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ОЛИЙ ВА ЎРТА МАХСУС ТАъЛИМ ВАЗИРЛИГИ

ГУЛИСТОН ДАВЛАТ УНИВЕРСИТЕТИ

Физика – математика факултети

“Математика” кафедраси

5130100- “Математика” таълим йўналиши бўйича бакалавр
даражасини олиш учун

САРИЕВА ЗУҲРАНИНГ

«АЛ ва КХК ўқувчиларига триганометрик функцияларни ўргатиш усуллари»

мавзусида

БИТИРУВ МАЛАКАВИЙ ИШИ

Раҳбар: __________ физ-мат.ф.н., доц. Х.Норжигитов
имзо ф.и.ш. илмий даража, илмий унвон

БМИ “Математика” кафедрасининг 20___ йил __ май №__ сонли йиғилишида кўриб чиқилди ва ҳимояга тавсия этилади.

Кафедра мудири__________ физ-мат.ф.н., доц. Х.Норжигитов
имзо ф.и.ш. илмий даража, илмий унвон

Физика математика факултети декани томонидан ҳимоя қилишга рухсат этилади.

Факултет декани__________ п.ф.н. доц. Ш. Аширов
имзо ф.и.ш. илмий даража, илмий унвон

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М У Н Д А Р И Ж А

КИРИШ

Hozirgi vaqtda trigonometrik funksiyalar yordamida echiladigan masalalar qadim zamonlarda paydo bo'lgan. Qadimdan bunday masalalarni echa bilishga jiddiy talablarni astronomiya qo'ygan. Astronomlarni sferada yotgan katta doiralarning yoylaridan tuzilgan sferik uchburchaklarning tomonlari bilan burchaklari orasidagi munosabatlar qiziqtirgan. Ular tekis uchburchaklarni "echish"ga doir masalalarga qaraganda murakkabroq masalalarni echishni yaxshigina uddalaganlar.

Bizning trigonometrik jadvallarimiz o'rnida qadimgi matematiklar berilgan uzunliklardagi yoylarni tortib turuvchi vatarlar jadvalini tuzishgan. Eramizdan ilgari III – II asrlarda grek matematiklari tomonidan tuzilgan bunday qadimiy jadvallar bizgacha etib kelmagan. Vatar uzunliklari haqidagi bizgacha saqlanib qolgan eng qadimiy jadval Aleksandriyalik astronom Ptolemey (eramizning II asri) tomonidan tuzilgan. Bu jadvallarda aylana vatarlarining uzunliklari 30^0 dan oralatib berilgan. Vatar uzunliklari uch xonali oltmishlik kasrlar shaklida, ya'ni $\frac{a}{60}+\frac{b}{60^2}+\frac{c}{60^3}$, bunda *a, b, c* sonlari 0 dan 59 gacha bo'lgan butun sonlardir.

Sin, cos, tg, ctg, sec, cosec trigonometrik funksiyalar aylanada o'tkazilgan kesmalar uzunliklarining nisbatlari sifatida V – X asr hind va arab matematiklarida uchraydi. Hind matematigi Ariabxata (V asrning ohiri) $Sin^2\alpha+Cos^2\alpha=1$ formulani va hatto yarim burchak sinusi, kosinusi va tangensi formulalarini bilar edi. Bu formulalar unga shu funksiyalarning jadvallarini tuzish uchun xizmat qilgan.

G'arbiy Yevropada trigonometriya XV – XVI asrlarda aktiv rivojlandi. Bunda bir qator natijalar fransuz matematigi F.Vietga (1540-1603) tegishlidir.

Differensial hisob paydo bo'lishi bilan trigonometrik funksiyalarning hosilalari uchun formulalar topildi. Bu formulalar asosan I.Ньютonga ma'lum edi. Bu formulalarning geometrik usul bilan chiqarilishini Kotesning (1682 - 1716) ishlaridan topish mumkin. Argument $-\infty$ dan $+\infty$ gacha o'zgarganda trigonometrik funksiyalarning qanday o'zgarishi haqidagi ochiq tasavvurlar D.Vallis (1616 - 1703) ning asarlarida uchraydi. Ammo, umuman aytganda, L.Eyler (1707 - 1783) gacha bo'lgan matematiklar bu xususida uncha katta izchillik ko'rsatmadilar va ba'zi masalalarga bog'liq ravishda trigonometrik funksiyalarning aniqlanish sohalarini turli usullar bilan cheklab qo'ydilar. Son argumentning sonli funksiyalari yoki kesma uzunliklarining burchak kattaligiga yoki yoy uzunligiga bog'liqligi deyilganda nima nazarda tutilishi ochiq emas edi.

Trigonometrik funksiyalar nazariyasi hozirga ko'rinishi L.Eyler asarlari, jumladan uning "CHeksiz kichiklar analiziga kirish" 1748 yildagi kitobida oldi.

Umuman olganda, matematikaning, xususan trigonometriyaning rivojida nafaqat chet el olimlari, balki o'zimizning buюk allomalarimiz ham o'zlarini hissalarini qo'shganlar. Bulardan Muhammad al-Xorazmiy, Ahmad Farg'oniy, Abu Rayhon Beruniy, Mirzo Ulug'bek, Ali Qushchi, G'iyosiddin Jamshid al-Koshiy kabilardir.

Юlduzlarning osmon sferasidagi koordinatalarini aniqlash, sayyoralarning harakatlarini kuzatish, Oy va Quyosh tutilishini oldindan aytib berish va boshqa ilmiy, amaliy ahamiyatga molik masalalar aniq hisoblarni, bu hisoblarga asoslangan jadvallar tuzishni taqozo etar edi. Ana shunday astronomik jadvallar SHarqda "Zij"lar deb atalgan.

Muhammad al-Xorazmiy, Abu Rayhon Beruniy, Mirzo Ulug'bek kabi olimlarimizning matematik asarlari bilan birga "Zij"lari ham mashhur bo'lgan, ular lotin va boshqa tillarga tarjima qilingan. Evropada matematikaning, astronomiyaning taraqqiyotiga salmoqli ta'sir o'tkazgan.

Beruniyning "Qonun ma'sudiy" asarida sinuslar jadvali 15 minut oraliq bilan, tangenslar jadvali 1^0 oraliq bilan 10^{-8} gacha aniqlikda berilgan. Nihoyatda aniq "Zij"lardan biri Mirzo Ulug'bekning "Zij"i – "Ziji Ko'ragoniy" dir. Bunda sinuslar jadvali 1 minut oraliq bilan, tangenslar jadvali 0^0 dan 45^0 gacha – 1 minut oraliq bilan, 46^0 dan 90^0 gacha esa – 5 minut oraliq bilan 10^{-10} gacha aniqlikda berilgan.

G'iyosiddin Jamshid al-Koshiy "Vatar va sinus" haqida risolasida $\sin 1^0$ ni verguldan so'ng 17 xona aniqligida hisoblaydi

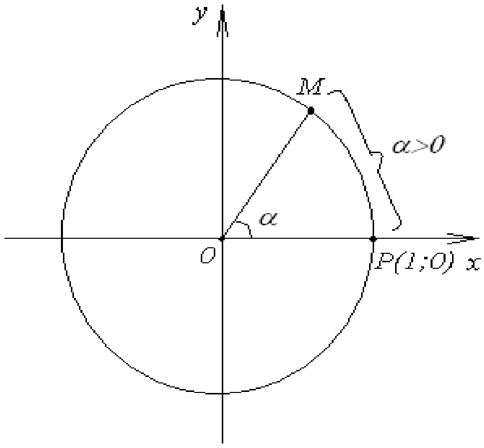
$$\sin 1^0 = 0{,}0174524643 7283512..$$

Buюk allomalarimiz qoldirgan izlarni o'z misollarimizda keng qo'llaymiz.

1-БОБ.TRIGONOMETRIЯ TA’RIFЛАРI ВА ТРИГОНОМЕТРИК ФОРМУЛАЛАР

§1.1. Таърифлар ва ишоралар

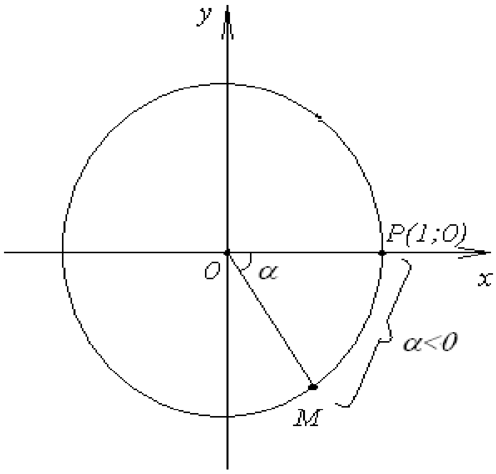
Koordinata tekisligida radiusi 1 ga teng va markazi koordinata boshida bo’lgan aylanani chizamiz. Bu aylana birlik aylana deyiladi.



a – rasm.

1. Aytaylik, $\alpha > 0$ bo’lsa, bu nuqta birlik aylana bo’ylab *R* nuqtadan soat milli yo’nalishiga qarama-qarshi harakat qilib, α uzunlikdagi yo’lni bosib o’tadi, deylik (a – rasm). Yo’lning oxirgi nuqtasini *M* bilan belgilaymiz.

Bu xolda *M* nuqta *R* nuqtani koordinata boshi atrofida α radian burchakka burish bilan hosil qilinadi deb ataymiz. 2. Aytaylik, $\alpha < 0$ bo’lsin. U holda α radian burchakka burish harakat soat milli yo’nalishida sodir bo’lganligini va nuqta α uzunlikdagi yo’lni bosib o’tganligini bildiradi.



b – rasm.

Geometriya kursida 0^0 dan 180^0 gacha bo'lgan burchaklar qaralgan. Birlik aylananing nuqtalarini koordinatalar boshi atrofida burishdan foydalanib, 180^0 dan katta burchaklarni, shuningdek manfiy burchaklarni ham qarash mumkin. Burish burchagini graduslarda ham, radianlarda ham berish mumkin. Masalan, R(1;0) nuqtani $\frac{\pi}{3}$ ga burish 60^0 ga burishni bildiradi, π ga burish 180^0 ga burishdir.

Nuqtani 360^0 dan katta burchakka va -360^0 dan kichik burchakka burishga oid misol ko'ramiz. Masalan, 810^0 burchakka burishda nuqta soat milli harakatiga qarama-qarshi ikkita to'la aylanishni va yana 90^0 yo'lni bosib o'tadi. Buni quyidagicha yozish mumkin: $810^0 = 2 \cdot 360^0 + 90^0$.

Agar -810^0 burchakka burish kerak bo'lsa, nuqta soat milli yo'nalishi -90^0 yo'lni bosadi.

R(1;0) nuqtani 810^0 burchakka burishda 90^0 ga burishdagi nuqtaning ayni o'zi hosil bo'ladi.

Har qanday gradusni son qiymati mavjuddir. Eng avvalo trigonometrik elementlariga ta'rif berib o'tsak.

1-ta'rif. α burchakning sinusi deb (1;0) nuqtani koordinatalar boshi atrofida α burchakka burish natijasida hosil bo'lgan nuqtaning ordinatasiga aytiladi ($\sin$ kabi belgilanadi)

$$y = \sin\alpha . \qquad (1)$$

2-ta'rif. α burchakning kosinusi deb (1;0) nuqtani koordinatalar boshi atrofida α burchakka burish natijasida hosil bo'lgan nuqtaning abssissasiga aytiladi ($\cos$ kabi belgilanadi)

$$x = \cos\alpha . \qquad (2)$$

3-ta'rif. α burchakning tangensi deb α burchak sinusini uning kosinusi nisbatiga aytiladi (tg kabi belgilanadi)

$$tg\alpha = \frac{\sin\alpha}{\cos\alpha} . \qquad (3)$$

$$ctg\alpha = \frac{\cos\alpha}{\sin\alpha} . \qquad (4)$$

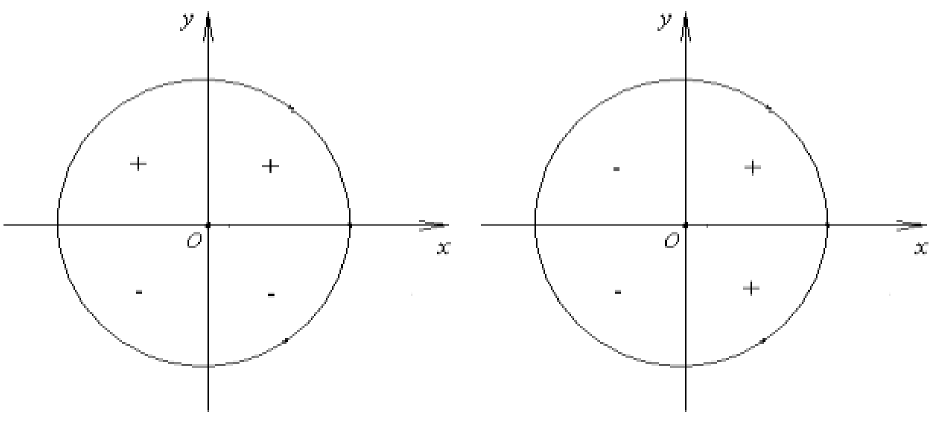
Sinus, kosinus, tangens, kotangensda ko'proq uchrab turadigan qiymatlari jadvalini keltiramiz.

Gradus	0^0	30^0	45^0	60^0	90^0	180^0	270^0	360^0
Radian	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
$tg\alpha$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	-	0	-	0
$ctg\alpha$	-	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	-	0	-

Har qanday trigonometrik elementlarni son qiymatini topish mumkin. Bundan tashqari ularni radiandan gradusga, gradusdan radianga aylantirish mumkin. U quyidagicha topiladi: $1rad=\left(\frac{180}{\pi}\right)^0$ va $\alpha rad=\left(\frac{180}{\pi}\alpha\right)^0$.

Aytaylik, (1;0) nuqta birlik aylana bo'ylab soat milli harakatiga qarama-qarshi harakat qilmoqda. Bu holda I chorakda joylashgan nuqtalarning ordinatalari va assissalari musbat. SHuning uchun, agar $0<\alpha<\frac{\pi}{2}$ bo'lsa, $\sin\alpha>0;$ $\cos\alpha>0$ bo'ladi.

II chorakda joylashgan nuqtalar uchun ordinatalar musbat, assissalar esa manfiy. SHuning uchun, agar $\frac{\pi}{2}<\alpha<\pi$ bo'lsa, $\sin\alpha>0;$ $\cos\alpha<0$ bo'ladi. SHunga o'xshash, III chorakda $\sin\alpha<0;$ $\cos\alpha<0$, IV chorakda esa $\sin\alpha<0;$



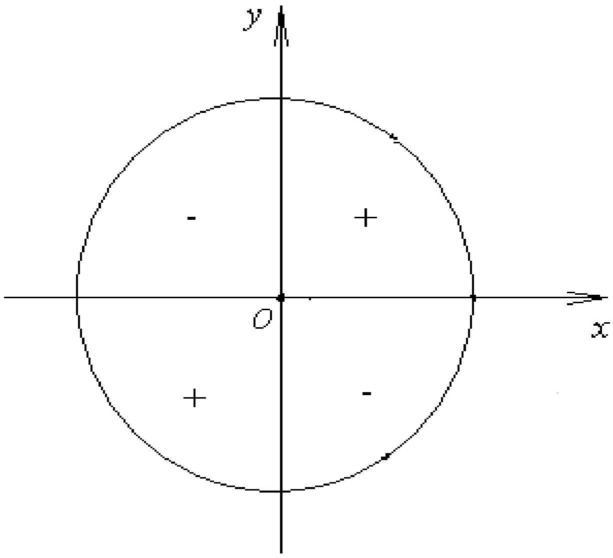
$\sin\alpha$ $\cos\alpha$

$\cos\alpha>0$.

Agar (1;0) nuqta soat milli yo'nalishida harakat qilsa, u holda ham sinus va kosinusning ishoralari nuqta qaysi chorakda joylashganiga qarab aniqlanadi.

Bizga ma'lumki, ta'rifga ko'ra $tg\alpha = \frac{\sin\alpha}{\cos\alpha}$. SHuning uchun, $\sin\alpha$ va $\cos\alpha$ bir xil ishoralarga ega bo'lsa, $tg\alpha > 0$, $\sin\alpha$ va $\cos\alpha$ qarama-qarshi ishoralarga ega bo'lsa, $tg\alpha < 0$ bo'ladi.

$ctg\alpha$ ning ishoralari $tg\alpha$ ning ishoralari bilan bir xil.



$tg\alpha$, $ctg\alpha$

§1.2. Тригонометрик формулалар

1.Sinus bilan kosinus orasidagi munosabatлар.

Aytaylik, birlik aylananing *M(x;u)* nuqtasi (1;0) nuqtani α burchakka burish natijasida hosil qilingan bo'lsin. U holda sinu va kosinus ta'rifiga ko'ra $x = \cos\alpha,\ \ y = \sin\alpha$ bo'ladi.

M nuqta birlik aylanaga tegishli, shuning uchun uning *(x;u)* koordinatalari $x^2 + y^2 = 1$ tenglamani qanoatlantiradi.

$$\sin^2\alpha + \cos^2\alpha = 1. \qquad (1)$$

(1) tenglik α ning istalgan qiymatida bajariladi va **<u>asosiy trigonometrik ayniyat</u>** deyiladi.

(1) tenglikdan sinusni kosinus orqali va aksincha kosinusni sinus orqali ifodalash mumkin:

$$\sin\alpha = \sqrt{1-\cos^2\alpha}, \qquad (2)$$

$$\cos\alpha = \sqrt{1-\sin^2\alpha}. \qquad (3)$$

Endi tangens bilan kotangens orasidagi bog’lanishni aniqlaymiz. Tangens va kotangens ta’rifiga qo’ra:

$$tg\alpha = \frac{\sin\alpha}{\cos\alpha}; \quad ctg\alpha = \frac{\cos\alpha}{\sin\alpha}.$$

Bu tenglamalarni ko’paytiramiz: $tg\alpha \cdot ctg\alpha = \frac{\sin\alpha}{\cos\alpha} \cdot \frac{\cos\alpha}{\sin\alpha} = 1$. Demak,

$$tg\alpha \cdot ctg\alpha = 1. \qquad (4)$$

(4) tenglikdan tangensni sotangens orqali va aksincha kotangensni tangens orqali ifodalash mumkin.

$$tg\alpha = \frac{1}{ctg\alpha}, \qquad (5)$$

$$ctg\alpha = \frac{1}{tg\alpha}. \qquad (6)$$

(4) – (6) tengliklar $\alpha \neq \frac{\pi}{2}k$ bo’lganda o’rinli bo’ladi.

Asosiy trigonometrik ayniyatdan va tangensning ta’rifidan foydalanib, tangens bilan kosinus orasidagi bog’liqlikni topamiz. $\cos\alpha \neq 0$ faraz qilib, $\sin^2\alpha + \cos^2\alpha = 1$ tenglikni ikkala qismini $\cos^2\alpha$ ga bo’lamiz:

$$\frac{\cos^2\alpha}{\cos^2\alpha} + \frac{\sin^2\alpha}{\cos^2\alpha} = \frac{1}{\cos^2\alpha},$$

$$1 + tg^2\alpha = \frac{1}{\cos^2\alpha}. \qquad (7)$$

Agar $\cos\alpha \neq 0$, ya’ni $\alpha \neq \frac{\pi}{2} + \pi k$ bo’lsa, (7) formula to’g’ri bo’ladi.

(7) formuladan tangensni kosinus va kosinusni tangens orqali ifodalash mumkin.

Bundan tashqari, asosiy trigonometrik ayniyatda kotangens bilan sinus orasidagi bog’liqlikni topish mumkin.

$$\sin^2\alpha + \cos^2\alpha = 1 \quad \Big| : \sin^2\alpha$$

$$\frac{\sin^2\alpha}{\sin^2\alpha} + \frac{\cos^2\alpha}{\sin^2\alpha} = \frac{1}{\sin^2\alpha},$$

$$1 + ctg^2\alpha = \frac{1}{\sin^2\alpha}. \qquad (8)$$

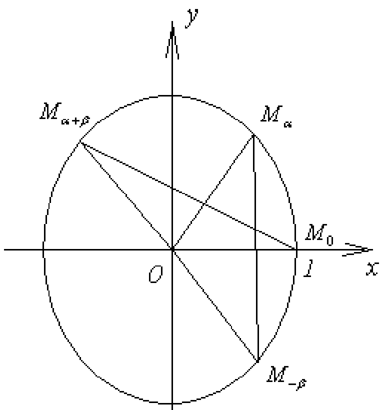
2. Қўшиш формулалари. Qo'shish formulalari deb $\cos(\alpha \pm \beta)$ va $\sin(\alpha \pm \beta)$ larni α va β burchaklarning sinus va kosinuslari orqali ifodalovchi formulalarga aytiladi.

Teorema. Ixtiyoriy α va β uchun quyidagi tenglik o'rinli bo'ladi

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta. \qquad (1)$$

Isbot: $M_0(1;0)$ nuqtani koordinatalar boshi atrofida α, $-\beta$, $\alpha+\beta$ radian burchaklarga burish natijasida mos ravishda M_α, $M_{-\beta}$ va $M_{\alpha+\beta}$ nuqtalar hosil bo'ladi.

Sinus, kosinus ta'rifiga ko'ra bu nuqtalar quyidagi koordinatalarga ega: $M_\alpha(\cos\alpha;\sin\alpha)$, $M_{-\beta}(\cos(-\beta);\sin(-\beta))$, $M_{\alpha+\beta}(\cos(\alpha+\beta);\sin(\alpha+\beta))$.



$\angle M_0OM_{\alpha+\beta}=\angle M_{-\beta}OM_\alpha$ bo'lgani uchun $M_0OM_{\alpha+\beta}$ va $M_{-\beta}OM_\alpha$ teng yonli uchburchaklar teng va ularning $M_0M_{\alpha+\beta}$ va $M_{-\beta}M_\alpha$ asoslari ham teng. Shuning uchun $(M_0M_{\alpha+\beta})^2=(M_{-\beta}M_\alpha)^2$. Geometriya kursidan ma'lum bo'lgan ikki nuqta orasidagi masofa formulasidan foydalanib, hosil qilamiz:

$$(1-\cos(\alpha+\beta))^2+(\sin(\alpha+\beta))^2=(\cos(-\beta)-\cos\alpha)^2+(\sin(\beta)-\sin\alpha)^2.$$

(1) formuladan foydalanib, bu tenglikni almashtiramiz:

$$1-2\cos(\alpha+\beta)+\cos^2(\alpha+\beta)+\sin^2(\alpha+\beta)=$$
$$=\cos^2\beta-2\cos\beta\cos\alpha+\cos^2\alpha+\sin^2\beta+2\sin\beta\sin\alpha+\sin^2\alpha$$

Asosiy trigonometrik ayniyatdan foydalanib, hosil qilamiz:

$2-2\cos(\alpha+\beta)=2-2\cos\alpha\cos\beta+2\sin\alpha\sin\beta$, bundan $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$.

(1) formuladan β ni $-\beta$ ga almashtirib, hosil qilamiz:

$$\cos(\alpha-\beta)=\cos\alpha\cos(-\beta)-\sin\alpha\sin(-\beta).$$

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta. \qquad (2)$$

Sinuslar uchun qo'shish formulasini keltirib chiqaramiz:

$$\sin(\alpha+\beta)=\cos\left(\frac{\pi}{2}-(\alpha+\beta)\right)=\cos\left(\left(\frac{\pi}{2}-\alpha\right)-\beta\right)=\cos\left(\frac{\pi}{2}-\alpha\right)\cos\beta+\sin\left(\frac{\pi}{2}-\alpha\right)\sin\beta=$$
$$=\sin\alpha\cos\beta+\cos\alpha\sin\beta.$$

Demak,

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta, \qquad (3)$$

$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta. (4)$$

$tg(\alpha+\beta)=\dfrac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}=\dfrac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}$. Bu kasrni surat va maxrajini $\cos\alpha\cos\beta$ ga bo'lib, quyidagi formulani hosil qilamiz:

$$tg(\alpha+\beta)=\frac{tg\alpha+tg\beta}{1-tg\alpha tg\beta}, \qquad (5)$$

$$tg(\alpha-\beta)=\frac{tg\alpha-tg\beta}{1+tg\alpha tg\beta}. \qquad (6)$$

3. Иккиланган бурчак учун формулалар. Qo'shish formulalaridan foydalanib, ikkilangan burchakning sinusi va kosinusi formulalarini keltirib chiqaramiz.

$\sin 2\alpha=\sin(\alpha+\alpha)=\sin\alpha\cos\alpha+\cos\alpha\sin\alpha=2\sin\alpha\cos\alpha$.

Demak,

$$\sin 2\alpha=2\sin\alpha\cos\alpha. \qquad (1)$$

$\cos 2\alpha=\cos(\alpha+\alpha)=\cos\alpha\cos\alpha-\sin\alpha\sin\alpha=\cos^2\alpha-\sin^2\alpha$.

$$\cos 2\alpha=\cos^2\alpha-\sin^2\alpha. \qquad (2)$$

$tg(\alpha+\beta)=\dfrac{tg\alpha+tg\beta}{1-tg\alpha tg\beta}$ bizga ma'lum. Biz $\beta=\alpha$ deb faraz qilib, tangensni ikkilangan burchagini topamiz:

$$tg 2\alpha=\frac{2tg\alpha}{1-tg^2\alpha}. \qquad (3)$$

4. Йиғинди ва айирмани кўпайтмага айлантириш.

Misol. Hisoblang: $\left(\sin\left(\alpha+\dfrac{\pi}{12}\right)+\sin\left(\alpha-\dfrac{\pi}{12}\right)\right)\sin\dfrac{\pi}{12}$.

Echish: qo'shish formulasi va ikkilangan burchak sinusi formulasidan foydalanib, quyidagiga ega bo'lamiz:

$$\left(\sin\left(\alpha+\frac{\pi}{12}\right)+\sin\left(\alpha-\frac{\pi}{12}\right)\right)\sin\frac{\pi}{12}=$$

$$=\left(\sin\alpha\cos\frac{\pi}{12}+\cos\alpha\sin\frac{\pi}{12}+\sin\alpha\cos\frac{\pi}{12}-\cos\alpha\sin\frac{\pi}{12}\right)\sin\frac{\pi}{12}=$$

$$=2\sin\alpha\cos\frac{\pi}{12}\sin\frac{\pi}{12}=\sin\alpha\sin\frac{\pi}{6}=\frac{1}{2}\sin\alpha.$$

Agar sinuslar yig'indisi formulasi

$$\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \qquad (1)$$

dan foydalanilsa, bu masalani soddaroq echish mumkin. SHu formula yordamida quyidagini hosil qilamiz:

$$\left(\sin\left(\alpha+\frac{\pi}{12}\right)+\sin\left(\alpha-\frac{\pi}{12}\right)\right)\sin\frac{\pi}{12}=2\sin\alpha\cos\frac{\pi}{12}\sin\frac{\pi}{12}=\frac{1}{2}\sin\alpha.$$

Endi (1) formula o'rinli ekanligini isbotlaymiz.

$\frac{\alpha+\beta}{2}=x$; $\frac{\alpha-\beta}{2}=y$ belgilash kiritamiz. U holda $x+y=\alpha$, $x-y=\beta$ va shuning uchun

$$\sin\alpha+\sin\beta=\sin(x+y)+\sin(x-y)=\sin x\cos y+\cos x\sin y+\sin x\cos y-\cos x\sin y=$$

$$=2\sin x\cos y=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}.$$

(1) formula bilan bir qatorda quyidagi sinuslar ayirmasi formulasi, kosinuslar yig'indisi va ayirmasi formulalaridan ham foydalaniladi:

$$\sin\alpha-\sin\beta=2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2} \qquad (2)$$

$$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \qquad (3)$$

$$\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \qquad (4)$$

5. Тарихий масалалар. Mashhur matematik Abul vafo Muhammad al-Buzjoniy (940 - 998) masalasi.

$\sin(\alpha-\beta)=\sqrt{\sin^2\alpha-\sin^2\alpha\sin^2\beta}-\sqrt{\sin^2\beta-\sin^2\alpha\sin^2\beta}$ bo'lishini isbotlang.

Isbot:

$$\sqrt{\sin^2\alpha-\sin^2\alpha\sin^2\beta}-\sqrt{\sin^2\beta-\sin^2\alpha\sin^2\beta}=\sqrt{\sin^2\alpha(1-\sin^2\beta)}-\sqrt{\sin^2\beta(1-\sin^2\alpha)}=$$

$$=\sqrt{\sin^2\alpha(\sin^2\beta+\cos^2\beta-\sin^2\beta)}-\sqrt{\sin^2\beta(\sin^2\alpha+\cos^2\alpha-\sin^2\alpha)}=$$

$$=\sqrt{\sin^2\alpha\cos^2\beta}-\sqrt{\sin^2\beta\cos^2\alpha}=\sin\alpha\cos\beta-\sin\beta\cos\alpha=\sin(\alpha-\beta).$$

Ayniyat isbotlandi. Eylerning quyidagi formulasini isbotlaylik:

$\sin(30^0+\alpha)=\cos\alpha-\sin(30^0-\alpha).$

Isbot:

$$\cos\alpha-\sin(30^0-\alpha)=\cos\alpha-(\sin 30^0\cos\alpha-\cos 30^0\sin\alpha)=$$

$$=\cos\alpha-\sin 30^0\cos\alpha+\cos 30^0\sin\alpha=\cos\alpha-\frac{1}{2}\cos\alpha+\frac{\sqrt{3}}{2}\sin\alpha=$$

$$=\frac{1}{2}\cos\alpha+\frac{\sqrt{3}}{2}\sin\alpha=\sin(30^0+\alpha).$$

Ayniyat isbotlandi.

Bu masalalardan ko'rinib turibdiki, bizning vatandosh allomalarimiz ham algebraga, shu qatori trigonometriyani yoritilishiga juda katta hissa qo'shganlar. Biz ulardan cheksiz fahrlanamiz!

6. Тригонометрик айниятлар

1. $\operatorname{tg}\alpha=\frac{\sin\alpha}{\cos\alpha}$

2. $ctg\alpha=\frac{\cos\alpha}{\sin\alpha}$

3. $\cos^2\alpha+\sin^2\alpha=1$

4. $\operatorname{tg}\alpha=\frac{1}{\operatorname{ctg}\alpha}$

5. $\operatorname{ctg}\alpha=\frac{1}{\operatorname{tg}\alpha}$

6. $\operatorname{tg}\alpha\cdot\operatorname{ctg}\alpha=1$

7. $1+\operatorname{ctg}^2\alpha=\frac{1}{\sin^2\alpha}$

8. $1+\operatorname{tg}^2\alpha=\frac{1}{\cos^2\alpha}$

9, 10. $\sin(\alpha\pm\beta)=\sin\alpha\cdot\cos\beta\mp\cos\alpha\cdot\sin\beta$

11, 12. $\cos(\alpha\pm\beta)=\cos\alpha\cdot\cos\beta\mp\sin\alpha\cdot\sin\beta$

13, 14. $\operatorname{ctg}(\alpha \pm \beta) = \dfrac{\operatorname{ctg}\alpha \cdot \operatorname{ctg}\beta \mp 1}{\operatorname{ctg}\beta \pm \operatorname{ctg}\alpha}$

15, 16. $\operatorname{tg}(\alpha \pm \beta) = \dfrac{\operatorname{tg}\alpha \pm \operatorname{tg}\beta}{1 \mp \operatorname{tg}\alpha \cdot \operatorname{tg}\beta}$

---

17. $\sin 2\alpha = 2\sin\alpha \cdot \cos\alpha$

18. $\cos 2\alpha = 1 - 2\sin^2\alpha$

19. $\cos 2\alpha = 2\cos^2\alpha - 1$

20. $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

21. $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$

22. $\sin 4\alpha = 8\cos^3\alpha \cdot \sin\alpha - 4\cos\alpha \cdot \sin\alpha$

23. $\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$

24. $\cos 4\alpha = 8\cos^4\alpha - 8\cos^2\alpha + 1$

25. $\operatorname{tg}2\alpha = \dfrac{2\operatorname{tg}\alpha}{1 - \operatorname{tg}^2\alpha}$

26. $\operatorname{ctg}2\alpha = \dfrac{\operatorname{ctg}\alpha - 1}{2\operatorname{ctg}\alpha}$

27. $\operatorname{tg}3\alpha = \dfrac{3\operatorname{tg}\alpha - \operatorname{tg}^3\alpha}{1 - 3\operatorname{tg}^2\alpha}$

28. $\operatorname{ctg}3\alpha = \dfrac{\operatorname{ctg}^3\alpha - 3\operatorname{ctg}\alpha}{3\operatorname{ctg}^2\alpha - 1}$

29. $\operatorname{tg}4\alpha = \dfrac{4\operatorname{tg}\alpha - 4\operatorname{tg}^3\alpha}{1 - 6\operatorname{tg}^2\alpha + \operatorname{tg}^4\alpha}$

30. $\operatorname{ctg}4\alpha = \dfrac{\operatorname{ctg}^4\alpha - 6\operatorname{ctg}^2\alpha + 1}{4\operatorname{ctg}^3\alpha - 4\operatorname{ctg}\alpha}$

---

31. $\sin\dfrac{\alpha}{2} = \sqrt{\dfrac{1 - \cos\alpha}{2}}$

32. $\cos\dfrac{\alpha}{2} = \sqrt{\dfrac{1 + \cos\alpha}{2}}$

33, 34. $\operatorname{tg}\dfrac{\alpha}{2} = \dfrac{\sin\alpha}{1 + \cos\alpha} = \dfrac{1 - \cos\alpha}{\sin\alpha}$

35, 36. $\operatorname{ctg}\frac{\alpha}{2}=\frac{\sin\alpha}{1-\cos\alpha}=\frac{1+\cos\alpha}{\sin\alpha}$

---

37. $\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}$

38. $\sin\alpha-\sin\beta=2\sin\frac{\alpha-\beta}{2}\cdot\cos\frac{\alpha+\beta}{2}$

39. $\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}$

40. $\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\cdot\sin\frac{\alpha-\beta}{2}$

41. $\operatorname{tg}\alpha+\operatorname{tg}\beta=\frac{\sin(\alpha+\beta)}{\cos\alpha\cdot\cos\beta}$

42. $\operatorname{tg}\alpha-\operatorname{tg}\beta=\frac{\sin(\alpha-\beta)}{\cos\alpha\cdot\cos\beta}$

43. $\operatorname{ctg}\alpha+\operatorname{ctg}\beta=\frac{\sin(\alpha+\beta)}{\sin\alpha\cdot\sin\beta}$

44. $\operatorname{ctg}\alpha-\operatorname{ctg}\beta=\frac{-\sin(\alpha-\beta)}{\sin\alpha\cdot\sin\beta}$

45. $\cos\alpha+\sin\alpha=\sqrt{2}\cdot\cos(45^0-\alpha)$

46. $\cos\alpha-\sin\alpha=\sqrt{2}\cdot\sin(45^0-\alpha)$

47. $\operatorname{tg}\alpha+\operatorname{ctg}\beta=\frac{\cos(\alpha-\beta)}{\cos\alpha\cdot\sin\beta}$

48. $\operatorname{tg}\alpha-\operatorname{ctg}\beta=\frac{-\cos(\alpha+\beta)}{\cos\alpha\cdot\sin\beta}$

49. $\operatorname{tg}\alpha-\operatorname{ctg}\alpha=-2\operatorname{tg}2\alpha$

---

50. $1+\cos\alpha=2\cos^2\frac{\alpha}{2}$

51. $1-\cos\alpha=2\sin^2\frac{\alpha}{2}$

52. $1+\sin\alpha=2\cos^2(45^0-\frac{\alpha}{2})$

53. $1-\sin\alpha=2\sin^2(45^0-\frac{\alpha}{2})$

---

54. $\sin^2\alpha=\frac{1-\cos2\alpha}{2}$

55. $\sin^3\alpha = \frac{1}{4}(3\sin\alpha - \sin 3\alpha)$

56. $\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$

57. $\cos^3\alpha = \frac{1}{4}(\cos 3\alpha + 3\cos\alpha)$

58. $\sin^4\alpha = \frac{1}{8}(\cos 4\alpha - 4\cos 2\alpha + 3)$

59. $\cos^4\alpha = \frac{1}{8}(\cos 4\alpha + 4\cos 2\alpha + 3)$

---

60. $\sin\alpha\cdot\cos\beta = \frac{1}{2}(\sin(\alpha+\beta) + \sin(\alpha-\beta))$

61. $\cos\alpha\cdot\cos\beta = \frac{1}{2}(\cos(\alpha+\beta) + \cos(\alpha-\beta))$

62. $\sin\alpha\cdot\sin\beta = \frac{1}{2}(\cos(\alpha-\beta) - \cos(\alpha+\beta))$

---

63. $\arcsin x = -\arcsin(-x) = \frac{\pi}{2} - \arccos x = \operatorname{arctg}\frac{x}{\sqrt{1-x^2}}$

64. $\arccos x = \pi - \arccos(-x) = \frac{\pi}{2} - \arcsin x = \operatorname{arcctg}\frac{x}{\sqrt{1-x^2}}$

65. $\operatorname{arcctg} x = -\operatorname{arctg}(-x) = \frac{\pi}{2} - \operatorname{arcctg} x = \arcsin\frac{x}{\sqrt{1+x^2}}$

66. $\operatorname{arcctg} x = \pi - \operatorname{arcctg}(-x) = \frac{\pi}{2} - \operatorname{arctg} x = \arccos\frac{x}{\sqrt{1+x^2}}$

67. $\arcsin(-x) = -\arcsin x$

68. $\arccos(-x) = \pi - \arccos x$

69. $\operatorname{arctg}(-x) = -\operatorname{arctg} x$

70. $\operatorname{arcctg}(-x) = \pi - \operatorname{arctg} x$

---

71. $\cos\alpha = \frac{1-\operatorname{tg}^2\frac{\alpha}{2}}{1+\operatorname{tg}^2\frac{\alpha}{2}}$

72. $\sin\alpha = \frac{2\operatorname{tg}\frac{\alpha}{2}}{1+\operatorname{tg}^2\frac{\alpha}{2}}$

---

73. $\sin x = a; |a| < 1$

$x = (-1)^n \arcsin a + \pi n, n \in Z$

74. $\cos x = a; |a| < 1$

$x = \pm \arccos a + 2\pi n, n \in Z$

---

75. $\operatorname{tg} x = a$

$x = \operatorname{arctg} a + \pi n, n \in Z$

76. $\operatorname{ctg} x = a$

$x = \operatorname{arcctg} a + \pi n, n \in Z$

---

77. при $|a| \le 1$

$0 \le \arccos a \le \pi$

$\cos(\arccos a) = a$

78. при $|a| \le 1$

$-\frac{\pi}{2} \le \arcsin a \le \frac{\pi}{2}$

$\sin(\arcsin a) = a$

79. при $\forall a$

$-\frac{\pi}{2} < \operatorname{arctg} a < \frac{\pi}{2}$

$\operatorname{tg}(\operatorname{arctg} a) = a$

80. при $\forall a$

$0 < \operatorname{arcctg} a < \pi$

$\operatorname{ctg}(\operatorname{arcctg} a) = a$

---

81. $\arcsin a + \arccos a = \frac{\pi}{2}$

82. $\operatorname{arctg} a + \operatorname{arcctg} a = \frac{\pi}{2}$

---

83. $\sin x = 0$

$x = \pi n, n \in Z$

84. $\cos x = 0$

$x = \frac{\pi}{2} + \pi n, n \in Z$

85. $\sin x = 1$

$x = \frac{\pi}{2} + 2\pi n, n \in Z$

86. $\cos x = 1$

$x = 2\pi n, n \in Z$

---

87. $\arcsin x = a$

$-\frac{\pi}{2} \le a \le \frac{\pi}{2}$

$x = \sin a$

88. $\arccos x = a$

$0 \le a \le \pi$

$x = \cos a$

2-БОБ. ТРИГОНОМЕТРИК ФУНКЦИЯЛАР ВА УЛАРНИНГ ХОССАЛАРИ

§2.1. *y = sin x, y = cos x   funksiyalar, ularning xossalaru va grafigi.*

Ta'rif: y = sinx va y = cosx funksiyalar mos ravishda sinus va kosinus deb ataladi.

Bu funksiyalarning aniqlanish sohalari barcha haqiqiy sonlar to'plamidan iborat, ya'ni D(y)= R.

Qiymatlar sohasi esa [-1; 1] kesmadan iborat, chunki birlik aylananing nuqtalari ordinata va absissalari –1 dan 1 gacha barcha qiymatlarni qabul qiladi. Demak,

D(sin) = D(cos) = R,   E(sin) = E(cos) = [-1;1]

Sinus va kosinus funksiyalarning ba'zi xossalarini eslatib o'tamiz, ya'ni $\forall x \in$ R uchun quyidagilar o'rinli bo'ladi.

1. sin (-x) = -sinx toq, cos (-x) = cosx juft.

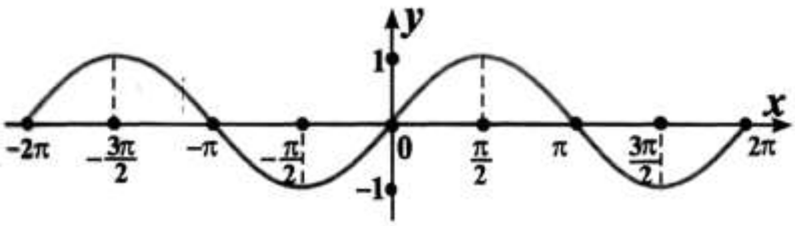
2. sin (2nπ +x) = sinx, cos (2nπ +x)= cosx $n \in z$ davriy $T = 2\pi$

Sinus va kosinus funksiya xossalaridan foydalanib ularning grafigini chizamiz.

1. y = sinx funksiyaning grafigi sinusoida deb ataladi. Sinx – davriy funksiya va uning asosiy davri 2π ga teng, toq funksiya. Funksiya davriy bo'lgani uchun uning grafigini [0;2π] kesmada yasaymiz.

Buning uchun OY o'qiga (0;-1) va (0;1) nuqtalarni , OX o'qida esa 2π ga teng nuqtani belgilaymiz. [0; 2π] kesmani va birlik aylanani teng qismlarga ajratamiz. Grafikning α absissali nuqtasini yasash uchun esa sinusning ta'rifidan foydalanamiz. Birlik aylanada P_α nuqtani belgilaymiz va OX o'qiga parallel to'g'ri chiziq o'tkazamiz bu to'g'ri chiziqning x = α chziq bilan kesishish nuqtasi ordinatasi izlanayotgan nuqta bo'ladi. Xuddi shu usulda qolgan nuqtalarni hosil qilamiz va bu nuqtalarni egri chiziq bilan tutashtirib y = sinx funksiyaning [0; 2π] kesmadagi grafigini hosil qilamiz. Funksiya davriy bo'lgani uchun hosil bo'lgan grafikning qismini $\pm 2\pi, \pm 4\pi; \pm 6\pi, \ldots$ parallel ko'chishlar yordamida y = sinx funksiyaning grafigi sinusoidani hosil qilamiz. Ordinatalar o'qining [-1;1] kesmasi sinuslar chizig'i ham deyiladi, bu kesma yordamida sinusning qiymatlari topiladi.

y=sin x funksiya

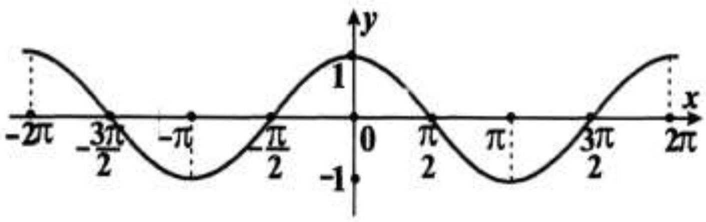


y=sin x funksiyaning xossalari

1. ***Aniqlanish sohasi:*** $D(y)=(-\infty;+\infty)$.
2. ***Qiymatlar sohasi:*** $E(y)=[-1;1]$.
3. ***Funksiya chegaralangan:*** $|\sin x| \le 1$.
4. ***Toq funksiya:*** $\sin(-x)=-\sin x$.
5. ***Davriy funksiya:*** $\sin(x+2\pi n)=\sin x;\ n\in Z$.
   Asosiy davri: $T(y)=2\pi$.
6. ***Funksiya quyidagi oraliqda o'suvchi:*** $[-\frac{\pi}{2}+2\pi k;\frac{\pi}{2}+2\pi k],\ k\in Z$.
   quyidagi oraliqda kamayuvchi: $[\frac{\pi}{2}+2\pi m;\frac{\pi}{2}+2\pi m],\ m\in Z$.

y = cosx funksiya grafigini yasash uchun cosx = $\sin(x+\frac{\pi}{2})$ ekanligidan foydalanamiz. Demak, kosinusning ixtiyoriy x_0 nuqtadagi qiymati sinusning $x_0+\frac{\pi}{2}$ nuqtadagi qiymatiga teng. Kosinus funksiya ham davriy ($T = 2\pi$) bo'lgani uchun grafigi sinus grafigini o x o'qining manfiy yo'nalishi $\frac{\pi}{2}$ masofa qadar parallel ko'chishdan iborat. Shu sababli y = cosx funksiyaning grafigi ham sinusoidadan iborat.

y=cos x funksiya



y=cos x funksiyaning xossalari

1. ***Aniqlanish sohasi:*** $D(y)=(-\infty;+\infty)$.
2. ***Qiymatlar sohasi:*** $E(y)=[-1;1]$.
3. ***Funksiya chegaralangan:*** $|\cos x| \le 1$.
4. ***Juft funksiya:*** $\cos(-x)=\cos x$.
5. ***Davriy funksiya:*** $\cos(x+2\pi n)=\cos x;\ n\in Z$.
   Asosiy davri: $T(y)=2\pi$.
6. ***Funksiya quyidagi oraliqda kamayadi:*** $[2\pi k;\pi+2\pi k],\ k\in Z$.
   quyidagi oraliqda o'sadi: $[-\pi+2\pi m;2\pi m],\ m\in Z$.

Bu grafiklardan y = sinx va y = cosx funksiyalarning o’sish va kamayish oraliqlarini yaqqol ko’rish mumkin.

y = sinx funksiya $(-\frac{\pi}{2}+2n\pi; \frac{\pi}{2}+2n\pi)$ o’sadi, $(\frac{\pi}{2}+2n\pi; \pi+2n\pi)$ kamayish.

y = cosx funksiya $((2n-1)\pi; 2n\pi)$ o’sadi, $(2n\pi; (2n+1)\pi)$ kamayadi.

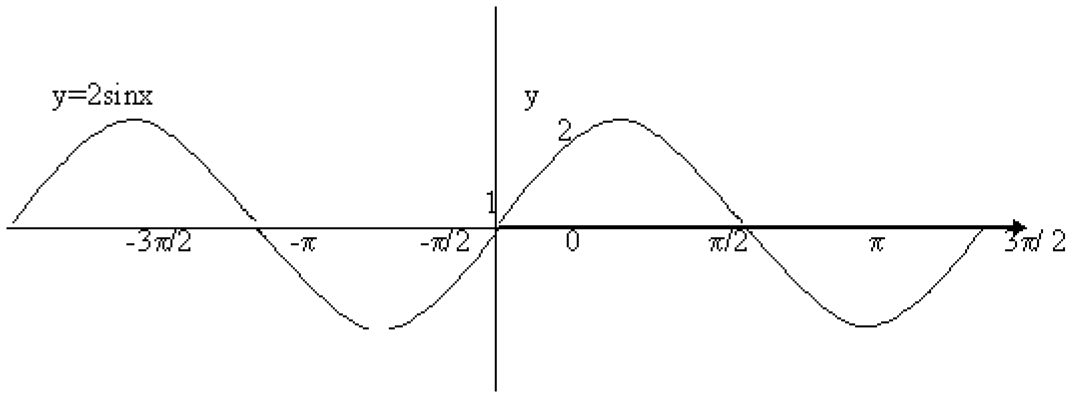
$$n \in Z$$

Ikkala funksiya uchun ham y=-1 min, y = 1 max qiymat .

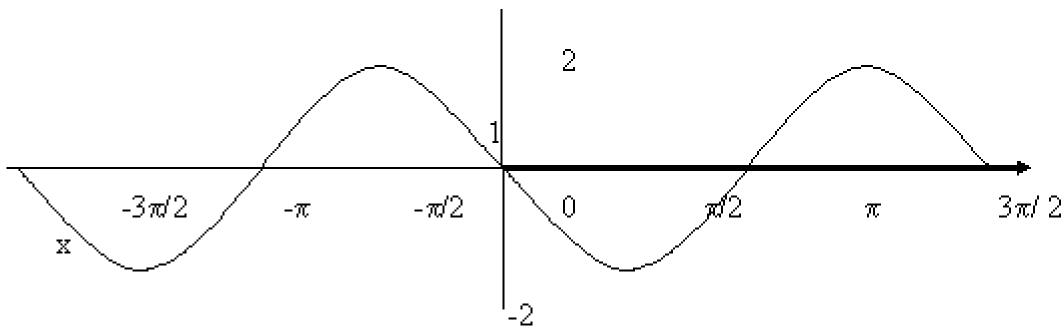
Misol. y = -2sinx funksiya grafigini yasang.

y = -2sinx funksiya grafigini yasash uchun y = sinx funksiya grafigidan foydalanamiz. Buning uchun sinusoidani OY o’qiga nisbatan 2 birlik cho’zib,

OX o’qi atrofida 180º ga burish kerak.



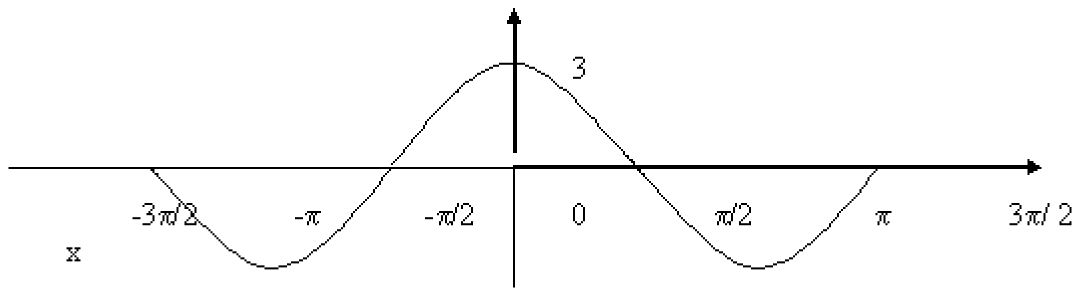
y=2sinx



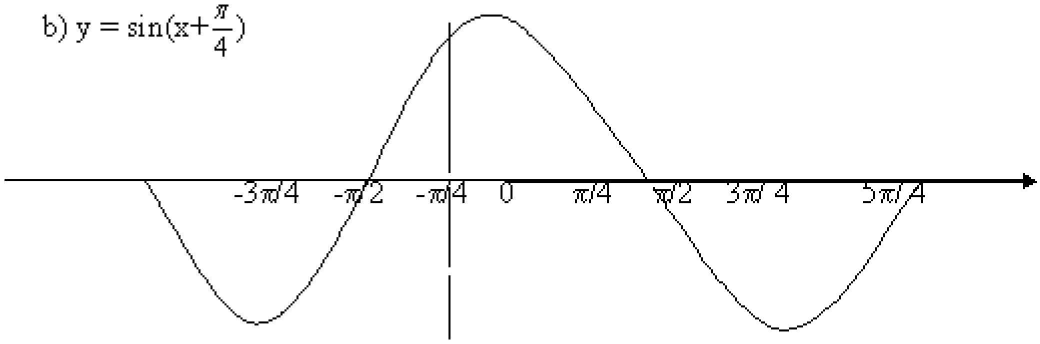
Mustahkamlash uchun mashqlar.

1.Funksiya grafigini yasang.

a) $y = 3\cos x$



b) $y = \sin(x + \frac{\pi}{4})$



1.67. Quyidagi funksiyalarni xossalarini aniqlang va grafigini yasang.

a) $y = \sin 3x$ d) $y = \frac{1}{3}\cos x$ j) $y = \sin x + 3$

b) $y = \cos 3x$ e) $y = 3\sin x$ i) $y = \cos x - 3$

c) $y = \frac{1}{3}\sin x$ f) $y = 3\cos x$ k) $y = 3\sin 0{,}5x + 1$

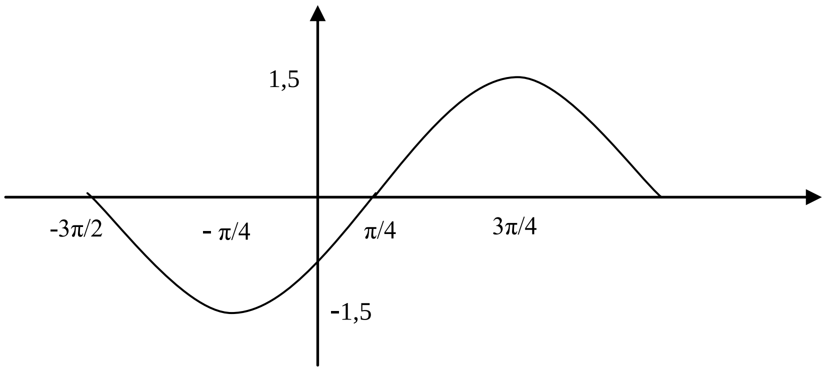
Test topshiriqlari.

1. $y = \sin(3x+1)$ funksiya davrini toping.

a) $\frac{2\pi}{3}$ b) π c) $\frac{\pi}{3}$ d) 2π e) 3π

2. $y = \cos\left(\frac{5x}{2} - \frac{5}{2}\right)$ funksiyaning eng kichik musbat davrini aniqlang.

a) $\frac{4\pi}{5}$ b) 2π c) π d) $\frac{2\pi}{5}$ e) $\frac{5\pi}{2}$



2. Rasmda quyidagi funksiyalardan qaysi birining grafigi tasvirlangan.

$-1,5$ sm $(2x+\frac{\pi}{4})$ D) $1,5 \sin(x-\frac{\pi}{4})$

$-1,5$sm $(2x-\frac{\pi}{4})$ E) $-1,5 \sin(x-\frac{\pi}{4})$

$1,5$sm $(2x+\frac{\pi}{4})$

3. $y = \sin\frac{x}{2}$ funksiya eng kichik musbat davrining $y = \cos 8x$ funksiya eng kichik musbat davriga nisbatini aniqlang.

a) 12 b) 14 c) 10 d) 18 e) 16

4. Quyidagi sonlardan eng kattasini aniqlang.

a) sin 170º b) sin 20º c) sin (-30º) d) sin (-250º) e)sin100º

§2.2. y = tgx, y = ctg x funksiyalar va ularning xossalari.

y = tgx funksiyaning xossalari.

1. D (y) = {x $|x \neq \frac{\pi}{2} + \kappa\pi$} chunki $y = tgx = \frac{\sin x}{\cos x}$ $\cos x + 0$ $x \neq k\pi + \frac{\pi}{2}$

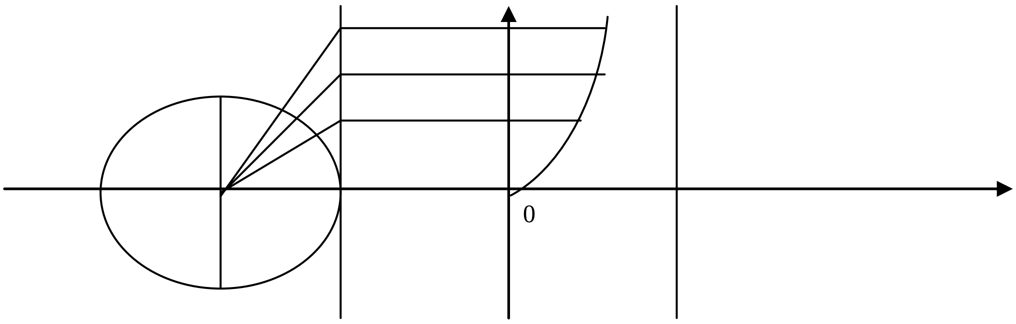
2. E(y) = ($-\infty;\infty$)

3. Funksiya toq $y = tgx = \frac{\sin x}{\cos x}$ $y(-x) = \frac{\sin(-x)}{\cos(-x)} = \frac{-\sin x}{\cos x} = -y$

4. Davriy T = π

Grafigini yasash uchun ($0;\frac{\pi}{2}$) oraliqda yasash so'ng uni koordinatalar boshiga nisbatan simmetrik akslantirish va absissalar o'qi bo'yicha $\pi\kappa$ ($\kappa \in Z$) qadar surish kerak.

Markazi $O_1(-1;0)$ nuqtada bo'lgan birlik aylananing AC = $\frac{\pi}{2}$yoki teng uzoqliklarda olingan B_1,B_2 … nuqtalar bilan bir nechta teng bo'lakka bo'lingan bo'lsin. Bu nuqtalar va O_1 nuqtadan o'tkazilgan $O_1B_1,O_1B_2,$… to'g'ri chiziqlar Al tangenslar chizig'I bilan T_1, T_2,.. nuqtalarda kesishsin. Chizmaga qaraganda $AT_1=\text{tg}\frac{\pi}{6}$, $AT_2=\text{tg}\ \frac{\pi}{3}$,… T_1, T_2 nuqtalardan o x o'qiga parallel va x = $\frac{\pi}{6},\frac{\pi}{3}$ nuqtalardan o y o'qiga o'tkazilgan parallel to'gri chiziqlarning kesishish nuqtalari bo'lsin. Ular ustidan o'tadigan to'g'ri chiziq y = tgx funksiyaning grafigi (tangensoida) bo'ladi.



Grafik x = $\frac{\pi}{2}$ to'g'ri chiziqqa tomon yaqinlshganida yuqoriga cheksiz ko'tariladi. Demak, y = tgx funksiya o'zining aniqlanish sohasida o'suvchidir.

y = ctg x funksiya xossalari.

1. D(y) ={x $|x \neq \kappa\pi$ } $\kappa \in Z$ ctgx = $\frac{\cos x}{\sin x}$ sinx $\neq 0$ x$\neq k\pi$, $\kappa \in Z$

2. E(y) = R = (- $\infty;\infty$)

3. Funksiya toq y = ctgx = $\frac{\cos x}{\sin x}$ y (-x)=$\frac{\cos(-x)}{\sin(-x)} = -\frac{\cos x}{\sin x} = -y$

4. Davriy T = π

y = ctgx funksiya grafigi ham xuddi yuqoridagi usulda yasaladi.

y = ctgx funksiya grafigi kotangensoida deb ataladi. y = ctgx funksiya o'zining aniqlanish sohasida kamayuvchi.

Mustaqil ishlash uchun mashqlar.1. Quyidagi funksiyalar grafiklarini yasang.

a) y = ctgx   d) y = 2ctgx   i) y = $|tg 2x|$

b) y = ctg (2x - $\frac{\pi}{3}$)   e) y = $|ctg 2x|$ k) y = tg2(x+$\frac{\pi}{4}$)

c) y = ctg $\frac{x}{2}$ j) y = $\frac{1}{2} tg 2x$ l) y = ctg $|x|$

m) y = tg $\left|x - \frac{\pi}{6}\right|$ n) y = $[ctgx]$ v) y = {ctgx}

p) y = [tg$\frac{x}{2}$]

2.Funksiya grafigini yasang.

1. y = tg (x+π)   2. y = 1+tgx

Test topshiriqlari. Quyidagi funksiyalar uchun eng kichik musbat davrini toping. y = tg3x, y = ctg6x, y = cos(3x+1) y = sin(6x+4)

a) $\frac{2\pi}{3}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) π e) 2π

2.Quyidagilardan qaysi biri toq.

a) y = x^4 *cos$\frac{x}{2}$ b) y = $|xctgx|$ c) y = sinx tg $\frac{x}{3}$ d) y = $|x| ctgx$ e) y = e^{x2}

3.Funksiyaning eng kichik musbat davrini toping.

y = 2sin $\frac{\pi x}{3} + 3\cos\frac{\pi x}{4} - tg\frac{x}{2}$

a) 12   b) 12π c) 2π d) 24π e) 24

4.y = ctg $\frac{x}{3} + tg\frac{x}{2}$ funksiya davrini toping.

a) 6π b) 2π c) 3π d) 12π e) 5π

5. Ushbu x = tg $\frac{5\pi}{7}$, y = sin $\frac{\pi}{6}$, z = tg $\frac{3\pi}{7}$ sonlar uchun quyidagilarni qaysi biri o'rinli.

a) z > y > x		b) x > z > y		c) y > x > z

d) x > y > z		e) y > z > x

§2.3. Айрим тригонометрик айниятларнинг исботлари.

1. Йиғиндини кўпайтмага айлантириш. Trigonometrik funksiyalar yig'indisini ko'paytmaga almashtirish formulalarini keltirib chiqaramiz. sinx ± siny = ? cosx ± cosy = ? tgx ± tgy = ? ctgx ± ctgy = ?

1) sinx ± siny = ?

Bizga ma'lumki ikki burchak yig'indisi va ayirmasining sinusi quyidagiga teng edi.

Sin $(\alpha+\beta)$ = sin $\alpha cos\beta$ + cos $\alpha \sin\beta$ (1)

Sin $(\alpha-\beta)$ = sin $\alpha cos\beta$ - cos $\alpha\sin\beta$ (2)

O'ng tomonlarini hadma-had qo'shib chiqamiz.

Sin $(\alpha+\beta)$ + Sin $(\alpha+\beta)$ = 2 sin $\alpha\cos\beta$ (3) bu erdan

$$\alpha+\beta=x$$
$$\alpha-\beta=y$$

almashtirish olamiz bunga $\alpha=\frac{x+y}{2}$ $\beta=\frac{x-y}{2}$ bu qiymatlarni (3) ga eltib qo'yamiz. va izlanayotgan formulani olamiz

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2} \quad (4)$$

2.sin x – sin y = ?

(1) – (2) = >

sin $(\alpha+\beta)$ – sin $(\alpha-\beta)$ = 2sin $\beta cos\alpha \Rightarrow \sin x+\sin y=2\cos\frac{x+y}{x}\sin\frac{x-y}{2}$ (5)

3. cos x + cos y = ?

cos $(\alpha+\beta)$ = cos $\alpha cos\beta-\sin\alpha\sin\beta$ (6)

cos $(\alpha-\beta)$ = $\cos\alpha\cos\beta-\sin\alpha\sin\beta$ (7)

(6) + (7) $\Rightarrow cos(\alpha+\beta)+\cos(\alpha-\beta)=2\cos\alpha\cos\beta \Rightarrow \cos x+\cos y=-2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$ (8)

4.cos x - cos y = ?

(6) – (7) cos($\alpha+\beta$) - cos($\alpha-\beta$) =

= $-2\sin\alpha\sin\beta \Rightarrow \cos x-\cos y=-2\sin\frac{x+y}{2}\sin\frac{x-y}{2}=2\sin\frac{x+y}{2}\sin\frac{y-x}{2}$ (9)

Misol: cos 45º + cos 15º =

$$2\cos\frac{45^0+15^0}{2}cos\frac{45^0-15^0}{2}=2\cos30^0\cos15^0=2\cdot\frac{\sqrt{3}}{2}\cdot\sqrt{\frac{\sqrt{3}+2}{4}}==\frac{\sqrt{3\sqrt{3}+6}}{2}$$

5. tg x + tg y = ?

tg x + tg y = $\frac{\sin x}{\cos x}+\frac{\sin y}{\cos y}=\frac{\sin x\cos y+\cos x\sin y}{\cos x\cos y}=\frac{\sin(x+y)}{}\cos x\cos y$

x,y $\neq\frac{\pi}{2}+k\pi \quad k\in z$

tg x – tg y = $\frac{\sin x}{\cos x}-\frac{\sin y}{\cos y}=\frac{\sin x\cos y-\sin y\cos x}{\cos x\cos y}=\frac{\sin(x-y)}{\cos x\cos y}$

x_1y$\neq\frac{\pi}{2}+k\pi \quad k\in z$

Quyidsagi formulalar ham shu tartibda keltirib chiqariladi.

ctg x + ctg y = $\frac{\sin(x+y)}{\sin x\sin y}$, x,y $\neq k\pi$, $k\in z$

ctg x – ctg y = $\frac{\sin(x-y)}{\sin x\sin y}$, x,y $\neq k\pi$, $k\in z$

Misol: u + v + w = π bo'lsa ctg u +ctg v - ctg w bo'lishini isbotlang.

Echish: ctg u +ctg v - ctg w = ctg u + ctg v – tg (π- (u+v) = ctg u +ctg v + tg(u+v) =

$$\frac{\sin(u+v)}{\sin u\sin v}+\frac{\sin(u+v)}{\cos(u+v)}=\frac{\sin(u+v)\cos(u+v)+\sin(u+v)\sin u\sin v}{\cos(u+v)\sin u\sin v}=\frac{\sin(u+v)(\cos(u+v)+\sin u\sin v)}{\cos(u+v)(\sin u\sin v)}=$$

$$=\frac{\sin(u+v)\cos u\cos v}{\cos(u+v)\sin u\sin v}=ctguctgvtg(u+v)=ctguctgvtg(\pi-w)=ctguctgvtgw$$

Mustahkamlash uchun mashqlar. Hisoblang.

1) cos 80ºcos 40ºcos 20º 4) $\frac{2cos80^0-\cos20^0}{\sin10^0}$

2) tg 35ºtg 55º 5. sin 76º - sin 26º

2. a) cos 43º + cos 37º b) sin 18º + cos 15º c) $\sin\frac{\pi}{5}-\cos\frac{\pi}{7}$

3. Ayniyatni isbotlang.

1) $\frac{\sin(t+s)-\sin(t-s)}{\sin(t+s)+\sin(t+s)} = ctgtctgs$

4. (1+sint sin s + cos t cos s) = 4 $\cos^2\frac{t-s}{2}$

tg 20º + tg 40º + tg 80º - tg 60º = 8cos 50º

Test topshiriqlari.

1. $\frac{cos6\alpha-\cos4\alpha}{\sin5\alpha}=?$

a) 2 sin α b) 2 cos α c) – 2cosα d) sinα e) –2sinα

2.Soddalashtiring. $\frac{\sin4\alpha-\sin6\alpha}{\cos5\alpha}$

a) sin 2α b) 2 sinα c) –2cosα d) –2sinα e) 2 cosα

3.sin 75º - sin 15º = ?

a) $\frac{\sqrt{2}}{2}$ b) $\frac{\sqrt{3}}{2}$ c) $\sqrt{2}$ d) $-\sqrt{2}$ e) $\frac{-2}{\sqrt{2}}$

4. $\frac{1+cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}}{1-\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}}=?$

a) tg $\frac{\alpha}{4}$ b) cos$\frac{\alpha}{4}$ c) - ctg$\frac{\alpha}{4}$ d) sin$\frac{\alpha}{4}$ e) – tg $\frac{\alpha}{4}$

5.cos $\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}=?$

a) -1/2 b) ¼ c) 1/3 d) $\frac{\sqrt{3}}{3}$ e) $\frac{-\sqrt{3}}{2}$

2.Кўпайтмани йиғиндига айлантириш. Trigonometrik funksiyalar uchun ko'paytmani yig'indiga keltirish, ya'ni $\sin\alpha\sin\beta$, $\cos\alpha\cos\beta$, $\mathrm{tg}\alpha\,\mathrm{tg}\beta$, $\mathrm{ctg}\alpha\,\mathrm{ctg}\beta$, $\sin\alpha\cos\beta$ larni qanday ko'rinishda bo'lishini ko'rib chiqamiz.

Bizga ma'lum bo'lgan 2 burchak yig'indisi va ayirmasi uchun formulalarni yozamiz.

Sin ($\alpha+\beta$) = $\sin\alpha\cos\beta$ (1)

Sin (α - β) = $\sin\alpha\cos\beta$ - $\cos\alpha\sin\beta$ (2)

Cos ($\alpha+\beta$) = $\cos\alpha\cos\beta$ - cos $\alpha\sin\beta$ (3)

Cos (α - β) = $\cos\alpha\cos\beta$ + $\sin\alpha\sin\beta$ (4)

(1) + (2) $\Rightarrow \sin(\alpha+\beta)+\sin(\alpha-\beta)=2\sin\alpha\cos\beta$

$\sin\alpha\cos\beta = \frac{1}{2}(\sin(\alpha+\beta)+\sin(\alpha-\beta)$ (5)

(1) $-$ (2) $\Rightarrow \sin(\alpha+\beta)-\sin(\alpha-\beta)=2\cos\alpha\sin\beta$

$\cos\alpha\sin\beta = \frac{1}{2}(\sin(\alpha+\beta)-\sin(\alpha-\beta)$ (6)

(2) + (4) $\Rightarrow \cos(\alpha+\beta)+\cos(\alpha-\beta)=2\cos\alpha\cos\beta$

$\cos\alpha\cos\beta = \frac{1}{2}(\cos(\alpha+\beta)+\cos(\alpha-\beta)$ (7)

(3) $-$ (4) $\Rightarrow \cos(\alpha+\beta)-\cos(\alpha-\beta)=-2\sin\alpha\sin\beta$

$\sin\alpha\sin\beta = \frac{1}{2}(\cos(\alpha-\beta)-\cos(\alpha+\beta)$ (8)

tg α tgβ ni topamiz. Buning uchun quyidagicha ish olib boramiz.

$$(8):(7)\Rightarrow \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}=\frac{\frac{1}{2}(\cos(\alpha-\beta)-\cos(\alpha+\beta))}{\frac{1}{2}(\cos(\alpha+\beta)+\cos(\alpha-\beta))}=\frac{\cos(\alpha-\beta)-\cos(\alpha+\beta)}{\cos(\alpha+\beta)+\cos(\alpha-\beta)} \quad (9)$$

$$(7):(8)\Rightarrow ctg\alpha ctg\beta=\frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{\cos(\alpha-\beta)-\cos(\alpha+\beta)} \quad (10)$$

Mustahkamlahs uchun misollar.

1. a) 2 sin 22º cos nº   b) sin x sin (x-1)

c. 4 sin 35º cos 25º sin 15º   d) 8 cos 3º cos 6º cos 12º cos 24º

2. sin x + sin 2x +sin 3x + sin 4x

3. $\sin 2x - \cos x - \sin 5x$

4. Isbotlang.

$\cos 9^\circ \cos 27^\circ \cos 63^\circ \cos 81^\circ + \cos 12^\circ \cos 24^\circ \cos 48^\circ \cos 96^\circ = 0$

$\cos 9^\circ \cos 27^\circ \cos 63^\circ \cos 81^\circ = \frac{1}{2}(\cos 90^\circ + \cos 72^\circ) \cdot \frac{1}{2}(\cos 90^\circ + \cos 36^\circ) = \frac{1}{4}\cos 72^\circ \cos 36^\circ$

$= \frac{1}{4}\sin 18^\circ \cos 36^\circ = \frac{1}{8}\cdot\frac{2\sin 18\cdot\cos 18\cdot\cos 36}{\cos 18} = \frac{1}{8}\frac{\sin 24\cos 24\cos 48 cjo 96}{\sin 12} = \frac{1}{4}\frac{\sin 48\cos 48\cos 96}{\sin 12} = \frac{1}{16}$

Test topshiriqlari.

1.$\sin 20^\circ \sin 40^\circ \sin 80^\circ$

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{\sqrt{3}}{3}$ d) $\frac{5}{4}$ e) $5\sqrt{3}$

2. $\frac{\sin^2 2{,}5\alpha - \sin^2 1{,}5\alpha}{\sin 4\sin\alpha + \cos 3\alpha\cos 2\alpha}$

a) $2\,\mathrm{tg}\,2\alpha$ b) $\mathrm{tg}\,2\alpha\,\mathrm{tg}\alpha$ c) $2\sin 2\alpha$ d) $4\cos^2\alpha$ e) $4\sin^2\alpha$

3. $8\sin^2\frac{15\pi}{16}\cdot\cos^{21}\frac{7\pi}{16} - 1$

a) $\frac{-\sqrt{2}}{2}$ b) $\frac{\sqrt{2}}{2}$ c) 1 d) ½ e) $\sqrt{2}$

$\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ$

a) $\frac{1}{2}$ b) 1/3 c) ¼ d) $1\sqrt{3}$ e) $\frac{1}{16}$

5.$\cos\frac{\pi}{5} cos \frac{2\pi}{5}$ a) $\frac{1}{2}$ b) $\frac{1}{8}$ c) $\frac{1}{4}$ d) $\frac{1}{12}$ e) $\frac{3}{4}$

6. $\sin 15^\circ$ ning qiymati $\cos 20^\circ \cos 40^\circ \cos 80^\circ$ ning qiymatidan qancha katta?

a) $\frac{1}{8}$ b) $\frac{5}{8}$ c) $\frac{3}{8}$ d) $\frac{7}{8}$ e) $\frac{1}{4}$

3-БОБ. ТРИГОНОМЕТРИК ТЕНГЛАМА ВА ТЕНГСИЗЛИКЛАР

§3.1. Энг содда тригонометрик тенгламалар.

1.cosx = a ko'rinishdagi tenglama

$\cos = a \ (-1 \leq a \leq 1)$

$x = \pm \arccos a + z\pi k, \ k \in Z$

Xususiy hollarda:

a) $\cos x = 1, \quad x = 2\pi k, \ k \in Z$

b) $\cos x = 0, \ x = \frac{\pi}{2} + \pi k, \ k \in Z$

c) $\cos x = \pi + 2\pi k, \ k \in Z$

1. $\cos^2 x = a \ (0 \leq a \leq 1)$ tenglamani echish:

$x = \pm \arccos \sqrt{a} + \pi k, \ k \in Z$

Misol: Tenglamani eching:

$\cos 2x - \frac{1}{2} = 0 \qquad \cos 2x = \frac{1}{2} \qquad 2x = \pm \arccos \frac{1}{2} + 2\pi k,$

bu erda $\arccos \frac{1}{2} = \frac{\pi}{3}$ bo'lgani uchun

$2x = \pm \frac{\pi}{3} + 2\pi k, \ k \in Z \qquad x = \pm \frac{\pi}{6} + \pi k, \ k \in Z$

2. **sinx = a (-1≤ a ≤ 1) ko'rinishidagi tenglama**

$x = (-1)^k \arcsin a + \pi k, \ k \in Z$

Xususiy hollarda:

a) $\sin x = 0, \ x = \pi k, \ k \in Z$

b) $\sin x = 1, \ x = \frac{\pi}{2} + 2\pi k, \ k \in Z$

c) $\sin x = -1, \ x = -\frac{\pi}{2} + 2\pi k, \ k \in Z$

2. $\sin^2 x = a \ (0 \leq a \leq 1)$ tenglama echimi:

$x = \pm \arcsin \sqrt{a} + \pi k,\ k € Z$

Misol: Tenglamani eching.

$\sin^2 x - \frac{1}{2} = 0$ $\sin^2 x = \frac{1}{2}$ $2x = (-1)^k \arcsin\frac{1}{2} + \pi k,\ k € Z$

bu erda: $\arcsin\frac{1}{2} = \frac{\pi}{6}$ bo'lgani uchun

$2x = (-1)^k \frac{\pi}{6} + \pi k,\ k € Z$ $x = (-1)^k \frac{\pi}{12} + \frac{\pi k}{2},\ k € Z$

3. **tgx = a tenglama echimi**: $x = \operatorname{arctg} x + \pi k,\ k € Z$

Xusuisy hollarda:

a) $\operatorname{tg} x = 0,\ x = \pi k,\ k € Z$

b) $\operatorname{tg} x = 1,\ x = \frac{\pi}{4} + \pi k,\ k € Z$

c) $\operatorname{tg} x = -1,\ x = -\frac{\pi}{4} + \pi k,\ k € Z$

3. $\operatorname{tg}^2 x = a$, bunda $a \in [0;\infty]$ $x = \pm \operatorname{arctg} \sqrt{a} + \pi k,\ k € Z$

$\operatorname{tg} x = a$ da $\operatorname{tg} x\ \frac{\sin x}{\cos x}$ bo'lgani uchun

$\cos \neq 0$, ya'ni $\operatorname{tg} x = a$ tenglama $\cos x = 0$ da, ya'ni $x = \pm\frac{\pi}{2} + \pi k$ da aniqlanmagan.

Misol: Tenglamani eching.

$\operatorname{tg}^2 - \frac{1}{3} = 0$ $\operatorname{tg}^2 x = \frac{1}{3}$

$x = \pm\operatorname{arctg}\frac{1}{\sqrt{3}} + \pi k,\ k € Z$ bu erda: $\operatorname{arctg} \frac{1}{\sqrt{3}} = \frac{\pi}{6}$ bo'lgani uchun

$x = \pm\frac{\pi}{6} + \pi k$, k €

Mustaqil ishlash. Tenglamalarni eching:

a) tg2x = 0

$2x = \pi k$, k € Z

$x = \frac{\pi k}{2}$, k € Z

b) tg3x = 0

$3x = \pi k$, k € Z

$x = \frac{\pi k}{3}$, k € Z

c) $tgx = \sqrt{3}$

$x = arctg\sqrt{3} + \pi k$

$x = \frac{\pi}{3}, + \pi k$, k € Z

Tenglama ildizini toping.

a)sin 4x = -1

$4x = (-1)^k \arcsin(-1) + \pi k$, k € Z

$4x = (-1)^{k+1}\frac{\pi}{2} + \pi k$, k € Z

$x = (-1)^{k+1}\frac{\pi}{8} + \frac{\pi k}{4}$, k € Z

b) $\sin x(2\sin x - \sqrt{2}) = 0$

1) sinx = 0, $x = \pi k$, k € Z

2) $\sin x = \frac{\sqrt{2}}{2}$

$x = (-1)^k \arcsin\frac{\sqrt{2}}{2} + \pi k$

$x = (-1)^k\frac{\pi}{4} + \pi k$, k € Z

Tenglamani eching.

a) $\frac{1 + cos2x}{\cos x} = 0$

$\cos x \neq 0$

$x \neq \frac{\pi}{2} + \pi k$, k € Z

1 + cos2x = 0

b) $\frac{\cos 3x}{tgx} = 0$

$tgx \neq 0$

$x \neq \pi k$, k € Z

cos3x = 0

$\cos 2x = -1$ $\quad$ $3x = \pm\frac{\pi}{2} + \pi k, k \in Z$

$2x = \pi + 2\pi k,$ $\quad$ $x = \pm\frac{\pi}{6} + \frac{\pi k}{3}, k \in Z$

$x = \frac{\pi}{2} + \pi k, k \in Z$

Tenglamaning echimlarini toping.

$tg3x\ (tg\frac{x}{2}+1)(tgx-1)(tg2x-\sqrt{3})(tg^2x-\frac{1}{3})=0$

Berilgan tenglamaning aniqlanish sohasini

topamiz.

$$\begin{cases} \cos 3x \neq 0 \\ \cos\frac{x}{2} \neq 0 \\ \cos x \neq 0 \\ \cos 2x \neq 0 \end{cases} \Rightarrow \begin{cases} 3x \neq \frac{\pi}{2} + \pi k \\ \frac{x}{2} \neq \frac{x}{2} + \pi k \\ x \neq \frac{\pi}{2} + \pi k \\ 2x \neq \frac{\pi}{2} + \pi k \end{cases} \Rightarrow \begin{cases} x \neq \frac{\pi}{6} + \frac{\pi k}{2}, k \in Z \\ x \neq \pi + 2\pi k, k \in Z \\ x \neq \frac{\pi}{2} + \pi k, k \in Z \\ x \neq \frac{\pi}{4} + \frac{\pi k}{2}, k \in Z \end{cases}$$

Berilgan tenglamani echish uchun bir ko’paytuvchini nolga tenglashtirb echamiz.

1) $tg\,3x = 0,$ $\quad 3x = \pi k,$ $\quad x = \frac{\pi k}{3},$ $\quad k \in Z$

2) $tg\frac{x}{2} + 1 = 0,$ $\quad tg\frac{x}{2} = -1,$ $\quad \frac{x}{2} = -\frac{\pi}{4} + \pi k$ $\quad x = -\frac{\pi}{2} + 2\pi k, k \in Z$

3) $tg\,x - 1 = 0,\ tgx = 1,\ x = \frac{\pi}{4} + \pi k\ \pi k$

4) tg 2x - $\sqrt{3}$ = 0, tg2x = $\sqrt{3}$, 2x = arctg$\sqrt{3}$ + πk $x = \frac{\pi}{6} + \frac{\pi k}{2}$. k € Z

Tenglamalarni eching.

1)tg2x = 0; 2x = πk; x = $\frac{\pi k}{2}$,k € Z 2) tg3x = 0; 3x = πk; x = $\frac{\pi k}{3}$, k € Z

3) tg ($\frac{x}{2}$ - 30°) = 0 4) tg (3x + 60) = $\sqrt{3}$

$\frac{x}{2}$ - 30° = 0° + πk 3x + 60° = 60° + πk

$\frac{x}{2}$ = 30° + πk 3x = πk x = $\frac{\pi k}{3}$, k € Z

x = 60° + 2 πk k € Z

5) tg4x = 3 6) sinx = $-\frac{\sqrt{2}}{2}$

4x = arctg 3 + πk x = $(-1)^k$arcsin$(-\frac{\sqrt{2}}{2}) +$ πk

x = $\frac{arctg3}{4} + \frac{\pi k}{4}$, k € Z x = $(-1)^{k+1}\frac{\pi}{4}$ + πk, k € Z

7) $8\cos^2 3x$ – cos3x = 0 8) cos 2x = $\frac{\sqrt{3}}{2}$

cos3x (8cos3x – 1) = 0 2x = $\pm\frac{\pi}{6}$ +2 πk

x = ± $\frac{\pi}{12}$, πk, k € Z

1)cos3x = 0 3x = $\frac{\pi}{2} +$ πk x = $\frac{\pi}{6} + \frac{\pi k}{3}$, k € Z

2) 8cos3x – 1 = 0 cos3x = $\frac{1}{8}$ x = $\pm\frac{1}{3}$ arccos $\frac{1}{8}$ + $\frac{2}{3}$ πk, k € Z

Tenglamani eching. 1) $\frac{cos\frac{x}{2}}{\cos x}=0$ $\qquad$ $\cos\frac{x}{2}=0$ $\quad$ $\cos x\neq 0$

$\frac{x}{2}=\pm\frac{\pi}{2}+\pi k$ $\quad x\neq\pm\frac{\pi}{2}\pi k,\ k \in Z$ $\qquad$ $x=\pm\pi+2\pi k,\ k\in Z$

2) $\frac{cos3x}{tgx}=0$ $\qquad$ $tgx\neq 0$ $\qquad$ $x\neq\pi k,\ k\in Z$

$\cos 3x=0$ $\qquad$ $3x=\pm\frac{\pi}{2}+\pi k$ $\qquad$ $x=\pm\frac{\pi}{6}+\frac{\pi k}{3},\ k\in Z$

§3.2. Trigonometrik tengsizliklarni echish.

Noma'lum trigonometrik funksiyalarning argumenti bo'lib qatnashgan tengsizliklar trigonometrik tengsizliklar deyiladi.

Sinx > a (sinx <a), sinx $\geq a$ (sinx $\leq a$), cos x>a (cosx < a),

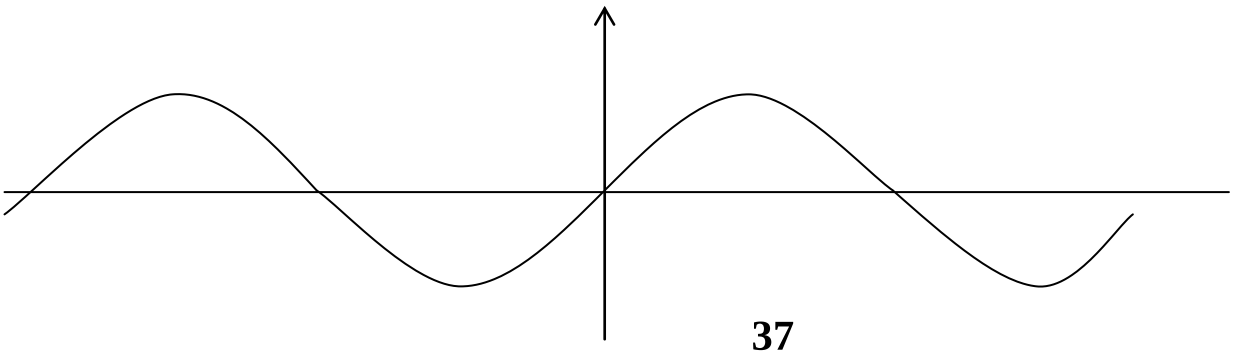
cosx$\geq a$ (cosx$\leq a$), tgx >a (tgx <a) , tgx $\geq a$ (tgx $\leq a$),

ctgx > a (ctgx>a), ctgx $\geq a$ (ctgx $\leq a$) ko'rinishdagi tengsizliklar eng sodda trigonometrik tengsizliklar hisoblanadi. Ba'zi trigonometrik tengsizliklarni echish bitta yoki bir nechta eng sodda trigonometrik tengsizliklarni echishga keltiradi.

Trigonometrik tengsizliklarni echishda birlik aylanadan yoki trigonometric funksiyalarning grafiklaridan foydalanish mumkin.

1. sinx > a tengsizlikni echamiz, bunda -1$\leq$ a $\leq$1

y = sinx va y = a funksiyalarning grafiklarini chizamiz.



x_1= arcsin a va x_2 = π - arcsin a.

Dastlab echimni [0 ; 2π] oraliqda topamiz.

arcsin a < x <π - arcsin a oraliqda y = sinx funksiya grafigi y = a funksiya grafigidan yuqorida yotishini ko’rishimiz mumkin, ya’ni sinx $\geq$a tengsizlikning [0 ; 2π] kesmadagi echimi: arcsin a < x <π - arcsin a . y = sinx funksiyaning $2\pi n\ (n \in z)$ davrli davriy funksiya ekanligini hisobga olsak, tengsizlikning barcha echimlar to’plamini quyidagi ko’rinishda yozishimiz mumkin:

$$\arcsin a + 2\pi n < x < \pi - \arcsin a + 2\pi n,\ (n \in z)$$

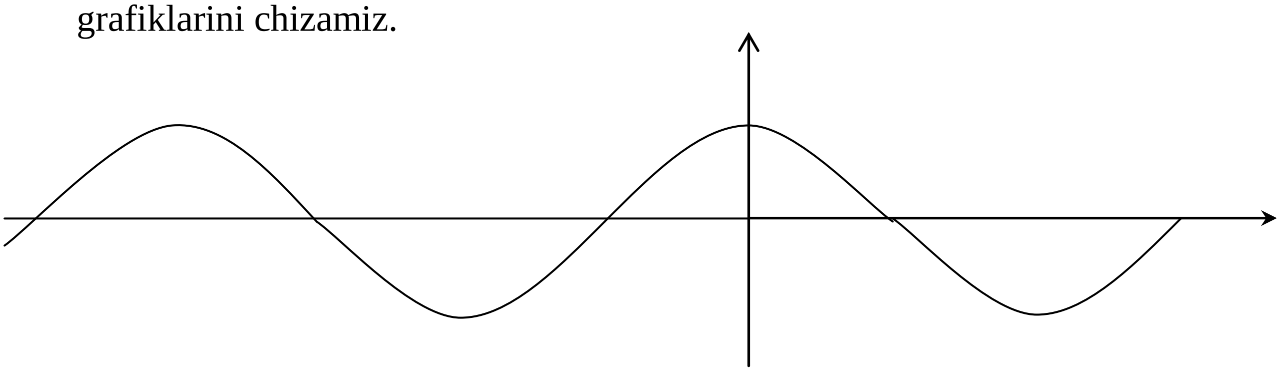
sin x >a tengsizlik a $\geq$ 1 da bajarilmaydi, a <-1 da esa barcha x larda bajariladi.

2. sinx< a tengsizlikni ham 1- rasmdan foydalanib, echamiz. x_0 = - π - arcsin a. Dastlab echimni [-$\frac{3\pi}{2}$;$\frac{\pi}{2}$] kesmada topamiz. (-π- arcsin a, arcsin a) oraliqda y = sinx funksiya grafigi y = a funksiya grfigidan pastda yotishini ko’rishimiz mumkin, ya’ni sinx $\leq$a tengsizlikning [-$\frac{3\pi}{2}$;$\frac{\pi}{2}$] kesmadagi echimi: (-π- arcsin a, arcsin a) Butun son o’qida tengsizlikning qolgan echimlari (-π- arcsin a, arcsin a) echimdan 2πn $(n \in z)$ uzoqliklarda joylashadi. Shunday qilib, tengsizlikning barcha echimlar to’plamini quyidagicha yozish mumkin:

$$-\pi - \arcsin a + 2\pi n \prec x \prec \arcsin a + 2\pi n, \quad n \in z$$

3. cos x > a tengsizlikni echamiz. a $\geq$1 da tengsizlik echimga ega emas, a < -1da esa x ning barcha qiymatlari tengsizlikni qanoatlantiradi. Biz –1 $\leq$a$\leq$1 bo’lgan holni qaraymiz.

Tengsizlikni grafik usuldan foydalanib echamiz. y = cos x va y = a funksiyalarning grafiklarini chizamiz.



y = cos x va funksiyalar grfiklari kesishgan nuqtalar x_0, x_1, x_2, x_3, x_4 ,…

[-π;π] kesmada echimni qaraymiz. (- arcos a; arcos a) oraliqda y = cosx funksiya grafigi y = a funksiya grafigidan yuqorida joylashgan, ya'ni cosx > a tengsizlikning echimi: - arccos a ≤ x≤ arccos a

y = cos x funksiyaning 2πn $(n \in z)$ davrli davriy funksiya ekanligini hisobga olsak, cosx > a tengsizlikning barcha echimlari to'plamini quyidagicha yozishimiz mumkin:

$$-\arccos a + 2\pi n < x < \arccos a + 2\pi n , \ (n \in z)$$

4.cosx < a tengsizlikni ham 2 – rasmdan foydalanib echamiz. Dastlab [0 ; 2π] kesmadagi echimni topamiz. (arccos a; 2π-arccos a) oraliqda y = cosx funksiya grafigi y = a to'g'ri chiziqdan pastga, ya'ni cosx < a tengsizlik o'rinli. Tengsizlikning qolgan echimlari bu echimdan 2πn $(n \in z)$ uzoqliklarda joylashadi. Shunday qilib, cosx < a tengsizlikning barcha echimlari to'plamini quyidagi ko'rinishda yozishimiz mumkin.

$$\arccos a + 2\pi n < x < 2\pi - \arccos a + 2\pi n , \ (n \in z)$$

5.tgx > a tengsizlikni grfikdan foydalanib echamiz. y = tg x va y = a funksiyalarning grafiklarini echamiz:

$\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$ oraliq uchun tgx > a tengsizlikning echimi quyidagicha (arctg a; $\frac{\pi}{2}$) tgx > a tengsizlikning echimlari to'plamini esa quyidagi ko'rinishda yozishimiz mumkin:

$$\text{arctg } a + \pi n < x < \frac{\pi}{2} + \pi n , \ (n \in z)$$

6. tg x < a ushbu tengsizlikning ham $\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$ oraliqdagi echimi ($-\frac{\pi}{2};arctga$). Buni grafikdan ko'rish mumkin. Qolgan echimlar bu echimdan πn $(n \in z)$ uzoqlikda joylashadi. Shunday qilib, tg x < a tengsizlikning barcha echimlari to'planishni

$$-\frac{\pi}{2} + \pi n \langle x \langle arctga + \pi n, \ (n \in z)$$

ko'rinishda ifodalash mumkin.

7. ctg > a tengsizlik berilgan bo'lsin. Ushbu tengsizlikni ham grafik yordamida echamiz. y = ctgx va y = a funksiyalarning grafiklarini chizamiz. $x \neq \pi n, \ (n \in z)$

Tengsizlikning (0; π) oraliqdagi echimini topamiz. (0; arctg a) oraliqda y = ctg x funksiya grafigi y = a to'g'ri chiziqdan yuqorida ya'ni ctgx > a tengsizlik o'rinli.

Tengsizlikning qolgan echimlari (0; arcctga) echimdan π n $(n \in z)$ uzoqlikda joylashadi. Shunday qilib, ctgx > a tengsizlikning barcha echimlari to'plamini quyidagicha ifodalash mumkin:

$$\pi n < x < \text{arcctg}a + \pi n, \ (n \in z)$$

8. ctgx < a 4-rasmdan ko'rish mumkinki , (0; π) oraliqda ctgx < a tengsizlikning echimi:

$$\text{arcctg}a < x < \pi$$

Tengsizlikning barcha echimlari to'plamini esa quyidagi ko'rinishda ifodalash mumkin:

arcctga + π n < x < $\pi + \pi$ n,

$(n \in z)$

Testlar.

1. tg (x + $\frac{\pi}{4}$) $\geq$ 1 tengsizlikni eching.

a) $\left[-\frac{\pi}{4}+\pi n; \frac{\pi}{2}+\pi n\right], \ (n \in z)$ b) $[\pi n; \infty] \ (n \in z)$

c) $\left[\frac{\pi}{4}+2\pi n; \frac{\pi}{2}+2\pi n\right], \ (n \in z)$ d) $\left[\pi n; \frac{\pi}{4}+\pi n\right), \ (n \in z)$

e) $\left[\frac{\pi}{4}+\pi n; \frac{\pi}{2}+\pi n\right), (n \in z)$

Echish: tg (x + $\frac{\pi}{4}$) $\geq$ 1 $\quad arctg1 + \pi n \leq x + \frac{\pi}{4} < \frac{\pi}{2} + \pi n, n \in Z.$

$\frac{\pi}{4} - \frac{\pi}{4} + \pi n \leq x < \frac{\pi}{2} - \frac{\pi}{4} + \pi n, n \in Z. \quad x \in \left[\pi n; \frac{\pi}{4} + \pi n\right), \ (n \in z).$

2. 2sin 2x $\geq$ ctg $\frac{\pi}{4}$ tengsizlikni eching.

a) $\left[\frac{\pi}{6}+2\pi n; \frac{5\pi}{6}+2\pi n\right], \ (n \in z)$ b) $\left(\frac{\pi}{12}+\pi n; \frac{5\pi}{12}+\pi n\right), \ (n \in z)$

c) $\left[\frac{\pi}{12}+\pi n; \frac{5\pi}{12}+\pi n\right], (n \in z)$ d) $\left[\frac{\pi}{12}+2\pi n; \frac{5\pi}{12}+2\pi n\right], \ (n \in z)$

e) $\left[-\frac{\pi}{3}+2\pi n, \frac{\pi}{3}+2\pi n\right], \ (n \in z)$

Echish: $2\sin 2x \geq 1 \quad \sin 2x \geq \frac{1}{2}$

$$\frac{\pi}{6}+2\pi n \leq 2x \leq \pi-\frac{\pi}{6}+2\pi n, n \in Z.$$

$$x \in \left[\frac{\pi}{12}+\pi n; \frac{5\pi}{12}+\pi n\right], (n \in z)$$

3.Mustahkamlash.

1. $\sin x > \frac{1}{2}$ tengsizlikni eching.

Echish. Tengsizlikni birlik aylanadan foydalanib echamiz: Rasmdan ko'rish mumkinki ,

$$\frac{\pi}{6}+2\pi n < x < \pi-\frac{\pi}{6}+2\pi n, \ (n \in z)$$

Javob: $\frac{\pi}{6}+2\pi n < x < \frac{5\pi}{6}+2\pi n \ (n \in z)$

2. $2\cos^2 x-1<\frac{1}{2}$ tengsizlikni eching.

Echish. $2\cos^2 x-1 = \cos 2x$ ekanligidan foydalanamiz. Demak, $\cos 2x <\frac{1}{2}$ tengsizlikni echishimiz kerak.

$\arccos \frac{1}{2} + 2\pi n < 2x < 2\pi - \arccos\frac{1}{2} + 2\pi n, \ (n \in z)$

Javob: $\frac{\pi}{6}+\pi n < x < \frac{5\pi}{6}+\pi n, (n \in z)$

3. x ning ($-\pi$; π) oraliqqa tegishli qanday qiymatlarida ($\cos x + 2,5$) ≥ 3 tengsizlik o'rinli bo'ladi?

Echish. Ma'lumki, $-1 \leq \cos x \leq 1$. Bundan kelib chiqadiki, $\cos x + 2,5 >3$ tengsizlik bilan almashtirsh mumkin. U holda quyidagilar o'rinli bo'ladi:

$$\cos x \geq 3 - 2,5 \qquad \cos x \geq \frac{1}{2}$$

$$-\frac{\pi}{3}+2\pi n \leq x \leq \frac{\pi}{3}+2\pi n \qquad (n \in z)$$

n = 0 da ($-\pi$; π) oraliqdagi echim hosil bo'ladi.

Javob: $-\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$ yoki $\left[-\frac{\pi}{3}; \frac{\pi}{3}\right]$

§3.3. Trigonometrik funksiyalarning hosilalari.

1. f (x) = sinx funksiyaning hosilasini topish masalasi berilgan bo'lsin. Quyidagicha ayirmali nisbat tuzamiz:

$$\frac{f(x+h)-f(x)}{h}=\frac{\sin(x+h)-\sin x}{h}=\frac{2\sin\frac{h}{2}\cdot\cos\left(x+\frac{h}{2}\right)}{h}=\frac{\sin\frac{h}{2}}{\frac{h}{2}}\cos(x+\frac{h}{2})$$

Bunda x,x+h $\in$D(f). Agar h$\to$0 bo'lsa, $x+\frac{h}{2}\to x$ va $\cos(x+\frac{h}{2})\to\cos x$ ekanligni ko'rishimiz mumkin. h$\to$0 da $\frac{\sin\frac{h}{2}}{\frac{h}{2}}\to 1$ bo'lishini ham ko'rsatish mumkin.

Bu uchun mikrokalkulyatordan foydalanish qulayroq. Hosilaning ta'rifiga asosan,

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\sin\frac{h}{2}}{\frac{h}{2}}\cdot\cos\left(x+\frac{h}{2}\right)=\cos x$$

Demak, $(\sin x)'=\cos x$

2. f (x)=cosx funksiyaning hosilasini ham ta'rifdan foydalanib hisoblaymiz:

$$f'(x)=(\cos x)'=\lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}=\lim_{h\to 0}\frac{-2\sin\left(x+\frac{h}{2}\right)\sin\frac{h}{2}}{h}=-\lim_{h\to 0}\frac{\sin\frac{h}{2}}{\frac{h}{2}}\sin\left(x+\frac{h}{2}\right)=$$

$$=\left|h\to da\frac{\sin\frac{h}{2}}{\frac{h}{2}}\to 1\right|=-\sin x$$

Demak, $(cosx)'=-\sin x$

3. f (x) = tg x funksiyaning hosilasini bo'linmaning hosilasidan foydalanib hisobalaymiz.

$$(tgx)'=\left(\frac{\sin x}{\cos x}\right)'=\frac{(\sin x)'\cdot\cos x-\sin x\cdot(\cos x)'}{\cos^2 x}=\frac{\cos^2 x+\sin^2 x}{\cos^2 x}=\frac{1}{\cos^2 x}$$

Demak, $(tgx)'=\frac{1}{\cos^2 x}$

Ma'lumki, $(kx+b)'=k$ munosabat o'rinli. Quyidagi formulalarni keltirishimiz mumkin.

$$\left(\sin(kx+b)\right)' = k\cdot\cos(kx+b)$$

$$\left(\cos(kx+b)\right)' = -k\sin(kx+b)$$

$$\left(tg(kx+b)\right)' = \frac{k}{\cos^2(kx+b)}$$

Mustaqil ishlash.

1.f (x) = ctg x funksiyaning hosilasini bo’linmaning hosilasidan foydalanib hisobalang

Echish.

$$(ctgx)' = \left(\frac{\cos x}{\sin x}\right)' = \frac{-\sin x\cdot\sin x-\cos x\cdot\cos x}{\sin^2 x} = -\frac{1}{\sin^2 x}$$

U holda $(ctg(kx+b))' = -\dfrac{k}{\sin^2(kx+b)}$ munosabat ham o’rinli.

O’quvchilarga mavzuga oid testlar yozilgan kartochkalarni tarqataman.

Testlar.

1. Agar f(x) = 5sinx + 3cosx bo’lsa, $f'(\frac{\pi}{4})$ ni hisoblang.

a) $-\sqrt{2}$ b) $\sqrt{2}$ c) $-2\sqrt{3}$ d) $4\sqrt{2}$ e) $4\sqrt{3}$

2. g(x) = $\frac{3x^2}{\pi} - 2tgx - \pi$ bo’lsa, $g'\left(\frac{\pi}{3}\right)$ni hisoblang.

a) $1\frac{1}{2}$ b) 10 c) $2\pi-8$ d) $\pi+4$ e) – 6

3. f(x) = $\cos^2$x – $\sin^2$x ning hosilasini hisoblang.

a) - $\sin^2$x - $\cos^2$x b) $\sin^2$x - $\cos^2$x c) -2 cos2x

d) –2sin 2x e) 2sin 2x

4. f(x) = 2cosx - $\frac{(\sqrt{\pi})^3}{\sqrt{x}} + \frac{\pi}{2}$, $f'(\pi)$ ni hisoblang.

a) $\frac{\sqrt{\pi}}{2}$ b) –1,5 c) 0,5 d) 2,5 e) $-\frac{\sqrt{\pi}}{3}$

5. f(x) = 0,5 tg 2x $f'\left(\frac{\pi}{6}\right)$ ni hisoblang.

a) $\frac{4}{3}$ b) $-\frac{1}{4}$ c) 4 d) 2 e) $-\frac{1}{2}$

6.Agar f(x) = $2\sqrt{3}\cos 4x - 2\cos x$ bo'lsa, $f'\left(\frac{\pi}{6}\right)$ni hisoblang.

a) –11 b) 13 c) $\sqrt{3}+1$ d) $\sqrt{3}-2$ e) – 13

7. y = cos 3xcos7x funksiyaning hosilasini hisoblang.

a) $y' = \frac{1}{3}\cos 3x \cdot \cos 7x$ b) $y' = \frac{1}{3}\cos 3x + \frac{1}{7}\cos 7x$

c) $y' = -5\sin 10x - 2\sin 4x$ d) $y' = -\frac{1}{20}\sin 10x + \frac{1}{8}\sin 4x$

e) $y' = 21\sin 3x \cdot \sin 7x$

Echish.

1. $f'(x) = 5\cos x - 3\sin x$

$f'(\frac{\pi}{4}) = 5\cos\frac{\pi}{4} - 3\sin\frac{\pi}{4} = \sqrt{2}.$ Javob:B

2. $g'(x) = \frac{3}{\pi} \cdot 2x - 2 \cdot \frac{1}{\cos^2 x}$

$g'(\frac{\pi}{3}) = \frac{3}{\pi} \cdot 2 \cdot \frac{\pi}{3}$ -2 $\cdot \frac{1}{\cos^2 \frac{\pi}{3}} = -6$ Javob:E

3. $f(x) = \cos^2 x - \sin^2 x = \cos 2x$

$f'(x) = -2\sin 2x$. Javob:D

4. $f'(x) = -2\sin x + \pi\sqrt{\pi} \cdot \frac{1}{2x\sqrt{x}}$

$f'(\pi) = -2\sin \pi + \pi\sqrt{\pi} \cdot \frac{1}{2\pi\sqrt{\pi}} = \frac{1}{2} = 0{,}5$ Javob:C

5. $f'(x) = \frac{1}{2} \cdot \frac{2}{\cos^2 2x} = \frac{1}{\cos^2 2x}$

$f'(\frac{\pi}{6}) = \frac{1}{\cos^2 \frac{\pi}{3}} = 4$ Javob:C

6. $$f'(x) = -2\sqrt{3}\cdot 4\sin 4x + 2\sin x$$
$$f'(\frac{\pi}{6}) = -2\sqrt{3}\cdot 4\sin\frac{2\pi}{3} + 2\sin\frac{\pi}{6} = -2\sqrt{3}\cdot 4\cdot\frac{\sqrt{3}}{2} + 2\cdot\frac{1}{2} = -11$$ Javob:A

7. $y = \cos 3x \cos 7x = \frac{1}{2}\cos 10x + \frac{1}{2}\cos 4x$

$y' = -\frac{10}{20}\sin 10x - \frac{4}{2}\sin 4x = -5\sin 10x - 2\sin 4x$ Javob:C

Mustahkamlash. Hosilalarni hisoblang.

1. $f(x) = 2\sin\left(\frac{x}{3}+3\right)$ $\qquad f'(x) = 2\cdot\frac{1}{3}\cos\left(\frac{x}{3}+3\right) = \frac{2}{3}\cos\left(\frac{x}{3}+3\right)$

2.f (x) = $\frac{\sin x - \cos x}{x}$ –

$$f'(x) = \frac{(\sin x - \cos x)'\cdot x - x'\cdot(\sin x - \cos x)}{x^2} = \frac{(\cos x + \sin x)\cdot x - (\sin - \cos x)}{x^2} =$$

$$\frac{1}{x}\cdot(\cos x + \sin x) + \frac{1}{x^2}(\cos x - \sin x).$$

3. $f(x) = tg(4x+5)$

$$f'(x) = \frac{(4x+5)'}{\cos^2(4x+5)} = \frac{4}{\cos^2(4x+5)}$$

4. $f(x) = ctg\left(1-\frac{x}{5}\right)$ $\qquad f'(x) = -\frac{\left(1-\frac{x}{5}\right)'}{\sin^2\left(1-\frac{x}{5}\right)} = -\frac{\frac{-1}{5}}{\sin^2\left(1-\frac{x}{5}\right)} = \frac{1}{5\sin^2\left(1-\frac{x}{5}\right)}$

5. $f(x) = \sin^4 2x$

$f'(x) = 4\cdot\sin^3 2x\cdot 2\cdot\cos 2x = 4\sin^2 2x\sin 4x$

1. Hosilalarni hisoblang.

a) $f(x) = \cos(3-2x) + 3\cos\frac{x-2}{3}$

Echish: $f'(x)=(-2)\cdot(\cos(3-2x))'\neq 3\cdot\frac{1}{3}\cdot(-\sin\frac{x-2}{3})=2\sin(3-2x)-\sin\frac{x-2}{3}$

b) $y=\sin^2 2x$

Echish: $y'=2\sin 2x\cdot 2\cos 2x=2\sin 4x$

c) $y = tg^2x$

Echish: $y'=2tgx\cdot\frac{1}{\cos^2 x}=\frac{2\sin x}{\cos x}\cdot\frac{1}{\cos^2 x}=\frac{2\sin x}{\cos^3 x}$

d) $y=tg3x+\frac{3}{\pi}\cos\frac{\pi}{3}+4x$

Echish: $y'=\frac{3}{\cos^2 3x}+4$

e) $y=\sin^2 2x+\cos^2 2x$

Echish:

$y=\sin^2 2x+\cos^2 2x=1$ $y'=0$

f) $y=x^2\cdot\cos 2x$

Echish: $y=(x^2)'\cos 2x+x^2\cdot(\cos 2x)'=2x\cos 2x-2x^2\sin 2x=2x(\cos 2x-x\sin 2x)$

Х У Л О С А

Фойдаланилган адабиётлар