrow\ (x-x_1)\,\vec{i}+(y-y_1)\,\vec{j}+(z-z_1)\,\vec{k}\ =\lambda\cdot[\,(x_2-x)\,\vec{i}+(y_2-y)\,\vec{j}+(z_2-z)\,\vec{k}\,]$$

$$x=\frac{x_1+\lambda x_2}{1+\lambda}\quad;\quad y=\frac{y_1+\lambda y_2}{1+\lambda}\quad;\quad z=\frac{z_1+\lambda z_2}{1+\lambda}$$

xususiy holda λ=1 bo'lsa, kesma o'rtasi koordinatalarini aniqlash formulasiga ega bo'lamiz:

$$x=\frac{x_1+x_2}{2}\quad;\quad y=\frac{y_1+y_2}{2}\quad;\quad z=\frac{z_1+z_2}{2}$$

Mavzuga doir yechimlari bilan berilgan topshiriqlardan namunalar

1.A(3,-2,1) va B(0,3,2) nuqtalar orasidagi masofani toping,

Yechish: Ikki nuqta orasidagi masofani toppish formulasi

$$|AB|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

dan foydalansak, quyidagilarga ega bo'lamiz:

$$|AB|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}=\sqrt{(0-3)^2+(3+2)^2+(2-1)^2}=\sqrt{9+25+1}=\sqrt{35}$$

2.A(2,-1,2) va B(3,4,2) nuqtalar bo'lsa, AB kesmani λ=3 nisbatda bo'luvchi N nuqtani koordinatalarini toping.

Yechish: $x=\frac{x_1+\lambda x_2}{1+\lambda}\quad;\quad y=\frac{y_1+\lambda y_2}{1+\lambda}\quad;\quad z=\frac{z_1+\lambda z_2}{1+\lambda}$ larga asosan

$$x=\frac{x_1+\lambda x_2}{1+\lambda}=\frac{2+3\cdot 3}{1+3}=\frac{11}{4} \qquad y=\frac{y_1+\lambda y_2}{1+\lambda}=\frac{-1+3\cdot 4}{1+3}=\frac{-11}{4}$$

$$z=\frac{z_1+\lambda z_2}{1+\lambda}=\frac{2+3\cdot 2}{1+4}=\frac{8}{4}=2$$ Demak: N ($\frac{11}{4}$,-$\frac{11}{4}$,2)

3.A(1,-1,-2) va B(-3,1,-2) nuqtalar bo'lsa, AB kesmani o'rtasini koordinatalarini toping.

Yechish: $x=\frac{x_1+x_2}{2}$; $y=\frac{y_1+y_2}{2}$; $z=\frac{z_1+z_2}{2}$ larga asosan

$$x=\frac{x_1+x_2}{2}=\frac{1+(-3)}{2}=\frac{-2}{2}=1 \qquad y=\frac{y_1+y_2}{2}=\frac{-1+1}{2}=0$$

$$z=\frac{z_1+z_2}{2}=\frac{-2+(-2)}{2}=\frac{-4}{2}=-2$$ Demak N (1,0 ,-2)

Mustaqil yechish uchun topshiriqlar

1.A(0,4,3) va B(2,1,-3) nuqtalar orasidagi masofani toping,

Javob: 7

2. A(5,-1,1) va B(3,2,0) nuqtalar orasidagi masofani toping,

Javob: $\sqrt{14}$

3.A(3,-2,4) va B(1,4,1) nuqtalar bo'lsa, AB kesmani λ=2 nisbatda bo'luvchi N nuqtani koordinatalarini toping.

Javob: $C(\frac{5}{3};2;2)$

4. A(3,-1,4) va B(2,-1,3) nuqtalar bo'lsa, AB kesmani λ=4 nisbatda bo'luvchi N nuqtani koordinatalarini toping.

Javob: $C(\frac{11}{5};-1;\frac{16}{5})$

5. A(0,-2,-3) va B(-1,1,0) nuqtalar bo'lsa, AB kesmani o'rtasini koordinatalarini toping.

Javob: $C(-\frac{1}{2};-\frac{1}{2};-\frac{3}{2})$

6. A(3,-3,-2) va B(1,1,4) nuqtalar bo'lsa, AB kesmani o'rtasini koordinatalarini toping.

Javob: $C(2;-1;-1)$

2.Ikki vektorning skalyar ko'paytmasi.

1-ta'rif. $\vec{a}$ va $\vec{b}$ vektorlarning skalyar ko'paytmasi deb, shunday songa aytiladiki, bu son shu vektorlar uzunliklari bilan ular orasidagi burchak kosinusi ko'paytmasiga teng bo'ladi va odatda $\vec{a}\,\vec{b}$ yoki $(\vec{a}\,\vec{b})$ ko'rinishda yoziladi.

Demak ta'rifga ko'ra $\vec{a}\,\vec{b}=|\vec{a}||\vec{b}|\cos\varphi$; $\varphi=\vec{a}\wedge\vec{b}$

Misol. $|\vec{a}|=3$, $|\vec{b}|=2$, $\varphi=60°$ bo'lsa $(\vec{a}\,\vec{b})=3\cdot 2\cdot\frac{1}{2}=3$

Skalyar ko'paytmaning xossalari.

1. $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$ o'rin almashtirish xossasi.

2. $(\vec{a}+\vec{b})\vec{c}=\vec{a}\,\vec{c}+\vec{a}\,\vec{c}$ taqsimot xossasi.

3. $\lambda\cdot(\vec{a}\cdot\vec{b})=(\lambda\cdot\vec{a})\cdot\vec{b}=\vec{a}\cdot(\lambda\cdot\vec{b})$ guruhlash xossasi.

4. Agar $\vec{a}$ va $\vec{b}$ vektorlar bir xil yo'nalishdagi kollinear vektorlar bo'lsa, $\bar{a}\bar{b}=|\bar{a}||\bar{b}|$ chunki $\cos 0^0=1$.

Agar qarama-qarshi yo'nalgan bo'lsa, $\bar{a}\bar{b}=-|\bar{a}||\bar{b}|$ chunki $\cos 180^0=-1$.

5. $\bar{a}\bar{a}=|\bar{a}||\bar{a}|\cos 0^0=|\bar{a}|^2$

6. $\vec{a}$ va $\vec{b}$ vektorlar o'zaro perpendikulyar bo'lsa , $\vec{a}\,\vec{b}=0$ bo'ladi.

Eslatma. 5- va 6- xossalardan foydalanib $\vec{i},\vec{j},\vec{k}$ birlik vektorlarning skalyar ko'paytmalarini ko'rsak

$$\vec{i}\cdot\vec{i}=\vec{j}\cdot\vec{j}=\vec{k}\cdot\vec{k}=1 \quad \text{ва}\ \vec{i}\cdot\vec{j}=\vec{i}\cdot\vec{k}=\vec{l}\cdot\vec{i}=\vec{j}\cdot\vec{k}=\vec{k}\cdot\vec{i}=\vec{k}\cdot\vec{j}=0$$

Tengliklarning o'rinli bo'lishi ma'lum.

Skalyar ko'paytmani vektorlar koordinatalari orqali ifodalash.

Agar $\vec{a}=\{x_1, y_1, z_1\}$, $\vec{b}=\{x_2, y_2, z_2\}$ vektorlar koordinatalari orqali berilgan bo'lsa,

$\vec{a}\cdot\vec{b}=\{x_1\vec{i}+y_1\vec{j}+z_1\vec{k})(x_2\vec{i}+y_2\vec{j}+z_2\vec{k})=x_1x_2+y_1y_2+z_1z_2$ (eslatmaga ko'ra)

Demak, koordinatalari bilan berilgan ikkita vektorning skalyar ko'paytmasi mos koordinatalari ko'paytmalarining yig'indisiga teng bo'ladi.

$\vec{a}$ va $\vec{b}$ vektorlar yig'indisi $\bar{a}\pm\bar{b}=\{x_1\pm x_2; y_1\pm y_2; z_1\pm z_2\}$ orqali aniqlanadi.

Ikki vektor orasidagi burchak va parallelik, perpendikulyarlik shartlari.

Agar $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchakni φ, bu vektorlarning skalyar ko'paytmasidan $\vec{a}\,\vec{b}=|\vec{a}||\vec{b}|\cos\varphi \Rightarrow$ $\cos\varphi=\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}$ (1)

ikki vektor orasidagi burchak kosinusini hisoblash formulasi kelib chiqadi.

Agar $\vec{a}=\{x_1, y_1, z_1\}$, $\vec{b}=\{x_2, y_2, z_2\}$ koordinatalari bilan berilgan bo'lsa,

$$\cos\varphi=\frac{x_1x_2+y_1y_2+z_1z_2}{\sqrt{x_1^2+y_1^2+z_1^2}\cdot\sqrt{x_2^2+y_2^2+z_2^2}} \qquad (2)$$

Agar $\vec{a}\perp\vec{b}$ bo'lsa, $\varphi=\dfrac{\pi}{2}$ bo'lib $\cos\varphi = 0$ bo'ladi va (2) dan

$$x_1x_2 + y_1y + z_1z_2 = 0 \qquad (3)$$

(3) ikki vektorning perpendikulyarlik sharti.

Agar $\vec{a}$ va $\vec{b}$ vektorlar parallel bo'lsa, u holda bu vektorlarning kollinearlik shartidan ya'ni $\vec{a}=\lambda\vec{b}$ tenglikdan

$x_1\vec{i} + y_1\vec{j} + z_1\vec{k} = \lambda(x_2\vec{i} + y_2\vec{j} + z_2\vec{k})\Rightarrow x_1=\lambda x_2$; $y_1=\lambda y_2$; $z_1=\lambda z_2$.

$$\frac{x_1}{x_2}=\frac{y_1}{y_2}=\frac{z_1}{z_2} \qquad (4)$$

(4) ikki vektorning parallelik sharti.

Mavzuga doir yechimlari bilan berilgan topshiriqlardan namunalar

1. **Agar** $|\vec{a}|=3$, $|\vec{b}|=2$, ular orasidagi burchak $\dfrac{2\pi}{3}$ ni tashkil qilsa. Quyidagilarni hisoblang.

   $a)$ $\vec{a}\cdot\vec{b}$, b) $\vec{a}^{\,2}$, c) $\vec{b}^{\,2}$, d) $(\vec{a}+\vec{b})^2$, e) $(\vec{a}-\vec{b})^2$

Yechish: $\vec{a}\vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\varphi$ ga asosan

$$a)\ \vec{a}\vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\varphi = 3 \cdot 2 \cdot \cos\frac{2\pi}{3} = 6 \cdot (-\frac{1}{2}) = -3$$

$$b)\ \vec{a}^2 = |\vec{a}| \cdot |\vec{a}| \cdot \cos\varphi = 3 \cdot 3 \cdot \cos 0 = 9 \cdot 1 = 9$$

$$c)\ \vec{b}^2 = |\vec{b}| \cdot |\vec{b}| \cdot \cos\varphi = 2 \cdot 2 \cdot \cos 0 = 4 \cdot 1 = 4$$

$$d)\ (\vec{a}+\vec{b})^2 = \vec{a}^2 + 2\vec{a}\vec{b} + \vec{b}^2 = |\vec{a}| \cdot |\vec{a}| + 2|\vec{a}| \cdot |\vec{b}| \cdot \cos\varphi + |\vec{b}| \cdot |\vec{b}| = 3 \cdot 3 + 2 \cdot 3 \cdot 2 \cdot \cos\frac{2\pi}{3} + 2 \cdot 2 = 7$$

$$e)\ (\vec{a}-\vec{b})^2 = \vec{a}^2 - 2\vec{a}\vec{b} + \vec{b}^2 = |\vec{a}| \cdot |\vec{a}| - 2|\vec{a}| \cdot |\vec{b}| \cdot \cos\varphi + |\vec{b}| \cdot |\vec{b}| = 3 \cdot 3 - 2 \cdot 3 \cdot 2 \cdot \cos\frac{2\pi}{3} + 2 \cdot 2 = 19$$

2. $\vec{a} = \{m;\ 3; 3\}$ va $\vec{b} = \{3;\ m; -8\}$ vektorlar berilgan. **m** ning qanday qiymatida bu vektorlar perpendikulyar bo'ladi.

Yechish: Bu ikki vektor perpendikulyar bo'lishi uchun $a_x b_x + a_y b_y + a_z b_z = 0$ shart balarilishi kerak. Shuning uchun vektorlarni mos koordinatalarini ko'paytirib qo'shish natijani 0 ga tenglash kerak. Yani

$$a_x b_x + a_y b_y + a_z b_z = 0 \quad \Rightarrow \quad m \cdot 3 + 3 \cdot m + 3 \cdot (-8) = 0 \quad \Rightarrow$$

$$\Rightarrow \quad 6m - 24 = 0 \quad \Rightarrow \quad 6m = 24 \quad \Rightarrow \quad m = 4$$

3. Agar $|\vec{a}|$=3, $|\vec{b}|$=4, ular orasidagi burchak 30^0 ni tashkil qilsa. $(2\vec{a}+\vec{b})\ va\ (\vec{a}-3\vec{b})$ vektorlarni skalyar ko'paytmasini hisoblang.

Yechish:
$$(2\vec{a}+\vec{b})\,(\vec{a}-3\vec{b}) = 2\vec{a}\cdot\vec{a} - 6\vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{b} - 3\vec{b}\cdot\vec{b} =$$
$$= 2\vec{a}\cdot\vec{a} - 5\vec{a}\cdot\vec{b} - 3\vec{b}\cdot\vec{b}$$

Bundan

$$2\vec{a}\cdot\vec{a} - 5\vec{a}\cdot\vec{b} - 3\vec{b}\cdot\vec{b} = 2\vec{a}\cdot\vec{a} - 5\vec{a}\cdot\vec{b}\cos\varphi - 3\vec{b}\cdot\vec{b} =$$
$$2 \cdot 3 \cdot 3 - 5 \cdot 3 \cdot 4\cos 30^0 - 3 \cdot 4 \cdot 4 = -30(1 + \sqrt{3})$$

4. $\vec{a}=\{1;\ 3;-2\}$ va $\vec{b}=\{3;\ -1;-5\}$ vektorlar berilgan.

Bu vektorlarni skalyar ko'paytmasini hisoblang.

Yechish:

$$\vec{a}\,\vec{b}=a_x b_x+a_y b_y+a_z b_z=1\cdot 3+3\cdot(-1)+(-2)\cdot(-5)=10$$

Mustaqil yechish uchun topshiriqlar

1. Agar $|\vec{a}|=2$, $|\vec{b}|=4$, ular orasidagi burchak $\frac{\pi}{3}$ ni tashkil qilsa. Quyidagilarni hisoblang.

$a)\ 2\vec{a}\cdot\vec{b}$, b) $(\vec{a}+\vec{b})^2$, c) $(\vec{a}-\vec{b})^2$

Javob: $a)$ 8, $b)$ 28, $c)$ 12

2. $\vec{a}=\{m;\ 2;-1\}$ va $\vec{b}=\{-4;\ m;-4\}$ vektorlar berilgan. **m** ning qanday qiymatida bu vektorlar perpendikulyar bo'ladi.

Javob: $m=2$

3. $\vec{a}=\{m;\ 3;-1\}$ va $\vec{b}=\{-5;\ m;16\}$ vektorlar berilgan. **m** ning qanday qiymatida bu vektorlar perpendikulyar bo'ladi.

Javob: $m=-8$

4. Agar $|\vec{a}|=2$, $|\vec{b}|=1$, ular orasidagi burchak 60^0 ni tashkil qilsa.

$(2\vec{a}+3\vec{b})$ *va* $(2\vec{a}+2\vec{b})$ vektorlarni skalyar ko'paytmasini hisoblang.

Javob: 32

5. Agar $|\vec{a}|=1$, $|\vec{b}|=3$, ular orasidagi burchak 45^0 ni tashkil qilsa.

$(2\vec{a}+2\vec{b})$ *va* $(2\vec{a}-3\vec{b})$ vektorlarni skalyar ko'paytmasini hisoblang.

Javob: $-50-\sqrt{2}$

6. $\vec{a}=\{2;\ 3;-1\}$ va $\vec{b}=\{-2;\ -1;4\}$ vektorlar berilgan.

Bu vektorlarni skalyar ko'paytmasini hisoblang.

Javob: -11

7. $\vec{a} = \{2;\ 1; -1\}$ va $\vec{b} = \{0;\ -1; 2\}$ vektorlar berilgan.

Bu vektorlarni skalyar ko'paytmasini hisoblang.

Javob: -3

3. Ikki vektorning vektor ko'paytmasi va uning geometrik ma'nosi.

Ta'rif. $\vec{a}$ vektorning $\vec{b}$ vektorga *vektor ko'paytmasi* deb , quyidagi shartlarni qanoatlantiruvchi shunday $\vec{c}$ vektorga aytiladiki:

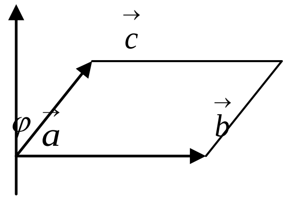
1. $\vec{c}$ vektorning moduli son jihatidan tomonlari $\vec{a}$ va $\vec{b}$ vektorlardan tuzilgan parallelogramning yuziga teng $|\vec{c}|=|\vec{a}||\vec{b}|\sin\varphi$, $\varphi=\widehat{\vec{a}\ \vec{b}}$

2. $\vec{c} \perp \vec{a}$, $\vec{c} \perp \vec{b}$.

3. $\vec{c}$ vektorning musbat yo'nalishi shundayki, agar $\vec{c}$ vektorning uchidan qaralsa, $\vec{a}$ vektordan $\vec{b}$ vektorgacha bo'lgan eng qisqa masofa soat strelkasi yo'nalishiga qarama-qarshi yo'nalishda bo'ladi.

Vektor ko'paytma $[\vec{a}\ \vec{b}]$ yoki $\vec{a}\times\vec{b}$ ko'rinishlarda belgilanadi.

Vektor ko'paytma moduli $|\vec{c}|=|[\vec{a}\ \vec{b}]|=|\vec{a}||\vec{b}|\sin\varphi$



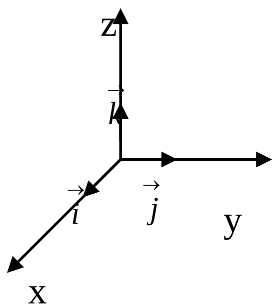
Vektor ko'paytmaning xossalari.

1. $[\vec{a}\ \vec{b}] = -[\vec{b}\ \vec{a}]$.

2. $\vec{a}$ va $\vec{b}$ vektorlar parallel bo'lsa , $\vec{a}\times\vec{b}=0$

3. $\lambda(\vec{a}\times\vec{b}) = (\lambda\vec{a})\times\vec{b}=\vec{a}\times(\lambda\vec{b})$

4. $\overline{a}\times(\overline{b}+\overline{c})=\overline{a}\times\overline{b}+\overline{a}\times\overline{c}$



Endi 1,2 xossalardan foydalanib $\vec{i},\vec{j},\vec{k}$ birlik vektorlarning vektor ko'paytmalarini chiqaraylik

2-xossaga ko'ra $\vec{i}\cdot\vec{i}=\vec{j}\cdot\vec{j}=\vec{k}\cdot\vec{k}=0$ ekanligi ravshan.

$|\vec{c}| = |[\vec{i}\,\vec{j}]| = |\vec{i}||\vec{j}|\sin\frac{\pi}{2}=1$

Ikkinchidan $\vec{i}\times\vec{j}=\vec{c}$ bu vektor $\vec{i}$ va $\vec{j}$ vektorlarga perpendikulyar bo'lib **z** o'qining musbat yo'nalishi bo'yicha yo'nalgan va $\vec{i}$ dan $\vec{j}$ gacha eng qisqa masofa soat strelkasiga qarshi yo'nalgan bo'ladi. Demak $\vec{c}=\vec{k}$ bo'ladi, $\vec{i}\text{ x }\vec{j}=\vec{k}$ xuddi shuningdek:

$$\vec{i}\text{ x }\vec{i}=0,\ \vec{i}\text{ x }\vec{j}=\vec{k},\ \vec{i}\text{ x }\vec{k}=-\vec{j},\ \vec{j}\text{ x }\vec{i}=-\vec{k},\ \vec{j}\text{ x }\vec{j}=0,$$

$$\vec{j}\text{ x }\vec{k}=\vec{i},\ \vec{k}\text{ x }\vec{i}=\vec{j},\ \vec{k}\text{ x }\vec{j}=-\vec{i},\ \vec{k}\text{ x }\vec{k}=0.$$

Koordinatalari bilan berilgan vektorlarning vektor ko'paytmasi.

$\vec{a}=\{x_1, y_1, z_1\}$ va $\vec{b}=\{x_2, y_2, z_2\}$ vektorlar berilgan bo'lsin.

$\vec{a}\text{ x }\vec{b}=(x_1\vec{i}+y_1\vec{j}+z_1\vec{k})\text{x}(x_2\vec{i}+y_2\vec{j}+z_2\vec{k})=(y_1z_2-z_1y_2)\vec{i}+(-x_1z_2+z_1x_2)\vec{j}+(x_1y_2-y_1x_2)\vec{k}=\begin{vmatrix} y_1 & z_1 \\ y_2 & z_2 \end{vmatrix}\vec{i}+\begin{vmatrix} z_1 & x_1 \\ z_2 & x_2 \end{vmatrix}\vec{j}+\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}\vec{k},$

$$\vec{a}\,x\,\vec{b}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$ ko'rinishda ham yozish mumkin

Mavzuga doir yechimlari bilan berilgan topshiriqlardan namunalar

1. $\vec{a}=2\vec{i}+3\vec{j}-\vec{k}$ va $\vec{b}=-\vec{i}+2\vec{j}$ vektorlarga qurilgan parallelogrammning yuzini toping.

Yechish: $\vec{a}$ va $\vec{b}$ vektorlarga qurilgan parallelogrammning yuzi shu vektorlar vektor ko'paytmasining moduliga teng.

Ya'ni $S=\left|\vec{a}\times\vec{b}\right|$. Vektor ko'paytmani topamiz:

$$\vec{a}\times\vec{b}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix},\quad \vec{a}=a_x\vec{i}+a_y\vec{j}-a_z\vec{k},\ \vec{b}=b_x\vec{i}+b_y\vec{j}-b_z\vec{k},$$

Bu yerda, $a_y=2,\ a_y=3,\ a_z=-1,\ b_x=-1,\ b_y=2,\ b_z=0$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 2 & 3 & -1\\ -1 & 2 & 0\end{vmatrix}=\vec{j}+4\vec{k}+3\vec{k}+2\vec{i}=2\vec{i}+\vec{j}+7\vec{k}$$

$S=\left|\vec{a}\times\vec{b}\right|=\sqrt{2^2+1^2+7^2}=\sqrt{54}$ kv.b

2. $\vec{a}$ va $\vec{b}$ vektorlar o'zaro perpendikulyar bo'lib, $|\vec{a}|=3$ va $|\vec{b}|=4$ bo'lsa, $\left|(2\vec{a}+\vec{b})\times(\vec{a}-3\vec{b})\right|$ ni hisoblang.

Yechish: $(2\vec{a}+\vec{b})\times(\vec{a}-3\vec{b})$ dan qavslarni ochib chiqamiz.

$$\left|(2\vec{a}+\vec{b})\times(\vec{a}-3\vec{b})\right|=\left|2\vec{a}\times\vec{a}-6\vec{a}\times\vec{b}-\vec{a}\times\vec{b}-3\vec{b}\times\vec{b}\right|=\left|2\vec{a}\times\vec{a}-7\vec{a}\times\vec{b}-3\vec{b}\times\vec{b}\right|=\left||\vec{a}|\times|\vec{a}|-7|\vec{a}|\times|\vec{b}|\sin\varphi-|\vec{b}|\times|\vec{b}|\right|$$

$$\left||\vec{a}|\times|\vec{a}|-7|\vec{a}|\times|\vec{b}|\sin\varphi-|\vec{b}|\times|\vec{b}|\right|=\left|0-7\cdot3\cdot4\cdot\sin 90^0-0\right|=\left|-84\cdot1\right|=\left|-84\right|=84$$

3.A(0,-3,-1) , B(2,1,2) va C(-1,3,0) nuqtalar berilgan bo'lsa, $\vec{AB}\times\vec{BC}$ ni hisoblang.

Yechish: $\vec{AB}\times\vec{BC}$, $\vec{AB}=\{2;4:3\}$, $\vec{BC}=\{-3;2;-2\}$

$$\vec{AB}\times\vec{BC}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 2 & 4 & 3\\ -3 & 2 & -2\end{vmatrix}=-8\vec{i}-9\vec{j}+4\vec{k}+12\vec{k}-6\vec{i}+4\vec{j}=-14\vec{i}-5\vec{j}+16\vec{k}$$

$$\vec{AB}\times\vec{BC}=\{-14;-5;16\}$$

Mustaqil yechish uchun topshiriqlar

1. $\vec{a}=\vec{i}-2\vec{j}+\vec{k}$ va $\vec{b}=2\vec{i}-3\vec{j}$ vektorlarga qurilgan parallelogrammning yuzini toping. **Javob:** $\sqrt{14}$

2. $\vec{a}=2\vec{i}-\vec{j}+2\vec{k}$ va $\vec{b}=-\vec{i}+2\vec{j}$ vektorlarga qurilgan parallelogrammning yuzini toping. **Javob:** $\sqrt{29}$

3. $\vec{a}$ va $\vec{b}$ vektorlar o'zaro perpendikulyar bo'lib, $|\vec{a}|=2$ va $|\vec{b}|=3$ bo'lsa,

$\left|(\vec{a}+2\vec{b})\times(\vec{a}-2\vec{b})\right|$ ni hisoblang.

Javob: -24

4. $\vec{a}$ va $\vec{b}$ vektorlar o'zaro perpendikulyar bo'lib, $|\vec{a}|=1$ va $|\vec{b}|=4$ bo'lsa,

$\left|(\vec{a}+\vec{b})\times(\vec{a}-2\vec{b})\right|$ ni hisoblang.

Javob: 12

5. A(1,-2,-1) , B(0,1,-3) va C(1,3,2) nuqtalar berilgan bo'lsa,

$\vec{AB}\times\vec{BC}$ ni hisoblang.

Javob: $19i+3j-5k$

6. A(0,-1,-2) , B(1,1,3) va C(-2,2,1) nuqtalar berilgan bo'lsa,

$\vec{AB}\times\vec{BC}$ ni hisoblang.

Javob: $-9i-13j+7k$

4. Uch vektorning aralash ko'paytmasi va uning geometrik ma'nosi.

$\vec{a}=\{x_1, y_1, z_1\}$, $\vec{b}=\{x_2, y_2, z_2\}$ va $\vec{c}=\{x_3, y_3, z_3\}$ vektorlar berilgan bo'lsa, bu vektorlarning *aralash ko'paytmasi* deb , $\vec{a}\times\vec{b}$ vektor ko'paytma bilan $\vec{c}$ vektorning skalyar ko'paytmasiga aytiladi va $(\vec{a}\times\vec{b})\vec{c}$ ko'rinishda yoziladi

$$\vec{a}\times\vec{b}=\begin{vmatrix} y_1 & z_1 \\ y_2 & z_2 \end{vmatrix}\vec{i}+\begin{vmatrix} z_1 & x_1 \\ z_2 & x_2 \end{vmatrix}\vec{j}+\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}\vec{k},\quad \vec{c}=x_3\vec{i}+y_3\vec{j}+z_3\vec{k},$$

$$(\vec{a}\times\vec{b})\vec{c}=\left(\begin{vmatrix} y_1 & z_1 \\ y_2 & z_2 \end{vmatrix}\vec{i}+\begin{vmatrix} z_1 & x_1 \\ z_2 & x_2 \end{vmatrix}\vec{j}+\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}\vec{k}\right)(x_3\vec{i}+y_3\vec{j}+z_3\vec{k})=$$

$$=\begin{vmatrix} y_1 & z_1 \\ y_2 & z_2 \end{vmatrix}x_3+\begin{vmatrix} z_1 & x_1 \\ z_2 & x_2 \end{vmatrix}y_3+\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}z_3=\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

Aralash ko'paytmaning geometrik ma'nosi qirralari berilgan $\vec{a},\vec{b},\vec{c}$ vektorlarning modullaridan tashkil topgan parallelipipedning hajmini ifodalaydi.

Fazodagi ixtiyoriy $\bar{a},\bar{b},\bar{c}$ vektorlarning komplanar vektorlar bo'lishi uchun ularning aralash ko'paytmasi **0** bo'lishi zarur va yetarli.

Mavzuga doir yechimlari bilan berilgan topshiriqlardan namunalar

1. $\vec{AB}=\{3;1;0\},\ \vec{BC}=\{4;-2;3\},\ \vec{CD}=\{-2;1;5\}$ veektorlarni aralash ko'paytmasini toping.

Yechish:

$$\vec{AB}\cdot\vec{BC}\cdot\vec{CD}=\begin{vmatrix} x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\\ x_3 & y_3 & z_3\end{vmatrix}=\begin{vmatrix} 3 & 1 & 0\\ 4 & -2 & 3\\ -2 & 1 & 5\end{vmatrix}=-30-6-9-20=-65$$

2.Uchlari O(0;0;0) , A(5;2;0), B(2;5;0) , C(1;2;4) nuqtalarda bo'lgan parallelpipedning hajmini toping.

Yechish: Parallelpiped $\vec{OA},\ \vec{OB},\ \vec{OC}$ vektorlarga qurilganligi uchun

$$\vec{OA}=\{5-0;2-0;0-0\}=\{5;2;0\}$$

$$\vec{OB}=\{2-0;5-0;0-0\}=\{2;5;0\}$$

$$\vec{OC}=\{1-0;2-0;4-0\}=\{1;2;4\}$$

$$V=\vec{a}\cdot\vec{b}\cdot\vec{c}=\begin{vmatrix} x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\\ x_3 & y_3 & z_3\end{vmatrix}\ \text{ga asosan}$$

$$V=(\vec{OA}\times\vec{OB})\cdot\vec{OC}=\begin{vmatrix} 5 & 2 & 0\\ 2 & 5 & 0\\ 1 & 2 & 4\end{vmatrix}=100+0+0-0-0-16=84\ \text{kub birlik.}$$

3.Uchlari A(-5;4;2) , B(-4;6;2), C(1;-5;3) , D(3;6;-4) nuqtalarda bo'lgan piramidaning hajmini toping.

Yechish: Piramidaning hajmi $\vec{AB},\ \vec{AC},\ \vec{AD}$ parallelpipedning hajmini oltidan bir qismiga tengligidan

$$V=\pm\frac{1}{6}(\vec{AB}\times\vec{AC})\cdot\vec{AD}=\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$ formula bilan aniqlanadi.

Buning uchun $\vec{AB},\ \vec{AC},\ \vec{AD}$ vektorlarni koordinatalarini topamiz:

$$\vec{AB}=\{-4+5;6-4;2-2\}=\{1;2;0\}$$

$$\vec{AC}=\{1+5;-5-4;3-2\}=\{6;-9;1\}$$

$$\vec{AD}=\{3+5;6-4;-4-2\}=\{8;2;-6\}$$

$$V=\pm\frac{1}{6}(\vec{AB}\times\vec{AC})\cdot\vec{AD}=\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}=\begin{vmatrix} 1 & 2 & 0 \\ 6 & -9 & 1 \\ 8 & 2 & -6 \end{vmatrix}=54+16-2+72=140\,kub.b$$

Mustaqil yechish uchun topshiriqlar

1. $\vec{AB}=\{2;1;-3\},\ \vec{BC}=\{1;-4;2\},\ \vec{CD}=\{-1;1;3\}$ veektorlarni aralash ko’paytmasini toping.

Javob: -22

2. $\vec{AB}=\{0;1;-5\},\ \vec{BC}=\{4;-2;2\},\ \vec{CD}=\{-3;2;3\}$ veektorlarni aralash ko’paytmasini toping.

Javob: -28

3. $\vec{AB}=\{4;2;1\},\ \vec{BC}=\{2;3;1\},\ \vec{CD}=\{-2;0;1\}$ veektorlarni aralash ko’paytmasini toping.

Javob: 10

4. $\vec{AB}=\{1;3;-4\},\ \vec{BC}=\{3;-1;3\},\ \vec{CD}=\{2;1;2\}$ veektorlarni aralash ko’paytmasini toping.

Javob: -25

5.Uchlari O(1;1;1) , A(6;3;1), B(3;6;1) , C(2;3;5) nuqtalarda bo'lgan parallelopipedning hajmini toping.

Javob: $V = 84 kub.b$

6. Uchlari A(-4;3;1) , B(-2;4;0), C(2;-4;1) , D(-2;3;-1) nuqtalarda bo'lgan piramidaning hajmini toping.

Javob: $V = \frac{13}{3} kub.b$

Fazoda tekislik. va to'g'ri chiziq

1.Fazoda tekislik tenglamalari:

Oxyz to'g'ri burchakli koordinatalar sistemasida har qanday tekislik tenglamasini x, y, z o'zgaruvchilarga nisbatan quyidagi chiziqli tenglama shaklida yo'zish mumkun:

$$Ax + By + Cz + D = 0$$

Bu tenglama tekislikning umumiy tenglamasi deyiladi. Bu yerda A, B, C kooefitsentlar berilgan tekislikka perpendikulyar bo'lgan va uning normal vektori deb ataluvchi $\vec{n}$ = {A, B, C} vektorning koordinatalaridir. Tekislikning fazodagi holati A, B, C kooefitsentlari va ozod hadining qiymatlariga bog'liq. Xususan , agar;

1. D = 0 bo'lsa, u xolda Ax + By + Cz = 0 va tekislik koordinatalar boshidan o'tadi.
2. C = 0 bo'lsa, u xolda Ax + By +D = 0 va tekislik Oz o'qiga parallel bo'ladi.
3. B = 0 bo'lsa, u xolda Ax + Cz +D = 0 va tekislik Oy o'qiga parallel bo'ladi.
4. A = 0 bo'lsa, u xolda By + Cz + D = 0 va tekislik Ox o'qiga parallel bo'ladi.
5. D = 0, C = 0 bo'lsa, u xolda Ax + By = 0 va tekislik Oz o'qi orqali o'tadi.
6. D = 0, B = 0 bo'lsa, u xolda Ax + Cz = 0 va tekislik Oy o'qi orqali o'tadi.
7. D = 0, A = 0 bo'lsa, u xolda By + Cz = 0 va tekislik Ox o'qi orqali o'tadi.
8. C = 0, B = 0 bo'lsa, u xolda Ax + D = 0 va tekislik Oyz koordinatalar tekisligiga parallel (yoki Ox o'qiga perpendikulyar) bo'ladi.
9. C = 0, A = 0 bo'lsa, u xolda By + D = 0 va tekislik Oxz koordinatalar tekisligiga parallel (yoki Oy o'qiga perpendikulyar) bo'ladi.

10.A = 0, B = 0 bo'lsa, u xolda Cz + D = 0 va tekislik Oxy koordinatalar tekisligiga parallel (yoki Oz o'qiga perpendikulyar) bo'ladi.

11.D = 0, A = 0, B = 0 bo'lsa, u xolda Cz = 0 yoki z = 0 va tekislik Oxy koordinatalar tekisligiga bilan ustma – ust tushadi.

12.D = 0, A = 0, C = 0 bo'lsa, u xolda By = 0 yoki y = 0 va tekislik Oxz koordinatalar tekisligiga bilan ustma – ust tushadi.

13.D = 0, B = 0, C = 0 bo'lsa, u xolda Ax = 0 yoki x = 0 va tekislik Oyz koordinatalar tekisligiga bilan ustma – ust tushadi.

Tekislikning tenglamalarini keltiramiz.

a) Berilgan $M_0(x_0;y_0;z_0)$ nuqtadan o'tuvchi va berilgan $\vec{n}=\{A;B;C\}$ normal vektorga ega tekislik tenglamasi: $A(x-x_0)+B(y-y_0)+C(z-z_0)=0$

b) Tekislikning kesmalarga nisbatan tenglamasi: $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

bunda, a, b, c – tekislikning mos koordinata o'qlaridan kesadigan kesmalari,

c) Berilgan uchta $M(x_1;y_1;z_1)$, $N(x_2;y_2;z_2)$, $P(x_3;y_3;z_3)$ nuqtadan o'tuvchi tekislik tenglamasi $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$

Mavzuga doir yechimlari bilan berilgan topshiriqlardan namunalar

1. $M_0(2:1:-1)$ nuqtadan o'tuvchi va $\vec{n}=\{1:-2:3\}$ normal vektorga ega tekislik tenglamasini tuzing.

Yechish: $A(x-x_0)+B(y-y_0)+C(z-z_0)=0$ tenglamadan foydalanamiz.

Bu yerda $M_0(x_0;y_0;z_0)$, $\vec{n}=\{A;B;C\}$

$$(x-2)+(-2)(y-1)+3(z+1)=0$$

$$x-2-2y+2+3z+3=0$$

Demak , izlanayotgan tekislik tenglamasi $x-2y+3z+3=0$ bo'ladi.

2. $M(-1;2;1)$, $N(2;3;-2)$, $P(3;4;2)$ nuqtalardan o'tuvchi tekislik tenglamasini tuzing.

Yechish:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$$ tenglamadan foydalanamiz.

Bu yerda $M(x_1;y_1;z_1)$, $N(x_2;y_2;z_2)$, $P(x_3;y_3;z_3)$

$$\begin{vmatrix} x+1 & y-2 & z-1 \\ 2+1 & 3-2 & -2-1 \\ 3+1 & 4-2 & 2-1 \end{vmatrix}=\begin{vmatrix} x+1 & y-2 & z-1 \\ 3 & 1 & -3 \\ 4 & 2 & 1 \end{vmatrix}=x+1-12y+24+6z-6-4z+4+6x+6-3y+6=$$

$$=7x-15y+2z+23$$

Demak , izlanayotgan tekislik tenglamasi $7x-15y+2z+23=0$ bo'ladi

Mustaqil yechish uchun topshiriqlar

1. $M_0(3:1:-2)$ nuqtadan o'tuvchi va $\vec{n}=\{2:-1:1\}$ normal vektorga ega tekislik tenglamasini tuzing.

Javob: $2x-y+z-3=0$

2. $M_0(-1:2:-3)$ nuqtadan o'tuvchi va $\vec{n}=\{3:-2:2\}$ normal vektorga ega tekislik tenglamasini tuzing.

Javob: $3x-2y+2z+13=0$

3. $M(-2;-1;0)$, $N(1;-2;-3)$, $P(2;2;3)$ nuqtalardan o'tuvchi tekislik tenglamasini tuzing.

Javob: $6x-21y+13z-9=0$

4. $M(2;3;2)$, $N(4;2;-1)$, $P(3;-2;3)$ nuqtalardan o'tuvchi tekislik tenglamasini tuzing.

Javob: $-16x-5y-9z+65=0$

Tekisliklarning o'zaro joylashuvi.

Tekisliklar $A_1x+B_1y+C_1z+D_1=0$ va $A_2x+B_2y+C_2z+D_2=0$ tenglamalar bilan berilgan bo'lsin. Ular orasidagi φ burchak quyidagi formula asosida hisoblanadi:

$$\cos\varphi = \frac{\vec{n_1}\cdot\vec{n_2}}{\left|\vec{n_1}\right|\cdot\left|\vec{n_2}\right|} = \frac{A_1\cdot A_2 + B_1\cdot B_2 + C_1\cdot C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2}\cdot\sqrt{A_2^2 + B_2^2 + C_2^2}}$$

bunda $\vec{n_1} = \{A_1; B_1; C_1\}$, $\vec{n_2} = \{A_2; B_2; C_2\}$ berilgan tekisliklarning normal vektori.

a) Agar tekisliklar perpendikulyar bo'lsa, u xolda $\vec{n}_1 \cdot \vec{n}_2 = 0$ yoki $A_1\cdot A_2 + B_1\cdot B_2 + C_1\cdot C_2 = 0$

b) Agar tekisliklar parallel bo'lsa, u xolda $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} \neq \frac{D_1}{D_2}$

c) Agar tekisliklar ustma-ust tushsa, u xolda $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} = \frac{D_1}{D_2}$

d) $M_0(x_0; y_0; z_0)$ nuqtadan $Ax + By + Cz + D = 0$ tekislikkacha bo'lgan d masofa

$d = \frac{\left|Ax_0 + By_0 + Cz_0 + D\right|}{\sqrt{A^2 + B^2 + C^2}}$ formula asosida hisoblanadi.

Mavzuga doir yechimlari bilan berilgan topshiriqlardan namunalar

1. Tekisliklar $x - 2y + 2z - 8 = 0$ $x + z - 6 = 0$ tenglamalar bilan berilgan bo'lsin. Ular orasidagi φ burchakni hisoblang.

Yechish: $\cos\varphi = \frac{\vec{n_1}\cdot\vec{n_2}}{\left|\vec{n_1}\right|\cdot\left|\vec{n_2}\right|} = \frac{A_1\cdot A_2 + B_1\cdot B_2 + C_1\cdot C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2}\cdot\sqrt{A_2^2 + B_2^2 + C_2^2}}$

bunda $\vec{n_1} = \{A_1; B_1; C_1\}$, $\vec{n_2} = \{A_2; B_2; C_2\}$ berilgan tekisliklarning normal vektori.

Bularga asosan:

$$\cos\varphi = \frac{\vec{n_1}\cdot\vec{n_2}}{\left|\vec{n_1}\right|\cdot\left|\vec{n_2}\right|} = \frac{A_1\cdot A_2 + B_1\cdot B_2 + C_1\cdot C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2}\cdot\sqrt{A_2^2 + B_2^2 + C_2^2}} = \frac{1\cdot 1 + (-2)\cdot 0 + 2\cdot 1}{\sqrt{1^2 + (-2)^2 + 2^2}\cdot\sqrt{1^2 + 0^2 + 1^2}} = \frac{3}{3\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Demak: $\cos\varphi = \frac{\sqrt{2}}{2}$, $\varphi = \arccos\frac{\sqrt{2}}{2}$, $\varphi = \frac{\pi}{4}$

2. Tekisliklar $3x + 2y + z - 1 = 0$ $x - 3z - 2 = 0$ tenglamalar bilan berilgan bo'lsin. Ularni perpendikulyar yoki parallelligini tekshiring.

Yechish: $A_1\cdot A_2 + B_1\cdot B_2 + C_1\cdot C_2 = 3\cdot 1 + 2\cdot 0 + 1\cdot(-3) = 0$

Shartga ko'ra $A_1\cdot A_2 + B_1\cdot B_2 + C_1\cdot C_2 = 0$ bo'lganligidan, tekisliklar perpendikulyar ekanligi kelib chiqdi.

3. Tekisliklar $3x+6y+9z-1=0$ $2x+4y-6z-2=0$ tenglamalar bilan berilgan bo'lsin. Ularni perpendikulyar yoki parallelligini tekshiring.

Yechish: $\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}$ parallel bo'ladi.

$\frac{3}{2}=\frac{6}{4}=\frac{-9}{-6}$,

$\frac{3}{2}=\frac{3}{2}=\frac{3}{2}$ bo'lgani uchun

Shartga ko'ra tekisliklar parallel ekanligi kelib chiqdi.

4. $M_0(1:-2:-2)$ nuqtadan $2x-2y+z+5=0$ tekislikkacha bo'lgan d masofani toping.

Yechish: $d=\frac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}}$, bunda $M_0(x_0;y_0;z_0)$, $\vec{n}=\{A;B;C\}$ formulaga asosan: $d=\frac{|2\cdot1+(-2)\cdot(-2)+1\cdot(-2)+5|}{\sqrt{1^2+(-2)^2+(-2)^2}}=\frac{|2+4-2+5|}{\sqrt{9}}=\frac{9}{3}=3$ bo'ladi.

5. $K(-1;1;-2)$ nuqtadan $M(1;-1;1)$, $N(-2;1;3)$, $P(4;-5;-2)$ nuqtalar orqali o'tuvchi tekislikkacha bo'lgan masofani toping.

Yechish:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$$ tenglamadan

foydalanib. $M(1;-1;1)$, $N(-2;1;3)$, $P(4;-5;-2)$ nuqtalardan o'tuvchi tekislik tenglamasini tuzamiz.

Bu yerda $M(x_1;y_1;z_1)$, $N(x_2;y_2;z_2)$, $P(x_3;y_3;z_3)$

$$\begin{vmatrix} x-1 & y+1 & z-1 \\ -2-1 & 1+1 & 3-1 \\ 4-1 & -5+1 & -2-1 \end{vmatrix}=\begin{vmatrix} x-1 & y+1 & z-1 \\ -3 & 2 & 2 \\ 3 & -4 & -3 \end{vmatrix}=-6x+6+6y+6+12z-12-6z+6+8x-8-9y-9=2x-3y+6z-11$$

Demak , izlanayotgan tekislik tenglamasi $2x-3y+6z-11=0$ bo'ladi.

$d=\frac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}}$, bunda $K(x_0;y_0;z_0)$, $\vec{n}=\{A;B;C\}$ formulaga asosan:

$d=\frac{|2\cdot(-1)+(-3)\cdot1+6\cdot(-2)+(-11)|}{\sqrt{2^2+(-3)^2+6^2}}=\frac{|-2-3-12-11|}{\sqrt{49}}=\frac{28}{7}=4$ uz. b. bo'ladi.

Mustaqil yechish uchun topshiriqlar

1. Tekisliklar $2x-2y+z-4=0$ $2x+2y-3=0$ tenglamalar bilan berilgan bo'lsin. Ular orasidagi φ burchakni hisoblang.

Javob: $\cos\varphi = 90^0$

2. Tekisliklar $2x+y+z-5=0$ $x+y-z-2=0$ tenglamalar bilan berilgan bo'lsin. Ular orasidagi φ burchakni hisoblang.

Javob: $\cos\varphi = \frac{\sqrt{2}}{3}$

3. Tekisliklar $3x+y+2z-1=0$ $2x-2y-2z-3=0$ tenglamalar bilan berilgan bo'lsin. Ularni perpendikulyar yoki parallelligini tekshiring.

Javob: Perpendikulyar

4. Tekisliklar $3x+6y+9z-1=0$ $-x-2y-3z-4=0$ tenglamalar bilan berilgan bo'lsin. Ularni perpendikulyar yoki parallelligini tekshiring.

Javob: Parallel

5. $M_0(0:3:4)$ nuqtadan $3x+y+2z+4=0$ tekislikkacha bo'lgan d masofani toping. **Javob:** $d = \frac{15}{\sqrt{14}}$

6. $K(1;1;1)$ nuqtadan $M(2;2;5)$, $N(3;2;2)$, $P(2;0;3)$ nuqtalar orqali o'tuvchi tekislikkacha bo'lgan masofani toping.

Javob: $d = \frac{12}{\sqrt{44}}$

7. $K(5;2;6)$ nuqtadan $M(0;1;-4)$, $N(1;8;3)$, $P(4;2;1)$ nuqtalar orqali o'tuvchi tekislikkacha bo'lgan masofani toping.

Javob: $d = \frac{28}{\sqrt{2138}}$

2.Fazoda to'g'ri chiziq tenglamasi:

To'g'ri chiziqning fazoda berilish usuliga qarab uning tenglamasi turlicha bo/lishi mumkun.

a) Berilgan $M_0(x_0;y_0;z_0)$, nuqtadan o'tuvchi va $\vec{s}=\{\ell;m;p;\}$ yo'naltiruvchi vektorga ega bo'lgan to'g'ri chiziqning kanonik shakldagi tenglamasi: $\frac{x-x_0}{\ell}=\frac{y-y_0}{m}=\frac{z-z_0}{p}$

b) To'g'ri chiziqning parametrik tenglamalari : $\begin{cases} x=x_0+\ell t \\ y=y_0+mt \\ z=z_0+pt \end{cases}$ bunda t – parameter.

c) Berilgan ikki $M_1(x_1;y_1;z_1)$, $M_2(x_2;y_2;z_2)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasi: $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$

d) Fazodagi to'g'ri chiziqning umumiy tenglamalari: $\begin{cases} A_1x+B_1y+C_1z+D_1=0 \\ A_2x+B_2y+C_2z+D_2=0 \end{cases}$

bunda $\frac{A_1}{A_2}\neq\frac{B_1}{B_2}\neq\frac{C_1}{C_2}$ Bu to'g'ri chiziqning yo'naltiruvchi vektori $\vec{s}$ ushbu

$\vec{s}=\vec{n}_1\times\vec{n}_2=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix}$ formula bo'yicha aniqlanadi.

Mavzuga doir yechimlari bilan berilgan topshiriqlardan namunalar

1. $M_0(2:-3:-2)$ nuqtadan o'tuvchi va $\vec{s}=\{1:-1:2\}$ vektorga parallel to'g'ri chiziq tenglamasini tuzing.

Yechish: $\frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{\ell}$ to'g'ri chiziqning kanonik tenglamasiga asosan,

Bu yerda $M_0(x_0;y_0;z_0)$, $\vec{s}=\{m;n;\ell\}$

$$\frac{x-2}{1}=\frac{y+3}{-1}=\frac{z+2}{-2}$$

$$x-2=-y-3=-\frac{z}{2}-1$$

Demak , izlanayotgan to'g'ri chiziqning tenglamasi $x-2=-y-3=-\frac{z}{2}-1$ bo'ladi.

2. $M_1(1;-2;3)$ va $M_2(4;3;2)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing.

Yechish: $\dfrac{x-x_1}{x_2-x_1}=\dfrac{y-y_1}{y_2-y_1}=\dfrac{z-z_1}{z_2-z_1}$ to'g'ri chiziqning tenglamasiga asosan,

Bu yerda $M_1(x_1;y_1;z_1)$, $M_2(x_2;y_2;z_2)$

$$\frac{x-1}{4-1}=\frac{y+2}{3+2}=\frac{z-3}{2-3}$$

$$\frac{x-1}{3}=\frac{y+2}{5}=\frac{z-3}{-1}$$

$$\frac{x-1}{3}=\frac{y+2}{5}=-z+3$$

Demak , izlanayotgan to'g'ri chiziqning tenglamasi $\dfrac{x-1}{3}=\dfrac{y+2}{5}=-z+3$ bo'ladi.

3. Fazodagi to'g'ri chiziqning umumiy tenglamalari: $\begin{cases} 2x-3y+4z-1=0 \\ -3x+y+2z+4=0 \end{cases}$ berilgan. To'g'ri chiziqning yo'naltiruvchi vektori $\vec{s}$ ni toping.

Yechish: Bu to'g'ri chiziqning yo'naltiruvchi vektori $\vec{s}$ ushbu

$$\vec{s}=\vec{n}_1\times\vec{n}_2=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix}$$

formula bilan aniqlanadi. Bu yerda $\vec{n}_1=\{A_1;B_1;C_1\}$, $\vec{n}_2=\{A_2;B_2;C_2\}$.

$$\vec{s}=\vec{n}_1\times\vec{n}_2=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -3 & 4 \\ -3 & 1 & 2 \end{vmatrix}=-6\vec{i}-12\vec{j}+2\vec{k}-9\vec{k}-4\vec{i}-4\vec{j}=-10\vec{i}-16\vec{j}-7\vec{k}$$

Demak , izlanayotgan $\vec{s}$ vektor $\vec{s}=\{-10;-16;-7\}$ bo'ladi.

Mustaqil yechish uchun topshiriqlar

1. $M_0(1:-1:3)$ nuqtadan o'tuvchi va $\vec{s}=\{2:3:-3\}$ vektorga parallel to'g'ri chiziq tenglamasini tuzing.

Javob: $\frac{x-1}{1}=\frac{y+1}{4}=\frac{z-3}{-6}$

2. $M_0(4;\ 2;\ 1)$ nuqtadan o'tuvchi va $\vec{s}=\{4:-2:1\}$ vektorga parallel to'g'ri chiziq tenglamasini tuzing.

Javob: $\frac{x-4}{4}=\frac{y-2}{-2}=\frac{z-1}{1}$

3. $M_1(2;-2;1)$ va $M_2(3;-1;-1)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing.

Javob: $\frac{x-2}{1}=\frac{y+2}{1}=\frac{z-1}{-2}$

4. $M_1(3;1;2)$ va $M_2(2;3;3)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing.

Javob: $\frac{x-3}{-1}=\frac{y-1}{2}=\frac{z-2}{1}$

5. Fazodagi to'g'ri chiziqning umumiy tenglamalari: $\begin{cases} 3x-3y+z-3=0 \\ -x+2y+2z+4=0 \end{cases}$ berilgan. To'g'ri chiziqning yo'naltiruvchi vektori $\vec{s}$ ni toping.

Javob: $\vec{s}=\{-8;-7;3\}$

6. Fazodagi to'g'ri chiziqning umumiy tenglamalari: $\begin{cases} 2x-2y+3z-5=0 \\ -4x+2y+4z+4=0 \end{cases}$ berilgan. To'g'ri chiziqning yo'naltiruvchi vektori $\vec{s}$ ni toping.

Javob: $\vec{s}=\{-14;-20;-4\}$

Tekislikdagi to'g'ri chiziqning tenglamalari:

$Ax+By+Cz+D=0$ va $z=0$ tekisliklarning kesishish chizig'i Oxy tekislikda yotuvchi $Ax+By+C=0$ to'g'ri chiziqdan iborat bo'ladi. Bu tenglama tekislikdagi to'g'ri chiziqning umumiy tenglamasi deyiladi. Berilgan to'g'ri chiziqqa perpendikulyar bo'lgan $\vec{n}$ = {A, B} vektor to'g'ri chiziqning normal vektori deb ataladi.

a) Berilgan $M_0(x_0; y_0)$ nuqtadan o'tuvchi va berilgan $\vec{n} = \{A; B\}$ normal vektorga ega to'g'ri chiziq tenglamasi: $A(x - x_0) + B(y - y_0) = 0$

b) To'g'ri chiziqning kanonik tenglamasi: $\frac{x - x_0}{m} = \frac{y - y_0}{n}$, bunda $\vec{s} = \{m; n;\}$ to'g'ri chiziqning yo'naltiruvchi vektori, $M_0(x_0; y_0)$ – to'g'ri chiziqda yotuvchi berilgan nuqta.

c) to'g'ri chiziqning burchak koeffitsentli tenglamasi: $y = kx + b$ bunda b - to'g'ri chiziqning Oy o'qidan kesadigan kesmasi, k = tg α (α - to'g'ri chiziq bilan Ox o'qining musbat yo'nalishi orasidagi burchak.

$M_0(x_0; y_0)$ nuqtadan o'tuvchi va k burchak koeffitsentli to'g'ri chiziqning tenglamasi: $(y - y_0) = k(x - x_0)$

d) to'g'ri chiziqning kesmalarga nisbatan tenglamasi: $\frac{x}{a} + \frac{y}{b} = 1$ bunda, a, b – to'g'ri chiziqning mos koordinata o'qlaridan kesadigan kesmalari,

e) Berilgan ikki $M_1(x_1; y_1)$, $M_2(x_2; y_2)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasi: $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$

Mavzuga doir yechimlari bilan berilgan topshiriqlardan namunalar

1. $M_0(-3:4)$ nuqtadan o'tuvchi va $\vec{s} = \{2:-1\}$ vektorga parallel to'g'ri chiziq tenglamasini tuzing.

Yechish: $\frac{x - x_0}{m} = \frac{y - y_0}{n}$ to'g'ri chiziqning kanonik tenglamasiga asosan,

Bu yerda $M_0(x_0; y_0)$, $\vec{s} = \{m; n;\}$

$$\frac{x+3}{2} = \frac{y-4}{-1}$$

Demak , izlanayotgan to'g'ri chiziqning tenglamasi $\frac{x+3}{2} = \frac{y-4}{-1}$ bo'ladi.

2. $M_1(2;-4)$ va $M_2(5;3)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing.

Yechish: $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}$ to'g'ri chiziqning tenglamasiga asosan,

Bu yerda $M_1(x_1;y_1)$, $M_2(x_2;y_2)$

$$\frac{x-2}{5-2}=\frac{y+4}{3+4}$$

$$\frac{x-2}{3}=\frac{y+4}{7}$$

Demak , izlanayotgan to'g'ri chiziqning tenglamasi $\frac{x-2}{3}=\frac{y+4}{7}$ bo'ladi.

3.Berilgan $2x-y+3=0$ tenglama bo'yicha k burchak koeffitsientini va koordinatalar o'qlaridan kesadigan a, b kesmalarini toping.

Yechish: Berilgan tenglamani $y=kx+b$ ko'rinishga keltiramiz: $y=2x+3$.

Bundan k=2, b=3 kelib chiqadi. Ox o'qidan kesadigan kesmasini toppish uchun y=0 deb yuqoridagi tenglamadan x ni topamiz: $2x-y+3=0 \Rightarrow 2x=-3 \Rightarrow x=-\frac{3}{2}$ bo'ladi.

Demak, $a=-\frac{3}{2}$, b=3, k=2

4. Berilgan $2x+2y-5=0$ to'g'ri chiziq abstsissalar o'qining musbat yo'nalishi bilan tashkil etgan burchakni toping.

Yechish: Berilgan tenglamani $y=kx+b$ ko'rinishga keltiramiz: $y=-x+\frac{5}{2}$.

Bundan k=-1 kelib chiqadi. $k=tg\alpha$, $\alpha-$to'g'ri chiziq **Ox** o'qining musbat yo'nalishi bilan tashkil etgan burchak. Shuning uchun: $tg\alpha=-1 \Rightarrow \alpha=arctg(-1)$ $\Rightarrow \alpha=135^0$

5. $M_1(3;-2)$ nuqtadan o'tuvchi va k burchak koeffitsientli to'g'ri chiziq tenglamasini tuzing.

Yechish: $(y-y_0)=k(x-x_0)$ to'g'ri chiziqning burchak koeffitsientli tenglamasidan quyidagilarga ega bo'lamiz: $(y-3)=k(x+2)$.

Mustaqil yechish uchun topshiriqlar

1. $M_0(-1:2)$ nuqtadan o'tuvchi va $\vec{s}=\{3:-4\}$ vektorga parallel to'g'ri chiziq tenglamasini tuzing. **Javob:** $\frac{x+1}{3}=\frac{y-2}{-4}$

2. $M_0(2:3)$ nuqtadan o'tuvchi va $\vec{s}=\{1:-3\}$ vektorga parallel to'g'ri chiziq tenglamasini tuzing.

Javob: $\frac{x-2}{1}=\frac{y-3}{-3}$

3. $M_1(1;-3)$ va $M_2(-2;4)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing.

Javob: $\frac{x-1}{-3}=\frac{y+3}{7}$

4. $M_1(5;2)$ va $M_2(2;6)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing.

Javob: $\frac{x-5}{-3}=\frac{y-2}{4}$

5. Berilgan $3x-y=0$ tenglama bo'yicha k burchak koeffitsientini va koordinatalar o'qlaridan kesadigan a, b kesmalarini toping.

Javob: $k=3, \quad a=0 \quad , \quad b=0$

6. Berilgan $5x+2y-8=0$ tenglama bo'yicha k burchak koeffitsientini va koordinatalar o'qlaridan kesadigan a, b kesmalarini toping.

Javob: $k=-\frac{5}{2}, \quad a=\frac{8}{5} \quad , \quad b=4$

7. Berilgan $\sqrt{3}x-y-5=0$ to'g'ri chiziq abstsissalar o'qining musbat yo'nalishi bilan tashkil etgan burchakni toping.

Javob: $\varphi=60^0$

8. $M_1(1;-3)$ nuqtadan o'tuvchi va k burchak koeffitsientli to'g'ri chiziq tenglamasini tuzing.

Javob: $(y-1)=k(x+3)$

9. $M_1(4;1)$ nuqtadan o'tuvchi va k burchak koeffitsientli to'g'ri chiziq tenglamasini tuzing.

Javob: $(y-1)=k(x-4)$

10. $M_1(5;-1)$ nuqtadan o'tuvchi va k burchak koeffitsientli to'g'ri chiziq tenglamasini tuzing.

Javob: $(y+1)=k(x-5)$

To'g'ri chiziqlarning joylashuvi.

To'g'ri chiziqlar $\frac{x-x_1}{\ell_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{p_1}$ va $\frac{x-x_2}{\ell_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{p_2}$ kanonik tenglamalar bilan berilgan bo'lsin. Bu to'g'ri chiziqlar orasidagi φ burchak quyidagi formula asosida hisoblanadi:

$$\cos\varphi=\frac{\vec{s_1}\cdot\vec{s_2}}{\left|\vec{s_1}\right|\cdot\left|\vec{s_2}\right|}=\frac{\ell_1\cdot\ell_2+m_1\cdot m_2+p_1\cdot p_2}{\sqrt{\ell_1^2+m_1^2+p_1^2}\cdot\sqrt{\ell_2^2+m_2^2+p_2^2}}$$

bunda $\vec{s_1}=\{\ell_1;m_1;\ p_1\}$, $\vec{s_2}=\{\ell_2;m_2;\ p_2\}$

berilgan to'g'ri chiziqlarning yo'naltiruvchi vektori.

a) Agar to'g'ri chiziqlar perpendikulyar bo'lsa, u xolda $\vec{s_1}\cdot\vec{s_2}=0$ yoki $\ell_1\cdot\ell_2+m_1\cdot m_2+p_1\cdot p_2=0$

b) Agar to'g'ri chiziqlar parallel bo'lsa, u xolda $\frac{\ell_1}{\ell_2}=\frac{m_1}{m_2}=\frac{p_1}{p_2}$

c) Agar to'g'ri chiziqlar ustma-ust tushsa, u xolda $\frac{\ell_1}{\ell_2}=\frac{m_1}{m_2}=\frac{p_1}{p_2}$

shu bilan birga $\frac{x_2-x_1}{\ell_1}=\frac{y_2-y_1}{m_1}=\frac{z_2-z_1}{p_1}$

d) Agar to'g'ri chiziqlar kesishsa, u xolda $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ \ell_1 & m_1 & p_1 \\ \ell_2 & m_2 & p_2 \end{vmatrix}=0$

e) Agar to'g'ri chiziqlar ayqash bo'lsa, u xolda $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ \ell_1 & m_1 & p_1 \\ \ell_2 & m_2 & p_2 \end{vmatrix}\neq 0$

e) $M_1(x_1;y_1;z_1)$ nuqtadan $\frac{x-x_0}{\ell}=\frac{y-y_0}{m}=\frac{z-z_0}{p}$ to'g'ri chiziqqacha bo'lgan d masofa quyidagi formula bo'yicha hisoblanadi. $d=\frac{\left|\vec{s}\times\overrightarrow{M_1M_0}\right|}{\left|\vec{s}\right|}$

bunda, $M_0(x_0; y_0; z_0)$ nuqta shu to'g'ri chiziqqa tegishli va $\vec{s} = \{\ell; m; p\}$ uning yo'naltiruvchi vektori.

Mavzuga doir yechimlari bilan berilgan topshiriqlardan namunalar

1.To'g'ri chiziqlar $\frac{x-3}{1} = \frac{y+2}{4} = z+4$ $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z+1}{2}$ tenglamalar bilan berilgan bo'lsin. Ular orasidagi φ burchakni hisoblang.

Yechish: $\cos\varphi = \frac{\vec{s_1} \cdot \vec{s_2}}{|\vec{s_1}| \cdot |\vec{s_2}|} = \frac{\ell_1 \cdot \ell_2 + m_1 \cdot m_2 + p_1 \cdot p_2}{\sqrt{\ell_1^2 + m_1^2 + p_1^2} \cdot \sqrt{\ell_2^2 + m_2^2 + p_2^2}}$

bunda $\vec{s_1} = \{\ell_1; m_1; p_1\}$, $\vec{s_2} = \{\ell_2; m_2; p_2\}$ berilgan to'g'ri chiziqlarning yo'naltiruvchi vektori.

Bularga asosan:

$\vec{s_1} = \{1; 4; 1\}$, $\vec{s_2} = \{-1; 2; 2\}$

$$\cos\varphi = \frac{\vec{s_1} \cdot \vec{s_2}}{|\vec{s_1}| \cdot |\vec{s_2}|} = \frac{1\cdot(-1) + 4\cdot 2 + 1\cdot 2}{\sqrt{1^2 + 4^2 + 1^2} \cdot \sqrt{(-1)^2 + 2^2 + 2^2}} = \frac{-1+8+2}{\sqrt{18}\cdot\sqrt{9}} = \frac{9}{9\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Demak: $\cos\varphi = \frac{\sqrt{2}}{2}$, $\varphi = \arccos\frac{\sqrt{2}}{2}$, $\varphi = \frac{\pi}{4}$

2. To'g'ri chiziqlar $\frac{x+7}{3} = \frac{y+4}{4} = \frac{z+3}{2}$ $\frac{x-21}{6} = \frac{y+5}{-4} = \frac{z-2}{-1}$ tenglamalar bilan berilgan bo'lsin. Ularni perpendikulyar yoki parallelligini tekshiring.

Yechish: To'g'ri chiziqlar $\ell_1 \cdot \ell_2 + m_1 \cdot m_2 + p_1 \cdot p_2 = 0$ bo'lsa perpendikulyar, $\frac{\ell_1}{\ell_2} = \frac{m_1}{m_2} = \frac{p_1}{p_2}$ bo'lsa parallel bo'lish shartiga ko'ra $\ell_1 \cdot \ell_2 + m_1 \cdot m_2 + p_1 \cdot p_2 = 0 \Rightarrow$ $3\cdot 6 + 4\cdot(-4) + 2\cdot(-1) = 0$ bo'lganligidan, to'g'ri chiziqlar perpendikulyar ekanligi kelib chiqdi.

3. To'g'ri chiziqlar $\frac{x+7}{3} = \frac{y+4}{-2} = \frac{z+3}{2}$ $\frac{x-21}{6} = \frac{y+5}{-4} = \frac{z-2}{4}$ tenglamalar bilan berilgan bo'lsin. Ularni perpendikulyar yoki parallelligini tekshiring.

Yechish: To'g'ri chiziqlar $\ell_1 \cdot \ell_2 + m_1 \cdot m_2 + p_1 \cdot p_2 = 0$ bo'lsa perpendikulyar, $\frac{\ell_1}{\ell_2} = \frac{m_1}{m_2} = \frac{p_1}{p_2}$ bo'lsa parallel bo'lish shartiga ko'ra $\frac{3}{6} = \frac{-2}{-4} = \frac{2}{4} \Rightarrow \frac{1}{2} = \frac{1}{2} = \frac{1}{2}$ bo'lganligidan, to'g'ri chiziqlar parallel ekanligi kelib chiqdi.

Mustaqil yechish uchun topshiriqlar

1. To'g'ri chiziqlar $\frac{x-1}{2} = \frac{y+1}{2} = \frac{z+3}{2}$ $\frac{x+2}{-2} = \frac{y-2}{1} = \frac{z+1}{2}$ tenglamalar bilan berilgan bo'lsin. Ular orasidagi φ burchakni hisoblang

Javob: $\cos\varphi = \frac{1}{3\sqrt{3}}$

2. To'g'ri chiziqlar $\frac{x-1}{2} = \frac{y+1}{4} = \frac{z+3}{-4}$ $\frac{x+1}{2} = \frac{y-2}{-2} = \frac{z+1}{2}$ tenglamalar bilan berilgan bo'lsin. Ular orasidagi φ burchakni hisoblang

Javob: $\cos\varphi = -\frac{1}{\sqrt{3}}$

3. To'g'ri chiziqlar $\frac{x+7}{2} = \frac{y+4}{4} = \frac{z+3}{-2}$ $\frac{x-21}{6} = \frac{y+5}{12} = \frac{z-2}{-6}$ tenglamalar bilan berilgan bo'lsin. Ularni perpendikulyar yoki parallelligini tekshiring.

Javob: Parallel

4. To'g'ri chiziqlar $\frac{x+7}{1} = \frac{y+4}{3} = \frac{z+3}{-3}$ $\frac{x-21}{3} = \frac{y+5}{2} = \frac{z-2}{3}$ tenglamalar bilan berilgan bo'lsin. Ularni perpendikulyar yoki parallelligini tekshiring.

Javob: Perpendikulyar.

Tekislik bilan to'g'ri chiziqlar

$Ax + By + Cz + D = 0$ tekislik bilan $\frac{x-x_0}{\ell} = \frac{y-y_0}{m} = \frac{z-z_0}{p}$ to'g'ri chiziq orasidagi φ burchak quyidagi formula asosida hisoblanadi:

$$\sin\varphi = \frac{\vec{n} \cdot \vec{s}}{|\vec{n}| \cdot |\vec{s}|} = \frac{\ell \cdot A + m \cdot B + p \cdot C}{\sqrt{\ell^2 + m^2 + p^2} \cdot \sqrt{A^2 + B^2 + C^2}}$$

. Bunda $\vec{s}$ = {l, m, p} to'g'ri chiziqning yo'naltiruvchi vektori, $\vec{n} = \{A; B; C\}$ tekislikning normal vektori.

a) Agar tekislik bilan to'g'ri chiziq perpendikulyar bo'lsa, u xolda $\vec{n}$ va $\vec{s}$ vektorlar kolleniar yoki $\frac{\ell}{A}=\frac{m}{B}=\frac{p}{C}$ bo'ladi.

b) Agar tekislik bilan to'g'ri chiziq parallel bo'lsa, u xolda $\vec{n}$ *va* $\vec{s}$ vektorlar parallel yoki $\ell\cdot A+m\cdot B+p\cdot C=0$ bo'ladi.

Mavzuga doir yechimlari bilan berilgan topshiriqlardan namunalar

1. $-x+2y+2z-5=0$ tekislik bilan $\frac{x+1}{1}=\frac{y-3}{1}=\frac{z}{4}$ to'g'ri chiziq tenglamalar bilan berilgan bo'lsin. Ular orasidagi φ burchakni hisoblang.

Yechish: $\sin\varphi=\frac{\vec{n}\cdot\vec{s}}{|\vec{n}|\cdot|\vec{s}|}=\frac{\ell\cdot A+m\cdot B+p\cdot C}{\sqrt{\ell^2+m^2+p^2}\cdot\sqrt{A^2+B^2+C^2}}$

bunda $\vec{s}=\{\ell; m;\ p\}$, $\vec{n}=\{A; B;\ C\}$ berilgan to'g'ri chiziq va tekislikning mos ravishda yo'naltiruvchi va normal vektorlari.

Bularga asosan:

$\vec{s}=\{1;1;\ 4\}$, $\vec{n}=\{-1;2;\ 2\}$

$$\sin\varphi=\frac{\vec{n}\cdot\vec{s}}{|\vec{n}|\cdot|\vec{s}|}=\frac{1\cdot(-1)+1\cdot2+4\cdot2}{\sqrt{1^2+1^2+4^2}\cdot\sqrt{(-1)^2+2^2+2^2}}=\frac{-1+2+8}{\sqrt{18}\cdot\sqrt{9}}=\frac{9}{9\sqrt{2}}=\frac{\sqrt{2}}{2}$$

Demak: $\sin\varphi=\frac{\sqrt{2}}{2}$, $\varphi=\arcsin\frac{\sqrt{2}}{2}$, $\varphi=\frac{\pi}{4}$

2. $3x-3y+2z-5=0$ tekislik bilan $\frac{x+1}{2}=\frac{y-2}{4}=\frac{z+1}{3}$ to'g'ri chiziq tenglamalar bilan berilgan bo'lsin.. Ularni perpendikulyar yoki parallelligini tekshiring.

Yechish: Agar tekislik bilan to'g'ri chiziq perpendikulyar bo'lsa, u xolda $\frac{\ell}{A}=\frac{m}{B}=\frac{p}{C}$ bo'ladi,

agar tekislik bilan to'g'ri chiziq parallel bo'lsa, u xolda $\ell\cdot A+m\cdot B+p\cdot C=0$ bo'ladi. $\ell\cdot A+m\cdot B+p\cdot C=2\cdot3+4\cdot(-3)+3\cdot2=6-12+6=0$ Shartga ko'ra parallel ekanligi kelib chiqdi.

3. $4x+y-3z+1=0$ tekislik bilan $\frac{x+13}{8}=\frac{y-1}{2}=\frac{z-4}{-6}$ to'g'ri chiziq tenglamalar bilan berilgan bo'lsin.. Ularni perpendikulyar yoki parallelligini tekshiring.

Yechish: Agar tekislik bilan to'g'ri chiziq perpendikulyar bo'lsa, u xolda $\frac{\ell}{A}=\frac{m}{B}=\frac{p}{C}$ bo'ladi,

agar tekislik bilan to'g'ri chiziq parallel bo'lsa, u xolda $\ell\cdot A+m\cdot B+p\cdot C=0$ bo'ladi. $\frac{\ell}{A}=\frac{m}{B}=\frac{p}{C}$ $\Rightarrow$ $\frac{8}{4}=\frac{2}{1}=\frac{-6}{-3}$ $\Rightarrow$ $2=2=2$ Shartga ko'ra perpendikulyar ekanligi kelib chiqdi.

Mustaqil yechish uchun topshiriqlar

1. $-2x+y+2z-5=0$ tekislik bilan $\frac{x+3}{2}=\frac{y-1}{2}=\frac{z+2}{-2}$ to'g'ri chiziq tenglamalar bilan berilgan bo'lsin. Ular orasidagi φ burchakni hisoblang.

Javob: $\sin\varphi=-\frac{1}{\sqrt{3}}$.

2. $2x-4y+4z+3=0$ tekislik bilan $\frac{x+4}{2}=\frac{y+2}{2}=\frac{z+1}{-2}$ to'g'ri chiziq tenglamalar bilan berilgan bo'lsin. Ular orasidagi φ burchakni hisoblang.

Javob: $\sin\varphi=-\frac{1}{\sqrt{3}}$.

3. $2x+3y-2z-4=0$ tekislik bilan $\frac{x+3}{6}=\frac{y-2}{9}=\frac{z-3}{-6}$ to'g'ri chiziq tenglamalar bilan berilgan bo'lsin.. Ularni perpendikulyar yoki parallelligini tekshiring.

Javob: Perpendikulyar.

4. $x-2y-z-1=0$ tekislik bilan $\frac{x-2}{3}=\frac{y-3}{1}=\frac{z+2}{1}$ to'g'ri chiziq tenglamalar bilan berilgan bo'lsin.. Ularni perpendikulyar yoki parallelligini tekshiring.

Javob: Parallel.

Tekislikdagi to'g'ri chiziqlar

Tekislikdagi to'g'ri chiziq $A_1x+B_1y+C_1=0$ va $A_2x+B_2y+C_2=0$ tenglamalar bilan berilgan bo'lsin. Ular orasidagi φ burchak quyidagi formula asosida hisoblanadi:

$$\cos\varphi = \frac{\vec{s_1}\cdot\vec{s_2}}{|\vec{s_1}|\cdot|\vec{s_2}|} = \frac{A_1\cdot A_2 + B_1\cdot B_2}{\sqrt{A_1^2 + B_1^2}\cdot\sqrt{A_2^2 + B_2^2}},$$

bunda $\vec{s_1} = \{A_1; B_1\}$, $\vec{s_2} = \{A_2; B_2\}$ berilgan to'g'ri chiziqlarning yo'naltiruvchi vektorlari.

a) Agar to'g'ri chiziqlar perpendikulyar bo'lsa, u xolda $A_1\cdot A_2 + B_1\cdot B_2 = 0$

b) Agar to'g'ri chiziqlar parallel bo'lsa, u xolda $\frac{A_1}{A_2} = \frac{B_1}{B_2}$

c) Agar to'g'ri chiziqlar ustma-ust tushsa, u xolda $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$

Tekislikdagi to'g'ri chiziqlar $y_1 = k_1 x + b_1$ va $y_2 = k_2 x + b_2$ tenglamalar bilan berilgan bo'lsin. Ular orasidagi φ burchak quyidagi formula asosida hisoblanadi:

$$tg\varphi = \frac{k_2 - k_1}{1 + k_1\cdot k_2}$$

Bu to'g'ri chiziqlarning perpendikulyarlik sharti $k_1\cdot k_2 = -1$ dan iborat, parallellik sharti esa $k_1 = k_2$ bo'ladi.

$M_0(x_0; y_0)$ nuqtadan $Ax + By + C = 0$ to'g'ri chiziqqacha bo'lgan d masofa $d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$ formula asosida hisoblanadi.

Mavzuga doir yechimlari bilan berilgan topshiriqlardan namunalar

1.To'g'ri chiziqlar $\frac{x+2}{1} = \frac{y+1}{3}$ $\frac{x+2}{1} = \frac{y-3}{-2}$ tenglamalar bilan berilgan bo'lsin. Ular orasidagi φ burchakni hisoblang.

Yechish: $\cos\varphi = \frac{\vec{s_1}\cdot\vec{s_2}}{|\vec{s_1}|\cdot|\vec{s_2}|} = \frac{\ell_1\cdot\ell_2 + m_1\cdot m_2}{\sqrt{\ell_1^2 + m_1^2}\cdot\sqrt{\ell_2^2 + m_2^2}}$

bunda $\vec{s_1} = \{\ell_1; m_1\}$, $\vec{s_2} = \{\ell_2; m_2\}$ berilgan to'g'ri chiziqlarning yo'naltiruvchi vektori.

Bularga asosan: $\vec{s_1} = \{1; 3\}$, $\vec{s_2} = \{1; -2\}$

$$\cos\varphi = \frac{\vec{s_1}\cdot\vec{s_2}}{|\vec{s_1}|\cdot|\vec{s_2}|} = \frac{1\cdot 1 + 3\cdot(-2)}{\sqrt{1^2+3^2}\cdot\sqrt{1^2+(-2)^2}} = \frac{1-6}{\sqrt{10}\cdot\sqrt{5}} = \frac{-5}{5\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

Demak: $\cos\varphi = -\frac{\sqrt{2}}{2}$, $\varphi = \arccos(-\frac{\sqrt{2}}{2})$, $\varphi = \frac{3\pi}{4}$

2. To'g'ri chiziqlar $\frac{x+2}{3} = \frac{y-1}{2}$ va $\frac{x+1}{2} = \frac{y+3}{-3}$ tenglamalar bilan berilgan bo'lsin. Ularni perpendikulyar yoki parallelligini tekshiring.

Yechish: To'g'ri chiziqlar $\ell_1\cdot\ell_2 + m_1\cdot m_2 = 0$ bo'lsa perpendikulyar, $\frac{\ell_1}{\ell_2} = \frac{m_1}{m_2}$ bo'lsa parallel bo'lish shartiga ko'ra $\ell_1\cdot\ell_2 + m_1\cdot m_2 = 0 \;\Rightarrow\; 3\cdot 2 + 2\cdot(-3) = 0$ bo'lganligidan, to'g'ri chiziqlar perpendikulyar ekanligi kelib chiqdi.

3. To'g'ri chiziqlar $\frac{x+7}{1} = \frac{y+4}{3}$ $\frac{x-21}{-3} = \frac{y+5}{-9}$ tenglamalar bilan berilgan bo'lsin. Ularni perpendikulyar yoki parallelligini tekshiring.

Yechish: To'g'ri chiziqlar $\ell_1\cdot\ell_2 + m_1\cdot m_2 = 0$ bo'lsa perpendikulyar, $\frac{\ell_1}{\ell_2} = \frac{m_1}{m_2}$ bo'lsa parallel bo'lish shartiga ko'ra $\frac{1}{-3} = \frac{3}{-9} \;\Rightarrow\; -\frac{1}{2} = -\frac{1}{3}$ bo'lganligidan, to'g'ri chiziqlar parallel ekanligi kelib chiqdi.

4. To'g'ri chiziqlar $2x + 2y - 5 = 0$ va $y = 0$ tenglamalar bilan berilgan bo'lsin. Ular orasidagi φ burchakni hisoblang.

Yechish: $tg\varphi = \frac{k_2 - k_1}{1 + k_1\cdot k_2}$ formulaga asosan hisoblanadi. Bu yerda $2x - 2y - 5 = 0$ tenglamadan $2y = 2x + 5 \;\Rightarrow\; y = x + \frac{5}{2}$, $y = 0$ tenglamadan $y = 0\cdot x + 0$ bo'ladi.

Bulardan $y = kx + b$ tenglamaga asosan $k_1 = 1$, $k_2 = 0$ kelib chiqadi.

Yuqoridagi formulaga asosan $tg\varphi = \frac{k_2 - k_1}{1 + k_1\cdot k_2} = \frac{0-1}{1+0\cdot 1} = \frac{-1}{1} = -1$,

$tg\varphi = -1$, $\varphi = arctg(-1)$, $\varphi = 135^0$

5. $x - 2y + 3 = 0$ va $3x - 6y - 2 = 0$ tenglamalar bilan berilgan to'g'ri chiziqlar bo'lsin . Ularni perpendikulyar yoki parallelligini tekshiring.

Yechish: $y_1 = k_1 x + b_1$ va $y_2 = k_2 x + b_2$ to'g'ri chiziq tenglamalar bilan berilgan bo'lsa, $k_1 = k_2$ da parallel, $k_1\cdot k_2 = -1$ da perpendikulyar bo'ladi.

Birinchi tenglamadan $x-2y+3=0 \Rightarrow y=\frac{1}{2}x+\frac{3}{2}$ $k_1=\frac{1}{2}$,

Ikkinchi tenglamadan $3x-6y-2=0 \Rightarrow y=\frac{1}{2}x-\frac{1}{3}$, $k_1=\frac{1}{2}$,

Demak, $\frac{1}{2}=\frac{1}{2} \Rightarrow k_1=k_2$ Bu ikki to'g'ri chiziq parallel bo'ladi.

6. $2x-y+1=0$ va $2x+4y-3=0$ tenglamalar bilan berilgan to'g'ri chiziqlar bo'lsin . Ularni perpendikulyar yoki parallelligini tekshiring.

Yechish: $y_1=k_1x+b_1$ va $y_2=k_2x+b_2$ to'g'ri chiziq tenglamalar bilan berilgan bo'lsa, $k_1=k_2$ da parallel, $k_1\cdot k_2=-1$ da perpendikulyar bo'ladi.

Birinchi tenglamadan $2x-y+1=0 \Rightarrow y=2x+1$ $k_1=2$,

Ikkinchi tenglamadan $2x+4y-3=0 \Rightarrow y=-\frac{1}{2}x-\frac{3}{4}$, $k_1=-\frac{1}{2}$,

$2\neq-\frac{1}{2} \Rightarrow k_1\neq k_2$ Bundan ikki to'g'ri chiziq parallel emas.

$2\cdot(-\frac{1}{2})=-1 \Rightarrow k_1\cdot k_2=-1$. Demak shartga ko'ra ikki to'g'ri chiziq perpendikulyar bo'ladi.

7. $M_1(4;-3)$ nuqtadan $2x+3y+5=0$ to'g'ri chiziqqacha bo'lgan masofani toping.

Yechish: Berilgan nuqtadan to'g'ri chiziqqacha bo'lgan masofa $d=\frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}$ formula bilan aniqlanadi. Bu yerda $M_0(x_0;y_0)$. A, B, C lar $Ax+By+C=0$ tenglamaning kooeffitsentlari va ozod hadi.

$$d=\frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}=\frac{|2\cdot4+3\cdot(-3)+5|}{\sqrt{4^2+(-3)^2}}=\frac{|4|}{\sqrt{25}}=\frac{4}{5}$$

Demak, berilgan nuqtadan to'g'ri chiziqqacha bo'lgan masofa: $d=\frac{4}{5}$ ga teng ekan.

Mustaqil yechish uchun topshiriqlar

1. To'g'ri chiziqlar $\frac{x-2}{3}=\frac{y-1}{-1}$ $\frac{x+2}{-1}=\frac{y-2}{2}$ tenglamalar bilan berilgan bo'lsin. Ular orasidagi φ burchakni hisoblang

Javob: $\varphi=120^0$.

2. To’g’ri chiziqlar $\frac{x-2}{2}=\frac{y-3}{-1}$ $\frac{x+3}{-2}=\frac{y-1}{1}$ tenglamalar bilan berilgan bo’lsin. Ular orasidagi φ burchakni hisoblang

Javob: $\varphi=180^0$.

3. To’g’ri chiziqlar $\frac{x+5}{2}=\frac{y+2}{-1}$ $\frac{x-1}{6}=\frac{y+3}{12}$ tenglamalar bilan berilgan bo’lsin. Ularni perpendikulyar yoki parallelligini tekshiring.

Javob: Perpendikulyar.

4. To’g’ri chiziqlar $\frac{x-1}{2}=\frac{y-3}{-4}$ va $\frac{x+1}{3}=\frac{y-4}{-6}$ tenglamalar bilan berilgan bo’lsin. Ularni perpendikulyar yoki parallelligini tekshiring.

Javob: Parallel.

5. $2x-2y+6=0$ va $6x-6y+4=0$ tenglamalar bilan berilgan to’g’ri chiziqlar bo’lsin . Ularni perpendikulyar yoki parallelligini tekshiring.

Javob: Parallel.

6. $3x-3y+3=0$ va $5x+5y-1=0$ tenglamalar bilan berilgan to’g’ri chiziqlar bo’lsin . Ularni perpendikulyar yoki parallelligini tekshiring.

Javob: Perpendikulyar.

7. $M_1(2;-1)$ nuqtadan $3x+y-2=0$ to’g’ri chiziqqacha bo’lgan masofani toping.

Javob: $d=\frac{3}{\sqrt{5}}$

3- bob. Matematik analiz.

Funktsiyaning xosilasi va uning tadbiqlari.

1.Funktsiya tushunchasi. Funktsiyani berilish usullari. Juft va toq funktsiyalar.

<u>1-Ta’rif</u> : Agar x miqdorining D soxadagi har bir qiymatiga biror usul yoki qonun bo’yicha uning biror E soxadagi aniq bir qiymati mos qo’yilsa, y o’zgaruvchi miqdor x o’zgaruvchi miqdorning <u>funktsiyasi</u> deyiladi.

x-erkli o’zgaruvchi, argument

y-bog’liq o’zgaruvchi, funktsiya.

Funktsiya quyidagi ko’rinishda belgilanadi:

$$y = f(x),\quad y = y(x),\quad y = \varphi(x) \text{ va hokazo.}$$

Agar $X = X_0$ qiymatida $y = f(x)$ funktsiyaning qiymati y_0 bo’lsa, uni quyidagicha belgilanadi:

$$y_0 = f(x_0)$$

<u>2-ta’rif:</u> O’zgaruvchi x ning $f(x)$ funktsiya ma’noga ega bo’ladigan qiymatlari to’plami funktsiyaning aniqlanish sohasi deyiladi va $D(f)$ bilan belgilanadi.

<u>3-ta’rif:</u> Funktsiyaning qabul qiladigan qiymatlari to’plami uning o’zgarish sohasi deyiladi va $E(f)$ shaklida belgilanadi.

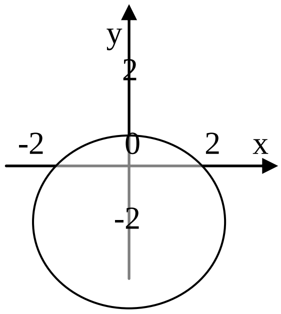
Misol 1: $y = \sqrt{4 - x^2}$ funktsiyaning aniqlanish va o’zgarish sohasini toping.

Yechish: $4 - x^2 \geq 0,$ bo’lganda funktsiya ma’noga egadir.

$x^2 \leq 4, \quad \Rightarrow \quad |x| \leq 2 \quad \Rightarrow$

$-2 \leq x \leq 2\ , \quad \Rightarrow \quad [-2;2]$

Demak, $D(f) = [-2;2], \qquad E(f) = [0;2]$



$y = f(x)$ funktsiyaning <u>grafigi</u> deb Oxy tekislikdagi koordinatalari $y = f(x)$ munosabat bilan bog’langan R(x, y) nuqtalar to’plamiga aytiladi.

Funktsiya turli <u>usullar</u> bilan berilshi mumkin:

1) Jadval usuli

2) Analitik usuli

3) Grafik usuli

Funktsiya <u>analitik</u> usulida berilganda x va y miqdorlar orasidagi bog'lanish formula orqali ifodalanadi.

Masalan, $y=x^2$; $y=(x-3)^2$.

Funktsiya o'z aniqlanish sohasining turli qismlarida turlicha formulalar orqali berilishi mumkin:

$$f(x)=\begin{cases} x^2, agar, x\in[-1;0] bo'lsa \\ x, agar, x\in[0;+\infty] bo'lsa \end{cases}$$

Funktsiya jadval usulda beriganda x va y miqdorlar orasidagi bog'lanish jadval ko'rinishda ifodalanadi:

x	x_1	x_2	……..	x_n
y	y_1	y_2	……..	y_n

Masalan, logarifmik, trigonometrik funktsiyalar jadvallari ma'lum.

Funktsiya grafik usulda berilganda uning grafigi ma'lum bo'lib, argumentning turli qiymatlariga mos keluvchi qiymatlari bevosita grafikdan topiladi.

<u>**1 - ta'rif**</u>. Agar x ning shu sohaga tegishli ixtiyoriy ikkita x_1 va x_2 qiymatlari uchun $x_1 < x_2$ bo'lganda $f(x_1) < f(x_2)$ tengsizlik o'rinli bo'lsa, f funktsiya D sohada o'<u>suvchi</u> deyiladi.

<u>**2 - ta'rif.**</u> Agar $x_1 < x_2$ bo'lganda $f(x_1) \le f(x_2)$ bo'lsa, funktsiya D sohada kamaymaydigan funktsiya deyiladi. $D(f)=[a;b]$ soha esa f funktsiyaning mos ravishda o'sish yoki kamayish oralig'i deyiladi.

<u>**3 - ta'rif.**</u> Agar $y=f(x)$ funktsiya har bir $x\in D(f)$ uchun $f(-x)=f(x)$ tenglik bajarilsa, u holda $y=f(x)$ funktsiya <u>juft</u> funktsiya deyiladi. Agar har bir

$x \in D(f)$ uchun $f(-x) = -f(x)$ tenglik bajarilsa, u holda $f(x)$ funktsiya toq funktsiya deyiladi.

Masalan: $y=x^2$, $y = \cos x$, $y=\ln(1-x^2)$ - juft funktsiyalar.

$y=x^3$; $y = \sin x$, $y = x - \frac{1}{x}$ - toq funktsiyalar.

Juft funktsiyaning grafigi ordinatalar o'qiga toq funktsiyaning grafigi koordinata boshiga nisbatan simmetrik bo'ladi.

4 - ta'rif. Agar $y = f(x)$ funktsiya har bir $x \in D(f)$ va $x \pm T \in D(f)$ uchun $f(x \pm T) = f(x)$ tenglik bajarilsa, u holda $y = f(x)$ funktsiya davriy funktsiya defiladi.

T - qandaydir haqiqiy son. Uning eng kichik musbat qiymati T_0 mavjud bo'lsa, unga f(x) funktsiyaning davri deyiladi.

Masalan: $y=x^{|\cos x|}$ funktsiya berilgan bo'lsin. Uning davrini topamiz. $\cos x = \pm\cos(x-T)$ tenglamani T ga nisbatan yechamiz.

$T_1 = (2n-1)\pi - 2x$; $T_2 = (2n-1)\pi$; $T_3 = 2n\pi - 2x$, $T_4 = 2kn - 2\pi$ larni topamiz.

T_1 va T_3 lar x ga bog'liq, demak, ular davr bo'la olmaydi. n = 0 bo'lganda $T_2 = \pi$ va $T_4 = 2\pi$ ga ega bo'lib, ularning eng kichik $T_2 = \pi$ berilgan funktsiyaning izlangan davri bo'ladi. Analitik usulda beriladigan funktsiyalar ichida elementar funktsiyalar muhim o'rin tutadi.

O'zgarmas funktsiya $y = c$

Darajali funktsiya $y=x^{\alpha}$ $\alpha - const$

Ko'rsatkichli funktsiya $y=a^x$ $a > 0,$ $a \neq 1$

Logarifmik funktsiya $y=\log^x$ $a > 0,$ $a \neq 1$

Trigonometrik funktsiyalar $y = \sin x,\ y = \cos x,\ y = tgx,\ y = ctgx$.

Teskari trigonometrik funktsiyalar: $y = \arcsin x,\ y = \arccos x,\ y = arctgx,\ y = arcctgx$

Murakkab funktsiya biror D sohada x o'zgaruvchining funktsiyasi $y = \varphi(x)$ berilgan bo'lib, uning o'zgarish sohasi K bo'lsin. K sohada $y = f(n)$ funktsiya berilgan bo'lsin. U holda **x** o'zgaruvchining K sohadagi aniq bir qiymati va bu qiymatga **y** o'zgaruvchining aniq bir qiymati mos keladi.

$$y = F(x) = f(\varphi(x))$$

Bunda $\varphi(x)$ funktsiya x o'zgaruvchining f va φ funktsiyalarida tuzilgan murakkab funktsiyasi deyiladi.

$y = \varphi(x)$ - oraliq o'zgaruvchi deyiladi.

Misol: $y = \lg(y)$ va $y = tgx$ bo'lsa, $y = \lg tgx$,

Funktsiyaning o'suvchiligi va kamayuvchiligi

Funktsiyaning o'suvchiligi sharti. $f(x)$ funktsiya (a,b) da $f'(x)$ hosilaga ega bo'lsin. $f(x)$ (a,b) da o'suvchi bo'lishi uchun $f'(x) \geq 0$ $(x \in (a,b))$ bo'lishi zarur va etarli.

Agar $f'(x) > 0$ bo'lsa, $f(x)$ funktsiya (a,b) da qat'iy o'suvchi bo'ladi.

Funktsiyaning kamayuvchiligi sharti. $f(x)$ funktsiya (a,b) da $f'(x)$ hosilaga ega bo'lsin $f(x)$ (a,b) da kamayuvchi bo'lishi uchun $f'(x) \leq 0$ $(x \in (a,b))$ bo'lishi zarur va etarli.

Agar $f'(x) < 0$ bo'lsa $f(x)$ funktsiya (a,b) da qat'iy kamayuvchi bo'ladi.

Mavzuga doir yechimlari bilan berilgan topshiriqlardan namunalar

1: $y = \dfrac{1}{\sqrt{x^2 - 3x + 2}}$ funktsiyaning aniqlanish va o'zgarish sohasini toping.

Yechish: $y = x^2 - 3x + 2$ $(-\infty; +\infty)$ *da aniqlangan*,

shu bilan birga $x^2 - 3x + 2 < 0,\quad x^2 - 3x + 2 \neq 0$ da aniqlanmagan.

$x^2 - 3x + 2 = 0$ tenglamani yechib $x_1 = 1,\quad x_2 = 2$ ildizlarni topamiz.

$x < 1$ *va* $x > 2$ oraliqda aniqlangan bo'ladi. $x \neq 1$ *va* $x \neq 2$ chunki, kasrning maxraji 0 bo'lishi mumkin emas.

Demak, aniqlanish sohasi $D(f)\ [-\infty;1) \cup (2;+\infty]$, o'zgarish sohasi $E(f)\ [1;2]$ bo'ladi.

2. $y = \sqrt{16 - x^2}$ funktsiyaning aniqlanish va o'zgarish sohasini toping.

Yechish: $16 - x^2 \geq 0,$ bo'lganda funktsiya ma'noga egadir.

$x^2 \leq 16, \quad \Rightarrow \quad |x| \leq 4 \quad \Rightarrow$

$-4 \le x \le 4$, $\Rightarrow$ $[-4;4]$

Demak, $D(f)=[-2;2]$, $E(f)=[0;2]$

3. $y = x^2 \sin 4x$ funksiyani juft yoki toqligini aniqlang.

Yechish: Berilgan funktsiya $f(-x)=f(x)$ tenglik bajarilsa juft, $f(-x)=-f(x)$ tenglik bajarilsa toq bo'iadi.

$y(-x)=(-x)^2 \sin 4(-x) = -x^2 \sin 4x$. Demak, shartga ko'ra funktsiya toq bo'ladi.

4. $y = 3\cos x - 1$ funksiyani juft yoki toqligini aniqlang.

Yechish: Berilgan funktsiya $f(-x)=f(x)$ tenglik bajarilsa juft, $f(-x)=-f(x)$ tenglik bajarilsa toq bo'iadi.

$y(-x)=3\cos(-x)-1=3\cos x-1$, chunki cosx funksiya juft . Demak, shartga ko'ra funktsiya juft bo'ladi.

5. $y = x^4 - x^2 + x$ funksiyani juft yoki toqligini aniqlang.

Yechish: Berilgan funktsiya $f(-x)=f(x)$ tenglik bajarilsa juft, $f(-x)=-f(x)$ tenglik bajarilsa toq bo'iadi. Agar bu shartlar ikkalasi ham bajarilmasa funktsiya na juft na toq bo'ladi.

$y(-x)=(-x)^4-(-x)^2+(-x)=x^4-x^2-x$. Demak, shartga ko'ra funktsiya najuft natoq bo'ladi.

6. $y=\sin 5x$ funktsiyani davrini aniqlang.

Yechish: $y=\sin x$ funktsiyani davri 2π ga teng. Shuning uchun, $5x = 2\pi$, $x=\frac{2}{5}\pi$ bo'ladi.

7. $y= x^2+2x-3$ funktsiyaning o'sish va kamayish oralig'ini aniqlang.

Yechish: $y= x^2+2x-3$ funktsiya $f'(x)>0$ da o'suvchi $f'(x)<0$ da kamayuvchi bo'ladi.

$y'=2x+2$, $2x+2>0$, $x>-1$; $2x+2<0$, $x<-1$;

Demak, $x>-1$ o'suvchi, $x<-1$ kamayuvchi.

Mustaqil yechish uchun topshiriqlar

1. $y=\frac{1}{\sqrt{x^2-5x+4}}$ funktsiyaning aniqlanish va o'zgarish sohasini toping.

Javob: $D(f)\ [-\infty;1)\cup(4;+\infty]$, o’zgarish sohasi $E(f)\ [1;4]$ bo’ladi..

2. $y=\sqrt{x^2-12x+20}$ funktsiyaning aniqlanish va o’zgarish sohasini toping.

Javob: $D(f)=[-\infty;2]$, $E(f)=[10;+\infty]$

3. $y=x^3\cos 3x$ funksiyani juft yoki toqligini aniqlang.

Javob: toq funksiya

4. $y=x^5\sin 4x$ funksiyani juft yoki toqligini aniqlang.

Javob: juft funksiya

5. $y=x^4-\sin x$ funksiyani juft yoki toqligini aniqlang.

Javob: najuft va natoq funksiya

6. $y=\cos 4x$ funktsiyani davrini aniqlang.

Javob: $\frac{\pi}{2}$

7. $y=x^2+x-20$ funktsiyaning o’sish va kamayish oralig’ini aniqlang.

Javob: $x>-\frac{1}{2}$ o’suvchi, $x<-\frac{1}{2}$ kamayuvchi.

2.Funktsiya limiti.

Natural sonlar to’plamida aniqlangan funksiya sonli ketma-ktlik deyiladi va u $\{X_n\}$ ko’rinishida belgilanadi.

Agar shunday **M** musbat son mavjud bo’lib, xar qanday natural son **n** uchun

$$|X_n|\le M$$

tengsizlik o’rinli bo’lsa, $\{X_n\}$ chegaralangan ketma-ketlik deyiladi.

Agar xar qanday natural son n uchun

$$|X_{n+1}|>X_n$$

tengsizlik bajarilsa, $\{X_n\}$ o’suvchi ketma-ketlik deyiladi.

Agar xar qanday natural son n uchun

$$|X_{n+1}|<X_n$$

tengsizlik bajarilsa, $\{X_n\}$ kamayuvchi ketma-ketlik deyiladi.

Faqat o’suvchi yoki kamayuvchi ketma-ketlik monoton ketma-ketlik deyiladi.

Agar istalgan $\varepsilon > 0$ son uchun shunday $N = N(\varepsilon) > 0$ son mavjud bo’lsaki, barcha $n > N$ lar uchun

$$|x_n - a| < \varepsilon$$

tengsizlik bajarilsa, o’zgarmas aa son $\{X_n\}$ ketma-ketlikning limiti deyiladi va bu quyidagicha yoziladi:

$$\lim_{n\to\infty} x_n = a$$

Agar $\{X_n\}$ ketma-ketlik limitga ega bo’lsa, u yaqinlashuvchi, aks holda uzoqlashuvchi ketma-ketlik deyiladi.

Xar qanday chegaralangan va monoton ketma-ketlik limitga ega.

Agar xar qanday $\varepsilon > 0$ son uchun shunday $\delta = \delta(\varepsilon) > 0$ son mavjud bo’lib, $|x - a| < \delta$ da $|f(x) - b| < \varepsilon$ tengsizlik bajarilsa, b soni f(x) funksiyaning $x \to a$ dagi limiti deyiladi va bunday yoziladi:

$$\lim_{x\to a} f(x) = b$$

Agar ixtiyoriy $n > 0$ uchun shunday $N = N(\varepsilon) > 0$ son mavjud bo’lib, barcha $|x| > N$ lar uchun $|f(x) - b| < \varepsilon$ tengsizlik bajarilsa, b soni f(x) funksiyaning $x \to \infty$ dagi limiti deyiladi va bunday yoziladi:

$$\lim_{x\to\infty} f(x) = b$$

Agar ixtiyoriy $M > 0$ uchun shunday $\delta = \delta(M) > 0$ mavjud bo’lib, $|x - a| < \delta$ da $|f(x)| > M$ tengsizlik bajarilsa, f(x) funksiya $x \to a$ da cheksiz katta deyiladi va bunday yoziladi:

$$\lim_{x\to a} f(x) = \infty$$

Agar $x \to a$ da $x > a$ bo’lsa, u holda $x \to a + 0$ belgi, agar $x \to a$ da $x < a$ bo’lsa, u holda $x \to a - 0$ belgi qo’llaniladi. $f(x)$ funksiyaning a nuqtadagi chap va o’ng limitlari deb mos ravishda

$$f(a-0) = \lim_{x\to a-0} f(x) \quad \text{va} \quad f(a+0) = \lim_{x\to a+0} f(x)$$ sonlarga aytiladi.

$f(x)$ funksiyaning $x \to a$ dagi limiti mavjud bo’lishi uchun $f(a-0) = f(a+0)$ bo’lishi zarur va yetarli.

Limitlar haqida quyidagi teoremalar o'rinli (limitga o'tish qoidalari):

a) Agar c o'zgarmas bo'lsa, $\lim\limits_{x\to a} c = c$

b) Agar $\lim\limits_{x\to a} f(x)$ va $\lim\limits_{x\to a} \varphi(x)$ mavjud bo'lsa,

$$\lim_{x\to a} (f(x)+\varphi(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} \varphi(x),$$

v) Agar $\lim\limits_{x\to a} f(x)$ va $\lim\limits_{x\to a} \varphi(x)$ mavjud bo'lsa,

$$\lim_{x\to a} (f(x)\cdot\varphi(x)) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} \varphi(x),$$

g) Agar $\lim\limits_{x\to a} f(x)$ va $\lim\limits_{x\to a} \varphi(x) \neq 0$ mavjud bo'lsa, u holda

$$\lim_{x\to a} \frac{f(x)}{\varphi(x)} = \frac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} \varphi(x)}$$ tengliklar o'rinli.

Agar bu teoremalarning shartlari bajarilmasa, u holda $\frac{\infty}{\infty}$, $\frac{0}{0}$, $0\cdot\infty$ ko'rinishidagi aniqmasliklar paydo bo'lishi mumkin. Bu aniqmasliklar ba'zi hollarda algebraik almashtirishlar yordamida ochiladi.

Ko'pgina limitlarni topishda quyidagi ma'lum formulalardan foydalaniladi:

$\lim\limits_{\alpha\to 0} \frac{\sin\alpha}{\alpha} = 1$ -birinchi ajoyib limit;

$\lim\limits_{x\to\infty} (1+\frac{1}{x})^x = \lim\limits_{\alpha\to 0} (1+\alpha)^{\frac{1}{\alpha}} = e$ -ikkinchi ajoyib limit.

Misollar yechganda quyidagi tengliklarni nazarda tutish foydali:

$$\lim_{x\to\infty} (1+\frac{k}{x})^x = \lim_{\alpha\to 0} (1+k\alpha)^{\frac{1}{\alpha}} = e^k$$

$$\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1$$

$$\lim_{x\to 0} \frac{e^x-1}{x} = 1$$

$$\lim_{x\to 0} \frac{a^x-1}{x} = \ln a, \quad a>0$$

Agar $\alpha(x)$ *va* $\beta(x)$ $x\to x_0$ xolda cheksiz kichik funksiyalar bo'lib, $\lim\limits_{x\to x_0} \frac{\alpha(x)}{\beta(x)} = 1$ bo'lsa, u holda ular ekvivalent deyiladi va $x\to x_0$ da $\alpha(x) \sim \beta(x)$ kabi belgilanadi.

Masalan, $\lim\limits_{\alpha\to 0} \frac{\sin\alpha}{\alpha} = 1$ shu sababli $x\to 0$ da $\sin x \sim x$.

Shunga o'xshash $x \to 0$ da quyidagi cheksiz kichik funksiyalar ekvivalentdir:

$$\arcsin x \sim x, \quad \cos x \sim \frac{x^2}{2}, \quad tgx \sim x, \quad arctgx \sim x,$$

$$e^x - 1 \sim x, \quad a^x - 1 \sim x \ln a, \quad \ln(1+x) \sim x, \quad (1+x)^m - 1 \sim mx.$$

Ikkita cheksiz kichik funksiyalar nisbatining limiti ularga ekvivalent cheksiz kichik funksiyalar nisbatining limitiga teng, ya'ni agar $x \to x_0$ da $\alpha(x) \sim \alpha_1(x)$, $\beta(x) \sim \beta_1(x)$ bo'lsa, u holda

$$\lim_{x \to x_0} \frac{\alpha(x)}{\beta(x)} = \lim_{x \to x_0} \frac{\alpha_1(x)}{\beta_1(x)}$$

$x \to x_0$ da $\alpha(x)$ va $\beta(x)$ cheksiz kichik funksiyalar bo'lsin. Bu funksiyalarni taqqoslash uchun ular nisbatining $x \to x_0$ dagi limiti hisoblanadi:

a)Agar $\lim\limits_{x \to x_0} \frac{\alpha(x)}{\beta(x)} = 0$ bo'lsa, u holda $\alpha(x)$ funksiya $\beta(x)$ ga nisbatan yuqori tartibli cheksiz kichik funksiya deyiladi va $\alpha = 0(\beta)$ kabi belgilanadi.

b)Agar $\lim\limits_{x \to x_0} \frac{\alpha(x)}{\beta(x)} = \infty$ bo'lsa, u holda $\alpha(x)$ funksiya $\beta(x)$ ga nisbatan quyi tartibli cheksiz kichik funksiya deyiladi. Ravshanki bu holda $\lim\limits_{x \to x_0} \frac{\beta(x)}{\alpha(x)} = 0$ yoki $\beta = 0(\beta)$.

c) Agar $\lim\limits_{x \to x_0} \frac{\alpha(x)}{\beta(x)} = A \neq 0$ va A chekli son bo'lsa, u holda $\alpha(x)$ va $\beta(x)$ bir xil tartibli cheksiz kichik funksiyalar deyiladi.

Xususan, agar A=1 bo'lsa, u holda ekvivalent cheksiz kichik funksiyalarga ega bo'lamiz.

g) Agar $\alpha(x)^k$ va $\beta(x)$ bir xil tartibli cheksiz kichik funksiyalar bo'lib, k>0 bo'lsa, u holda $\beta(x)$ cheksiz kichik funksiya $\alpha(x)$ ga nisbatan **k**-tartibga ega deyiladi.

Mavzuga doir yechimlari bilan berilgan topshiriqlardan namunalar

1. $\lim\limits_{x\to\infty}\dfrac{5x^2+4x-1}{2x^2-3x}$ ni hisoblang.

Yechish: kasrni surat va maxraji ∞ ka intiladi. Ya'ni $\dfrac{\infty}{\infty}$ ko'rinishdagi aniqmaslikka egamiz. Kasrni surat va maxrajini x^2 ga bo'lamiz.

$$\lim_{x\to\infty}\frac{\frac{5x^2}{x^2}+\frac{4x}{x^2}-\frac{1}{x^2}}{\frac{2x^2}{x^2}-\frac{3x}{x^2}}=\lim_{x\to\infty}\frac{5+\frac{4}{x}-\frac{1}{x^2}}{2-\frac{3}{x}}=\frac{5}{2}$$

2. $\lim\limits_{x\to 2}\dfrac{3x+1}{4x^2+5}$ ni hisoblang.

Yechish: $\lim\limits_{x\to 2}\dfrac{3x+1}{4x^2+5}=\dfrac{3\cdot 2+1}{4\cdot 4+5}=\dfrac{7}{21}=\dfrac{1}{3}$

3. $\lim\limits_{x\to 3}\left(\dfrac{6}{x^2-9}-\dfrac{1}{x-3}\right)$ ni hisoblang.

Yechish: $\infty-\infty$ ko'rinishdagi aniqmaslikka egamiz. Shuning uchun shakl almashtiramiz:

$$\lim_{x\to 3}\left(\frac{6}{x^2-9}-\frac{1}{x-3}\right)=\lim_{x\to 3}\frac{6-(x+3)}{(x-3)(x+3)}=\lim_{x\to 3}\frac{3-x}{(x-3)(x+3)}=\lim_{x\to 3}\frac{-(x-3)}{(x-3)(x+3)}=-\lim_{x\to 3}\frac{-1}{x+3}=-\frac{1}{6}$$

4. $\lim\limits_{x\to 2}\dfrac{\sqrt{6-x}-2}{\sqrt{8x-7}-3}$ ni hisoblang.

Yechish: $\dfrac{0}{0}$ ko'rinishdagi aniqmaslikka egamiz. Shuning uchun shakl almashtiramiz:

$$\lim_{x\to 2}\frac{\sqrt{6-x}-2}{\sqrt{8x-7}-3}=\lim_{x\to 2}\frac{(\sqrt{6-x}-2)(\sqrt{6-x}+2)(\sqrt{8x-7}+3)}{(\sqrt{8x-7}-3)(\sqrt{8x-7}+3)(\sqrt{6-x}+2)}=\lim_{x\to 2}\frac{(6-x-4)(\sqrt{8x-7}+3)}{(8x-7-9)(\sqrt{6-x}+2)}=$$

$$=\lim_{x\to 2}\frac{-(x-2)(\sqrt{8x-7}+3)}{8(x-2)(\sqrt{6-x}+2)}=\lim_{x\to 2}\frac{-(\sqrt{8x-7}+3)}{8(\sqrt{6-x}+2)}=\frac{-6}{32}=-\frac{3}{16}$$

5. $\lim\limits_{x\to 1}\dfrac{3x^2-4x+1}{\sqrt{x+8}-\sqrt{10-x}}$ ni hisoblang.

Yechish: $\dfrac{0}{0}$ ko'rinishdagi aniqmaslikka egamiz. Shuning uchun shakl almashtiramiz:

$$\lim_{x\to1}\frac{3x^2-4x+1}{\sqrt{x+8}-\sqrt{10-x}}=\lim_{x\to1}\frac{(3x-1)(x-1)}{\sqrt{x+8}-\sqrt{10-x}}=\lim_{x\to1}\frac{(3x-1)(x-1)(\sqrt{x+8}+\sqrt{10-x})}{(\sqrt{x+8}-\sqrt{10-x})(\sqrt{x+8}-\sqrt{10-x})}=$$

$$=\lim_{x\to1}\frac{(3x-1)(x-1)(\sqrt{x+8}+\sqrt{10-x})}{x+8-10+x}=\lim_{x\to1}\frac{(3x-1)(x-1)(\sqrt{x+8}+\sqrt{10-x})}{2(x-1)}=$$

$$=\lim_{x\to1}\frac{(3x-1)(\sqrt{x+8}+\sqrt{10-x})}{2}=\lim_{x\to1}\frac{(3\cdot1-1)(\sqrt{1+8}+\sqrt{10-1})}{2}=\frac{2\cdot6}{2}=6$$

6. $\lim\limits_{x\to5}\dfrac{\sqrt{2x+1}-\sqrt{x+6}}{x^2-8x+15}$ ni hisoblang.

Yechish: $\dfrac{0}{0}$ ko’rinishdagi aniqmaslikka egamiz. Shuning uchun shakl almashtiramiz:

$$\lim_{x\to5}\frac{\sqrt{2x+1}-\sqrt{x+6}}{x^2-8x+15}=\lim_{x\to5}\frac{(\sqrt{2x+1}-\sqrt{x+6})(\sqrt{2x+1}+\sqrt{x+6})}{(x-5)(x-3)}=\lim_{x\to5}\frac{(2x+1-x-6)}{(x-5)(x-3)}=$$

$$=\lim_{x\to5}\frac{(x-5)}{(x-5)(x-3)}=\lim_{x\to5}\frac{1}{(x-3)}=\frac{1}{5-3}=\frac{1}{2}$$

7. $\lim\limits_{x\to2}\dfrac{x^2-5x+6}{2x^2-9x+10}$ ni hisoblang.

Yechish: $\dfrac{0}{0}$ ko’rinishdagi aniqmaslikka egamiz. Shuning uchun shakl almashtiramiz:

$$\lim_{x\to2}\frac{x^2-5x+6}{2x^2-9x+10}=\lim_{x\to2}\frac{(x-2)(x-3)}{(2x-5)(x-2)}=\lim_{x\to2}\frac{x-3}{2x-5}=\frac{2-3}{2\cdot2-5}=\frac{-1}{-1}=1$$

8. $\lim\limits_{x\to0}\dfrac{1-\cos^2 x}{xtgx}$ ni hisoblang.

Yechish: $\dfrac{0}{0}$ ko’rinishdagi aniqmaslikka egamiz. Shuning uchun 1-ajoyib limitdan foydalanamiz::

$$\lim_{x\to0}\frac{1-\cos^2 x}{xtgx}=\lim_{x\to0}\frac{\sin^2 x}{x\dfrac{\sin x}{\cos x}}==\lim_{x\to0}\frac{\sin x\cdot\cos x}{x}=\lim_{x\to0}\frac{\sin x}{x}\cdot\lim_{x\to0}\cos x=1\cdot1=1$$

9. $\lim\limits_{x\to0}\dfrac{\sin 3x-\sin x}{4x}$ ni hisoblang.

Yechish: $\dfrac{0}{0}$ ko’rinishdagi aniqmaslikka egamiz. Shuning uchun 1-ajoyib limitdan foydalanamiz:

$$\lim_{x\to0}\frac{\sin 3x-\sin x}{4x}=\lim_{x\to0}\frac{2\cos 2x\cdot\sin x}{4x}=\lim_{x\to0}\frac{\sin x}{x}\cdot\lim_{x\to0}\frac{2\cos 2x}{4}=1\cdot\lim_{x\to0}\frac{\cos 2x}{2}=1\cdot\frac{1}{2}=\frac{1}{2}$$

10. $\lim\limits_{x\to+\infty}\left(\dfrac{3x+5}{3x-7}\right)^{4x+3}$ ni hisoblang.

Yechish: Suratini maxrajiga bo'lib $\frac{3x+5}{3x-7}=\frac{3x-7+12}{3x-7}=1+\frac{12}{3x-7}$ ni hosil qilamiz. Bunda 1^{∞} ko'rinishdagi aniqmaslikka egamiz. Shuning uchun 2-ajoyib limitdan foydalanamiz::

$$\lim_{x\to\infty}\left(\frac{3x+5}{3x-7}\right)^{4x+3}=\lim_{x\to\infty}\left(1+\frac{12}{3x-7}\right)^{4x+3}=\lim_{x\to\infty}\left[\left(1+\frac{12}{3x-7}\right)^{\frac{3x-7}{12}}\right]^{\frac{12(4x+3)}{3x-7}}.$$

$x\to\infty$ da $\frac{12}{3x-7}\to 0$ bo'lgani sababli 2-ajoyib limitga asosan:

$$\lim_{x\to+\infty}\left(1+\frac{12}{3x-7}\right)^{4x+3}=e$$

$$\lim_{x\to\infty}\frac{12(4x+3)}{3x-7}=\lim_{x\to\infty}\frac{12(4+\frac{3}{x})}{3-\frac{7}{x}}=16$$ ekanini hisobga olib,

$$\lim_{x\to+\infty}\left(\frac{3x+5}{3x-7}\right)^{4x+3}=e^{16}$$

11. $x\to 0$ da $y=\sqrt{1+x\sin x}-1$ cheksiz kichik funksiyaning x ga nisbatan tartibini aniqlang.

Yechish. $\frac{y}{x^k}$ nisbatning $x\to 0$ dagi limitini qaraymiz va **k** ning bu limit mavjud va noldan farqli bo'ladigan qiymatini aniqlaymiz:

$$\lim_{x\to0}\frac{y}{x^k}=\lim_{x\to0}\frac{\sqrt{1+x\sin x}-1}{x^k}=\lim_{x\to0}\frac{(\sqrt{1+x\sin x}-1)(\sqrt{1+x\sin x}+1)}{x^k(\sqrt{1+x\sin x}+1)}=\lim_{x\to0}\frac{1+x\sin x-1}{x^k(\sqrt{1+x\sin x}+1)}=\lim_{x\to0}\frac{x\sin x}{x^k(\sqrt{1+x\sin x}+1)}=$$

$$=\lim_{x\to0}\frac{\frac{\sin x}{x}}{x^{k-2}(\sqrt{1+x\sin x}+1)}=\frac{1}{2}\lim_{x\to0}x^{2-k}$$ chunki $\lim_{x\to0}\frac{\sin x}{x}=1$, $\lim_{x\to0}(\sqrt{1+x\sin x}+1)=2$

Ravshanki k=2 da $\lim_{x\to0}x^{2-k}=1$. Demak, $\lim_{x\to0}\frac{y}{x^k}=\frac{1}{2}$. Shunday qilib, **y** va $\mathbf{x^2}$ cheksiz kichik miqdorlarning tartibi bir xil. Shu sababli **y** miqdor **x** cheksiz kichik miqdorga nisbatan ikkinchi tartibli (k=2) cheksiz kichik miqdor bo'ladi.

Mustaqil yechish uchun topshiriqlar

1. $\lim_{x\to\infty}\frac{8x^2-7x+5}{2+x-4x^2}$ ni hisoblang.

Javob: -2

2. $\lim\limits_{x\to\infty} \frac{5x^3-2x^2+3}{3-4x-10x^3}$ ni hisoblang.

Javob: $-\frac{1}{2}$

3. $\lim\limits_{x\to3} \frac{2x+2}{4x^2+4}$ ni hisoblang.

Javob: $\frac{1}{5}$

4. $\lim\limits_{x\to1} \frac{5x+3}{3x^3+5}$ ni hisoblang.

Javob: 1

5. $\lim\limits_{x\to4} (\frac{8}{x^2-16}-\frac{1}{x-4})$ ni hisoblang.

Javob: $-\frac{1}{8}$

6. $\lim\limits_{x\to5} (\frac{10}{x^2-25}-\frac{1}{x-5})$ ni hisoblang.

Javob: $-\frac{1}{10}$

7. $\lim\limits_{x\to4} \frac{3-\sqrt{1+2x}}{\sqrt{x}-2}$ ni hisoblang.

Javob: $-\frac{8}{9}$

8. $\lim\limits_{x\to-3} \frac{2x^2+3x-9}{\sqrt{x+10}-\sqrt{4-x}}$ ni hisoblang.

Javob: $-9\sqrt{7}$

9. $\lim\limits_{x\to2} \frac{\sqrt{5-x}-\sqrt{x+1}}{x^2+5x-14}$ ni hisoblang.

Javob: $-\frac{1}{9}\sqrt{3}$

10. $\lim\limits_{x\to2} \frac{3x^2-x-10}{x^2+3x-10}$ ni hisoblang.

Javob: $\frac{11}{7}$

11. $\lim\limits_{x\to0} \frac{\cos 5x-\cos x}{2x^2}$ ni hisoblang.

Javob: 6

12. $\lim\limits_{x\to+\infty} (\frac{2x+4}{2x-6})^{3x+1}$ ni hisoblang.

Javob: e^{15}

3.Funktsiya hosilasi.

$y=f(x)$ funktsiyaning x_0 nuqtadagi orttirmasi Δy ning argument orttirmasi Δx ga nisbatining Δx nolga intilgandagi limiti mavjud bo’lsa, bu limit $y=f(x)$ funktsiyaning x_0 nuqtadagi hosilasi deyeiladi.

Hosilaning belgilanishi:

y' yoki f ' (x_0) yoki $\frac{dy}{dx}$ yoki $\frac{df}{dx}$

Shunday qilib, ta’rifga ko’ra:

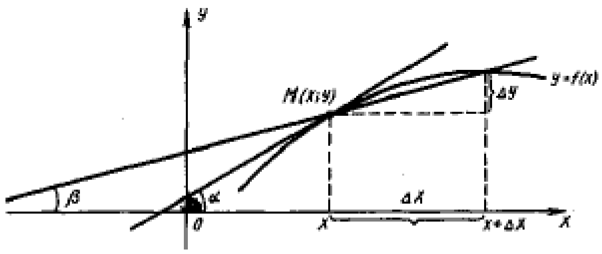
$$f'(x_0)=\lim_{\Delta x\to 0}\frac{y}{x}=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}\frac{\Delta y}{\Delta x}$$

Agar $y=f(x)$ funksiya $\mathbf{x_0}$ nuqtada hosilaga ega bo’lsa, u holda f(x) funksiya $\mathbf{x_0}$ nuqtada differensiallanuvchi deyeiladi.

Hosilani topish jarayoni differensiallash deyeiladi.

4.Hosilaning geometrik ma’nosi.

Geometrik nuqtai nazardan $y=f(x)$ funksiyaning $\mathbf{x_0}$ nuqtadagi hosilasi uning grafigiga $M(x_0;f(x_0))$ nuqtada o’tkazilgan urunmaning **Ox** o’qining musbat yo’nalishi bilan hosil qilgan burchagining tangensiga teng.



$y=f(x)$ egri chiziqqa $M(x_0;y_0)$ nuqtada o’tkazilgan urunma tenglamasi ushbu ko’rinishga ega: $y-y_0=f'(x_0)(x-x_0)$. bunda $y_0=f(x_0)$

$y=f(x)$ funksiya grafigiga urunish nuqtasi $M(x_0;y_0)$ da o’tkazilgan normalning tenglamasi ushbu ko’rinishga ega: $y-y_0=-\frac{1}{f'(x_0)}(x-x_0)$,

agar $f'(x_0)\neq 0$ bo’lsa, $x=x_0$, agar $f'(x_0)=0$ bo’lsa.

$y=f_1(x)$ $y=f_2(x)$ egri chiziqlar $M(x_0;y_0)$ nuqtada kesishsin, bu nuqtadagi ular orasidagi burchak deb $M(x_0;y_0)$ da ularga o'tkazilgan urunmalar orasidagi burchakka aytiladi. va u quyidagi formuladan topiladi: $tg\varphi=\frac{f'_2(x_0)-f'_1(x_0)}{1+f'_1(x_0)\cdot f'_2(x_0)}$

X – erkli o'zgaruvchi, $u=u(x)$ va $\vartheta=\vartheta(x)$ differensiallanuvchi funksiyalar, C – o'zgarmas son bo'lsin, u holda quyidagi differensiallash qoidalari o'rinli:

1. $C'=0$ 3. $(u\pm\vartheta)'=u'\pm\vartheta'$ 5. $(u\cdot\vartheta)'=u'\cdot\vartheta+u\cdot\vartheta'$

2. $(C\cdot u)'=Cu'$ 4. $x'=1$ 6. $(\frac{u}{\vartheta})'=\frac{u'\cdot\vartheta-u\cdot\vartheta'}{\vartheta^2}$ 7. $(\frac{C}{\vartheta})'=-\frac{C\cdot\vartheta'}{\vartheta^2}$

8. Agar $y=f(u)$ va $u=\varphi(x)$ yani $y=f(\varphi(x))$ murakkab funksiya bo'lsa u holda $y'=f'(u)\cdot\varphi'(x)$

9. Agar $y=f(x)$ va $x=\varphi(y)$ o'zaro teskari funksiyalar bo'lsa, u holda $f'(x)=\frac{1}{\varphi'(y)}$

Hosilalar jadvali:

1. $(u^\alpha)'=\alpha u^{\alpha-1}\cdot u'$	7. $(\log_a u)'=\frac{1}{u\cdot\ln a}\cdot u'$
2. $(\sqrt{u})'=\frac{1}{2\sqrt{u}}\cdot u'$	8. $(\ln u)'=\frac{1}{u}\cdot u'$
3. $(\frac{1}{u})'=-\frac{1}{u^2}\cdot u'$	9. $(\sin u)'=\cos u\cdot u'$
4. $(a^u)'=a^u\cdot\ln a\cdot u'$	**10.** $(\cos u)'=-\sin u\cdot u'$
5. $(e^u)'=e^u\cdot u'$	**11.** $(tgu)'=\frac{1}{\cos^2 u}\cdot u'$
6. $(u^\vartheta)'=\vartheta\cdot u^{\vartheta-1}\cdot u'+u^\vartheta\cdot\ln u\cdot u'$	**12.** $(ctgu)'=-\frac{1}{\sin^2 u}\cdot u'$
13. $(\sec u)'=\frac{\sin u}{\cos^2 u}\cdot u'=tgu\cdot\sec u\cdot u'$	**16.** $(\arccos u)'=-\frac{1}{\sqrt{1-u^2}}\cdot u'$
14. $(\cos ecu)'=-\frac{\cos u}{\sin^2 u}\cdot u'=-ctgu\cdot\cos ecu\cdot u'$	**17.** $(arctgu)'=\frac{1}{1+u^2}\cdot u'$
15. $(\arcsin u)'=\frac{1}{\sqrt{1-u^2}}\cdot u'$	**18.** $(arcctgu)'=-\frac{1}{1+u^2}\cdot u'$

5.Logarifmik hosila

$y=f(x)$ funksiyaning logarifmik hosilasi deb, shu funksiyaning logarifmidan olingan hosilaga aytiladi, yani: $(\ln y)'=\frac{y'}{y}=\frac{f'(x)}{f(x)}$. Funksiyani oldindan logarifmlashdan foydalanish ba'zan uning hosilasini topishni osonlashtiradi. Funksiyani logarifmlash va differensiallashni ketma-ket qo'llash logarifmik differensiallash deyeiladi.

6.Yuqori tartibli hosila.

$y=f(x)$ funktsiyaning Ikkinchi tartibli yoki Ikkinchi hosilasi deb uning birinchi tartibli hosilasidan olingan hosilaga, yani $(y')'$ ga aytiladi.

Ikkinchi tartibli hosila quyidagilarning biri bilan belgilanadi.

y'' yoki $f''(x)$ yoki $\frac{d^2y}{dx^2}$

$y=f(x)$ funktsiyaning **n-** tartibli yoki **n-** hosilasi deb uning **(n-1)** - tartibli hosilasidan olingan hosilaga, yani $(y^{(n-1)})'$ ga aytiladi. **n-** tartibli hosila quyidagilarning biri bilan belgilanadi.

y^n yoki $f^n(x)$ yoki $\frac{d^ny}{dx^n}$. Belgilashlarga ko'ra $y^n=(y^{(n-1)})'$

X o'zgaruvchining **y** funksiyasi oshkormas shaklda **F(x;y) = 0** tenglama bilan berilgan bo'lsa, u xolda y' hosilani topish uchun **F(x;y) = 0** tenglikning ikkala qismini **x** bo'yicha differensiallab, so'ngra hosil bo'lgan y' ga nisbatan chiziqli tenglamadan hosilani topish kerak. Ikkinchi va undan yuqoriroq tartibli hosilalar ham shu kabi topiladi.

7.Funksiyaning differentsiali

$y=f(x)$ funksiyaning differensiali deb, uning orttirmasining erkli o'zgaruvchi **x** ning orttirmasiga nisbatan chiziqli bo'lgan bosh qismiga aytiladi.

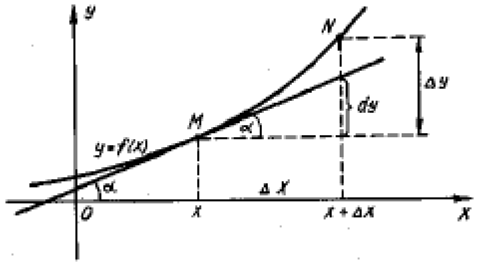
$y=f(x)$ funksiyaning differensiali **dy** bilan belgilanadi. Funksiyaning differensiali uning hosilasi bilan erkli o'zgaruvchi orttirmasining ko'paytmasiga teng:

$dy=f'(x)\cdot\Delta x$ yoki $dy=y'\cdot\Delta x$

ravshanki, $dx=\Delta x$. Shu sababli $dy=f'(x)\cdot dx$ yoki $dy=y'\cdot dx$

Differensial geometrik jihatdan $y = f(x)$ funksiya grafigiga $M(x; y)$ nuqtada o'tkazilgan urunma ordinatasining orttirmasiga teng.

Funksiyaning differensiali **dy** uning **Δy** orttirmasidan **Δx** ga nisbatan yuqori tartibli cheksiz kichik miqdoriga farq qiladi.



Agar $u = u(x)$ va $\vartheta = \vartheta(x)$ funksiyalar differensiallanuvchi bo'lsa, u holda differensialning ta'rifi va differensiallash qoidalaridan bevosita differensialning asosiy xossalariga ega bo'lamiz: 1. $d(C) = 0 \quad - o'zgarmas$

2. $d(Cu) = Cdu$

3. $d(u \pm \vartheta) = du \pm d\vartheta$

4. $d(u \cdot \vartheta) = ud\vartheta + \vartheta du$

5. $d(\frac{u}{\vartheta}) = \frac{\vartheta du - ud\vartheta}{\vartheta^2}$, bunda $v \neq 0$

6. $df(u) = f'(u)du$

8.Yuqori tartibli differensial.

$y = f(x)$ funktsiyaning Ikkinchi tartibli differensiali deb uning birinchi tartibli differensialidan olingan differensialga aytiladi va $d^2 y = d(dy)$ kabi belgilanadi.

$y = f(x)$ funktsiyaning **n-** tartibli differensiali deb uning **(n-1)** - tartibli differensialidan olingan differensialga aytiladi, yani $d^n y = d(d^{n-1} y)$.

$y = f(x)$ funktsiya berilgan bo'lib, bunda **x**-erkli o'zgaruvchi bo'lsa, u xolda uning yuqori tartibli differensiallari ushbu formulalar bo'yicha hisoblanadi.

$d^2 y = y''dx^2$, $\quad d^3 y = y'''dx^3$, --------, $\quad d^n y = y^{(n)} dx^n$

Funksiyaning **dy** differensiali uning Δy orttirmasidan Δx= dx ga nisbatan yuqori tartibli cheksiz kichik miqdorga farq qiladi, shu sababli $\Delta y \approx$ dy yoki

$f(x + \Delta x) - f(x) \approx f'(x)\Delta x$ bundan

$f(x+\Delta x) \approx f(x) + f'(x)\Delta x$ formulaga ega bo'lamiz, bu formula funksiya qiymatlarini taqribiy hisoblashlarda qo'llaniladi.

9.Lopital qoidasi ($\frac{0}{0}$, $\frac{\infty}{\infty}$ ko'rinishdagi aniqmasliklarni ochish)

$f(x)$ *va* $\varphi(x)$ funksiyalar $\mathbf{x_0}$ nuqtaning biror atrofida ($\mathbf{x_0}$ nuqtaning o'zidan tashqari) differensiallanuvchi va $\varphi'(x) \neq 0$ bo'lsin. Agar $\lim\limits_{x \to x_0} f(x) = \lim\limits_{x \to x_0} \varphi(x) = 0$

yoki $\lim\limits_{x \to x_0} f(x) = \lim\limits_{x \to x_0} \varphi(x) = \infty$ bo'lib, $\lim\limits_{x \to x_0} \frac{f'(x)}{\varphi'(x)}$ mavjud bo'lsa, u xolda

$\lim\limits_{x \to x_0} \frac{f(x)}{\varphi(x)} = \lim\limits_{x \to x_0} \frac{f'(x)}{\varphi'(x)}$ bo'ladi.

$x \to \infty$ da ham Lopital qoidasi o'rinli**.**

$0 \cdot \infty$ yoki $\infty - \infty$ shaklidagi aniqmasliklar algebraik almashtirishlar orqali $\frac{0}{0}$, $\frac{\infty}{\infty}$ ko'rinishdagi aniqmasliklarga keltirilib, so'ngra Lopital qoidasidan foydalaniladi.

0^0, ∞^0, 1^∞ shaklidagi aniqmasliklar logarifmlash orqali $\frac{0}{0}$, $\frac{\infty}{\infty}$ ko'rinishdagi aniqmasliklarga keltirilib, so'ngra Lopital qoidasidan foydalaniladi.

10.Funktsiya monotonligi, qavariqligi, ekstremumlari.

Agar $(a;b)$ oraliqning $x_2 > x_1$ tengsizlikni qanoatlantiruvchi ikkita ixtiyoriy x_1 va x_2 nuqtalari uchun $f(x_2) > f(x_1)$ tengsizlik bajarilsa, $f(x)$ funksiya $(a;b)$ oraliqda **o'suvchi** deyiladi.

Agar $(a;b)$ oraliqning $x_2 > x_1$ tengsizlikni qanoatlantiruvchi ikkita ixtiyoriy x_1 va x_2 nuqtalari uchun $f(x_2) < f(x_1)$ tengsizlik bajarilsa, $f(x)$ funksiya $(a;b)$ oraliqda **kamayuvchi** deyiladi.

Oraliqda o'suvchi yoki kamayuvchi funksiyalar **monoton** funksiyalar deyiladi.

M o n o t o n l i k n i n g z a r u r i y sh a r t l a r i :

1. Agar $(a;b)$ oraliqda differensiallanuvchi $y = f(x)$ funksiya o'suvchi bo'lsa,u holda $f'(x) > 0$.

2. Agar $(a;b)$ oraliqda differensiallanuvchi $y = f(x)$ funksiya kamayuvchi bo'lsa,u holda $f'(x) < 0$.

M o n o t o n l i k n i n g y e t a r l i l i k sh a r t l a r i :

1. Agar $(a;b)$ oraliqda differensiallanuvchi $y = f(x)$ funksiya musbat hosilaga ega bo'lsa, ya'ni $f'(x) > 0$, u holda funksiya shu oraliqda **o'suvchi funksiya** bo'ladi.
2. Agar $(a;b)$ oraliqda differensiallanuvchi $y = f(x)$ funksiya manfiy hosilaga ega bo'lsa, ya'ni $f'(x) < 0$, u holda funksiya shu oraliqda **kamayuvchi funksiya** bo'ladi.

Funksiyaning birinchi tartibli hosilasi nolga teng yoki uzilishga ega bo'ladigan nuqtalari **kritik nuqtalar** deyiladi.

Eng sodda hollarda $y = f(x)$ funksiyaning aniqlanish sohasini chekli sondagi kritik nuqtalar bilan chegaralangan monotonlik oraliqlarga bo'lish mumkin.

Agar x_0 nuqtaning shunday atrofi mavjud bo'lsaki, bu atrofning har qanday $x \neq x_0$ nuqtasi uchun $f(x) < f(x_0)$ tengsizlik o'rinli bo'lsa, u holda $y = f(x)$ funksiya x_0 nuqtada **maksimumga erishadi** deyiladi.

Agar x_0 nuqtaning shunday atrofi mavjud bo'lsaki, bu atrofning har qanday $x \neq x_0$ nuqtasi uchun $f(x) > f(x_0)$ tengsizlik o'rinli bo'lsa, u holda $y = f(x)$ funksiya x_0 nuqtada **minimumga erishadi** deyiladi.

Funksiya maksimum yoki minimumga erishadigan nuqtalar uning ***ekstremum* nuqtalari** deyiladi. Funksiyaning ekstremum nuqtalaridagi qiymatlari funksiyaning *ekstremal* (*maksimal* yoki *minimal*) *qiymatlari* deyiladi.

E k s t r e m u m n i n g z a r u r i y sh a r t i . Agar $y = f(x)$ funksiya x_0 nuqtada ekstremumga ega bo'lsa, u holda $f'(x_0)$ nolga teng yoki mavjud bo'lmaydi.

Ammo har qanday kritik nuqta ham ekstremum nuqtasi bo'lavermaydi.

E k s t r e m u m n i n g n i n g y e t a r l i l i k sh a r t i . Agar x_0 nuqta $y = f(x)$ funksiyaning kritik nuqtasi bo’lib, funksiyaning hosilasi bu nuqtadan o’tishda ishorasini *o’zgartirsa,* u holda x_0 bu funksiyaning *ekstremum nuqtasi* bo’ladi, shu bilan birga:

1. Agar x_0 nuqtadan chapdan o’ngga o’tishda $f'(x)$ o’z ishorasini musbatdan manfiyga o’zgartirsa, u holda x_0 nuqtada funksiya maksimumga erishadi.
2. Agar x_0 nuqtadan chapdan o’ngga o’tishda $f'(x)$ o’z ishorasini manfiydan musbatga o’zgartirsa, u holda x_0 nuqtada funksiya minimumga erishadi.

Shunday qilib, monotonlik oraliqlarini va funksiya ekstremumini toppish uchun oldin funksiyaning aniqlanish sohasini kritik nuqtalar yordamida monotonlik oraliqlariga bo’lish va ularda hosila ishorasini tekshirish kerak.

Shundan keyin monotonlik va ekstremumning yetarlilik shartlaridan foydalanib, o’sish va kamayish oraliqlarini, maksimum va minimum nuqtalarini topish hamda funksiyaning bu nuqtalardagi qiymatlarini hisoblab, natijalarni tegishli jadvalga yozish kerak.

11.Funksiyaning qavariqligi va botiqligi. Egilish nuqtalari. Asimptotalar.

$y = f(x)$ funksiyaning grafigi $(a;b)$ oraliqning istalgan nuqtasida o’tkazilgan urunmadan pastda yotsa, u holda funksiya grafigi qavariq deyiladi.

$y = f(x)$ funksiyaning grafigi $(a;b)$ oraliqning istalgan nuqtasida o’tkazilgan urunmadan yuqorida yotsa, u holda funksiya grafigi botiq deyiladi.

Funksiya grafigining qavariq qismini botiq qismidan ajratuvchi $M_0(x_0; f(x_0))$ nuqta grafikning egilish nuqtasi deyiladi.

Funksiya grafigining qavariq yoki botiq bo’lishining yetarlilik shartlari.

Agar $(a;b)$ oraliqda differensiallnuvchi $y = f(x)$ funksiyaning Ikkinchi tartibli hosilasi manfiy, yani $f''(x) < 0$ bo’lsa, u holda bu oraliqda funksiya grafigi qavariq bo’ladi.

Agar $(a;b)$ oraliqda differensiallnuvchi $y=f(x)$ funksiyaning Ikkinchi tartibli hosilasi musbat, yani $f''(x)>0$ bo'lsa, u holda bu oraliqda funksiya grafigi botiq bo'ladi.

Qavariqlik oralig'ini botiqlik oralig'idan ajratib turuvchi egilish nuqtasidan o'tishda funksiyaning ikkinchi tartibli hosilasi ishorasini o'zgartiradi. Bunday nuqtalarda funksiyaning ikkinchi tartibli hosilasi yo nolga teng , yoki mavjud bo'lmaydi.

$f''(x)=0$ yoki $f''(x)$ mavjud bo'lmaydigan nuqtalar ikkinchi tur kritik nuqtalar deyiladi.

Eegilish nuqtalari mavjud bo'lishining etarlilik sharti. Agar x_0 nuqta $y=f(x)$ funksiya uchun Ikkinchi tur kritik nuqta bo'lsa va $f''(x)$ ikkinchi tartibli hosila bu nuqtadan o'tishda ishorasini o'zgartirsa , u holda bu funksiya grafigining x_0 abstsissali nuqtasi egilish nuqtasi bo'ladi.

Demak , funksiya grafigining qavariqlik va botiqlik oraliqlarini, egilish nuqtalarini topish uchun oldin funksiya aniqlanish sohasini ikkinchi tur kritik nuqtalar bilan oraliqlarga bo'lish va bu oraliqlarda ikkinchi tartibli hosila ishorasini tekshirish kerak. Shundan keyin etarlilik shartlaridan foydalanib, qavariqlik va botiqlik oraliqlari va egilish nuqtalari aniqlanadi.

Agar $y=f(x)$ funksiya grafigidagi nuqta shu grafik bo'ylab cheksiz uzoqlashganda undan biror to'g'ri chiziqqacha bo'lgan masofa nolga intilsa, bu to'g'ri chiziq $y=f(x)$ funksiya grafigining asimptotasi deb ataladi.

Agar $\lim\limits_{x\to\infty} f(x)=\infty$ bo'lsa $x=a$ to'g'ri chiziq $y=f(x)$ funksiya grafigining vertikal asimptotasi deb ataladi

Agar $k=\lim\limits_{x\to\infty}\dfrac{f(x)}{x}$ va $b=\lim\limits_{x\to\infty}(f(x)-kx)$ limitlar mavjud bo'lsa, u hoda $y=kx+b$ to'g'ri chiziq $y=f(x)$ funksiyaning og'ma asimptotasi deb ataladi.

Xususan, k = 0 da gorizontal asimptotaga ega bo'ladi.

12.Funktsiyani to'la tekshirish va grafigini yasash.

Funksiyaning xossalarini tekshirish va uning grafigini yasashda quyidagilarni bajarish maqsadga muvofiq:

1) Funksiyaning aniqlanish sohasi va uzilish nuqtalari topiladi; funksiyaning chegaraviy nuqtalaridagi qiymatlari (yoki unga mos limitlari) hisoblanadi.

2) Funksiyaning toq-juftligi, davriyligi tekshiriladi.

3) Funksiyaning nollari va ishora turg`unlik oraliqlari aniqlanadi.

4) Asimptotalar topiladi.

5) Funksiya ekstremumga tekshiriladi, uning monotonlik oraliqlari aniqlaniladi.

6) Funksiya grafigining burilish nuqtalari, qavariqlik va botiqlik oraliqlari topiladi.

Mavzuga doir yechimlari bilan berilgan topshiriqlardan namunalar

1. $y=\frac{2x+3}{x+1}$ funktsiyaning hosilasini hosilani ta’rifidan foydalanib hisoblang.

Yechish: $f'(x_0)=\lim\limits_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim\limits_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$ formulaga asosan hisoblaymiz**.**

Buning uchun Δy ni hisoblaymiz.

$$\Delta y=\frac{2(x+\Delta x)+3}{x+\Delta x+1}-\frac{2x+3}{x+1}=\frac{2x+2\Delta x+3}{x+\Delta x+1}-\frac{2x+3}{x+1}=\frac{(2x+2\Delta x+3)(x+1)-(2x+3)(x+\Delta x+1)}{(x+\Delta x+1)(x+1)}=$$

$$\frac{2x^2+2x+2x\Delta x+2\Delta x+3x+3-2x^2-2x\Delta x-2x-3x-3\Delta x-3}{(x+\Delta x+1)(x+1)}=-\frac{\Delta x}{(x+\Delta x+1)(x+1)}$$

$$\frac{\Delta y}{\Delta x}=\frac{-\frac{\Delta x}{(x+\Delta x+1)(x+1)}}{\Delta x}=-\frac{1}{(x+\Delta x+1)(x+1)}$$

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\left(-\frac{1}{(x+\Delta x+1)(x+1)}\right)=-\frac{1}{(x+0+1)(x+1)}=-\frac{1}{(x+1)^2}$$

Demak, $y'=-\frac{1}{(x+1)^2}$

2. $y=\frac{2x+3}{x+1}$ funktsiyaning hosilasini hosilani qoidasidan foydalanib hisoblang.

Yechish: $(\frac{u}{\vartheta})'=\frac{u'\cdot\vartheta-u\cdot\vartheta'}{\vartheta^2}$ formuladan foydalanamiz.

$$(\frac{u}{\vartheta})'=\frac{u'\cdot\vartheta-u\cdot\vartheta'}{\vartheta^2}=\frac{(2x+3)'(x+1)-(x+1)'(2x+3)}{(x+1)^2}=\frac{2x+2-2x-3}{(x+1)^2}=-\frac{1}{(x+1)^2}$$

Demak, $y' = -\frac{1}{(x+1)^2}$

3. $y = \cos 2x$ funktsiyaning hosilasini hosilani jadvalidan foydalanib hisoblang.

Yechish: $(\cos u)' = -\sin u \cdot u'$ formuladan foydalanamiz.

$y' = (\cos 2x)' = -\sin 2x \cdot (2x)' = -2\sin 2x$

4. $y = 2^{3x}$ funktsiyaning hosilasini hosilani jadvalidan foydalanib hisoblang.

Yechish: $(a^u)' = a^u \cdot \ln a \cdot u'$ formuladan foydalanamiz.

$y' = (2^{3x})' = 2^{3x} \cdot \ln 2 \cdot (3x)' = 3 \cdot 2^{3x} \cdot \ln 2$

5. $y = \ln 3x$ funktsiyaning hosilasini hosilani jadvalidan foydalanib hisoblang.

Yechish: $(\ln u)' = \frac{1}{u} \cdot u'$ formuladan foydalanamiz.

$y' = (\ln 3x)' = \frac{(3x)'}{3x} = \frac{3}{3x} = \frac{1}{x}$

6. $y = \ln x + 2x^4 - 3x + 5$ funktsiyaning 3 tartibli hosilasini hisoblang.

Yechish: $y^n = (y^{(n-1)})'$ formuladan foydalanamiz.

Buning uchun berilgan funktsiyani 1 – tartibli hosilasini hisoblaymiz, yana 1-tartibli hosilasidan 1-tartibli hosilasini hisoblab 2-tartibli hosilasini hisoblaymiz, so’ngra 2-tartibli hosilasidan 1-tartibli hosilasini hisoblab, 3-tartibli hosilasini topamiz.

$y' = (\ln x + 2x^4 - 3x + 5)' = (\ln x)' + (2x^4)' - (3x)' + 5' = \frac{1}{x} + 8x^3 - 3$

$y'' = (y')' = (\frac{1}{x} + 8x^3 - 3)' = -\frac{1}{x^2} + 16x^2$

$y''' = (y'')' = (-\frac{1}{x^2} + 16x^2)' = \frac{2x}{x^4} + 32x$

Demak , $y''' = \frac{2x}{x^4} + 32x$

7. $y = x^3 - 2x + 5$ funktsiyaning birinchi tartibli differensialini toping.

Yechish: $dy = y' \cdot dx$ formuladan foydalanamiz

$dy = d(x^3 - 2x + 5) = (x^3 - 2x + 5)' dx = (3x^2 - 2)dx$

8. $y = 3x^5 + 2x$ funktsiyaning uchinchi tartibli differensialini toping.

Yechish: $d^3 y = y''' \cdot d\, x^3$ formuladan foydalanamiz

$dy = y'dx = (3x^5 + 2x)'dx = (15x^4 + 2)dx$

$d^2y = d(dy) = y''dx^2 = (y')'dx^2 = (15x^4 + 2)'dx^2 = 60x^3dx^2$

$d^3y = d(d^2y) = y'''dx^3 = (y'')'dx^3 = (60x^3)'dx^3 = 180x^2dx^3$

Demak, $d^3y = 180x^2dx^3$

9. $\lim\limits_{x\to 0}\frac{e^x - e^{-x}}{\ln(1+x)}$ funktsiyani Lopital qoidasidan foydalanib limitini hisoblang.

Yechish: $\lim\limits_{x\to x_0}\frac{f'(x)}{\varphi'(x)}$ formuladan foydalanamiz,

$$\lim_{x\to 0}\frac{e^x - e^{-x}}{\ln(1+x)} = \lim_{x\to 0}\frac{(e^x - e^{-x})'}{(\ln(1+x))'} = \lim_{x\to 0}\frac{e^x + e^{-x}}{\frac{1}{1+x}} = \frac{1+1}{1} = \frac{2}{1} = 2$$

10. $y = x(x^2 - 1)$ funksiyani tekshiring va grafigini chizing.

Yechish. 1) aniqlanish sohasi - haqiqiy sonlar to`plami. Uzilish nuqtalari yo`q. Funksiyaning chegaraviy qiymatlari: $\lim\limits_{x\to\infty} x(x^2-1) = +\infty$; $\lim\limits_{x\to\infty} x(x^2-1) = -\infty$;

2) funksiya davriy emas, toq funksiya

3) funksiyaning uchta noli bor: x=0; x=-1; x=1. Ushbu $x(x^2-1) > 0$ tengsizlikni yechamiz, uning yechimi $(-1;0)\cup(1;+\infty)$ to`plamdan iborat. Demak, funksiya $(-1;0)\cup(1;+\infty)$ to`plamda musbat va $(-\infty;-1)\cup(0;1)$ to`plamda manfiy qiymatlar qabul qiladi.

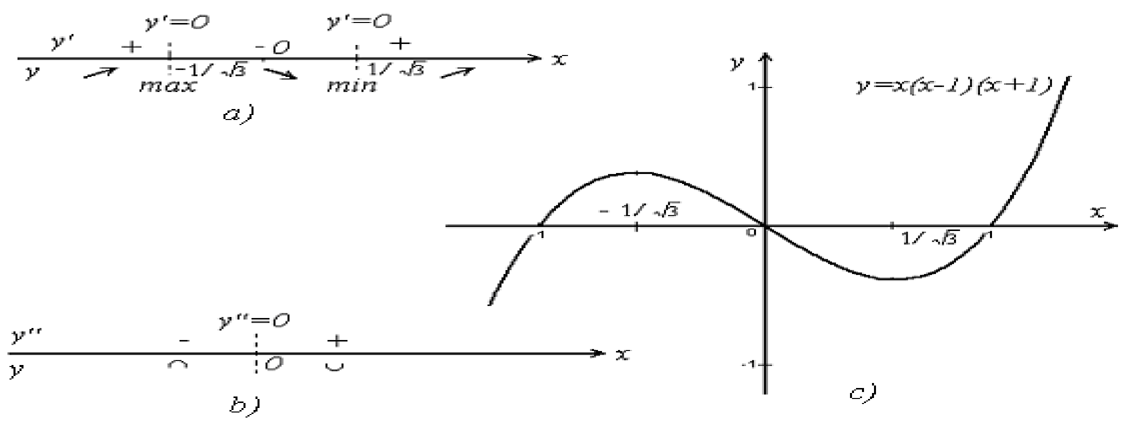
4) og`ma asimptotaning burchak koeffitsientini topamiz: $k = \lim\limits_{x\to\infty}\frac{y}{x} = \lim\limits_{x\to\infty}(x^2-1) = \infty$. Demak, og`ma asimptota mavjud emas. Vertikal asimtotalar ham mavjud emas (chunki, uzilish nuqtalari yo`q).

5) Funksiya hosilasini topamiz: $y' = 3x^2 - 1$. Hosilani nolga tenglashtirib statsionar nuqtalarini topamiz: $y' = 0$ yoki $3x^2 - 1 = 0$, bundan $x = -\frac{1}{\sqrt{3}}$; $x = \frac{1}{\sqrt{3}}$. Ushbu (1-a-rasm) sxemani chizamiz, va intervallar metodidan foydalanib funksiya hosilasining ishoralarini aniqlaymiz. Bundan funksiya $(-\infty;-\frac{1}{\sqrt{3}})$ va $(\frac{1}{\sqrt{3}};+\infty)$ intervallarda monoton o`suvchi,

$(-\frac{1}{\sqrt{3}};\frac{1}{\sqrt{3}})$ intervalda monoton kamayuvchi; $x=-\frac{1}{\sqrt{3}}$ nuqtada maksimumga, $x=\frac{1}{\sqrt{3}}$ nuqtada minimumga ega ekanligi kelib chiqadi. Ekstremum nuqtalarida funksiya qiymatlarini hisoblaymiz: agar $x_{\max}=-\frac{1}{\sqrt{3}}$ bo`lsa, u holda $y_{\max}=\frac{2}{3\sqrt{3}}$; agar $x_{\min}=\frac{1}{\sqrt{3}}$ bo`lsa, u holda $y_{\max}=-\frac{2}{3\sqrt{3}}$.

6) Ikkinchi tartibli hosilani topamiz: $y''=6x$. Ikkinchi tartibli hosilani nolga tenglashtirib $y''=6x=0,\ x=0$, ekanligini topamiz. Sxemani (1-b-rasm) chizamiz va hosil bo`lgan intervallarda ikkinchi tartibli hosila ishoralarini aniqlaymiz. Bundan $x=0$ nuqtada burilish mavjud, $(-\infty;0)$ da funksiya grafigi qavariq, $(0;+\infty)$ da botiq ekanligini topamiz. Burilish nuqtasi ordinatasini topamiz: $y(0)=0$.

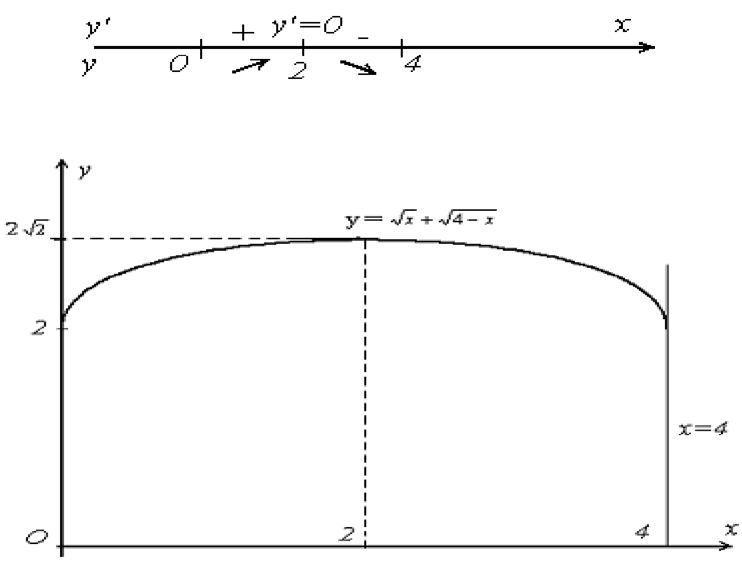
Funksiya grafigi 1–c-rasmda keltirilgan.



1-rasm

11. $y=\sqrt{x}+\sqrt{4-x}$ funksiyani tekshiring va grafigini chizing.

Yechish. 1) Aniqlanish sohasi – $[0;4]$kesma. Funksiyaning



chegaraviy qiymatlarini topamiz: agar $x=0$ bo`lsa, u holda $y=2$; agar $x=4$ bo`lsa, $y=2$. Funksiyaning uzilish nuqtalari yo`q.

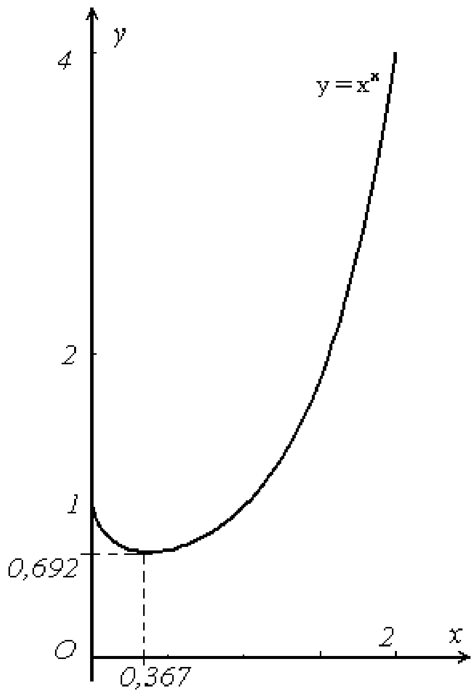
2) Funksiya toq ham, juft ham emas, davriy ham emas. 2-rasm

3) funksiyaning nollari yo`q,

4) Og`ma asimptotalari yo`q, chunki aniqlanish sohasi kesmadan iborat.

5) Hosilasini topamiz: $y'=\dfrac{\sqrt{4-x}-\sqrt{x}}{2\sqrt{x}\cdot\sqrt{4-x}}$.

Hosilani nolga tenglashtirib, kritik (statsionar) nuqtani topamiz: $x=2$. 2-rasmdagi sxemani chizamiz. Bundan funksiya $(0;2)$ intervalda o`suvchi, $(2;4)$ intervalda kamayuvchi, $x=2$ nuqtada funksiya maksimumga yerishishi kelib chiqadi. Maksimum nuqtasining ordinatasi $y_{\max}=2\sqrt{2}$.



3-rasm

6) Ikkinchi tartibli hosilani topamiz: $y''=-\dfrac{1}{4}\cdot\dfrac{(4-x)^{3/2}+x^{3/2}}{x^{3/2}(4-x)^{3/2}}$. $(0;4)$ intervalda ikkinchi tartibli hosila manfiy, demak bu intervalda funksiya grafigi qavariq bo`ladi.

Funksiya grafigi 2–rasmda chizilgan. Shuni aytib o`tish kerakki, $\lim\limits_{x\to 0+} y=+\infty$, $\lim\limits_{x\to 4-0} y=-\infty$ bo`lganligi sababli, funksiya grafigi $(0;2)$ nuqtada ordinatalar o`qiga, $(4;2)$ nuqtada $x=4$ to`g`ri chiziqqa urinadi.

12. $y=x^x$. funksiyani tekshiring va grafigini chizing.

Yechish. Avval funksiyani quyidagicha yozib olamiz: $y=x^x=e^{x\ln x}$.

1) funksiyaning aniqlanish sohasi barcha musbat sonlar to`plami. Chegaraviy qiymatlari: $\lim\limits_{x\to 0+} e^{x\ln x}=1$, $\lim\limits_{x\to+\infty} e^{x\ln x}=+\infty$. Uzilish nuqtalari yo`q.

2) Funksiya juft ham, toq ham, davriy ham emas.

3) Funksiyaning nollari mavjud emas.

4) Og`ma asimptotasini izlaymiz: $k = \lim\limits_{x\to+\infty} \frac{x^{x\ln x}}{x} = +\infty$, demak og`ma asimptota yo`q.

5) Hosilasini topamiz: $y' = x^x(\ln x + 1)$. $y' = 0$ tenglamadan $x = e^{-1} \approx 0.367$. Funksiya $(0;\frac{1}{e})$ intervalda kamayuvchi, $(\frac{1}{e};+\infty)$ intervalda o`suvchi bo`ladi. $x = e^{-1}$ nuqtada funksiya minimumga ega, uning ordinatasi $y_{\min} = 0{,}692$.

6) Ikkinchi tartibli hosilani topamiz: $y'' = x^x(\ln x+1)^2 + \frac{1}{x}$. Ikkinchi tartibli hosila $(0;+\infty)$ intervalda musbat, demak funksiya bu intervalda botiq.

Funksiyaning $x = 0$ nuqta atrofida tekshiramiz. $\lim\limits_{x\to 0+} y' = \lim\limits_{x\to 0+} x^x(\ln x + 1) = -\infty$, bundan funksiya grafigi $(0;1)$ nuqtada ordinatalar o`qiga urinishi kelib chiqadi.

Funksiya grafigi 3–rasmda berilgan.

13. $f(x) = x + \ln(x^2 - 1)$ funksiyani to`la tekshiring va grafigini chizing.

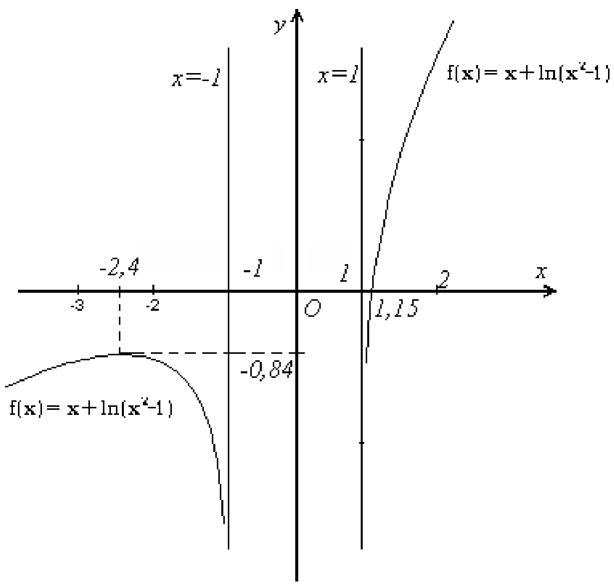
Yechish. 1) Funksiya $x^2 - 1 > 0$, ya`ni $(-\infty;-1)$ va $(1;+\infty)$ oraliqlarda aniqlangan va uzluksiz. Funksiyaning chegaraviy qiymatlarini izlaymiz:

$$\lim_{x\to 1+0} f(x) = \lim_{x\to 1+0}(x + \ln(x^2 - 1)) = -\infty$$

Demak, funksiya grafigi ikkita x=-1 va x=1 vyertikal asimptotalarga ega.

2) funksiya toq ham, juft ham, davriy ham emas.

3) funksiya $(-\infty;-1)$ intervalda manfiy, $(1;+\infty)$ intervalda yagona noli mavjud, uni topish uchun taqribiy hisoblash metodlaridan foydalaniladi, natijada $x_0 \approx 1{,}15$ ekanligini aniqlashimiz mumkin. Demak, funksiya (1;1,15) intervalda manfiy, (1,15, +∞) oraliqda musbat.



4) Og`ma asimptotalarini izlaymiz:

$$k=\lim_{x\to\pm\infty}\frac{y}{x}=\lim_{x\to\pm\infty}(1+\frac{\ln(x^2-1)}{x})=1,$$

$b=\lim\limits_{x\to\pm\infty}(y-kx)=\lim\limits_{x\to\pm\infty}\ln(x^2-1)=+\infty$, demak og`ma asimptota mavjud emas.

5) Funksiya hosilasi $y'=1+2x(x^2-1)$

4-rasm

funksiyaning aniqlanish sohasida mavjud, shu sababli uning kritik nuqtalari faqat statsionar nuqtalardan iborat bo`ladi. Bunda $y'=0$ tenglama yechimlari $x_1=-1-\sqrt{2}$ va $x_2=-1+\sqrt{2}$ bo`lib, $x_2=-1+\sqrt{2}$ funksiyaning aniqlanish sohasiga tegishli emas.

Shunday qilib, yagona kritik nuqta mavjud va $(-\infty;-1)$ oraliqqa tegishli. $(1;+\infty)$ oraliqda $y'>0$ va funksiya o`suvchi bo`ladi. $x_1=-1-\sqrt{2}$ nuqtada maksimum mavjud. Uning ordinatasi

$f(-1-\sqrt{2})=-1-\sqrt{2}+\ln(2+2\sqrt{2})\approx-0.84$ ga teng.

6) Ikkinchi tartibli hosilani topamiz: $y''=-\dfrac{2(x^2+1)}{(x^2-1)^2}$. Bundan $y''<0$, demak grafik qavariq. Funksiya grafigi 4-rasmda berilgan.

Mustaqil yechish uchun topshiriqlar

1. $y=\dfrac{3x-1}{x+2}$ funktsiyaning hosilasini hosilani ta'rifidan foydalanib hisoblang.

Javob: $\dfrac{7}{(x+2)^2}$

2. $y=\dfrac{5x-2}{x+3}$ funktsiyaning hosilasini hosilani ta'rifidan foydalanib hisoblang.

Javob: $\dfrac{17}{(x+3)^2}$

3. $y=\dfrac{3x-1}{x+2}$ funktsiyaning hosilasini hosilani qoidasidan foydalanib hisoblang.

Javob: $\dfrac{7}{(x+2)^2}$

4. $y=\frac{5x-2}{x+3}$ funktsiyaning hosilasini hosilani qoidasidan foydalanib hisoblang.

Javob: $\frac{17}{(x+3)^2}$

5. $y=\sin 3x$ funktsiyaning hosilasini hosilani jadvalidan foydalanib hisoblang.

Javob: $3\cos 3x$

6. $y=\sin 3x^2$ funktsiyaning hosilasini hosilani jadvalidan foydalanib hisoblang.

Javob: $6x\cos 3x^2$

7. $y=3^{2x}$ funktsiyaning hosilasini hosilani jadvalidan foydalanib hisoblang.

Javob: $2{\cdot}3^{2x}\ln 3$

8. $y=2^x$ funktsiyaning hosilasini hosilani jadvalidan foydalanib hisoblang.

Javob: $2^x\ln 2$

9. $y=\ln 2x^2$ funktsiyaning hosilasini hosilani jadvalidan foydalanib hisoblang.

Javob: $\frac{2}{x}$

10. $y=\ln(3x^2+2x)$ funktsiyaning hosilasini hosilani jadvalidan foydalanib hisoblang.

Javob: $\frac{6x+2}{3x^2+2x}$

11. $y=\cos x+2x^3-3x$ funktsiyaning 3 tartibli hosilasini hisoblang.

Javob: $\sin x+12$

12. $y=\cos 2x+3x^3-x^2$ funktsiyaning 3 tartibli hosilasini hisoblang.

Javob: $6\sin 2x+18$

13. $y=2x^3-3x^2+5x$ funktsiyaning birinchi tartibli differensialini toping.

Javob: $dy=6x^2-6x+5)dx$

14. $y=\ln x-3x^3+x$ funktsiyaning birinchi tartibli differensialini toping.

Javob: $dy=(\frac{1}{x}-9x^2+1)dx$

15. $y=\cos x+2x^4+2x$ funktsiyaning uchinchi tartibli differensialini toping.

Javob: $d^3y=(\sin x+48x)d\,x^3$

16. $y=\sin 2x+2x^3+2$ funktsiyaning uchinchi tartibli differensialini toping.

Javob: $d^3 y = (-8\cos 2x + 12)d\,x^3$

17. $\lim\limits_{x\to 0} \arcsin x \cdot ctgx$ funktsiyani Lopital qoidasidan foydalanib limitini hisoblang.

Javob: 1

18. $\lim\limits_{x\to 0}(\frac{1}{x^2} - ctg^2 x)$ funktsiyani Lopital qoidasidan foydalanib limitini hisoblang.

Javob: $\frac{2}{3}$

19. $y = \frac{x-1}{x^2-2x}$ funksiyani tekshiring va grafigini chizing.

20. $y = \frac{x}{\ln x}$ funksiyani tekshiring va grafigini chizing.

Boshlang'ich funktsiya va aniqmas integral.

1.Boshlang'ich funksiya.

Biror oraliqda aniqlangan $f(x)$ funksiya uchun bu oraliqning hamma qiymatlarida

$$F'(x) = f(x) \text{ yoki } dF(x) = f(x)dx$$

shart bajarilsa, u holda $F(x)$ funksiya $f(x)$ funksiyani boshlang'ich funksiyasi deyiladi.

Agar $f(x)$ funksiya $F(x)$ boshlang'ich funksiyaga ega bo'lsa, u holda $F(x)+C$ $f(x)$ funksiyaning hamma boshlang'ich funksiyalari to'plami bo'ladi, bunda C – ixtiyoriy o'zgarmas. Shunga ko'ra berilgan $f(x)$ funksiyaning har qanday ikkita boshlang'ich funksiyasi bir – biridan ixtiyoriy o'zgarmasga farq qiladi.

$f(x)$ (yoki $f(x)$dx ifoda) dan olingan aniqmas integral deb, bu funksiyaning barcha $F(x)+C$ boshlang'ich funksiyalari to'plamiga aytiladi va bunday belgilaniladi: $\int f(x)dx = F(x)+C$

Aniqmas integralni toppish jarayoni integrallash deyiladi.

Aniqmas integralning asosiy xossalari (integrallash qoidalari)

a) $(\int f(x)dx)' = f(x)$

b) $d(\int f(x)dx)' = f(x)dx$

c) $d\int F(x) = F(x) + C$

d) $\int kf(x)dx = k\int f(x)dx$ (k- o'zgarmas)

e) $\int (f(x) \pm \varphi(x))dx = \int f(x)dx \pm \int \varphi(x)dx$

f) Agar $\int f(x)dx = F(x) + C$ va $y = \varphi(x)$ har qanday differensiallanuvchi funksiya bo'lsa, u holda ; $\int f(u)d\vartheta = F(u) + C$

2.Aniqmas integrallar jadvali:

$\int du = u + C$	$\int \cos udu = -\sin u + C$
$\int u^{\alpha} du = \dfrac{u^{\alpha+1}}{\alpha+1} + C, \quad \alpha \neq -1$	$\int \dfrac{du}{\cos^2 u} = tgu + C$
$\int \dfrac{du}{u} = \ln\lvert u\rvert + C$	$\int \dfrac{du}{\sin^2 u} = -ctgu + C$
$\int \dfrac{du}{\sqrt{u}} = 2\sqrt{u} + C$	$\int tgudu = -\ln\lvert\cos u\rvert + C$
$\int a^u du = \dfrac{a^u}{\ln a} + C$	$\int ctgudu = \ln\lvert\sin u\rvert + C$
$\int e^u du = e^u + C$	$\int \dfrac{du}{\sin u} = \ln\left\lvert tg\dfrac{u}{2}\right\rvert + C$
$\int \sin udu = \cos u + C$	$\int \dfrac{du}{\cos u} = \ln\left\lvert tg(\dfrac{\tau}{4} + \dfrac{u}{2}\right\rvert + C$

3.Integrallashning asosiy usullari.

O'zgaruvchini almashtirish usuli

Aniqmas integralda **o'zgaruvchini almashtirish** quyidagicha amalga oshiriladi:

a) $x = \varphi(t)$, bunda $\varphi(t)$– yangi o'zgaruvchi **t** ning differensiallanuvchi funksiyasi bo'lsin. Bu holda o'zgaruvchini almashtirish formulasi ushbu ko'rinishga ega:

$$\int f(x)dx = \int f(\varphi(t))\varphi(t)dt;$$

b) $\psi(x) = t$, bunda **t** – yangi o'zgaruvchi. Bu holda o'zgaruvchini almashtirish formulasi ushbu ko'rinishga ega:

$$\int f(\psi(x))\psi'(x)dx = \int f(t)dt.$$

Ikkala holda ham integrallashdan keyin o'zgaruvchi *x* ga qaytish kerak.

Bo'laklab integrallash usuli

$$\int ud\vartheta = u\vartheta - \int \vartheta du$$

formulaga asoslanadi, bunda **u** va **v — x** ning integrallanuvchi funksiyalari.

Bu usul har xil sinfdagi funksiyalar ko'paytmalarini integrallashda foydalaniladi:

$$\int P_n(x)e^{\alpha x}dx\ , \qquad \int P_n(x)\cos\alpha x dx\ , \qquad \int P_n(x)\sin\alpha x dx,$$

$$\int P_n(x)arctgx dx, \quad \int P_n(x)\sin x dx,$$

$$\int P_n(x)\cos x dx, \quad \int P_n(x)\ln x dx$$

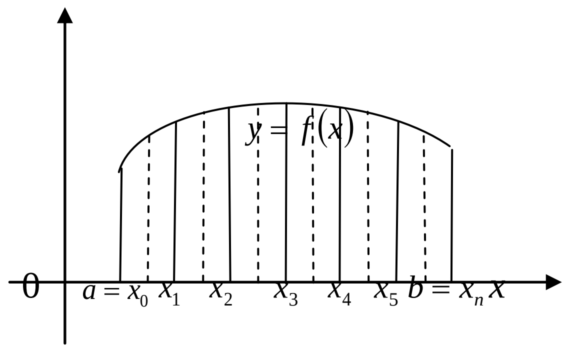
Dstlabki uchta integralda **u** uchun $P_n(x)$ ko'phad qabul qilinadi, oxirgi to'rtta integralda esa **u** uchun *arc tgx, arc sinx, arc cosx, lnx* qabul qilinadi.

Ba'zi hollarda bo'laklab integrallash formulasini bir necha marta qo'llash zarur bo'ladi.

4. Aniq integral va uning tadbiqlari.

1.Aniq integral tushunchasi.

Ixtiyoriy $y = f(x)$ funksiya biror $[a, b]$ oraliqda berilgan bo`lib, u uzluksiz bo`lsin. $[a, b]$ oraliqda n ta $x_0, x_1, x_2, x_3, \ldots, x_n$ ketma- ket nuqtalar olamiz. U holda, bu nuqtalar $[a, b]$ oraliqni n ta qismga ajratadi. Bunda $a = x_0$ va $b = x_n$ deb olamiz. Hosil bo`lgan elementar kesmalarni quyidagicha ifodalaymiz:



$[x_0, x_1],[x_1, x_2],\ldots,[x_{n-1}, x_n].$

$[x_0, x_1]$ kesmada $\xi_1, [x_1, x_2]$da $\xi_2, [x_2, x_3]$ da ξ_3 va hokazo, $[x_{n-1}, x_n]$da

$\xi_1 \quad \xi_2 \quad \xi_3 \quad \xi_4 \quad \xi_5 \quad \xi_n$

ξ_n nuqta olamiz. U holda, quyidagi yig`indi o`rinli bo`ladi:

$$S_n = f(\xi_1)(x_1 - x_0) + f(\xi_2)(x_2 - x_1) + \ldots + f(\xi_n)(x_n - x_{n-1}) \qquad (1)$$

yoki
$$\sum_{k=1}^{k=n} f(\xi_k)(x_k - x_{k-1}). \qquad (2)$$

$x_1 - x_0 = \Delta x_1, \quad x_2 - x_1 = \Delta x_2, \ldots, x_n - x_{n-1} = \Delta x_n$ belgilashlar kiritamiz. U holda (1) va (2) ni quyidagicha yozish mumkin:

$S_n = f(\xi_1)\Delta x_1 + f(\xi_2)\Delta x_2 + \ldots + f(\xi_n)\Delta x_n$ yoki

$$S_n = \sum_{k=n}^{n} f(\xi_k)\Delta x_k = f(\xi_1)\Delta x_1 + f(\xi_2)\Delta x_2 + \ldots + f(\xi_n)\Delta x_n. \qquad (3)$$

(3) ga $f(x)$ funksiyaning $[a, b]$ oraliqdagi ***integral yig`indisi*** deyiladi.

***Ta`rif:*** $f(x)$ funksiyaning $[a, b]$ kesmadag ***aniq integrali*** deb integral yig`indining elementar kesmalardan eng kattasining uzunligi $\lambda = 0$ bo`lgandagi limitiga aytiladi va quyidagi ko`rinishda ifodalanadi:

$$\int_a^b f(x)dx = \lim_{\lambda \to 0} \sum_{k=1}^{n} f(\xi_k)\Delta x_k. \qquad (4)$$

Bunda a - integralning quyi, b - yuqori chegarasidir. Integralning o`qilishi: «Integral a dan b gacha, ef iks de iks».

Agar $f(x)$ funksiya $[a, b]$ oraliqda uzluksiz bo`lsa, u holda integral yig`indi S_n chekli limitga ega bo`ladi, ya`ni qarralayotgan $f(x)$ funksiya $[a, b]$ da integrallanuvchi bo`lib, integral yig`indining limiti $[a, b]$ oraliqning bo`linish usuliga va har bir elementar kesmadagi $\xi$ nuqtaning olinishiga bog`liq bo`lmaydi.

2. Aniq integralning xossalari

1- xossa. Har qanday M o`zgarmas son uchun quyidagi tenglik o`rinli:

$$\int_a^b M dx = M(b - a). \qquad (1)$$

Isboti: $f(x) = M$ funksiyaning $[a, b]$ dagi integral yig`indisini qaraydik:

$$\sum_{i=1}^{n} f(\xi_1)\Delta x_i = \sum_{i=1}^{n} M \cdot \Delta x = M \sum_{i=1}^{n} \Delta x_i = M(b - a)$$

Demak, (1) tenglik o`rinli ekan.

2- xossa. O`zgarmas sonni integral belgisi oldiga chiqarish mumkin:

$$\int_a^b Mf(x)dx = M\int_a^b f(x)dx. \qquad (2)$$

Isboti: $Mf(x)$ funusiyaning $[a,b]$ dagi integral yig`indisi uchun quyidagi o`rinlidir:

$$\sum_{i=1}^{n} Mf(\xi_i)\cdot \Delta x_i = M\sum_{i=1}^{n} f(\xi_i)\cdot \Delta x_i.$$

Shuning uchun $\lim\limits_{n\to\infty}\sum_{i=1}^{n} Mf(\xi_i)\cdot \Delta x_i = M\lim\limits_{n\to\infty}\sum_{i=1}^{n} f(\xi_i)\cdot \Delta x_i = M\int_a^b f(x)dx.$

Demak, $Mf(x)$ funksiya $[a,b]$ oraliqda integrallanuvchi bo`lib, (2) formula o`rinli ekan.

3-xossa. Agar $f(x)$ va $g(x)$ funksiyalar $[a,b]$ oraliqda integrallanuvchi bo`lsa, ularning algebraik yig`indisi ham shu oraliqda integrallanuvchi bo`ladi, ya`ni:

$$\int_a^b [f(x)\pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx. \qquad (3)$$

Isboti: $\int_a^b [f(x)\pm g(x)]dx = \lim\limits_{n\to\infty}\sum_{i=1}^{n}[f(\xi_i)\pm g(\xi_i)]\Delta x_i = \lim\limits_{n\to\infty}\sum_{i=1}^{n} f(\xi_i)\Delta x_i \pm$

$$\pm\lim_{n\to\infty}\sum_{i=1}^{n} g(\xi_i)\Delta x_i = \int_a^b f(x)dx \pm \int_a^b g(x)dx.$$

4-xossa. Agar aniq integralning chegaralari o`zaro almashtirilsa, uning ishorasi qarama –qarshiga o`zgaradi:

$$\int_a^b f(x)dx = -\int_b^a f(x)dx. \qquad (4)$$

Isboti talabalarga havola qilinadi.

5-xossa. Chegaralari o`zaro teng, ya`ni $a=b$ bo`lgan aniq integral nolga teng:

$$\int_b^a f(x)dx = 0. \qquad (5)$$

Isboti talabalarga havola qilinadi.

6-xossa. Agar $f(x)$ fnksiya $[a, b]$da musbat bo`lib, $a < b$ bo`lsa, quyidagi tengsizlik o`rinli bo`ladi:

$$\int_a^b f(x)dx \geq 0. \qquad (6)$$

Isboti: $[a, b]$ oraliq ixtiyoriy $[x_{i-1}, x_i]$ elementar kesmalarga ajratilganda va ξ_i nuqta $[x_{i-1}, x_i]$ da ixtiyoriy tanlanganda $f(\xi_i) \geq 0$ va $\Delta x_i > 0$ bo`ladi. U holda,

$$\sum_{i=1}^{n} f(\xi_i)\Delta x_i \geq 0.$$

Bundan, $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n} f(\xi_i)\Delta x_i = \int\limits_a^b f(x)dx \geq 0.$

7-xossa. Agar $[a, b]$ oraliqda $a < b$ bo`lganda $f(x) \geq \varphi(x)$ bo`lsa,

$$\int_a^b f(x)dx \geq \int_a^b \varphi(x)dx. \qquad (7)$$

o`rinli bo`ladi.

Isboti: Shartga asosan $f(x) \geq \varphi(x)$. U holda, uni $[a, b]$da integrallaymiz:

$$\int_a^b [f(x) - \varphi(x)]dx \geq 0.$$

3-xossaga asosan $\int\limits_a^b f(x)dx - \int\limits_a^b \varphi(x)dx \geq 0.$

8-xossa. Agar $[a, b]$ oraliqda $a < b$ bo`lib, $m$ va $M$ lar $f(x)$ funksiyaning shu oraliqdagi eng kichik va eng katta qiymatlari bo`lsa, quyidagi o`rinli bo`ladi:

$$m(b-a) \leq \int_a^b f(x)dx \leq M(b-a). \qquad (8)$$

Isboti: Shartga asosan

$$m \leq f(x) \leq M.$$

Tengsizlikni $[a, b]$ oraliqda integrallaymiz:

$$\int_a^b m dx \le \int_a^b f(x)dx \le \int_a^b M dx.$$

U holda, 2- xossaga va 1- xossaga asosan

$$m(b-a) \le \int_a^b f(x)dx \le M(b-a).$$

9-xossa. (O`rta qiymat haqidagi teorema). Agar $f(x)$ funksiya $[a,b]$ oraliqda uzluksiz bo`lsa, bu oraliqda shunday c nuqta mavjud bo`ladiki, uning uchun

$$\int_a^b f(x)dx = f(c)(b-a). \qquad (9)$$

tenglik o`rinli bo`ladi.

Isboti: $[a,b]$ oraliqda m va M lar $f(x)$ funksiyaning eng kichik hamda eng katta qiymatlari bo`lsin. U holda, 8 –xossaga asosan

$$m(b-a) \le \int_a^b f(x)dx \le M(b-a).$$

Bundan, $m \le \frac{1}{b-a}\int_a^b f(x)dx \le M$. $f(x)$ funksiya uzluksiz bo`lganligi sababli u $[a,b]$ oraliqdagi barcha qiymatlarni qabul qiladi. U holda, $c$ $[a,b]$ dagi $x_1$ va $x_2$ nuqtalar orasida yotadi, ya`ni $x_1 < c < x_2$. Bundan

$$a < c < b.$$

Demak, $\frac{1}{b-a}\int_a^b f(x)dx = f(c)$ yoki $\int_a^b f(x)dx = (b-a)f(c).$

3. N`yuton-Leybnis formulasi

Faraz qilaylik, $f(x)$ funksiya $[a,b]$ oraliqda uzluksiz bo`lsin. U holda, funksiya shu oraliqda boshlang`ich funksiyaga ega bo`ladi. Boshlang`ich funksiyalaridan biri $Q(x)$ bo`lib, u quyidagidan iborat bo`lsin:

$$Q(x) = \int_a^x f(t)dt, \text{ bunda } a \le x \le b. \qquad (1)$$

Shu oraliqda $f(x)$ funksiyaning boshqa boshlang`ichi $F(x)$ ham mavjud bo`lsin. U holda, bu boshlang`ich funksiyalar bir – biridan biror $c$ o`zgarmas songa farq qilishi ma`lum, ya`ni

$$\int_a^x f(t)dt = F(x)+c. \qquad (2)$$

Agar $x=a$ bo`lsa, (1) tenglik hamda 5- xossaga asosan quyidagiga ega bo`lamiz:

$$\int_a^a f(t)dt = F(a)+c. \qquad (3)$$

$$F(a)+c=0, \quad c=-F(a). \qquad (4)$$

(4) ni (2) ga qo`ysak

$$\int_a^x f(t)dt = F(x)-F(a). \qquad (5)$$

hosil bo`ladi. $x=b$ deb olsak

$$\int_a^b f(x)dx = F(b)-F(a). \qquad (6)$$

(6) formulaga ***Nyuton –Leybnis formulasi*** deyiladi.

$F(b)-F(a)$ ayirmani quyidagi ko`rinishlarda yozish qabul qilingan.

$$F(x)\Big|_a^b \text{ yoki } [F(x)]_a^b$$

U holda, (6) formula bunday ifodalanadi:

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b)-F(a). \qquad (7)$$

Yuqori chegarasi o`zgaruvchidan iborat bo`lgan (5) aniq integralni hisoblashning Nyuton –Leybnis usuli quyidagi ko`rinishda bo`ladi:

$$\int_a^x f(t)dt = F(t)\Big|_a^x = F(x)-F(a). \qquad (8)$$

Shuningdek, quyi chegarasi o`zgaruvchidan iborat bo`lgan aniq integral ifodasi esa quyidagicha bo`ladi:

$$\int_x^b f(t)dt = F(t)\Big|_x^b = F(b) - F(x). \qquad (9)$$

$\int_a^b f(x)dx$ aniq integralni hisoblashda quyidagi bosqich ishlari ketma – ket bajariladi:

1. Quyidagi aniqmas integral topiladi:

$$\int f(x)dx = F(x) + c.$$

2. $F(x)+c$ ning $x=b$ dagi qiymati topiladi, ya`ni $F(b)+c.$
3. $F(x)+c$ ning $x=a$ dagi qiymati hisoblanadi, ya`ni $F(a)+c.$
4. $[F(b)+c]-[F(a)+c]=F(b)-F(a)$ ayirama topiladi.

O`rta qiymat haqidagi teorema

Teorema. Agar $f(x)$ funksiya $[a, b]$ kesmada uzluksiz bo`lsa, u holda, shu kesmada shunday $c$ nuqta mavjud bo`ladiki, uning uchun

$$\int_a^b f(x)dx = f(c)(b-a) \qquad (1)$$

tenglik o`rinli bo`ladi.

Isboti: Faraz qilaylik, $a<b$ bo`lsin. U holda, funksiyaning berilgan $[a, b]$ kesmadagi eng katta qiymati $M$ va eng kichik qiymati $m$ bo`lsin, ya`ni

$$m \le f(x) \le M\ . \qquad (2)$$

$[a, b]$ da (2) tengsizlikni integrallaymiz:

$$\int_a^b m dx \le \int_a^b f(x)dx \le \int_a^b M dx.$$

Bundan, $$m(b-a) \le \int_a^b f(x)dx \le M(b-a). \qquad (3)$$

(3)ni $b-a>0$ ga hadma – had bo`lamiz:

$$m \le \frac{1}{b-a}\int_a^b f(x)dx \le M \,. \qquad (4)$$

Berilgan $f(x)$ funksiya $[a, b]$ da uzluksiz bo`lganligi uchun qo`yi va yuqori chegara oralig`idagi (ya`ni [m, M]) istalgan qiymatni qabul qiladi. U holda, $[a, b]$ da shunday c nuqta mavjud bo`ladiki, $f(c)=\frac{1}{b-a}\int_a^b f(x)dx$ bo`lishini ta`minlaydi. Bu esa (1) formuladan iborat. Teorema isbot bo`ldi.

4. Aniq integralni o`zgaruvchini almashtirish (o`rniga qo`yish) usuli bilan hisoblash

Aniqmas integralni o`zgaruvchini almashtirish usulida yechishdan ma`lumki, agar integrallash qoidalari, xossalari yoki formulalar yordamida integrallash qiyinlik tug`dirsa integral ostidagi funksiyaga yangi o`zgaruvchi kiritish lozim. Aniq integralni hisoblashda ham shu usul qo`llaniladi.

$\int_a^b f(x)dx$ ni o`zgaruvchini almashtirish usulida hisoblash talab qilinsin. Yangi $t$ o`zgaruvchini kiritaylik. U holda, $x=\varphi(t)$. $\varphi(t)$ funksiya $[a, b]$ kesmada uzluksiz va differensiallanuvchi bo`lsin. Agarda $t$ o`zgaruvchi $[\alpha, \beta]$ kesmada o`zgarganda $x$ o`zgaruvchi $[a, b]$ da o`zgarsa, ya`ni $\varphi(\alpha)=a,\ \varphi(\beta)=b,$ hamda $f(\varphi(t))$ murakkab funksiya $[\alpha, \beta]$ kesmada uzluksiz va aniqlangan bo`lsa, quyidagi formula o`rinli bo`ladi:

$$\int_a^b f(x)dx = \int_\alpha^\beta f(\varphi(t))\cdot\varphi'(t)dt. \qquad (1)$$

(1) formulaga o`zgaruvchini almashtirish usulida integral formulasi deyiladi.

$F(x)$ funksiya $f(x)$ ning boshlang`ichi bo`lsin. U holda, $F(\varphi(t))$ funksiya $f(\varphi(t))\cdot\varphi'(t)$ ning boshlang`ichi bo`ladi. Shuning uchun

$$\int_\alpha^\beta f(\varphi(t))\cdot\varphi'(t)dt = F(\varphi(\beta))-F(\varphi(\alpha))=F(b)-F(a)=\int_a^b f(x)dx.$$

Demak, (1) formula hosil bo`ldi.

Yuqoridagilarni umumlashtirib, o`zgaruvchini almashtirish usulida integrallashni quyidagi ketma – ketlikda bajarish tavsiya qilinadi:

1. Imkoni bo`lsa, integral ostida berilgan ifodani soddalashtirish.
2. Yangi o`zgaruvchini kiritish ($x=\varphi(t)$).
3. Integralning yangi chegaralarini aniqlash.
4. Hosil bo`lgan integralni hisoblash.

5. Aniq integralni bo`laklab integrallash

$u(x)$ va $v(x)$ funksiyalar berilgan bo`lib, ular $[a, b]$ kesmada uzluksiz hosillalar ( $u'(x)$ va $v'(x)$ ) ga ega bo`lsin. U holda, bizga ma`lum bo`lgan

$$(u(x)v(x))' = u'(x)v(x)+u(x)v'(x) \qquad (1)$$

tenglikni $[a, b]$ kesmada integrallaymiz:

$$\int_a^b [u(x)v(x)]' dx = \int_a^b u'(x)v(x)dx + \int_a^b u(x)v'(x)dx . \qquad (2)$$

(2)ni quyidagi ko`rinishda ham ifodalash mumkin:

$$\int_a^b (uv)' dx = \int_a^b u' v dx + \int_a^b u v' dx. \qquad (3)$$

(3)ning chap tomonini quyidagicha yozish mumkin:

$$\int_a^b (uv)' dx = uv\Big|_a^b . \qquad (4)$$

U holda (3):

$$uv\Big|_a^b = \int_a^b u' v dx + \int_a^b u v' dx \qquad (5)$$

yoki
$$\int_a^b u dv = uv\Big|_a^b - \int_a^b v du. \qquad (6)$$

hosil bo`ladi. Bu formulaga ***aniq integralni bo`laklab integrallash formulasi*** deyiladi.

6.Aniq integralning tadbiqlari.

Tekis shakl yuzini *h*isoblash.

***a*)** Yuqorida egri chiziqli trapetsiyaning yuzini hisoblash uchun $S=\int_a^b f(x)dx$ formulani chiqardik. Bu erda *f(x) [a;b]* kesmada uzluktsiz va manfiy bo'lmagan funktsiyadir.

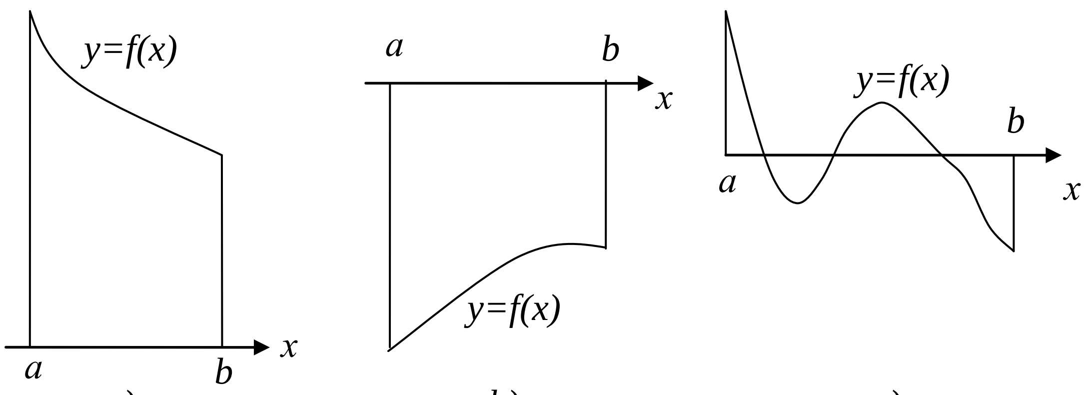
***b*)** Agar *f(x)*funktsiya [a;b] kesmada uzluktsiz va musbat bo'lmasa,

$$S=-\int_a^b f(x)dx$$

formula egri chiziqli trapetsiya yuzining qiymatini to'g'ri beradi (agar «-» ishora integral oldiga qo'yilmasa, yuza qiymati manfiy bo'lib qolar edi).

c) Agar f(x)funktsiya [*a*,*b*] oraliqda uzluktsiz va ishorasini o'zgartirsa, u holda egri chiziqli trapetsiya yuzining qiymati uchun

$S=\int_a^b |f(x)|dx$ formula o'rinlidir

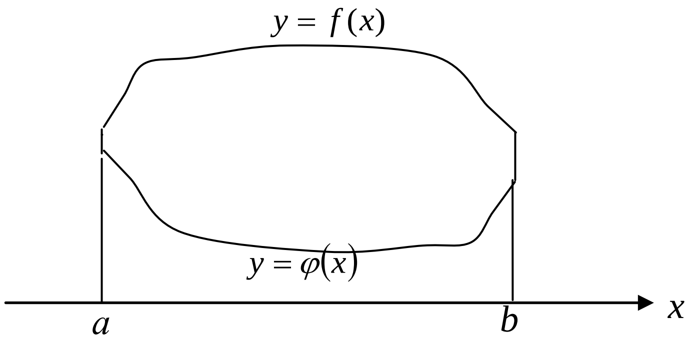


a) *b)* *c)*

d) Tekis shakl yuqoridan y=f(x) uzluktsiz funktsiya, quyidan esa y=φ(x) uzluksiz funktsiya grafiklari bilan [a;b] kesmada chegaralangan bo'lsa, uning yuzi uchun

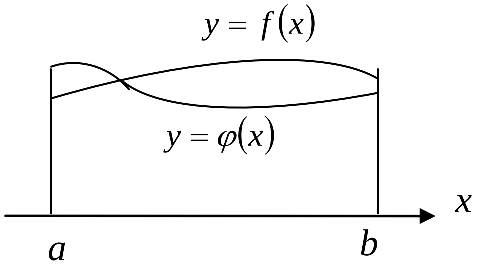
$$S=\int_a^b [f(x)-\varphi(x)]dx$$

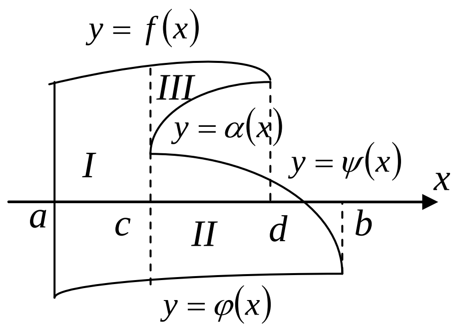
formula o'rinlidir, bu erda $\forall x\in[a;b]$, $\varphi(x)\le f(x)$.



e) Agar tekis shakl [a;b] kesmada y=f(x) va y=φ(x) uzluktsiz funktsiyalarning garfiklari bilan chegaralangan bo'lib, ular kesishsa, bu shakl yuzi uchun

$S=\int_a^b |f(x)-\varphi(x)|dx$ formula o'rinlidir





f) Agar tekis shakl murakkabroq bo'lib, yuqoridagi hollardan birortasiga ham to'g'ri kelmasa, uni bo'laklarga shunday ajratish kerakki, har bir bo'lakka yuqoridagi formulalardan biri to'g'ri kelsin. Masalan, o'ngdagi rasmdagi shaklni qarasak, uni uchta I,II va III bo'laklarga ajratilsa, bu rasmdan ko'rinadiki,

$$S_1=\int_a^c [f(x)-\varphi(x)]dx,\quad S_{II}=\int_c^b [\psi(x)-\varphi(x)]dx,\quad S_{III}=\int_c^d [f(x)-\alpha(x)]dx$$

larni hisoblab, tekis shakl yuzi uchun $S=S_I Q_{II} Q S_{III}$ ni olamiz.

Bunday egri chiziqli sektor yuzini hisoblash masalasini qo'yib, *[α;β]* kesmani ixtiyoriycha qilib, n ta bo'laklarga $\alpha=\varphi_0<\varphi_1<....<\varphi_{n=}\beta$ bo'lib har bir bo'linish nuqtasining qutb radiuslarini o'tkazsak, egri chiziqli sektor n ta bo'laklarga bo'linadi. Bu bo'laklardan k-siga mos keluvchi $[\varphi_{k-1};\ \varphi_k]$ kesmaga tegishli ψ_k ni olib, $f(\psi_k)$ ni hisoblab, bu bo'lak yuzi Δ S_k ning taqribiy qiymati sifatida radiusi $f(\psi_k)$ ga markaziy burchagi $\Delta\varphi_k q \varphi_k-\varphi_{k-1}$ ga teng bo'lgan doiraviy sektor yuzini qabul qilamiz. U holda, bu ishni barcha bo'laklar uchun bajargach, egri chiziqli sektor yuzi S ning taqribiy qiymati uchun

$$S \approx \sum_{k=1}^{n} \frac{1}{2} f^2(\psi_k) \cdot \Delta\varphi_k$$

ga ega bo'lamiz. Bu taqribiy tenglikning o'ng tomonidagi ifoda $\frac{1}{2} f^2(\varphi)$ funktsiyaning *[α;β]* kesma bo'yicha integral yig'indisi ekanligidan $\lambda = \max\limits_k \Delta\varphi_k \to 0$ dagi limitga o'tish natijasida

$$S = \frac{1}{2}\int_{\alpha}^{\beta} f^2(\varphi)d\varphi$$

ni olamiz. bu egri chiziqli sektorning yuzini hisoblash formulasidir.

Parallel kesimlarining yuzi bo'yicha jismning hajmini hisoblash.

Aytaylik, jism *Ox* o'qi yo'nalishi bo'yicha *[a;b]* kesmaga joylashgan bo'lib, $\forall x \in [a;b]$ nuqtada uning abssissalar o'qiga perpendikulyar kesimining yuzi S *(x)* uzluktsiz funktsiya orqali ifodalansin (1-rasmga qarang), ya'ni uning parallel kesimlarining yuzi har bir *x* uchun ma'lum bo'lsin. Uning hajmini hisoblash uchun *[a;b]* ni ixtiyoriycha qilib *n* ta bo'laklarga bo'lamiz va i-bo'lakka mos keluvchi x_{i-1} va x_i nuqtalar orqali o'tkazilgan Ox o'qqa perpendikulyar kesimlar orasida qolgan qismini asosi S(ξ_i) (bu yerda $\xi_i \in [x_{i-1}; x_i]$) va balandligi Δx_i bo'lgan silindr bilan almashtirib, bu bo'lakcha hajmining taqribiy qiymati uchun

$$\Delta v_i \approx S(\xi_i)\ \Delta x_i$$

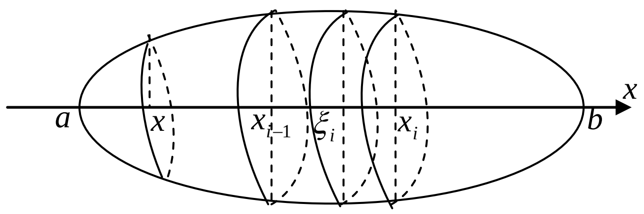
ni qabul qilamiz. Bu ishni barcha bo'lakchalar uchun bajarib, jism hajmining taqribiy qiymati uchun

$$V \approx \sum_{i=1}^{n} S(\xi_i)\Delta x_i$$

ga ega bo'lamiz. Bu taqribiy formulaning o'ng qismi *S(x)* funktsiyaning *[a;b]* oraliq bo'yicha integral yig'indisidir. Demak, $\lambda = \max\limits_i \Delta x_i \to 0$ dagi limitga o'tish natijasida

$$V = \int_a^b S(x)dx \qquad 1$$

ni olamiz. Bu *parallel kesimlarining yuzi ma'lum bo'lganda jismning hajmini hisoblash formulasidir.* $S(x)$ funktsiyani $[a;b]$ kesmada uzluktsiz deb faraz qilamiz.



1 –rasm.

Aylanish jismining hajmi.

Aytaylik, *[a;b]* kesmada manfiy bo'lmagan uzluktsiz yqf(x) funktsiya grafigi, *xqa, xqb, yq0* to'g'ri chiziqlar bilan chegaralangan egri chiziqli trapetsiya Ox o'qi atrofida aylanishidan jism hosil bo'lgan bo'lsin. Shu aylanish jismining hajmini topaylik

Agar *x∈[a;b]* nuqtadagi Ox o'qiga perpendikulyar kesimni qarasak, radiusi *yqf(x)* bo'lgan doiradan iboratdir ya'ni uning yuzi uchun *S(x)qπy²qπf²(x)* ni olamiz. Buni (1) ga qo'yib,

$$V = \pi\int_a^b y^2 dx = \pi\int_a^b f^2(x)dx \qquad 2$$

ga ega bo'lamiz. Bu *yuqorida aytilgan aylanish jismi hajmining formulasidir*

Mavzuga doir yechimlari bilan berilgan topshiriqlardan namunalar

1. $\int \frac{1+2x^2}{x^2(1+x^2)}dx$ aniqmas integralni toping.

Yechish: Integral ostidagi funktsiyaning shaklini almashtiramiz.

$$\int \frac{1+2x^2}{x^2(1+x^2)}dx = \int \frac{1+x^2+x^2}{x^2(1+x^2)}dx = \int \left(\frac{1+x^2}{x^2(1+x^2)} + \frac{x^2}{x^2(1+x^2)}\right)dx = \int \left(\frac{1}{x^2} + \frac{1}{1+x^2}\right)dx = \int \frac{1}{x^2}dx + \int \frac{1}{1+x^2}dx = -\frac{1}{x} + arctgx + C$$

2. $\int 4\cos^2\frac{x}{2}dx$ aniqmas integralni toping.

Yechish: Integral ostidagi funktsiyaning darajani pasaytirish formulasidan foydalanib shaklini almashtiramiz. $2\cos^2\frac{x}{2}=1+\cos x$

$$\int 4\cos^2\frac{x}{2}dx=\int 2(1+\cos x)dx=2\int dx+2\int(1+\cos x)dx=2\int dx+2\int\cos xdx=2x+2\sin x+C$$

3. $\int 3^x\cdot e^{3x}dx$ aniqmas integralni toping.

Yechish:

$$\int 3^x\cdot e^{3x}dx=\int(3\cdot e^3)^x dx=\frac{(3\cdot e^3)^x}{\ln 3\cdot e^3}+C$$

4. $\int\frac{dx}{\sqrt{2x-7}}$ aniqmas integralni toping.

Yechish: Differensial ostiga kiritish usulidan foydalanamiz: $dx=\frac{1}{2}d(2x-7)$

$$\int\frac{dx}{\sqrt{2x-7}}=\frac{1}{2}\int\frac{d(2x-7)}{\sqrt{2x-7}}=\frac{2}{2}\sqrt{2x-7}+C=\sqrt{2x-7}+C$$

5. $\int\frac{3x^2-8x}{x^3-4x^2+2}dx$ aniqmas integralni toping.

Yechish: Differensial ostiga kiritish usulidan foydalanamiz:

$$(2x^2-8x)dx=d(x^3-4x^2+2)$$

$$\int\frac{3x^2-8x}{x^3-4x^2+2}dx=\int\frac{d(x^3-4x^2+2)}{x^3-4x^2+2}=\ln\left|x^3-4x^2+2\right|+C$$

6. $\int\frac{dx}{x\sqrt{2x-4}}$ aniqmas integralni toping.

Yechish: Yangi o’zgaruvchi kiritish usulidan foydalanamiz:

$$\begin{cases}2x-4=t^2, & t=\sqrt{2x-4}\\ x=\frac{1}{2}(t^2+4), & dx=tdt\end{cases}$$

$$\int\frac{tdt}{\frac{1}{2}(t^2+4)\cdot t}=2\int\frac{dt}{4+t^2}=\frac{2}{2}arctg\frac{t}{2}+C=arctg\frac{\sqrt{2x-4}}{2}+C$$

6. $\int arctgxdx$ aniqmas integralni toping.

Yechish: Bo’laklab integrallash usulidan foydalanamiz: $\begin{cases}u=arctgx, & du=\frac{dx}{1+x^2}\\ d\vartheta=dx, & x=\vartheta\end{cases}$

$$\int arctgxdx=x\cdot arctgx-\int\frac{xdx}{1+x^2}=x\cdot arctgx-\frac{1}{2}\int\frac{d(1+x^2)}{1+x^2}=x\cdot arctgx-\frac{1}{2}\ln\left|1+x^2\right|+C$$

7. $\int \frac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^4 x} dx$ aniqmas integralni toping.

Yechish: Integral ostidagi funktsiya cosx ga nisbatan toq funktsiya, shuning uchun sinx=t belgilashdan foydalanamiz:

$$\int \frac{\cos^3 x + \cos^5 x}{\cos^2 x + \cos^4 x} dx = \int \frac{\cos^2 x(1+\cos^2 x)\cos x}{\sin^2 x + \sin^4 x} dx = \begin{cases} \sin x = t, \quad \cos^2 x = 1 - t^2 \\ \cos x dx = dt, \end{cases} =$$

$$= \int \frac{(1-t^2)(2-t^2)}{t^2+t^4} dt = \int \frac{t^4 - 3t^2 + 2}{t^4 + t^2} dt = \int (1 + \frac{2}{t^2} - \frac{6}{1+t^2}) dt = t - \frac{2}{t} - 6arctgt + C$$

Demak: $\int \frac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^4 x} dx = t - \frac{2}{t} - 6 \cdot arctgt + C = \sin x - \frac{2}{\sin x} - 6 \cdot arctg(\sin x) + C$

8. $\int_e^{e^2} \frac{dx}{x \ln x}$ aniq integralni hisoblang.

Yechish:

$$\int_e^{e^2} \frac{dx}{x \ln x} = \int_e^{e^2} \frac{d(\ln x)}{\ln x} = \ln|\ln x| \Big|_e^{e^2} = \ln(\ln e^2) - \ln(\ln e) = \ln(2\ln e) - \ln 1 = \ln(2 \cdot 1) - 0 = \ln 2$$

9. $\int_0^{\frac{\pi}{2}} \sin^3 x dx$ aniq integralni hisoblang.

Yechish:

$$\int_0^{\frac{\pi}{2}} \sin^3 x dx = \int_0^{\frac{\pi}{2}} \sin^2 x \cdot \sin x dx = -\int_0^{\frac{\pi}{2}} (1-\cos^2 x) d\cos x = -\int_0^{\frac{\pi}{2}} d\cos x + \int_0^{\frac{\pi}{2}} \cos^2 x d\cos x =$$

$$= -\cos x \Big|_0^{\frac{\pi}{2}} + \frac{1}{3}\cos^3 x \Big|_0^{\frac{\pi}{2}} = -(\cos\frac{\pi}{2} - \cos 0) + \frac{1}{3}(\cos^3\frac{\pi}{2} - \cos^3 0) = 1 - \frac{1}{3} = \frac{2}{3}$$

10. $\int_1^e x \cdot \ln^2 x dx$ aniq integralni hisoblang.

Yechish:

Bo’laklab integrallash usulidan foydalanamiz:

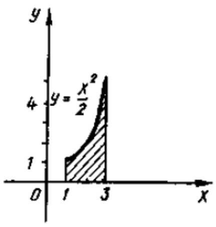
$$\int_1^e x\cdot\ln^2 xdx = \begin{cases} u=\ln^2 x, & du = 2\ln x\dfrac{dx}{x} \\ d\vartheta = xdx, & \vartheta = \dfrac{x^2}{2} \end{cases} = \frac{1}{2}x^2\cdot\ln^2 x\Big|_1^e - \int_1^e x\ln xdx =$$

$$= \begin{cases} u=\ln x, & du = \dfrac{dx}{x} \\ d\vartheta = xdx, & \vartheta = \dfrac{x^2}{2} \end{cases} = \frac{1}{2}e^2 - \left(\frac{x^2}{2}\cdot\ln x\Big|_1^e - \frac{1}{2}\int_1^e xdx\right) = \frac{1}{2}e^2 - \frac{1}{2}e^2 + \frac{1}{2}\int_1^e xdx =$$

$$= \frac{x^2}{4}\Big|_1^e = \frac{1}{4}(e^2-1)$$

11. $y=\frac{x^2}{2}$ parabola, x=1 , x=3 to’g’ri chiziq va Ox o’q bilan chegaralangan figurani yuzini hisoblang.

Yechish: Avval shaklni chizamiz:



Izlanayotgan yuza quyidagi formula bilan hisoblanadi.

$$S=\int_a^b ydx = \int_1^3 \frac{x^2}{2}dx = \frac{x^3}{6}\Big|_1^3 = \frac{27}{6}-\frac{1}{6} = \frac{26}{6} = \frac{13}{3} = 4\frac{1}{3}$$

Mustaqil yechish uchun topshiriqlar

1. $\int\frac{x-\arcsin x}{\sqrt{1-x^2}}dx$ aniqmas integralni toping.

Javob: $-\sqrt{1-x^2}-\frac{1}{2}\arcsin^2 x + C$

2. $\int\frac{dx}{\sqrt{2x-3}}$ aniqmas integralni toping.

Javob: $\sqrt{2x-3}+C$

3. $\int\frac{dx}{(x+1)\sqrt{x^2+x-1}}$ aniqmas integralni toping.

Javob: $\arccos\frac{x+3}{\sqrt{5}(x+1)}+C$

4. $\int (x^2+1)\cos x dx$ aniqmas integralni toping.

Javob: $2x\cos x + (x^2-1)\sin x + C$

5. $\int\limits_3^8 \frac{xdx}{\sqrt{x+1}}$ aniqmas integralni toping.

Javob: $\frac{32}{3}$

6. $\int\limits_1^2 \frac{dx}{2x+1}$ aniqmas integralni toping.

Javob: $\frac{1}{2}\ln 3$

7. $\int\limits_1^6 \frac{dx}{1+\sqrt{3x-2}}$ aniqmas integralni toping.

Javob: $\frac{2}{3}(3+\ln\frac{2}{5})$

8. $x = 2 - y - y^2$ egri chiziq va Oy o'q bilan chegaralangan figurani yuzini hisoblang.

Javob: $\frac{9}{2}$

FOYDALANILGAN ADABIYOTLAR

№	Muallif, adabiyot nomi, turi, nashriyot, yili, xajmi
1.	Yorqulov R., Jumaev M. ”Oliy matematika”,T.,2008y.
2.	Hamedova N.A. va bosh. ”Matematika”. OO’Yu uchun darslik,T.,Turon iqbol, 2007y.
3.	Hamedova N.A., Sadikova A.V., Laktayeva I.Sh. ”Matematika” – Gumanitar yo’nalishlar talabalari uchun o’quv qo’llanma. T, ”Jaxon-Print” 2007y.
4	Soatov Y.O.“Oliy matematika” I , I I I , I I I qism,Toshkent, 1994 y.
5	Tadjieva Z.G. “Matematikadan tarixiy materiallardan foydalanish”. T.: 2003y.
6.	Nazarov R.N., Toshpo’latov B.T., Dusumbetov A.F. “Algebra va sonlar nazariyasi”. T., O’qituvchi. 2- qism 1993., I I qism 1995y.
7.	Minorskiy V. “Oliy matematikadan masalalar to’plami”. T.: “O’qituvchi”,1988y.
8.	Hikmatov A., Toshmetov O’, Karasheva G. “Matematik anlizdan mashqlar va masalalar to’plami”,T.,1987y.
9.	Azlarov T.A., Mansurov X. “Matematik analiz” 1-2 qism. T.: “O’qituvchi”, 1974y.
10.	Feller V. Vvedenie v teoriyu veroyatnostey i ee prilojeniya, 1,2-tom. M.: Mir, 1984.
11.	Gnedenko B.V. Kurs teorii veroyatnostey. M.: URSS, 2005.
12.	Sirojiddinov S.X., Mamatov M.M. Ehtimollar nazariyasi va matematik statistika. T., 1972.
13.	Rasulov A.S., Raimova G.M., Sarimsakova X.Q. Ehtimollar nazariyasi va matematik statistika. T. 2005.