MATEMATIKA 11

ALGEBRA VA ANALIZ ASOSLARI
GEOMETRIYA
I QISM

Umumiy o‘rta ta’lim maktablarining 11-sinflari va o‘rta maxsus, kasb-hunar ta’limi muassasalari uchun darslik

O‘zbekiston Respublikasi Xalq ta’limi vazirligi tomonidan tasdiqlangan

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Darslikning “Algebra va analiz asoslari” bo‘limida ishlatilgan belgilar va ularning talqini:

△	– masalani yechish (isbotlash) boshlandi	▲	– masalani yechish (isbotlash) tugadi
!	– nazorat ishlari va test (sinov) mashqlari	?	– savol va topshiriqlar
■	– asosiy ma’lumot	*	– murakkabroq mashqlar

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Algebra va analiz asoslari

I BOB. HOSILA VA UNING TATBIQLARI

1–2 O‘ZGARUVCHI MIQDORLAR ORTTIRMALARINING NISBATI VA UNING MA’NOSI. URINMA TA’RIFI. FUNKSIYA ORTTIRMASI

O‘zgaruvchi miqdorlar orttirmalarining nisbati

Turli o‘lchov birliklariga ega bo‘lgan ikkita o‘zgaruvchi miqdor nisbatini hisoblash inson hayotida tez-tez uchrab turadi.

Masalan, avtomashinaning ***tezligi*** uning yurgan yo‘lining vaqtga nisbati km/soat yoki m/s larda o‘lchanadi, yoqilg‘i sarflashi esa km/litr yoki 100 km/litr larda o‘lchanadi.

Xuddi shunday, basketbolchining mahorati bir o‘yinda to‘plagan ochkolar soni bilan belgilanadi.

Misol. O‘quv ishlab chiqarish majmuasida 11-sinf o‘quvchilari orasida matn terishning sifati va tezligi bo‘yicha sinov o‘tkazilmoqda.

Karim 3 minut mobaynida 213 ta so‘zni terib, 6 ta imloviy xatoga, Nargiza esa 4 minut mobaynida 260 ta so‘zni terib, 7 ta imloviy xatoga yo‘l qo‘ygani ma’lum bo‘ldi. Ularning natijalarini solishtiring.

△ Har bir o‘quvchi uchun tegishli nisbatlarni tuzamiz:

Karim:

matn terishning tezligi $\frac{213 \text{ ta so‘z}}{3 \text{min}} = 71 \ \frac{\text{so‘z}}{\text{min}}$;

matn terishning sifati $\frac{6 \text{ ta xato}}{213 \text{ ta so‘z}} \approx 0{,}0282 \frac{\text{xato}}{\text{so‘z}}$.

Nargiza:

matn terishning tezligi $\frac{260 \text{ ta so‘z}}{4 \text{min}} = 65 \ \frac{\text{so‘z}}{\text{min}}$;

matn terishning sifati $\frac{7 \text{ ta xato}}{260 \text{ ta so‘z}} \approx 0{,}0269 \frac{\text{xato}}{\text{so‘z}}$.

Demak, Karim matnni Nargizaga nisbatan tezroq tergan bo‘lsa-da, Nargiza bu ishni sifatliroq bajargan. ▲

Mashqlar

1. Puls chastotasini tekshirish uchun barmoqlar uchi arteriya tomiri o‘tadigan joyga qo‘yiladi va zarbalarni his qilish uchun shu joy bosiladi.

Madina pulsni o‘lchaganda bir minutda 67 ta zarbani his qildi.

a) Puls chastotasining ma’nosini tushuntiring. U qanday kattalik (belgi)?

b) Har soatda Madinaning yuragi necha marta uradi?

2. Karim uyida 14 bet matn terib, 8 ta imloviy xatoga yo‘l qo‘ydi. Agar 1 betda o‘rtacha 380 ta so‘z bo‘lsa:

a) Karimning matn terish sifatini aniqlang va yuqoridagi misolda olinяgan natija bilan solishtiring. Karimning matn terish sifati yaxshilandimi?

b) Karim 100 ta so‘z terganda o‘rtacha qancha xato qiladi?

3. Ma’ruf 12 soat ishlab 148 m 20 cm, Murod esa 13 soat ishlab 157 m 95 cm ariq tozaladi. Ularning mehnat unumdorligini solishtiring.

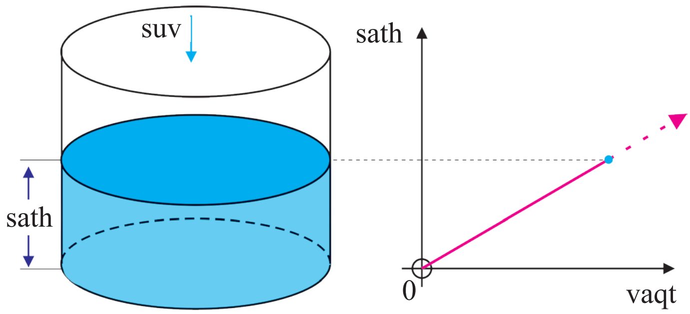
4. Avtomashina yangi shina protektorining chuqurligi 8 mm ni tashkil qiladi. 32178 km yurilganidan so‘ng yemirilish natijasida shina protektorining chuqurligi 2,3 mm bo‘lgani ma’lum bo‘ldi.

a) 1 km masofa yurilganda shina protektori chuqurligi qanday o‘zgaradi?

b) 10000 km masofa yurilganda-chi?

5. Madina Qarshi shahridan soat 11:43 da chiqib, soat 15:49 da Guliston shahriga yetib keldi. Agar u 350 km masofa yurgan bo‘lsa, uning o‘rtacha tezligi necha $\frac{\text{km}}{\text{soat}}$ bo‘ldi?

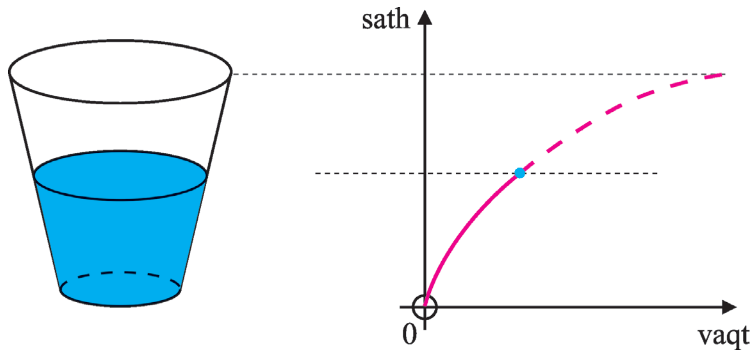
Misol. Silindr shaklidagi idish suv bilan bir xil tezlikda to‘ldirilmoqda. Bunda silindrik idish ichiga vaqtga proportsional bo‘lgan suv (hajmi) quyilayotgani bois suv sathining (balandligining) vaqtga nisbatan bog‘lanishi chiziqli funksiya ko‘rinishida bo‘ladi (1-rasmga qarang).



1-rasm.

Bu holda idishdagi suv sathining vaqtga bo‘lgan nisbati (ya’ni sathning *o‘zgarish tezligi*) o‘zgarmas son bo‘lib qolaveradi.

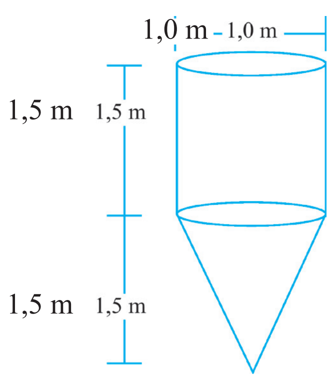
Endi boshqa shakldagi idishni qaraymiz (2-rasm):



2-rasm.

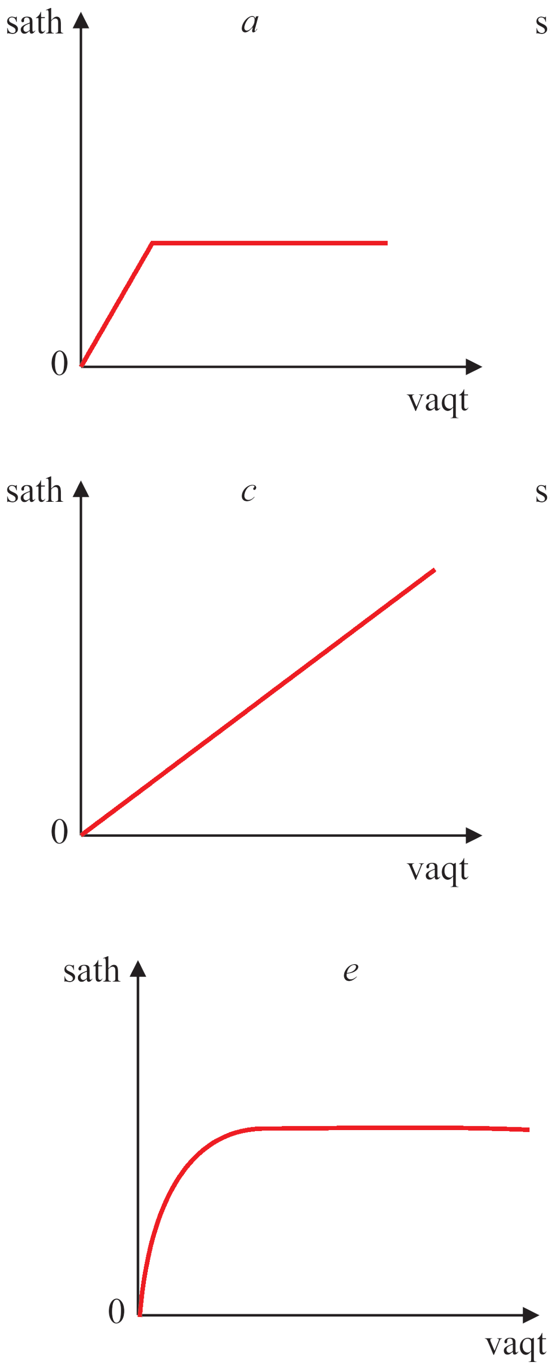
2- rasmda suv sathining o‘zgarish tezligining vaqtga nisbatan bog‘lanishi aks ettirilgan.

1-savol. 3-rasmda suv quyishga mo‘ljallangan idish tasvirlangan.



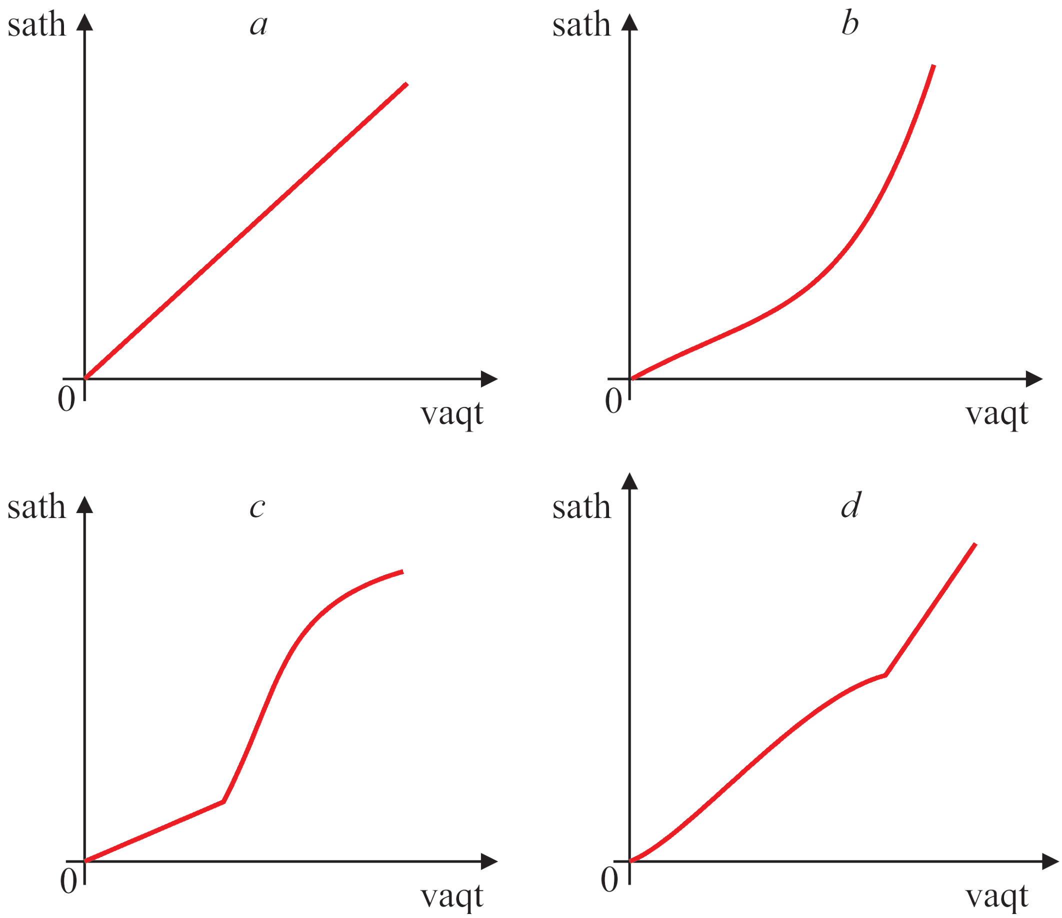
3-rasm.

Boshida unda suv yo‘q edi. Keyin u “bir sekundda bir litr” tezlikda to‘ldirila boshlandi. Suv sathining vaqtga nisbatan o‘zgarishi 4-rasmdagi qaysi grafikda to‘g‘ri ko‘rsatilgan?



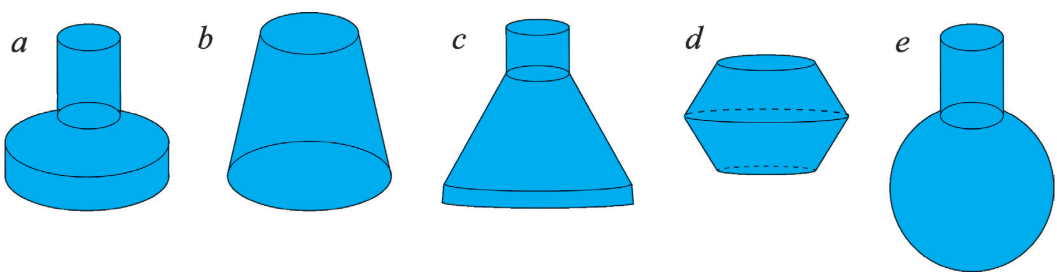
4-rasm.

2-savol. Suv sathining vaqtga nisbatan o'zgarishi 5-rasmdagi grafiklarda berilgan:



5-rasm.

Ular 6-rasmdagi qaysi idishlarga mos keladi?



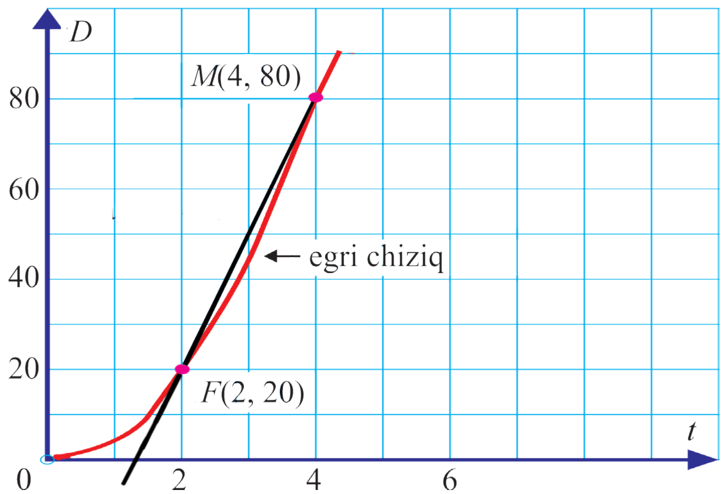
6-rasm.

O‘zgarishning o‘rtacha tezligi

Ikkita o‘zgaruvchi miqdorning bir-biriga bog‘lanishi chiziqli funksiya ko‘rinishida bo‘lsa, bu miqdorlar orttirmalarining nisbati o‘zgarmas son bo‘ladi.

Ikkita o‘zgaruvchi miqdorning bir-biriga bog‘lanishi chiziqli funksiya ko‘rinishida bo‘lmasa, biz bu o‘zgaruvchi miqdorlarning berilgan oraliqdagi o‘rtacha nisbatini topa olamiz. Agar oraliqlar turlicha olinsa, hisoblangan o‘rtacha nisbatlar ham turlicha bo‘ladi.

1-misol. Moddiy nuqtaning vaqtga nisbatan to‘g‘ri chiziq bo‘ylab harakat qonuni grafikda tasvirlangan (7- rasm). *FM* kesuvchining burchak koeffitsiyentini toping.



7-rasm.

△ Grafikda t=2 sekundga mos bo‘lgan F nuqtani va undan farqli (masalan, t=4 sekundga mos bo‘lgan) M nuqtani belgilaylik. $2 \le t \le 4$ vaqt oralig‘ida o‘rtacha tezlik $\frac{(80-20)\,m}{(4-2)\,s} = 30\frac{\text{m}}{\text{s}}$ ga teng ekanligini topamiz. Ko‘rinib turibdiki, *FM* kesuvchining burchak koeffitsiyenti 30 ga teng ekan. ▲

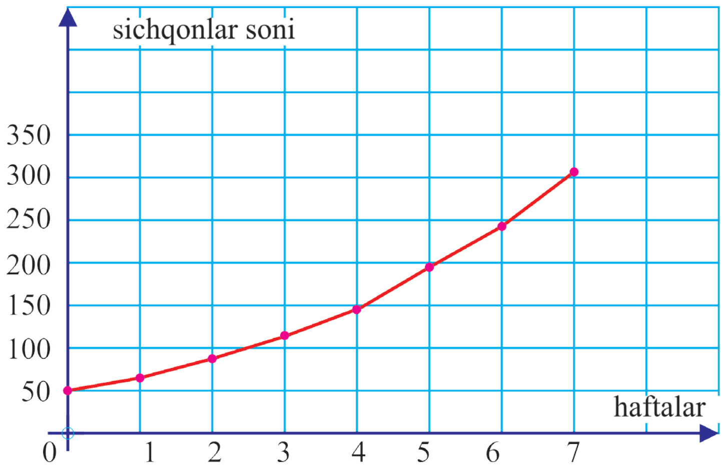
Savol. F nuqtani qo‘zg‘almas hisoblab, t ning quyida berilgan qiymatlariga mos bo‘lgan M nuqtalar uchun *FM* kesuvchilarning burchak koeffitsiyentlarini hisoblab, jadvallarni to‘ldiring:

t	burchak koeffitsiyenti
0	
1,5	
1,9	
1,99	

t	burchak koeffitsiyenti
3	
2,5	
2,1	
2,01	

Qanday xulosaga keldingiz?

2-misol. Populatsiyadagi sichqonlar soni haftalar kechishi bilan quyidagicha o‘zgaradi (8-rasm):



8-rasm.

3- va 6- hafta oralig‘ida sichqonlar soni o‘rtacha qanday o‘zgargan? 7 haftalik vaqt oralig‘da-chi?

△ Sichqonlar populatsiyasining o‘sish tezligi

$\frac{(240-110)\text{ ta sichqon}}{(6-3)\text{ ta hafta}} \approx 43\frac{\text{sichqon}}{\text{hafta}}$, ya’ni 3- va 6- hafta oralig‘ida sichqonlar soni haftasiga o‘rtacha 43 taga ko‘paygan.

Xuddi shunday 7 haftada $\frac{(315-50)\text{ ta sichqon}}{(7-0)\text{ ta hafta}} \approx 38\frac{\text{sichqon}}{\text{hafta}}$.

7 hafta oralig‘ida sichqonlar soni haftasiga o‘rtacha 38 taga ko‘paygan. ▲

Umumiy holda: x miqdor a dan b gacha o‘zgarganda $y=f(x)$ miqdor o‘zgarishining **o‘rtacha tezligi**

$$\frac{f(b)-f(a)}{b-a}$$

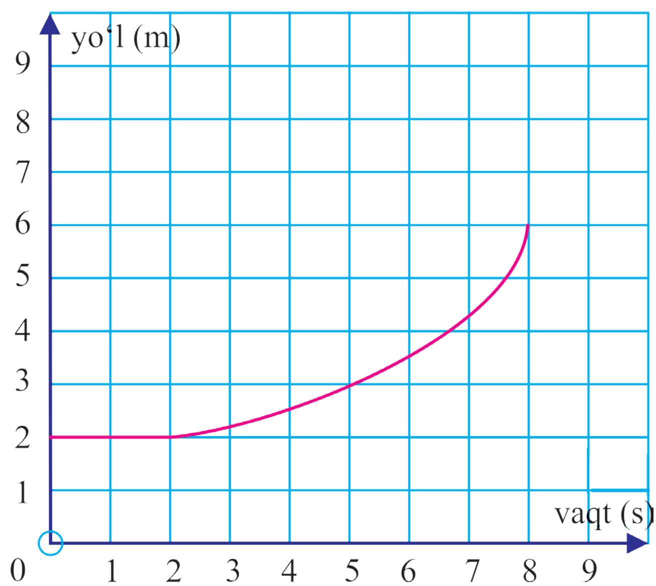
orttirmalar nisbatiga teng, bu yerda $f(b)-f(a)$ – funksiya orttirmasi, $b-a$ esa argument orttirmasi.

$h=b-a$ deb belgilasak, o‘rtacha tezlik $\frac{f(a+h)-f(a)}{h}$ ko‘rinishni oladi.

$\dfrac{f(a+h)-f(a)}{h}$ kasr suratini $y=f(x)$ *funksiyaning* argumenti x ning h *orttirmasiga* mos keluvchi *orttirmasi* deb atash qabul qilingan. Kasrning o‘zi esa *ayirmali nisbat* deb atashadi.

Mashqlar

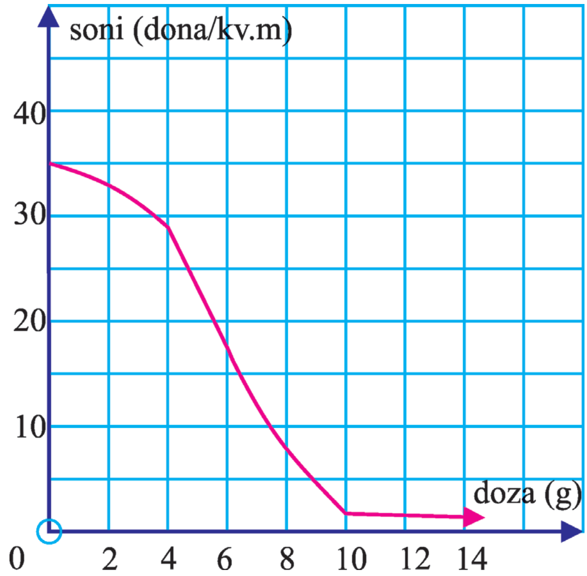
6. Nuqtaning to‘g‘ri chiziq bo‘ylab yurgan yo‘li vaqtga qanday bog‘langanligi 9-rasmdagi grafikda tasvirlangan.



9-rasm.

Nuqtaning
a) dastlabki 4 sekund;
b) so‘nggi 4 sekund;
c) 8 sekund mobaynidagi o‘rtacha tezligini toping.

7. 1) Dalaga turli miqdordagi (dozadagi) dori bilan ishlov berilganda 1 m^2 da mavjud bo‘lgan zararli hasharotlar sonining o‘zgarishi 10-rasmdagi grafikda ko‘rsatilgan.



10-rasm.

a) 1) doza 0 grammdan 10 grammgacha oshirilsa; 2) 4 grammdan 7 grammgacha oshirilsa, 1 m^2 da mavjud bo‘lgan zararli hasharotlar sonining o‘zgarishini toping.

b) doza 10 grammdan 14 grammgacha oshirilsa, qanday hodisa ro‘y beradi?

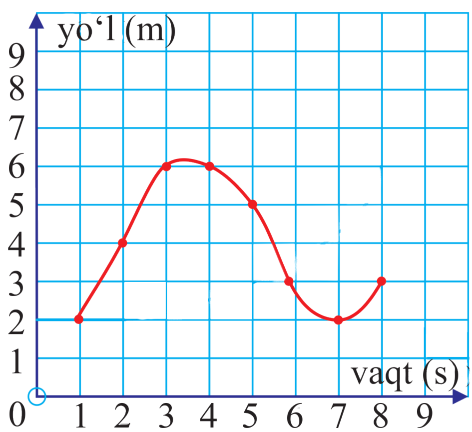
2) Moddiy nuqtaning to‘g‘ri chiziq bo‘yicha harakat qonuni $s(t)$ ning grafigi rasmda berilgan.

a) s(2), s(3), s(5), s(7) sonlar nechaga teng?

b) Qaysi oraliqlarda funksiya o‘suvchi?

c) Qaysi oraliqda funksiya kamayuvchi?

d) s(3)–s(1), s(5)–s(4), s(7)–s(6), s(8)–s(6) orttirmalarni hisoblang.



3–4 LIMIT HAQIDA TUSHUNCHA

x ning qiymatlari 2 dan kichik bo‘lib, 2 ga yaqinlasha borganda $f(x)=x^2$ funksiyaning qiymatlari jadvalini qaraylik:

x	1	1,9	1,99	1,999	1,9999
$f(x)$	1	3,61	3,9601	≈ 3,996 00	≈ 3,999 60

Jadvaldan ko‘rinib turibdiki, x ning qiymatlari 2 ga qancha yaqin bo‘laversa (*yaqinlashsa*), $f(x)$ funksiyaning mos qiymatlari ham 4 soniga yaqinlashaveradi.

Bunday holatda x argument (o‘zgaruvchi) 2 ga *chapdan yaqinlashganda* $f(x)$ ning qiymatlari 4 soniga *yaqinlashadi* deymiz.

Endi x ning qiymatlari 2 dan katta bo‘lib, 2 ga yaqinlasha borganida $f(x)=x^2$ funksiyaning qiymatlari jadvalini qaraylik:

x	3	2,1	2,01	2,001	2,0001
$f(x)$	9	4,41	4,0401	≈ 4,004 00	≈ 4,000 40

Bunday holatda x argument 2 ga *o‘ngdan yaqinlashganda,* $f(x)$ funksiya qiymatlari 4 soniga *yaqinlashadi* deymiz.

Yuqoridagi ikki holatni umumlashtirib, x argument 2 ga *yaqinlashganda,* $f(x)$ ning qiymatlari 4 soniga *yaqinlashadi* deymiz va buni quyidagicha yozamiz:

$$\lim_{x\to 2} x^2 = 4.$$

Bu yozuv shunday o‘qiladi: x argument 2 ga yaqinlashganda, $f(x) = x^2$ funksiyaning *limiti* 4 ga teng.

Umumiy holda *funksiya limiti* tushunchasiga quyidagicha yondashiladi:

$x \neq a$ bo‘lib, uning qiymatlari a soniga yaqinlashsa, $f(x)$ ning mos qiymatlari A soniga *yaqinlashsin.* Bu holda A sonni x a ga *yaqinlashganda* $f(x)$ funksiyaning *limiti* deyiladi va bunday belgilanadi:

$$\lim_{x\to a} f(x) = A.$$

Ayrim hollarda mazkur holatni x ning qiymatlari a ga *intilganda* $f(x)$ funksiya A ga *intiladi,* deymiz.

$\lim\limits_{x \to a} f(x) = A$ yozuv oʻrniga $x \to a$ da $f(x) \to A$ yozuv ham qoʻllaniladi.

Eslatma. x ning qiymati a ga *intilganda* $x \neq a$ sharti bajarilishining muhimligini aytib oʻtish joiz.

Misol. $x \to 0$ boʻlganda $f(x) = \frac{5x + x^2}{x}$ funksiyaning limitini toping.

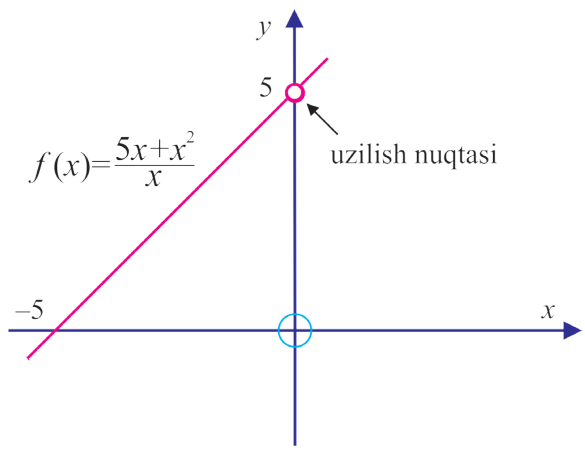
△ $x \neq 0$ sharti bajarilmasin, ya’ni $x=0$ boʻlsin. $x=0$ qiymatni $f(x)$ ga bevosita qoʻyib koʻrsak, $\frac{0}{0}$ koʻrinishdagi *aniqmaslikka* ega boʻlamiz.

Boshqa tomondan, $f(x) = \frac{x(5+x)}{x}$ boʻlgani uchun bu funksiya ushbu

$$f(x) = \begin{cases} 5 + x, & \text{agar } x \neq 0 \text{ boʻlsa} \\ \text{aniqlanmagan}, & \text{agar } x = 0 \text{ boʻlsa}, \end{cases}$$

koʻrinishni oladi.

$y=f(x)$ funksiyaning grafigi (0; 5) koordinatali nuqtasi “olib tashlangan” $y=x+5$ toʻgʻri chiziq koʻrinishida boʻladi (11-rasm):



11-rasm.

(0; 5) koordinatali nuqta $y = f(x)$ funksiyaning *uzilish nuqtasi* deyiladi.

Koʻrinib turibdiki, bu nuqtadan farqli boʻlgan nuqtalarda x ning qiymatlari 0 ga *yaqinlashganda* $f(x)$ funksiyaning mos qiymatlari 5 ga yaqinlashadi, ya’ni uning *limiti* mavjud:

$$\lim_{x \to 0} \frac{5x + x^2}{x} = 5. \ ▲$$

Amalda, funksiya limitini topish uchun, lozim bo‘lsa, tegishli soddalashtirishlarni bajarish maqsadga muvofiq.

1-misol. Limitlarni hisoblang:

a) $\lim\limits_{x\to2} x^2$; b) $\lim\limits_{x\to0} \frac{x^2+3x}{x}$; c) $\lim\limits_{x\to3} \frac{x^2-9}{x-3}$.

△ a) x ning qiymatlari 2 ga *yaqinlashganda* x^2 ning qiymatlari 4 ga yaqinlashadi, ya’ni $\lim\limits_{x\to2} x^2 = 4$.

b) $x \neq 0$ bo‘lgani uchun

$$\lim_{x\to0} \frac{x^2+3x}{x} = \lim_{x\to0} \frac{x(x+3)}{x} = \lim_{x\to0}(x+3) = 3.$$

c) $x \neq 3$ bo‘lgani uchun

$$\lim_{x\to3} \frac{x^2-9}{x-3} = \lim_{x\to3} \frac{(x+3)(x-3)}{x-3} = \lim_{x\to3}(x+3) = 6. \ ▲$$

Mashqlar

Limitni hisoblang (**8–11**):

8. a) $\lim\limits_{x\to3}(x+4)$; b) $\lim\limits_{x\to-1}(5-2x)$; c) $\lim\limits_{x\to4}(3x-1)$

d) $\lim\limits_{x\to2}(5x^2-3x+2)$; e) $\lim\limits_{h\to0} h^2(1-h)$; f) $\lim\limits_{x\to0}(x^2+5)$.

9. a) $\lim\limits_{x\to5} 5$; b) $\lim\limits_{h\to2} 7$; c) $\lim\limits_{x\to0} c$, c – o‘zgarmas son.

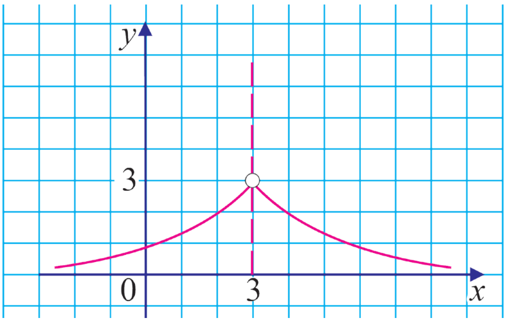
10. a) $\lim\limits_{x\to1} \frac{x^2-3x}{x}$; b) $\lim\limits_{h\to2} \frac{h^2+5h}{h}$; c) $\lim\limits_{x\to0} \frac{x-1}{x+1}$; d) $\lim\limits_{x\to0} \frac{x}{x}$.

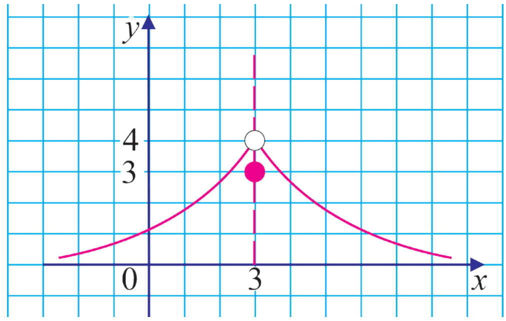
11. a) $\lim\limits_{x\to0} \frac{x^2-3x}{x}$; b) $\lim\limits_{x\to0} \frac{x^2-5x}{x}$; c) $\lim\limits_{x\to0} \frac{2x^2-x}{x}$.

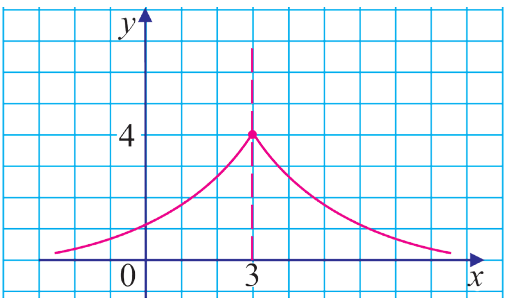
d) $\lim\limits_{h\to0} \frac{2h^2+6h}{h}$; e) $\lim\limits_{h\to0} \frac{3h^2-4h}{h}$; f) $\lim\limits_{h\to0} \frac{h^3-8h}{h}$;

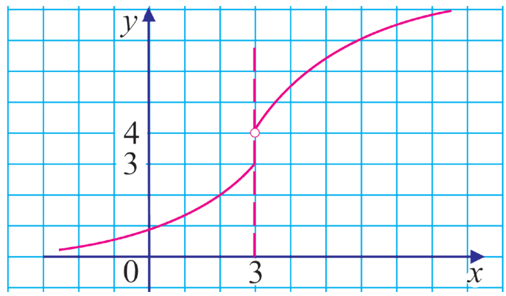
g) $\lim\limits_{x\to1} \frac{x^2-x}{x-1}$; h) $\lim\limits_{x\to2} \frac{x^2-2x}{x-2}$; i) $\lim\limits_{x\to3} \frac{x^2-x-6}{x-3}$.

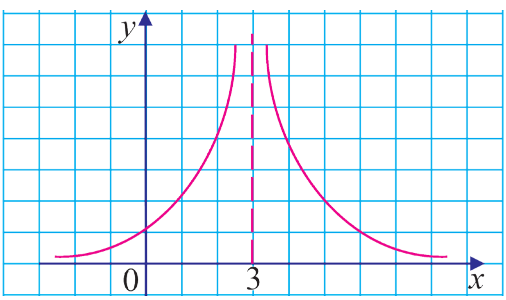
12. Quyidagi funksiyalardan qaysi biri $x \to 3$ da limitga ega? Shu limitni toping.

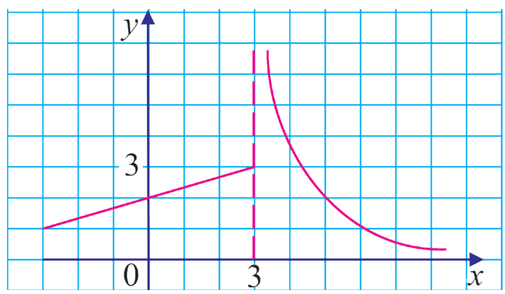


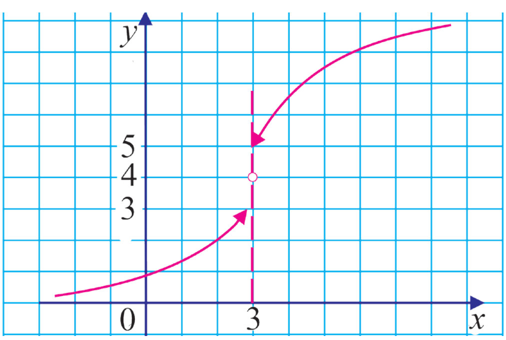


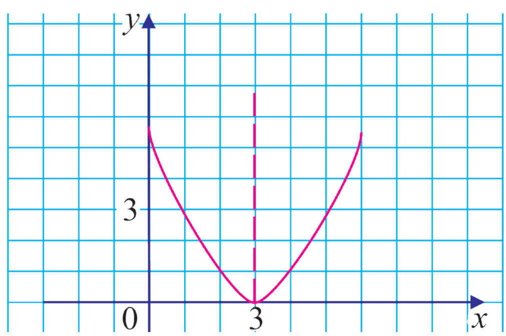






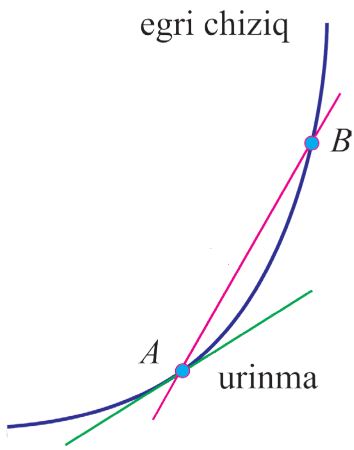




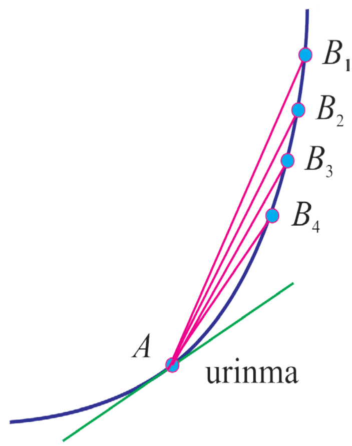


5–6 HOSILA, UNING GEOMETRIK VA FIZIK MA'NOSI

12-rasmda egri chiziq, kesuvchi va urinma tasvirlangan.



12-rasm.

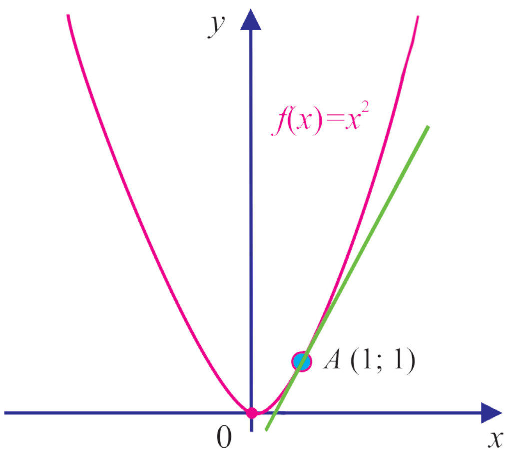


13-rasm.

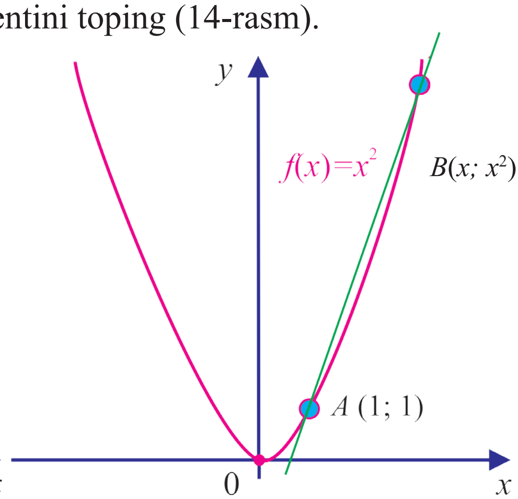
B nuqta B_1, B_2, ... holatlarni ketma-ket qabul qilib, A nuqtaga *egri chiziq bo'ylab* yaqinlashsa (13-rasm), mos kesuvchilarning egri chiziqqa A nuqtada o'tkazilgan urinma holatini olishga intilishini *intuitiv tarzda* qabul qilamiz.

Bu holda, ravshanki, AB to'g'ri chiziqning burchak koeffitsiyenti urinmaning burchak koeffitsiyentiga yaqinlashadi.

1-misol. $f(x) = x^2$ funksiyaning grafigiga $A(1;\ 1)$ nuqtada urinadigan to'g'ri chiziqning burchak koeffitsiyentini toping (14-rasm).



14-rasm.



15-rasm.

△ $f(x) = x^2$ funksiyaning grafigiga tegishli ixtiyoriy $B(x, x^2)$ nuqtani qaraylik (15-rasm).

AB to‘g‘ri chiziqning burchak koeffitsiyenti

$\frac{f(x)-f(1)}{x-1}$ yoki $\frac{x^2-1}{x-1}$ ga teng.

B nuqta A nuqtaga egri chiziq bo‘ylab yaqinlashganda, x ning qiymati 1 ga yaqinlashadi, bunda $x \neq 1$.

Demak, AB to‘g‘ri chiziqning burchak koeffitsiyenti urinmaning burchak koeffitsiyenti k ga yaqinlashadi, ya’ni:

$$k = \lim_{x\to 1}\frac{x^2-1}{x-1} = \lim_{x\to 1}\frac{(x+1)(x-1)}{x-1} = \lim_{x\to 1}(x+1) = 2\cdot$$

Shunday qilib, $k = 2$. ▲

$y = f(x)$ funksiya berilgan bo‘lsin. Uning grafigiga tegishli bo‘lgan $A(x; f(x))$ va $B(x+h; f(x+h))$ nuqtalarni qaraylik (16-rasm).

AB to‘g‘ri chiziqning burchak koeffitsiyenti

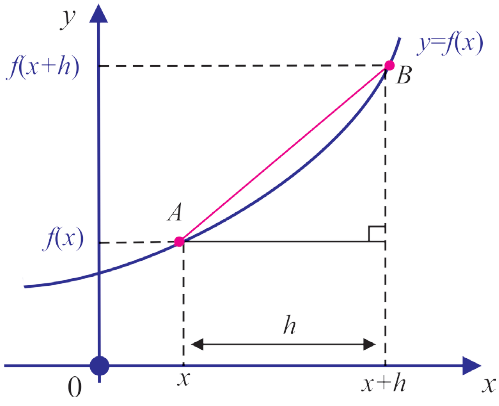
$$\frac{f(x+h)-f(x)}{x+h-x} = \frac{f(x+h)-f(x)}{h}$$

ayirmali nisbatga teng.

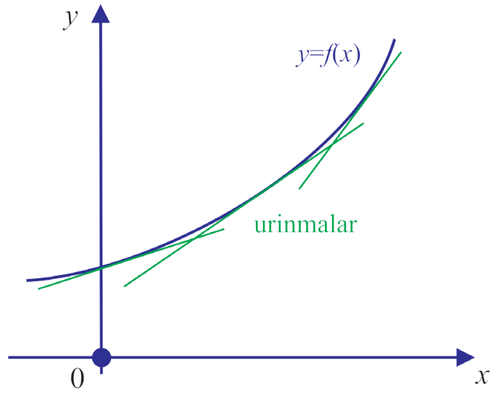
B nuqta A nuqtaga egri chiziq bo‘ylab yaqinlashganda $h \to 0$, ya’ni h orttirma nolga intiladi, AB kesuvchi esa funksiya grafigiga A nuqtada o‘tkazilgan urinmaga intiladi.

Shu bilan birga, AB to‘g‘ri chiziqning burchak koeffitsiyenti urinmaning burchak koeffitsiyentiga yaqinlashadi.

Boshqacha aytganda, h ning qiymati 0 ga intilganda ixtiyoriy $(x; f(x))$ nuqtada o‘tkazilgan urinmaning burchak koeffitsiyenti $\frac{f(x+h)-f(x)}{h}$ ayirmali nisbatning limit qiymatiga, ya’ni $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ qiymatga teng bo‘ladi.



16-rasm.



17-rasm.

x ning mazkur limit mavjud bo‘lgan ixtiyoriy qiymatiga funksiya grafigiga $(x, f(x))$ nuqtada o‘tkazilgan urinmaning burchak koeffitsiyentining yagona qiymatini mos qo‘yish mumkin (17-rasm).

Demak, $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ formula yangi funksiyani ifodalaydi.

Mana shu funksiya $y=f(x)$ funksiyaning **hosilaviy funksiyasi**, yoki sodda qilib **hosilasi** deb ataladi.

Ta’rif. $y=f(x)$ funksiyaning **hosilasi** deb quyidagi limitga (agar u mavjud bo‘lsa) aytiladi:

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}. \qquad (1)$$

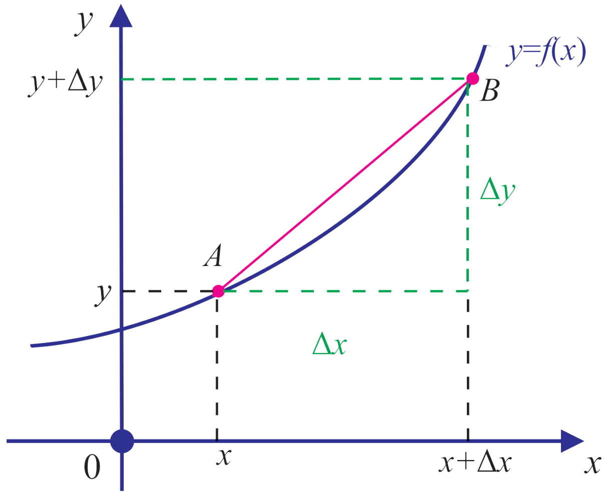
Odatda $y=f(x)$ funksiyaning hosilasi $f'(x)$ kabi belgilanadi.
Hosilani topish amali *differensiallash* deyiladi.

$f'(x)$ belgilash o‘rniga $\dfrac{dy}{dx}$ kabi belgilash ham qabul qilingan.

Bu belgilashning “kasr” ko‘rinishda ekanligini quyidagicha tushuntirish mumkin.

Agar orttirmalarni $h=\Delta x$, $f(x+\Delta x)-f(x)=\Delta y$ deb belgilasak,

$f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ dan quyidagiga ega bo‘lamiz (18-rasm): $f'(x)=\lim\limits_{\Delta x\to 0}\dfrac{\Delta y}{\Delta x}=\dfrac{dy}{dx}$.



18-rasm.

Yuqoridagi mulohazalardan shunday xulosaga kelamiz: $y = f(x)$ funksiya hosilasining x_0 nuqtadagi qiymati funksiya grafigiga shu nuqtada o‘tkazilgan urinmaning burchak koeffitsiyentiga teng. Hosilaning *geometrik ma’nosi* shundan iboratdir.

2-misol. Moddiy nuqta $s=s(t)$ (s – metrlarda, t – sekundlarda o‘lchanadi) qonunga muvofiq to‘g‘ri chiziq bo‘ylab harakat qilmoqda. Shu moddiy nuqtaning vaqtning t momentidagi (paytidagi) tezligi $v(t)$ ni toping.

△ Ma’lumki, oniy tezlik nuqtaning kichik vaqt oralig‘i Δt dagi o‘rtacha tezligi $v(t) = \dfrac{s(t+\Delta t) - s(t)}{\Delta t}$ ga taqriban teng. Δt nolga intilganda oniy tezlik va o‘rtacha tezlik orasidagi farq ham nolga intiladi. Demak, moddiy nuqtaning t momentdagi oniy tezligi

$$v(t) = \lim_{\Delta t \to 0} \frac{s(t+\Delta t) - s(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = s'(t). \ ▲$$

Shunday qilib, t momentdagi oniy tezlik nuqtaning harakat qonuni $s(t)$ funksiyadan olingan hosilaga teng ekan.

Hosilaning *fizik ma’nosi* ana shundan iborat. Umuman aytganda, *hosila funksiyaning o‘zgarish tezligidir.*

Misollar

Hosila ta’rifidan foydalanib, funksiyalarning hosilasini toping:

1. $f(x)=x^2$; **2.** $f(x)=5$; **3.** $f(x)=x^3-7x+5$;

4. $f(x)=x^4$; **5.** $f(x)=\frac{1}{x}$; **6.** $f(x)=\sqrt{x}$; 7. $f(x)=\sqrt[3]{x}$.

△ **1.** $h\neq 0$ bo‘lgani uchun

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{(x+h)^2+x^2}{h}=$$

$$=\lim_{h\to 0}\frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h}=\lim_{h\to 0}\frac{\cancel{h}(2x+h)}{\cancel{h}}=\lim_{h\to 0}(2x+h)=2x.$$

2. $h\neq 0$ bo‘lgani uchun $f(x+h)=5$, $f(x+h)-f(x)=5-5=0$,

$$\frac{f(x+h)-f(x)}{h}=\frac{0}{h}=0 \quad \text{Demak, } f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=0.$$

3. $h\neq 0$ bo‘lgani uchun

$f(x+h)=(x+h)^3-7(x+h)+5=x^3+3x^2h+3xh^2+h^3-7x-7h+5$;

$f(x+h)-f(x)=x^3+3x^2h+3xh^2+h^3-7x-7h+5-x^3+7x-5=$

$=3x^2h+3xh^2+h^3-7h$.

$$\frac{f(x+h)-f(x)}{h}=\frac{3x^2h+3xh^2+h^3-7h}{h}=3x^2+3xh+h^2-7.$$

$h\to 0$ da $3xh+h^2\to 0$ bo‘lgani uchun

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=3x^2-7.$$

4. Qisqa ko‘paytirish formulalariga ko‘ra $a^4-b^4=(a-b)(a+b)(a^2+b^2)$.

Demak, $(x+h)^4-x^4=(x+h-x)(x+h+x)((x+h)^2+x^2)=$

$=h(2x+h)(2x^2+2xh+h^2)=2hx(2x+h)(x+h)+h^3(2x+h)=$

$=2hx(2x^2+h(3x+h))+h^3(2x+h)$; $h\to 0$ bo‘lsa,

$2h^2x(3x+h)\to 0$ va $h^3(2x+h)\to 0$ bo‘lgani uchun

$$\lim_{h\to 0}\frac{(x+h)^4-x^4}{h}=\lim_{h\to 0}(4x^3+2hx(3x+h))+h^2(2x+h))=4x^3.$$

Demak, $f'(x)=(x^4)'=4x^3$.

5. $f(x)=\frac{1}{x}$, $x\neq 0$ bo‘lsin,

$$f(x+h)-f(x)=\frac{1}{x+h}-\frac{1}{x}=\frac{x-(x+h)}{(x+h)x}=-\frac{h}{(x+h)x},$$

$$\frac{f(x+h)-f(x)}{h}=\frac{-1}{(x+h)x}.$$

$h\to 0$ da $x+h\to x$ bo'lgani uchun $f'(x)=-\frac{1}{x^2}$ bo'ladi.

6. $f(x)=\sqrt{x}$, $x>0$, $x+h>0$ bo'lsin, $\frac{f(x+h)-f(x)}{h}=\frac{\sqrt{x+h}-\sqrt{x}}{h}$ ayirmali nisbatni tuzamiz va uni soddalashtiramiz:

$$\frac{f(x+h)-f(x)}{h}=\frac{\left(\sqrt{x+h}-\sqrt{x}\right)\left(\sqrt{x+h}+\sqrt{x}\right)}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=$$

$$=\frac{(x+h)-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=\frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=\frac{1}{\sqrt{x+h}+\sqrt{x}}.$$

$h\to 0$ da $\sqrt{x+h}\to\sqrt{x}$ bo'lgani uchun $f'(x)=\frac{1}{2\sqrt{x}}$ bo'ladi.

7. Ayirmali nisbatni tuzamiz:

$$\frac{f(x+h)-f(x)}{h}=\frac{\sqrt[3]{x+h}-\sqrt[3]{x}}{h}=\frac{\left(\sqrt[3]{x+h}-\sqrt[3]{x}\right)(\sqrt[3]{(x+h)^2}+\sqrt[3]{(x+h)x}+\sqrt[3]{x^2})}{h(\sqrt[3]{(x+h)^2}+\sqrt[3]{(x+h)x}+\sqrt[3]{x^2})}=$$

$$=\frac{x+h-x}{h(\sqrt[3]{(x+h)^2}+\sqrt[3]{(x+h)x}+\sqrt[3]{x^2})}=\frac{h}{h(\sqrt[3]{(x+h)^2}+\sqrt[3]{(x+h)x}+\sqrt[3]{x^2})}=$$

$$=\frac{1}{\sqrt[3]{(x+h)^2}+\sqrt[3]{(x+h)x}+\sqrt[3]{x^2}}.$$

$h\to 0$ da $\frac{1}{\sqrt[3]{(x+h)^2}+\sqrt[3]{(x+h)x}+\sqrt[3]{x^2}}\to\frac{1}{3\sqrt[3]{x^2}}$. Demak, $\left(\sqrt[3]{x}\right)'=\frac{1}{3\sqrt[3]{x^2}}$.

Javob: **1.** $2x$. **2.** 0. **3.** $3x^2-7$. **4.** $4x^3$. **5.** $-\frac{1}{x^2}$. **6.** $\frac{1}{2\sqrt{x}}$. **7.** $\frac{1}{3\sqrt[3]{x^2}}$.▲

Eslatish joizki, x miqdor x dan $x+h$ gacha o‘zgarganda $y=f(x)$ miqdor o‘zgarishining **o‘rtacha tezligi**

$$\frac{f(x+h)-f(x)}{h}$$

ayirmali nisbatga teng.

Bundan $\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$ ifoda $y=f(x)$ miqdor o‘zgarishining **oniy tezligini** bildiradi.

Mashqlar

13. Quyidagi funksiyaning hosilasi nimaga teng?

a) $f(x)=x^3$; b) $f(x)=x^{-1}$; c) $f(x)=x^{\frac{1}{2}}$; d) $f(x)=c$.

14. Jadvalni daftaringizga ko‘chiring va to‘ldiring:

a)

$f(x)$	$f'(x)$
x^1	
x^2	
x^3	
x^{-1}	
$x^{\frac{1}{2}}$	

b) Fikringizcha, $y=x^n$ funksiya hosilasi nimaga teng (bu yerda n – ratsional son) ?

15. Ta’rifdan foydalanib, funksiya hosilasini toping:

a) $f(x)=2x + 3$; b) $f(x)=3x^2 + 5x + 1$; c) $f(x)=2x^3 + 4x^2+6x - 1$.

16*. Daftaringizga ko‘chiring va to‘ldiring:

a) $f(x)=ax + b$ uchun $f'(x) = ...$;

b) $f(x)=ax^2 + bx +c$ uchun $f'(x) = ...$;

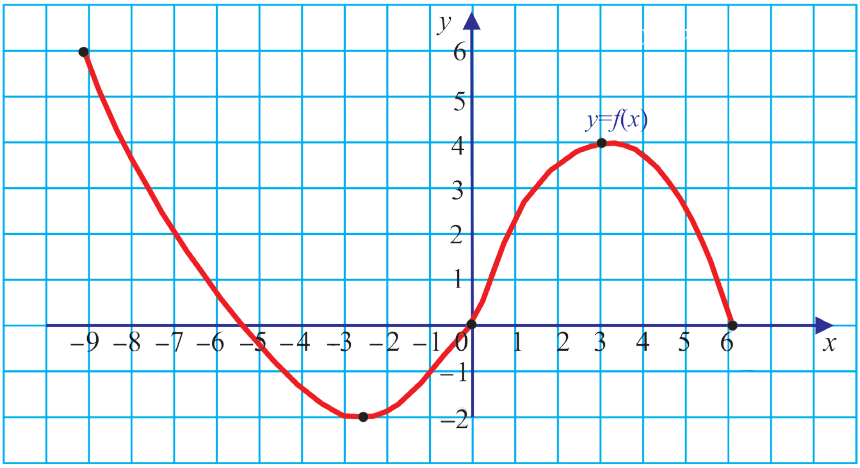
c) $f(x)=ax^3 + bx^2 + cx + d$ uchun $f'(x) = ...$

17*. Quyidagi tasdiqlarni isbotlang:

a) $f(x) = cg(x)$ bo‘lsa, u holda $f'(x) = cg'(x)$;

b) $f(x) = g(x) + h(x)$ bo‘lsa, u holda $f'(x) = g'(x) + h'(x)$.

18*. Funksiya grafigiga qarab hosilalar qiymatlarini solishtiring:



a) $f'(-7)$ va $f'(-2)$; c) $f'(-9)$ va $f'(0)$;
b) $f'(-4)$ va $f'(2)$; d) $f'(-1)$ va $f'(5)$.

19. 1) Yuqoridagi funksiya grafigiga qarab ushbu shartlarni qanoatlantiradigan x_1, x_2 nuqtalarni toping (x_1, x_2 – Ox o‘qidagi nuqtalar: –9, –8, ..., 5, 6):

a) $f'(x_1) > 0,\ f'(x_2) > 0$; b) $f'(x_1) < 0,\ f'(x_2) > 0$;

c) $f'(x_1) < 0,\ f'(x_2) < 0$; d) $f'(x_1) > 0,\ f'(x_2) < 0$.

2) Grafikka qarab ushbu savollarga javob bering:

a) funksiya qaysi oraliqda o‘suvchi? qaysi oraliqda kamayuvchi?
b) funksiyaning [0; 3], [3; 6], [–9; –6] oraliqlaridagi orttirmalarini hisoblang.

3) Funksiya qaysi nuqtada eng katta, qaysi nuqtada eng kichik qiymatni qabul qiladi?

4) Funksiya qaysi nuqtalarda nolga aylanyapti?

5) Qaysi oraliqda funksiya musbat qiymatlarni qabul qilyapti?

6) Qaysi oraliqda funksiya manfiy qiymatlarni qabul qilyapti?

7–9 HOSILANI HISOBLASH QOIDALARI

Agar $f(x)$ va $g(x)$ funksiyalarning har biri hosilaga ega bo‘lsa, u holda quyidagi differensiallash qoidalari o‘rinlidir:

1. Yig‘indining hosilasi hosilalar yig‘indisiga teng:

$$(f(x)+g(x))'=f'(x)+g'(x). \quad (1)$$

2. Ayirmaning hosilasi hosilalar ayirmasiga teng:

$$(f(x)-g(x))'=f'(x)-g'(x). \quad (2)$$

1-misol. Funksiyaning hosilasini toping:

1) $f(x)=x^3+x^2-x+10$; 2) $f(x)=\sqrt{x}-\frac{1}{\sqrt{x}}$.

△ Hosilani topishda 1, 2-qoidalaridan va hosilalar jadvalining 1, 3-bandlaridan foydalanamiz, ya’ni:

1) $f'(x)=(x^3)'+(x^2)'-(x)'+10=3x^2+2x-1$;

2) $f'(x)=\left(x^{\frac{1}{2}}\right)'-\left(x^{-\frac{1}{2}}\right)'=\frac{1}{2}x^{-\frac{1}{2}}+\frac{1}{2}x^{-\frac{3}{2}}$.

Javob: 1) $3x^2+2x-1$; 2) $\frac{1}{2}x^{-\frac{1}{2}}+\frac{1}{2}x^{-\frac{3}{2}}$. ▲

3. O‘zgarmas ko‘paytuvchini hosila belgisidan tashqariga chiqarish mumkin:

$$(cf(x))'=c\cdot f'(x), \quad c - \text{o‘zgarmas son}. \quad (3)$$

2-misol. Funksiyaning hosilasini toping:

1) $f(x)=7x^3-5x^2+4$; 2) $f(x)=3\sqrt{x}+\frac{5}{x}-x^3$.

△ Hosilani topishda 1, 2, 3-qoidalaridan va hosilalar jadvalining 1, 3-bandlaridan foydalanamiz, ya’ni:

1) $f'(x)=(7x^3-5x^2+4)'=(7x^3)'-(5x^2)'+(4)'=21x^2-10x$;

2) $f'(x)=\left(3\sqrt{x}+\frac{5}{x}-x^3\right)'=3\left(\sqrt{x}\right)'+5\cdot\left(\frac{1}{x}\right)'-(x^3)'=\frac{3}{2\sqrt{x}}-\frac{5}{x^2}-3x^2$.

Javob: 1) $21x^2-10x$; ; 2) $\frac{3}{2\sqrt{x}}-\frac{5}{x^2}-3x^2$. ▲

4. Ko‘paytmaning hosilasi:

$$(f(x)g(x))'=f'(x)g(x)+f(x)g'(x). \qquad (4)$$

3- misol. Funksiyaning hosilasini toping:

1) $f(x)=(2x+4)(3x+1)$; 2) $f(x)=(3x^2+4x+1)(2x+6)$; 3) $f(x)=\sqrt[3]{x}\cdot(x^2-5x)$.

△ Hosilani topishda 1, 3, 4-qoidalaridan va hosilalar jadvalining 1-, 3- bandlaridan foydalanamiz, ya’ni:

1) $f'(x)=((2x+4)(3x+1))'=(2x+4)'(3x+1)+(2x+4)(3x+1)'=$
$=2(3x+1)+3(2x+4)=6x+2+6x+12=12x+14;$

2) $f'(x)=((3x^2+4x+1)(2x+6))'=(3x^2+4x+1)'(2x+6)+$
$+(3x^2+4x+1)(2x+6)'=(6x+4)(2x+6)+2(3x^2+4x+1)=18x^2+52x+26;$

3) $f'(x)=\left(\sqrt[3]{x}\cdot\left(x^2-5x\right)\right)'=\left(\sqrt[3]{x}\right)'\left(x^2-5x\right)+\sqrt[3]{x}\left(x^2-5x\right)'=$

$=\dfrac{1}{3\sqrt[3]{x^2}}\left(x^2-5x\right)+\sqrt[3]{x}(2x-5)=\dfrac{x^2-5x}{3\sqrt[3]{x^2}}+(2x-5)\sqrt[3]{x}=\dfrac{x^2-5x+3(2x-5)\sqrt[3]{x^3}}{3\sqrt[3]{x^2}}=$

$=\dfrac{x^2-5x+6x^2-15x}{3\sqrt[3]{x^2}}=\dfrac{7x^2-20x}{3\sqrt[3]{x^2}}=\dfrac{x(7x-20)}{3\sqrt[3]{x^2}}=\dfrac{\sqrt[3]{x}}{3}(7x-20).$

Javob: 1) $12x+14;$; 2) $18x^2+52x+26$; 3) $\dfrac{\sqrt[3]{x}}{3}(7x-20)$. ▲

5. Bo‘linmaning hosilasi:

$$\left(\frac{f(x)}{g(x)}\right)'=\frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}, \quad \text{bunda} \quad g(x)\neq 0. \qquad (5)$$

4- misol. Funksiyaning hosilasini toping:

1) $f(x)=\dfrac{x+1}{x-2}$; 2) $f(x)=\dfrac{3x+7}{x-5}$; 3) $f(x)=\dfrac{\sqrt{x}}{5x-7}$.

△ Hosilani topishda 1, 3, 5-qoidalaridan va hosilalar jadvalining 1, 3- bandlaridan foydalanamiz, ya’ni:

1) $f'(x)=\left(\dfrac{x+1}{x-2}\right)'=\dfrac{(x+1)'(x-2)-(x+1)(x-2)'}{(x-2)^2}=\dfrac{x-2-(x+1)}{(x-2)^2}=-\dfrac{3}{(x-2)^2};$

2) $f'(x)=\left(\dfrac{3x+7}{x-5}\right)'=\dfrac{(3x+7)'(x-5)-(3x+7)(x-5)'}{(x-5)^2}=$

$=\dfrac{3(x-5)-(3x+7)\cdot 1}{(x-5)^2}=\dfrac{3x-15-3x-7}{(x-5)^2}=-\dfrac{22}{(x-5)^2};$

3) $f'(x)=\left(\dfrac{\sqrt{x}}{5x-7}\right)'=\dfrac{(\sqrt{x})'\cdot(5x-7)-\sqrt{x}\cdot(5x-7)'}{(5x-7)^2}=$

$$=\frac{\frac{1}{2\sqrt{x}}(5x-7)-\sqrt{x}\cdot 5}{(5x-7)^2}=\frac{5x-7-10x}{2\sqrt{x}(5x-7)^2}=-\frac{7+5x}{2\sqrt{x}(5x-7)^2}.$$

Javob: 1) $-\frac{3}{(x-2)^2}$; 2) $-\frac{22}{(x-5)^2}$; 3) $-\frac{7+5x}{2\sqrt{x}(5x-7)^2}$. ▲

5- misol. Funksiyalarning hosilasini toping:

1) $f(x)=\sin x$; 2) $f(x)=\cos x$; 3) $f(x)=\operatorname{tg} x$.

△ 1) Ayirmali nisbatni topishda sinuslar ayirmasini ko‘paytmaga keltirish formulasidan foydalanamiz:

$$\frac{\sin(x+h)-\sin x}{h}=\frac{2\sin\frac{h}{2}\cos\frac{2x+h}{2}}{h}=\frac{\sin\frac{h}{2}}{\frac{h}{2}}\cos\frac{2x+h}{2}.$$

$h\to 0$ da $\frac{\sin\frac{h}{2}}{\frac{h}{2}}\to 1,\ \ \cos\frac{2x+h}{2}\to\cos x$ ekanini isbotlash mumkin.

Demak, $(\sin x)'=\cos x$.

2) Ayirmali nisbatni topishda kosinuslar ayirmasini ko‘paytmaga keltirish formulasidam foydalanamiz:

$$\frac{\cos(x+h)-\cos x}{h}=-\frac{2\sin\frac{h}{2}\sin\frac{2x+h}{2}}{h}=-\frac{\sin\frac{h}{2}}{\frac{h}{2}}\sin\frac{2x+h}{2}=-\frac{\sin\frac{h}{2}}{\frac{h}{2}}\cdot\sin(x+\frac{h}{2}).$$

$h\to 0$ da; $\sin(x+\frac{h}{2})\to\sin x$ ekanini isbotlash mumkin.

Demak, $(\cos x)'=-\sin x$.

3) Hosilani topishning 5-qoidasi hamda shu misolning 1-, 2-qism javoblaridan foydalanib, $f(x)=\operatorname{tg}x=\frac{\sin x}{\cos x}$ funksiyaning hosilasini topamiz:

$$f'(x)=(\operatorname{tg}x)'=\left(\frac{\sin x}{\cos x}\right)'=\frac{(\sin x)'\cos x-\sin x(\cos x)'}{\cos^2 x}=$$

$$=\frac{\cos x\cos x-\sin x(-\sin x)}{\cos^2 x}=\frac{\cos^2 x+\sin^2 x}{\cos^2 x}=\frac{1}{\cos^2 x}.$$

Javob: 1) $(\sin x)'=\cos x$; 2) $(\cos x)' = -\sin x$; 3) $(\mathrm{tg}x)' = \dfrac{1}{\cos^2 x}$. ▲

Hosilani hisoblashda differensiallash qoidalari va quyidagi jadvaldan foydalanish maqsadga muvofiqdir.

Hosilalar jadvali

№	*Funksiyalar*	*Hosilalar*
1	c – o‘zgarmas	0
2	$kx+b$, k, b – o‘zgarmaslar	k
3	x^p, p – o‘zgarmas	px^{p-1}
4	$\sin x$	$\cos x$
5	$\cos x$	$-\sin x$
6	$\mathrm{tg}x$	$\dfrac{1}{\cos^2 x}$
7	$\mathrm{ctg}x$	$-\dfrac{1}{\sin^2 x}$
8	a^x, $a>0$	$a^x \ln a$
9	e^x	e^x
10	$\ln x$	$1/x$
11	$\lg x$	$\dfrac{1}{x \cdot \ln 10}$
12	$\log_a x$, $a>0$, $a=1$	$\dfrac{1}{x \cdot \ln a}$

? Savol va topshiriqlar

1. Hosilani hisoblash qoidalarini ayting. Har bir qoidaga misol keltiring.
2. Hosilalar jadvalining 4-, 5- bandlarini isbotlang.
3. Funksiyaning $x=x_0$ nuqtadagi hosilasi nima-yu, hosilaviy funksiya nima? Ularning qanday farqi bor? Misollarda tushuntiring.

Mashqlar

Hosilani toping (**20–22**):

20. 1) $y = x^4$; 2) $y = \frac{1}{x^2}$; 3) $y = \frac{1}{x^3}$.

21. 1) $y = x^4 - x^2 + x$; 2) $y = \frac{1}{x} + x$; 3) $y = x^3 + \sqrt[3]{x}$;

4) $y = x^4 + x^3 + x^2 - x - \frac{1}{x} - \frac{1}{x^2}$.

22. 1) $y = (x-1)(x^2-5)$; 2) $y = \frac{x^2-4}{x-2}$;

3) $y = (x^4 - \sqrt{x})(x^2 + x)$; 4) $y = \frac{\sqrt{x}+1}{x-1}$.

23. Moddiy nuqtaning berilgan t_0 vaqtdagi tezligini hisoblang:

1) $s(t) = t^3 - 2t^2 + t$; $t_0 = 5$; 2) $s(t) = 5t + t^3 + \sqrt{t}$, $t_0 = 4$.

24. Funksiyaning abssissasi berilgan nuqtadagi hosilasini hisoblang:

1) $f(x) = x^2 + 5x - 3$, $x_0 = 1$; 3) $f(x) = 2\sqrt{x} + x^3 + \frac{1}{2}$, $x_0 = 4$;

2) $f(x) = 4 - 3x$, $x_0 = -2$; 4) $f(x) = x^2 + \lg 2$, $x_0 = 1$.

Hosilani toping (**25–29**):

25. 1) $y = 2x^3 - 4x^2 + 5$; 3) $y = \frac{4}{x} + \frac{x}{4}$;

2) $y = 7x^2 - 2x + \sqrt{7}$; 4) $y = x^2 + \frac{1}{x^2}$.

26. 1) $y = (x-2)(x+2)$; 3) $y = \frac{x^2-9}{x-3}$;

2) $y = (x+2)^3$; 4) $y = x^2 + \lg 7 + \sin\frac{\pi}{9}$.

27. 1) $y = x^8 + 7x^2 + 5x$; 2) $y = 2x^8 + x^6$;

3) $y = \frac{x^4}{x^6-1}$; 4) $y = \frac{x^2+x+1}{x^3-1}$;

5) $y = x^{-2} + \frac{1}{x}$; 6) $y = x^4 - 4x$;

7) $y = \sqrt[5]{x^4} + \sqrt[3]{x^2}$; 8) $y = (x^5 + x^{-5})(x^2 + x^{-2})$.

28. 1) $f(x)=x^5+5x^4+4x^3+3x^2+2x+1$; 2) $f(x)=\sin^2 x+\cos^2 x$;

3) $f(x)=\dfrac{x}{\cos x}$; 4) $f(x)=\text{tg}x$; 5) $y=8^x$;

6) $y=\log_2 x+\log_2 3$; 7) $y=2^x x$; 8) $y=x\ln x$;

9) $y=e^x\cos x$; 10) $y=2e^x-\ln x+\dfrac{1}{x}$.

29. 1) $y=2^x\sin x$; 2) $y=e^x(\cos x+\sin x)$; 3) $y=x\,\text{tg}x$;

4) $y=\dfrac{\ln x}{x}$; 5) $y=3\sin^2 x$; 6) $y=5x+\sqrt{x}+\sqrt[3]{x}$;

7) $y=(x+1)(\ln x+1)$; 8) $y=(2+x)^3$; 9) $y=(3x+5)^6+2019$.

30. Moddiy nuqtaning berilgan t_0 vaqtdagi tezligini toping:

1) $s(t)=t^2+5t+1$, $t_0=1$; 2) $s(t)=4t^3+\dfrac{1}{t}+1$, $t_0=1$.

31. Funksiyaning berilgan nuqtadagi hosilasini toping:

1) $f(x)=(x+1)^3$, $x_0=-1$; 2) $f(x)=\sin x$, $x_0=\dfrac{\pi}{2}$.

32. Hosilani toping:

1) $y=2\sin x$; 2) $y=\sqrt{3}-\text{tg}x$; 3) $y=-3\cos x$; 4) $y=\text{tg}x-\text{ctg}x$;

5) $y=4x-\cos x$; 6) $y=x^2\sin x$; 7) $y=\dfrac{x}{\sin x}$; 8) $y=x\sin x+\cos x$.

33. Funksiyaning x_0 nuqtadagi hosilasini hisoblang:

1) $f(x)=\dfrac{2x+1}{3x-5}$, $x_0=2$; 2) $f(x)=\text{tg}x-x+2$, $x_0=\dfrac{\pi}{4}$;

3) $f(x)=x(\lg x-1)$, $x_0=10$; 4) $f(x)=\text{tg}x-\dfrac{1}{2}\ln x$, $x_0=\dfrac{\pi}{4}$.

34. Hosilani nolga aylantiradigan nuqtani toping:

1) $f(x)=x^4-4x$; 2) $f(x)=\text{tg}x-x$;

3) $f(x)=x^8-2x^4+3$; 4) $f(x)=\log_2 x-\dfrac{x}{\ln 2}$.

10–12 MURAKKAB FUNKSIYANING HOSILASI

Murakkab funksiya. $y=(x^2+3x)^4$ funksiyani qaraylik. Agar biz $g(x)=x^2+3x$, $f(x)=x^4$ belgilashlarni kiritsak, $y=(x^2+3x)^4$ funksiya $y=f(g(x))$ ko'rinishini oladi. Biz $y=f(g(x))$ funksiyani *murakkab funksiya* deymiz.

1-misol. Agar $f(x)=x^2$ va $g(x)=\frac{x-2}{x+3}$ bo'lsa, quyidagilarni toping:

1) $f(g(2))$; 2) $f(g(-4))$; 3) $g(f(1))$;

4) $f((-4))$; 5) $f(f(1))$ 6) $g(g(-1))$.

△ Berilgan funksiyalardan foydalanib, hisoblashlarni bajaramiz:

1) $f(g(x))=f\left(\frac{x-2}{x-3}\right)$, bundan $f(g(2))=f\left(\frac{2-2}{2+3}\right)=f(0)=0^2=0$;

2) $f(g(-4))=f\left(\frac{-4-2}{-4+3}\right)=f(6)=6^2=36$;

3) $g(f(1))=g(1^2)=g(1)=\frac{1-2}{1+3}=-\frac{1}{4}$;

4) $g(f(-4))=g((-4)^2)=g(16)=\frac{16-2}{16+3}=\frac{14}{19}$;

5) $f(f(1))=f(1^2)=f(1)=1^2=1$;

6) $g(g(-1))=g\left(\frac{-1-2}{-1+3}\right)=g\left(-\frac{3}{2}\right)=\frac{-\frac{3}{2}-2}{-\frac{3}{2}+3}=\frac{-3,5}{1,5}=-\frac{7}{3}$.

Javob: 1) 0; | 2) 36; | 3) $-\frac{1}{4}$; | 4) $\frac{14}{19}$; | 5) 1; | 6) $-\frac{7}{3}$. ▲

Murakkab funksiyaning hosilasi uchun ushbu formula o'rinli:

$$(f(g(x)))'=f'(g(x))\cdot g'(x) \qquad (1)$$

2-misol. Funksiyaning hosilasini toping (k, b – o‘zgarmas sonlar):

1) $f(x) = (kx + b)^n$; 2) $f(x) = \sin(kx + b)$;
3) $f(x) = \cos(kx + b)$; 4) $f(x) = \mathrm{tg}(kx + b)$.

△ 1) $f(t) = t^n$ va $t(x) = kx + b$ funksiyalarga (1) formulani qo‘llaymiz:

$$((kx+b)^n)' = (t^n)' \cdot (kx+b)' = nt^{n-1} \cdot k = n \cdot k \cdot (kx + b)^{n-1}.$$

2) $f(t) = \sin t$ va $t(x) = kx+b$ funksiyalarga (1) formulani qo‘llaymiz:

$$(\sin(kx+b))' = (\sin t)' \cdot (kx+b)' = k \cdot \cos t = k \cdot \cos(kx + b).$$

3) $f(t) = \cos t$ va $t(x) = kx + b$ funksiyalarga (1) formulani qo‘llaymiz:

$$(\cos(kx + b))' = (\cos t)' \cdot (kx+b)' = -k \cdot \sin t = -k \cdot \sin(kx + b).$$

4) $f(t) = \mathrm{tg}\, t$ va $t(x) = kx+b$ funksiyalarga (1) formulani qo‘llaymiz:

$$(\mathrm{tg}(kx + b))' = (\mathrm{tg}\, t)' \cdot (kx + b)' = \frac{1}{\cos^2 t} \cdot k = \frac{k}{\cos^2(kx+b)}.$$

Javob: 1) $((kx + b)^n)' = n \cdot k \cdot (kx + b)^{n-1}$; 2) $(\sin(kx + b))' = k \cdot \cos(kx+b)$;
3) $(\cos(kx+b)' = -k \cdot \sin(kx+b)$; 4) $(\mathrm{tg}(kx+b)' = \dfrac{k}{\cos^2(kx+b)}$. ▲

3-misol. $f(x) = \sin 8x \cdot e^{(3x+2)}$ funksiya hosilasini toping.

△ Hosilani topishning 4-qoidasi hamda (1) formulani qo‘llab hosilani topamiz:

$$f'(x) = (\sin 8x e^{(3x+2)})' = (\sin 8x)' e^{3x+2} + \sin 8x \cdot (e^{3x+2})' = \cos 8x e^{3x+2} \cdot (8x)' +$$
$$+ \sin 8x e^{3x+2} \cdot (3x+2)' = e^{3x+2} \cdot (8\cos 8x + 3\sin 8x).$$

Javob: $e^{3x+2} \cdot (8\cos 8x + 3\sin 8x)$. ▲

4-misol. $h(x) = (x^3 + 1)^5$ funksiyaning $x_0 = 1$ nuqtadagi hosilasini toping.

△ (1) formuladan foydalanib hosilani hisoblaymiz:

$$h'(x) = 5(x^3+1)^4(x^3+1)' = 5(x^3+1)^4 3x^2 = 15x^2(x^3+1)^4.$$

Demak, $h'(1) = 15(1^3+1)^4 \cdot 1^2 = 15 \cdot 16 = 240$. *Javob*: 240. ▲

5-misol. $f(x) = 2^{\cos x}$ funksiyaning hosilasini toping.

△ (1) formuladan foydalanib hosilani hisoblaymiz:

$f'(x) = 2^{\cos x} \ln 2 (\cos x)' = -\sin x\, 2^{\cos x} \ln 2$. *Javob*: $-\sin x\, 2^{\cos x} \ln 2$. ▲

6-misol. $f(x) = \mathrm{tg}^5 x$ funksiyaning hosilasini toping.

△ (1) formuladan foydalanib hosilani hisoblaymiz:

$$f'(x) = 5\,\mathrm{tg}^4 x\,(\mathrm{tg}\,x)' = 5\mathrm{tg}^4 x \frac{1}{\cos^2 x}.$$

Javob: $\dfrac{5\mathrm{tg}^4 x}{\cos^2 x}$. ▲

7-misol. $h(x) = 3^{\cos x} \cdot \log_7(x^3+2x)$ funksiyaning hosilasini toping.

△ $f(x) = 3^{\cos x}$ va $g(x) = \log_7(x^3+2x)$ belgilashlarni kiritib, (1) formulani – murakkab funksiya hosilasini topish formulasini qo‘llaymiz:

$$f'(x) = (3^{\cos x})' = 3^{\cos x}\ln 3 \cdot (\cos x)' = -3^{\cos x}\ln 3 \cdot \sin x,$$

$$g'(x) = (\log_7(x^3+2x))' = \frac{1}{(x^3+2x)\ln 7} \cdot (x^3+2x)' = \frac{3x^2+2}{(x^3+2x)\ln 7}$$

hamda $h(x)$ funksiyani 2 ta funksiyaning ko‘paytmasi deb qaraymiz:

$$h'(x) = (3^{\cos x} \cdot \log_7(x^3+2x))' = (3^{\cos x})' \cdot \log_7(x^3+2x) +$$

$$+\, 3^{\cos x} \cdot (\log_7(x^3+2x))' = -3^{\cos x} \cdot \ln 3 \cdot \sin x \cdot \log_7(x^3+2x) + \frac{3^{\cos x}(3x^2+2)}{(x^3+2x)\ln 7}.$$

Javob: $-3^{\cos x} \cdot \ln 3 \cdot \sin x \cdot \log_7(x^3+2x) + \dfrac{3^{\cos x}(3x^2+2)}{(x^3+2x)\ln 7}$. ▲

? Savol va topshiriqlar

1. Murakkab funksiya deb nimaga aytiladi? Misol keltiring.
2. Murakkab funksiyaning aniqlanish sohasi qanday topiladi?
3. Murakkab funksiya hosilasini topish formulasini yoza olasizmi?
4. Murakkab funksiya hosilasini topishni 1–2 ta misolda ko‘rsating.

Mashqlar

35. Agar $f(x)=x^2-1$ bo‘lsa, ko‘rsatilgan funksiyalarni toping:

1) $f(\frac{1}{x})$; 2) $f(2x)$; 3) $f(x^2-1)$; 4) $f(x+1)-f(x-1)$.

36. Agar $f(x)=\frac{x+1}{x-1}$ bo‘lsa, ko‘rsatilgan funksiyalarni toping:

1) $f(\frac{1}{x})$; 2) $f(\frac{1}{x^2})$; 3) $f(x-1)$; 4) $f(x+1)$.

37. Agar $f(x)=x^2$, $g(x)=x-1$ bo‘lsa, quyidagilarni toping:

1) $f(g(x))$; 2) $f(f(x))$; 3) $g(g(x))$; 4) $g(f(x))$.

38. Agar $f(x)=x^3$, $g(x)=x^2+1$ bo‘lsa, quyidagilarni toping:

1) $\frac{f(x^2)}{g(x)-1}$; 2) $f(x)+3g(x)+3x-2$;

3) $f(g(x))$; 4) $g(f(x))$.

Tenglikdan foydalanib, $f(x)$ ni toping (**39–42**):

39. $f(x+1)=x^2-1$. **40*.** $f(x)+3\cdot f(\frac{1}{x})=\frac{1}{x}$.

41. $f(x+3)=x^2-4$. **42*.** $2f(x)+f(\frac{1}{x})=x$.

Hosilani toping (**43–44**):

43. 1) $f(x)=(3x-2)^5$; 2) $f(x)=e^{\sin x}$; 3) $f(x)=(4-3x)^7$;

4) $f(x)=\sin^2 x$; 5) $f(x)=\frac{1}{(2x+9)^3}$; 6) $f(x)=\ln(4x-1)$;

7) $f(x)=\sqrt{4x-5}$; 8) $f(x)=(2x-1)^{10}$; 9) $f(x)=\cos^8 x$.

44*. 1) $e^{\sin x}\cdot \text{tg}\frac{1}{x}$; 2) $3^{\text{ctg}x}\cdot\log_a\cos x$; 3) $\ln\cos x$;

4) $(x^2-5x+4)^3\cdot 10^{\text{tg}x}$; 5) $7^{\log 3x}\cdot(x^3-2x+1)^3$; 6) $3^{\cos x}\cdot(x^2-8x+4)^2$;

7) $\text{ctg}x\cdot\ln(x^2+x)$; 8) $x^2\cos^{30}x+4$; 9) $5\ln x\cdot\text{ctg}x$.

13–14 FUNKSIYA GRAFIGIGA O‘TKAZILGAN URINMA VA NORMAL TENGLAMALARI

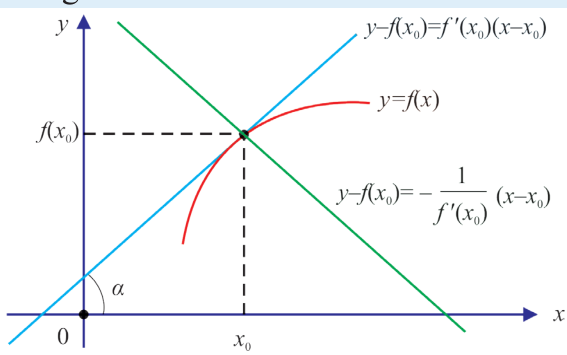
Urinma tenglamasi. $y = f(x)$ funksiyaga grafigining $(x_0; f(x_0))$ nuqtasidan o‘tuvchi urinma tenglamasini topamiz (19-rasm). Urinma to‘g‘ri chiziq bo‘lgani uchun uning umumiy ko‘rinshi $y = kx + b$ bo‘ladi. Hosilaning geometrik ma’nosiga ko‘ra $k = \text{tg}\alpha = f'(x_0)$, ya’ni urinma teglamasi $y = f'(x_0)x + b$ ko‘rinishini oladi. Bu urinma $(x_0;\ f(x_0))$ nuqtadan o‘tgani uchun $f(x_0) = f'(x_0)x_0 + b$ bo‘ladi, bundan $b = f(x_0) - f'(x_0)x_0$. Topilgan b ni urinma tenglamasiga qo‘yib,

$$y = f'(x_0)x + f(x_0) - f'(x_0)x_0 \quad \text{yoki}$$

$$y - f(x_0) = f'(x_0)(x - x_0) \qquad (1)$$

tenglamani hosil qilamiz.

$y - f(x_0) = f'(x_0)(x - x_0)$ tenglama $(x_0; f(x_0))$ nuqtada $y = f(x)$ funksiyaga o‘tkazilgan urinma tenglamasi bo‘ladi.



19-rasm.

1-misol. $f(x) = x^2 - 5x$ funksiya grafigiga $x_0 = 2$ abssissali nuqtada o‘tkazilgan urinma tenglamasini yozing.

△ Avval funksiyaning va funksiyadan olingan hosilaning $x_0=2$ nuqtadagi qiymatini topamiz:

$$f(x_0) = f(2) = 2^2 - 5\cdot 2 = -6, \quad f'(x) = 2x - 5, \quad f'(2) = 2\cdot 2 - 5 = -1.$$

Topilganlarni (1) tenglamaga qo‘yib, urinma tenglamasini hosil qilamiz:

$y - (-6) = -1\cdot(x - 2)$ yoki $y = -x - 4.$ *Javob:* $y = -x - 4.$ ▲

2-misol. $f(x)=x^3-2x^2$ funksiya grafigiga $x_0=1$ abssissali nuqtada o'tkazilgan urinma tenglamasini yozing.

△ Avval funksiyaning va funksiyadan olingan hosilaning $x_0=1$ nuqtadagi qiymatini topamiz:

$$f(x_0)=f(1)=1^3-2\cdot 1^2=-1, \quad f'(x)=3x^2-4x, \quad f'(1)=3\cdot 1^2-4\cdot 1=-1.$$

Topilganlarni (1) tenglamaga qo'yib, urinma tenglamasini hosil qilamiz:

$y-(-1)=-1(x-1)$ yoki $y=-x$. *Javob:* $y=-x$. ▲

Agar $y=f(x)$ funksiya grafigining x_0 abssissali nuqtasida o'tkazilgan urinma $y=kx+b$ to'g'ri chiziqqa parallel bo'lsa, $f'(x_0)=k$ bo'ladi. Bu shart orqali funksiyaning berilgan to'g'ri chiziqqa parallel bo'lgan urinmasi topiladi.

3-misol. $f(x)=x^2-3x+4$ funksiya uchun $y=2x-1$ to'g'ri chiziqqa parallel bo'lgan urinma tenglamasini yozing.

△ Urinmaning berilgan to'g'ri chiziqqa parallellik shartiga ko'ra, $f'(x_0)=2$ yoki $2x_0-3=2$ tenglamani hosil qilamiz. Bu tenglamada $x_0=2{,}5$ bo'lgani uchun urinma abssissasi $x_0=2{,}5$ bo'lgan nuqtadan o'tadi. Hisoblashlarni bajaramiz:

$$f(x_0)=f(2{,}5)=2{,}5^2-3\cdot 2{,}5+4=6{,}25-7{,}5+4=2{,}75$$

$$f'(x_0)=f(2{,}5)=2.$$

Endi urinma tenglamasini topamiz:

$$y-2{,}75=2(x-2{,}5) \text{ yoki } y=2x-2{,}25.$$

Javob: $y=2x-2{,}25$. ▲

4-misol. $f(x)=x^3-2x^2+3x-2$ funksiya grafigiga $x_0=4$ abssissali nuqtada o'tkazilgan urinma tenglamasini tuzing va urinma bilan Ox o'qining musbat yo'nalishi tashkil qilgan burchakning sinusini toping.

△ Avval funksiyaning va funksiyadan olingan hosilaning $x_0=4$ nuqtadagi qiymatini topamiz:

$$f(x_0)=f(4)=3\cdot 4^3-2\cdot 4^2+3\cdot 4-2=170, \quad f'(x)=3x^2-4x+3,$$

$$f'(4)=3\cdot 4^2-4\cdot 4+3=35.$$

Topilganlarni (1) tenglamaga qo'yib, urinma tenglamasini hosil qilamiz:

$y-170=35(x-4)$ yoki $y=35x+30$.

Hosilaning geometrik ma'nosiga ko'ra $\operatorname{tg}\alpha=35$, bundan

$$\sin\alpha=\frac{1}{\sqrt{1+\mathrm{ctg}^2\alpha}}=\frac{1}{\sqrt{1+\frac{1}{\mathrm{tg}^2\alpha}}}=\frac{\mathrm{tg}\alpha}{\sqrt{1+\mathrm{tg}^2\alpha}}=\frac{35}{\sqrt{1+35^2}}=\frac{35}{\sqrt{1226}}.$$

Javob: $y=35x+30$; $\sin\alpha=\frac{35}{\sqrt{1226}}$. ▲

5*-misol. $f(x)=x^2$ parabolaga abssissasi x_0 bo‘lgan A nuqtada o‘tkazilgan urinma Ox o‘qini $\frac{1}{2}x_0$ nuqtada kesib o‘tadi. Shu da’voni isbotlang.

△ $f'(x)=2x$, $f(x_0)=x_0^2$, $f'(x_0)=2x_0$.

Urinma tenglamasi (1) ga ko‘ra $y=2x_0\cdot x-x_0^2$ bo‘ladi. Uning Ox o‘qi bilan kesisish nuqtasi $\left(\frac{x_0}{2};0\right)$ ekani ravshan. Bundan $y=x^2$ parabolaga abssissasi x_0 bo‘lgan A nuqtada o‘tkazilgan urinmani yasash usuli kelib chiqadi: A nuqta va $\left(\frac{x_0}{2};0\right)$ nuqta orqali o‘tuvchi to‘g‘ri chiziq $y=x^2$ parabolaga A nuqtada urinadi.

Normal tenglamasi. $y=f(x)$ funksiya grafigiga $x=x_0$ abssissali nuqtada o‘tkazilgan urinmaga $x=x_0$ nuqtada perpendikular bo‘lgan

$$y-f(x_0)=-\frac{1}{f'(x_0)}(x-x_0) \qquad (2)$$

to‘g‘ri chiziqqa $y=f(x)$ funksiya grafigining x_0 abssissali nuqtasida o‘tkazilgan normal deyiladi (19- rasm).

6-misol. $f(x)=x^5$ funksiya grafigiga $x_0=1$ abssissali nuqtada o‘tkazilgan normal tenglamasini tuzing.

△ Hosila formulasiga ko‘ra $f'(x)=5x^4$ bo‘ladi. Funksiya va uning hosilasining $x_0=1$ nuqtadagi qiymatlarini hisoblaymiz: $f(1)=1^5=1$ va $f'(1)=5\cdot1^4=5$. Bu qiymatlarni normalning tenglamasiga qo‘yamiz va $y-1=-\frac{1}{5}(x-1)$ yoki $y=-\frac{1}{5}x+\frac{6}{5}$ tenglamani hosil qilamiz.

Javob: $y=-\frac{1}{5}x+\frac{6}{5}$. ▲

Eslatma: $f(x)=x^5$ funksiya grafigiga $x_0=1$ abssissali nuqtada o'tkazilgan urinma tenglamasi $y=5x-4$ bo'ladi (isbotlang!). Urinma va normalning burchak koeffitsiyenti ko'paytmasi $5\cdot(-\frac{1}{5})=-1$ ekaniga e'tibor bering.

? Savol va topshiriqlar

1. $y=f(x)$ funksiya grafigiga x_0 abssissali nuqtada o'tkazilgan urinma tenglamasini yozing.

2. $y=f(x)$ funksiya grafigiga x_0 abssissali nuqtada o'tkazilgan normal tenglamasini yozing.

3. Berilgan funksiyaning biror to'g'ri chiziqqa parallel bo'lgan urinmasi qanday topiladi? Misolda tushuntiring.

Mashqlar

45. Funksiya grafigiga abssissasi $x_0=1$; $x_0=-2$; $x_0=0$ bo'lgan nuqtada o'tkazilgan urinma tenglamasini yozing:

1) $f(x)=2x^2-5x+1$; | 2) $f(x)=3x-4$; | 3) $f(x)=6$;
4) $f(x)=x^3-4x$; | 5) $f(x)=e^x$; | 6) $f(x)=2^x$;
7) $f(x)=2^x+\ln 2$; | 8) $f(x)=\sin x$; | 9) $f(x)=\cos x$;
10) $f(x)=\cos x-\sin x$; 11) $f(x)=e^x x$; | 12) $f(x)=x\cdot\sin x$.

46. Funksiya uchun $y=7x-1$ to'g'ri chiziqqa parallel bo'lgan urinma tenglamasini yozing:

1) $f(x)=x^3-2x^2+6$; 2) $f(x)=4x^2-5x+3$; 3) $f(x)=8x-4$.

47. Berilgan $f(x)$ va $g(x)$ funksiyalarning urinmalari parallel bo'ladigan nuqtalarni toping:

1) $f(x)=3x^2-5x+4$, $g(x)=4x-5$;
2) $f(x)=8x+9$, $g(x)=-5x+8$;
3) $f(x)=7x+11$, $g(x)=7x-9$;
4) $f(x)=x^3-8$, $g(x)=x^2+5$;
5) $f(x)=x^3+x^2$, $g(x)=5x-7$;
6) $f(x)=x^4+11$, $g(x)=x^3+10$.

48. Funksiya grafigiga abssissasi a) $x_0 = 1$; b) $x_0 = -2$; d) $x_0 = 0$ bo‘lgan nuqtada o‘tkazilgan normal tenglamasini toping:

1) $f(x)=3x^2-5x+1$; 2) $f(x)=3x-40$; 3) $f(x)=7$;

4) $f(x)=x^3-10x$; 5) $f(x)=e^x$; 6) $f(x)=12^x$;

7) $f(x)=\sin x$; 8) $f(x)=\cos x$; 9) $f(x)=\cos x - \sin x$;

10) $f(x)=e^{\pi x}$; 11) $f(x)=x\cdot\cos x$; 12) $f(x)=x\cdot\sin x$.

Nazorat ishi namunasi

I variant

1. $f(x)=x^3+2x^2-5x+3$ funksiya uchun $x_0=2$ va $\Delta x=0,1$ bo‘lganda funksiya orttirmasining argument orttirmasiga nisbatini toping.

2. $f(x)=-8x^2+4x+1$ funksiyaning $x_0=-3$ nuqtadagi hosilasini hisoblang.

3. $f(x)=x^3-7x^2+8x-5$ funksiya grafigiga $x_0=-4$ abssissali nuqtada o‘tkazilgan urinma tenglamasini yozing.

4. Moddiy nuqta $s(t)=8t^2-5t+6$ qonuniyat bilan harakatlanmoqda. Agar t – sekund, s – metrlarda o‘lchanadigan bo‘lsa, nuqtaning $t_0=8$ sekunddagi oniy tezligini toping.

5. Ko‘paytmaning hosilasini toping: $(3x^2-5x+4)\cdot e^x$.

II variant

1. Bo‘linmaning hosilasini toping: $\dfrac{x^2-5x+6}{x+1}$.

2. Murakkab funksiyaning hosilasini toping: $\mathrm{ctg}^{15}x$.

3. $f(x)=\sqrt{x\sqrt{x}}$ funksiyaning $x_0=\dfrac{1}{16}$ nuqtadagi hosilasini hisoblang.

4. $f(x)=\ln(x+1)$ funksiya grafigiga $x=0$ nuqtada o‘tkasilgan urinma tenglamasini yozing.

5. $s(t)=0,5t^2-6t+1$ qonuniyati bilan harakatlanayotgan moddiy nuqtaning $t=16$ sekunddagi oniy tezligini toping. (t – sekundda, s – metrlarda o‘lchanadi).

15–17 MASALALAR YECHISH

49. $y=f(x)$ funksiya uchun x_0 va x nuqtalarga mos h va Δy ni hisoblang:

1) $f(x)=4x^2-3x+2$, $x_0=1$, $x=1{,}01$; | 2) $f(x)=(x+1)^3$, $x_0=0$, $x=0{,}1$.

50. Agar $x_0=3$ va $\Delta x=0{,}03$ bo‘lsa, berilgan funksiyalar uchun: a) funksiya orttirmasini; b) funksiya orttirmasining argument orttirmasiga nisbatini toping:

1) $f(x)=7x-5$; 2) $f(x)=2x^2-3x$; | 3) $f(x)=x^3+2$; | 4) $f(x)=x^3+4x$.

51. Agar $x_0=2$ va $\Delta x=0{,}01$ bo‘lsa, berilgan funksiyalar uchun: a) funksiya orttirmasini; b) funksiya orttirmasining argument orttirmasiga nisbatini toping:

1) $f(x)=-4x+3$; | 2) $f(x)=-8$; | 3) $f(x)=x^2+10x$; | 4) $f(x)=x^3-10$.

52. $x\to 0$ bo‘lsa, funksiya qaysi songa intiladi:

1) $f(x)=x^3-2x^2+3x+4$; 2) $f(x)=x^5-6x^4+8x-7$;

3) $f(x)=(x^2-5x+1)(x^3-7x^2-11x+6)$;

4) $f(x)=\dfrac{x^2-x-19}{x^2+7x-28}$; 5) $f(x)=\dfrac{x^3-8x}{x^3+x^2+x+1}$?

53. Funksiyaning hosilasini toping:

1) $y=17x$; 2) $y=29x-3$; 3) $y=-15$; 4) $y=16x^2-3x$;

5) $y=-5x+40$; 6) $y=18x-x^2$; 7) $y=x^2+15x$;

8) $y=16x^3+5x^2-2x+14$; 9) $y=3x^3+2x^2+x$.

54. Funksiyaning hosilasini: a) $x=-3$; b) $x=1{,}1$; c) $x=0{,}4$; d) $x=-0{,}2$ nuqtalarda hisoblang:

1) $y=15x$; 2) $y=9x+3$; 3) $y=-20$; 4) $y=5x^2+x$;

5) $y=-8x+4$; 6) $y=8x-x^2$; 7) $y=x^2+25x$; | 8) $y=x^3+5x^2-2x+4$.

55. $y=f(x)$ funksiya hosilasini ta‘rifga ko‘ra toping:

1) $f(x)=2x^2+3x+5$; 3*) $f(x)=\dfrac{x+1}{x}$;

2) $f(x)=(x+2)^3$; 4*) $f(x)=\dfrac{x^2+1}{x}$.

56. $y=f(x)$ funksiyaning x_0 nuqtadagi hosilasini toping:

1) $f(x)=4x^3+3x^2+2x+1$, $x_0=1$;

2) $f(x)=\frac{1}{3}x^3+\sin 22°$, $x_0=-1$;

3) $f(x)=(2x+1)(\sqrt{x}-1)$, $x_0=4$;

4) $f(x)=\frac{x^3-1}{x^2+1}$, $x_0=-3$.

57. Moddiy nuqta $s(t)=\frac{4}{3}t^3-t+5$ qonuniyat bilan harakatlanmoqda (s metrda, t – sekundda). Moddiy nuqtaning 2-sekunddagi tezligini toping.

58. Funksiyaning hosilasini toping:

1) $y=\frac{1}{\sqrt{x}}+2\sqrt{x}$;

2) $y=\sqrt[3]{x}+2x^3$;

3*) $y=\sqrt[5]{x}+x\cdot \text{tg}x-\log_3 x$;

4) $y=(2x+3)^3$;

5*) $y=x\cdot\ln x\cdot(x+1)$;

6) $y=(x+\sqrt{x})(\sqrt{x}-2)$;

7) $y=\frac{x+2}{\sin x}$;

8) $y=10^x+\log_2 5+\cos 15°$;

9) $y=3^{-x}\cdot\sin x$;

10*) $y=\text{tg}x\cdot\cos x+7^x\cdot x^7$;

11) $f(x)=\frac{1}{4}x^4-8x^2+3$;

12) $f(x)=\frac{\sqrt{2}}{2}x-\sin x+5$;

13) $f(x)=x^{10}-80x$;

14) $f(x)=8x-\frac{2^x}{\ln 2}$.

59. Funksiya hosilasining x_0 nuqtadagi qiymatini hisoblang:

1) $f(x)=\frac{1}{\cos x}$, $x_0=0$;

2) $f(x)=(x^2+3x)\ln x$, $x_0=1$;

3) $f(x)=\frac{\text{arctg}x}{1+x^2}$, $x_0=1$;

4) $f(x)=e^x(x-\ln 2)$, $x_0=\ln 2$.

60*. $f'(x)>0$ tengsizlikni yeching:

1) $f(x)=x\cdot\ln 27-3^x$;

2) $f(x)=\sin x-2x$;

61. Moddiy nuqta $s(t)=\frac{1}{3}t^3-\frac{3}{2}t^2+2t$ qonuniyat bilan harakatlanmoqda.

Moddiy nuqtaning tezligi qachon nolga teng bo‘ladi? Buning ma’nosi nima?

62. Hosilani toping: 1) $y=x^5-x^4+x$; | 2) $y=\frac{1}{x^2}-x$; | 3) $y=x^4+\sqrt[5]{x}$.

63. Moddiy nuqtaning t_0 vaqtdagi tezligini toping:

1) $x(t)=t^4-2t^3+t$, $t_0=-5$; | 2) $x(t)=-5t+t^2-\sqrt{t}$, $t_0=4$.

Hosilani toping (**64–66**):

64. 1) $y=(x+2)(x^2-5x)$; | 2) $y=\frac{x^2-3x}{x+8}$; | 3) $y=(x^4+\sqrt{x})(x^3-5x)$;

4) $y=2x^3+4x^2+5x$; | 5) $y=\frac{14}{x}-\frac{x}{14}$; | 6) $y=7x^2+12x+\sqrt{2019}$.

65*. 1) $y=\frac{x^8}{x^{10}-1}$; 2) $y=\frac{x^3+x+1}{x^5+7}$; 3) $y=(x^{10}+x^{-10})(x^8+x^{-8})$.

66*. 1) $y=\frac{3^x\cdot\sin x}{\cos x}$; 2) $y=e^{5x}(\cos x-\sin x)$;

3) $y=x\text{ctg}x$; 4) $y=\frac{\ln x}{x^2}$.

67*. Hosilani x_0 nuqtada hisoblang:

1) $f(x)=\frac{5x+1}{13x-5}$, $x_0=-2$; 2) $f(x)=\text{ctg}x-2x+2$, $x_0=\frac{-\pi}{4}$;

3) $f(x)=x^2(\lg x-1)$, $x_0=1$; 4) $f(x)=\text{ctg}x-\frac{1}{20}\ln x$, $x_0=1$.

68*. Murakkab funksiyaning hosilasini toping:

1) $x^2\cdot\sin x$; 2) $\log_{15}\cos x$; 3) $\ln\text{ctg}x$;

4) $\text{tg}^{35}x$; 5) $e^{\text{ctg}x}$; 6) $23^{\cos x}$;

7) $35^{\sin x}$; 8) $(x^2-10x+7)\ln\cos x$;

9) $\frac{x^5-6x+4}{e^x}$; 10) $e^{-3x}(x^4-3x^2+2)$; 11) $\ln\text{tg}x$;

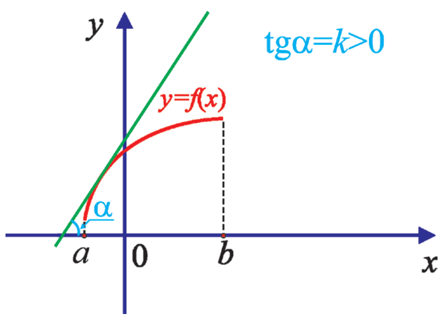
12) $\frac{x^3+7x+1}{e^{2x}}$; 13) $e^{5x}(x^5+8x+11)$; 14) $\ln\cos 2x$.

18–21 HOSILA YORDAMIDA FUNKSIYANI TEKSHIRISH VA GRAFIKLARNI YASASH

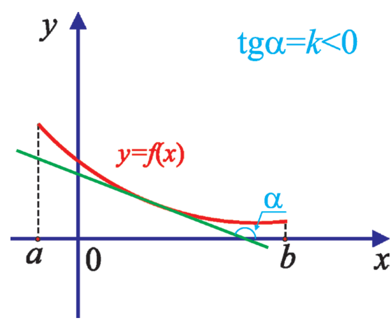
Funksiyaning o'sishi va kamayishi. O'suvchi va kamayuvchi funksiyalar bilan tanishsiz. Endi funksiyaning o'sish va kamayish oraliqlarini aniqlash uchun hosila tushunchasidan foydalanamiz.

1-teorema. $y = f(x)$ funksiya $(a; b)$ oraliqda aniqlangan va hosilasi mavjud bo'lsin. Agar $x \in (a; b)$ uchun $f'(x) > 0$ bo'lsa, $y = f(x)$ funksiya $(a; b)$ oraliqda o'suvchi funksiya bo'ladi (20-rasm).

2-teorema. $y = f(x)$ funksiya $(a; b)$ oraliqda aniqlangan va hosilasi mavjud bo'lsin. Agar $x \in (a; b)$ uchun $f'(x) < 0$ bo'lsa, $y = f(x)$ funksiya $(a; b)$ oraliqda kamayuvchi funksiya bo'ladi (21-rasm).



20-rasm.



21-rasm.

1, 2- teoremalarni isbotsiz qabul qilamiz.

1-misol. Funksiyaning o'sish va kamayish oraliqlarini toping: $f(x) = 2x^3 - 3x^2 - 12x + 3$.

△ Bu funksiya $(-\infty; +\infty)$ oraliqda aniqlangan. Uning hosilasi:

$$f'(x) = 6x^2 - 6x - 12 = 6(x-2)(x+1).$$

$f'(x) > 0$, $f'(x) < 0$ tengsizliklarni oraliqlar usuli bilan yechib, $(-\infty; -1)$ va $(2; +\infty)$ oraliqlarda funksiyaning o'sishi hamda $(-1; 2)$ oraliqda funksiyaning kamayishini bilib olamiz.

Javob: $(-\infty; -1)$ va $(2; +\infty)$ oraliqlarida funksiya o'sadi; $(-1; 2)$ oraliqda esa funksiya kamayadi. ▲

2-misol. Funksiyaning o'sish va kamayish oraliqlarini toping: $f(x) = x + \frac{1}{x}$.

△ Bu funksiya $(-\infty;0)\cup(0;+\infty)$ oraliqda aniqlangan. Uning hosilasi: $f'(x)=1-\frac{1}{x^2}$; $f'(x) > 0$, ya'ni $1-\frac{1}{x^2}>0$ tengsizlikni oraliqlar usuli bilan yechib, hosilaning $(-\infty; -1)$ va $(1; +\infty)$ oraliqlarda musbatligini topamiz. Xuddi shuningdek, $f'(x)<0$, ya'ni $1-\frac{1}{x^2}<0$ tengsizlikni oraliqlar usuli bilan yechib, bu tengsizlik $(-1; 0)$ va $(0; 1)$ oraliqlarda bajarilishini bilib olamiz.

Javob: funksiya $(-\infty; -1)$ va $(1; +\infty)$ oraliqlarda o'sadi; funksiya $(-1; 0)$ va $(0; 1)$ oraliqlarda esa kamayadi. ▲

Funksiyaning statsionar nuqtalari. $y = f(x)$ funksiya $(a; b)$ oraliqda aniqlangan bo'lsin.

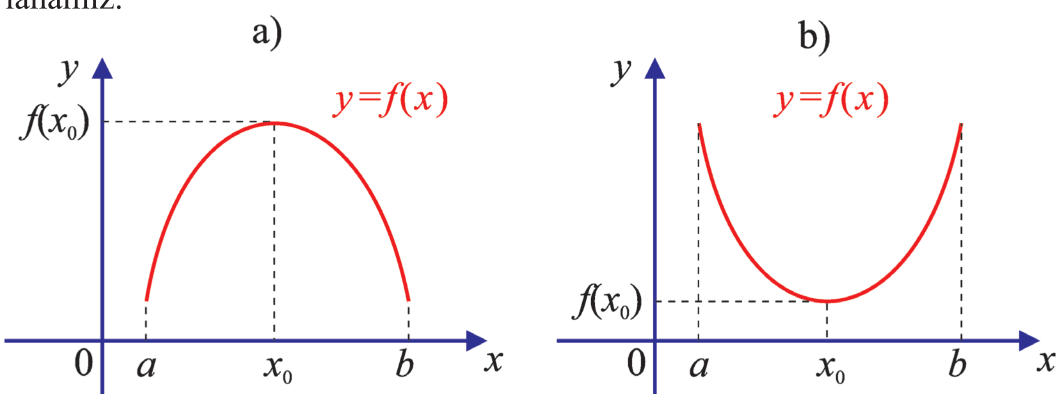
1-ta'rif. $y = f(x)$ funksiyaning hosilasi 0 ga teng bo'ladigan nuqtalar *statsionar nuqtalar* deyiladi.

3-misol. Funksiyaning statsionar nuqtalarini toping: $f(x)=2x^3-3x^2-12x+3$.

△ Funksiyaning hosilasini topib, uni nolga tenglaymiz: $f'(x) = 6x^2-6x-12 = 0$. Bu tenglamani yechib funksiyaning statsionar nuqtalari $x_1=-1$, $x_2=2$ ekanini topamiz.

Javob: funksiyaning statsionar nuqtalari $x_1= -1, x_2= 2$. ▲

Funksiyaning lokal maksimum va lokal minimumlari. Funksiyaning lokal maksimum va lokal minimumlarini aniqlash uchun hosiladan foydalanamiz.



22- rasm.

3-teorema. $f(x)$ funksiya $(a; b)$ oraliqda aniqlangan va $f'(x)$ mavjud; $(a; x_0)$ oraliqda $f'(x)>0$ va $(x_0; b)$oraliqda $f'(x)<0$ bo'lsin, $x_0\in(a; b)$. U holda x_0 nuqta $f(x)$ funksiyaning lokal maksimumi bo'ladi (22-*a* rasm).

4-teorema. $f(x)$ funksiya $(a;\ b)$oraliqda aniqlangan va $f'(x)$ mavjud; $(a; x_0)$ oraliqda $f'(x)<0$ va $(x_0; b)$ oraliqda $f'(x)>0$ bo‘lsin, $x_0 \in (a,\ b)$.

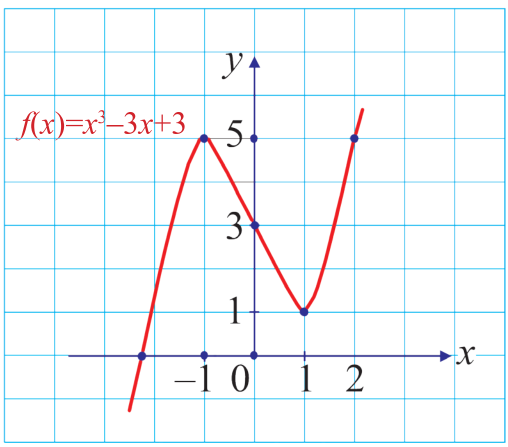
U holda x_0 nuqta $f(x)$ funksiyaning lokal minimumi bo‘ladi (22-*b* rasm).

3, 4-teoremalarni isbotsiz qabul qilamiz.

2-ta‘rif. Funksiyaning lokal maksimum va lokal minimumlariga uning *ekstremumlari* deyiladi.

4-misol. Funksiyaning lokal maksimum va lokal minimum nuqtalarini toping: $f(x)=x^3-3x+3$.

△ Funksiyaning hosilasini topamiz: $f'(x)=3x^2-3=3(x-1)(x+1)$. Hosila barcha nuqtalarda aniqlangan va $x=\pm 1$ nuqtalarda nolga aylanadi. Shuning uchun $x=\pm 1$ nuqtalar funksiyaning kritik nuqtalaridir. Oraliqlar usulidan foydalanib $(-\infty;\ -1)$ va $(1;\ +\infty)$ oraliqlarda $f'(x)>0$, $(-1;\ 1)$ oraliqda esa $f'(x)<0$ ekanini aniqlaymiz. Demak, $x=-1$ lokal maksimum va $x=1$ lokal minimum nuqtalari ekan (23-rasm).



23-rasm.

Javob: $x=-1$ lokal maksimum va $x=1$ lokal minimum nuqta. ▲

Funksiyaning eng katta va eng kichik qiymatlari bilan 10-sinfdan tanishmiz.

f(*x*) funksiya [*a; b*] kesmada aniqlangan va (*a*; *b*) da hosilasi mavjud bo‘lsin. Uning eng katta qiymatini topish qoidasi shunday:

1) funksiyaning bu oraliqdagi barcha statsionar nuqtalari topiladi;

2) funksiyaning statsionar, chegaraviy *a* va *b* nuqtalardagi qiymatlari hisoblanadi;

3) bu qiymatlarning eng kattasi funksiyaning shu oraliqdagi eng katta qiymati deyiladi.

Funksiyaning eng kichik qiymati ham shu kabi topiladi.

5-misol. $f(x)=x^3+4{,}5x^2-9$ funksiyaning $[-4; 2]$ oraliqdagi eng katta va eng kichik qiymatlarini toping.

△ Funksiyaning hosilasini topamiz: $f'(x)=3x^2+9x$. Hosilani nolga tenglab, funksiyaning statsionar nuqtalarini topamiz: $f'(x)=3x(x+3)=0$, $x_1=0$ va $x_2=-3$. Funksiyaning topilgan $x_1=0$, $x_2=-3$ hamda $a=-4$, $b=2$ nuqtalardagi qiymatlarini topamiz:

$f(0)=0^3+4{,}5\cdot0^2-9=-9$, $f(-3)=(3)^3+4{,}5\cdot(-3)^2-9=4{,}5$,

$f(-4)=(-4)^3+4{,}5\cdot4^2-9=-1$, $f(2)=2^3+4{,}5\cdot2^2-9=17$.

Demak, funksiyaning eng katta qiymati 17 va eng kichik qiymati –9 ekan.

Javob: funksiyaning eng katta qiymati 17 va eng kichik qiymati –9. ▲

Hosila yordamida funksiyani tekshirish va grafigini yasash. Funksiya grafigini yasashni quyidagi ketma-ketlikda amalga oshiramiz.

Funksiyaning:

1) aniqlanish sohasini;

2) statsionar nuqtalarini;

3) o'sish va kamayish oraliqlarini;

4) lokal maksimum va lokal minimumlarini hamda funksiyaning shu nuqtalardagi qiymatlarini topamiz;

5) topilgan ma'lumotlarga ko'ra funksiyaning grafigini yasaymiz.

Grafikni yasashda funksiya grafigini koordinata o'qlari bilan kesisish va boshqa ayrim nuqtalarini topish maqsadga muvofiq.

6-misol. $f(x)=x^3-3x$ funksiyani hosila yordamida tekshiring va uning grafigini yasang.

1. Funksiya $(-\infty; +\infty)$ oraliqda aniqlangan.

2. Statsionar nuqtalarini topamiz:

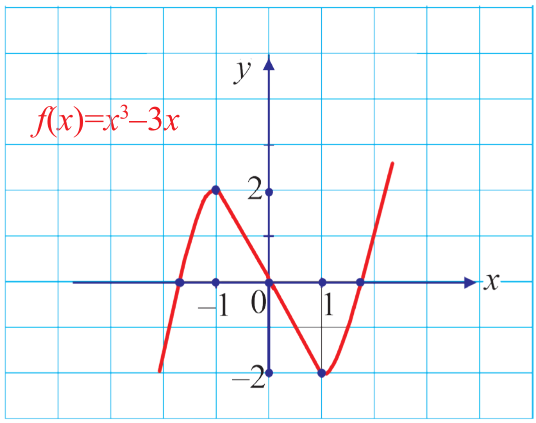
$f'(x)=(x^3-3x)'=3x^2-3=0$. $x_1=1$ va $x_2=-1$ statsionar nuqtalardir.

3. Funksiyaning o'sish va kamayish oraliqlarini topamiz:

$(-\infty; -1)\cup(1; +\infty)$ oraliqlarda $f'(x)>0$ bo'lgani uchun $f(x)$ funksiya shu oraliqlarda o'sadi va $(-1; 1)$ oraliqda $f'(x)<0$ bo'lgani uchun $f(x)=x^3-3x$ funksiya $(-1; 1)$ oraliqda kamayadi.

4. $x=-1$ bo‘lganda funksiya lokal maksimum $f(-1)=(-1)^3-3\cdot(-1)=2$ ga va $x=1$ bo‘lganda funksiya lokal minimum $f(1)=1^3-3\cdot 1=-2$ ga ega.

5. Funksiyaning Ox o‘qi bilan kesisish nuqtalarini topamiz: $x^3-3x=x(x^2-3)=0$. Bundan $x=0$ yoki $x^2-3=0$ tenglamani hosil qilamiz. Tenglamani yechib $x_1=0$, $x_2=\sqrt{3}$, $x_3=-\sqrt{3}$ funksiya grafigining Ox o‘qi bilan kesisish nuqtalarini topamiz. Natijada 24- rasmdagi grafikni hosil qilamiz.



24-rasm.

Savol va topshiriqlar

1. Funksiyaning o‘sish va kamayish oraliqlari qanday topiladi?
2. Funksiyaning statsionar nuqtasiga ta‘rif bering.
3. Funksiyaning lokal maksimum va lokal minimumlari qanday topiladi?
4. Funksiyaning eng katta va eng kichik qiymatlari qanday topiladi?
5. Hosila yordamida funksiyaning grafigini yasash bosqichlarini ayting va bitta misolda tushuntiring.
6. Funksiyaning statsionar nuqtalari uning ekstremum nuqtalari bo‘lishi shartmi? Misollar keltiring.
7. $f(x)=\frac{1}{4}x^4-\frac{1}{2}x^2$ funksiyani hosila yordamida tekshiring va grafigini yasang.

Mashqlar

69. Funksiyaning o'sish va kamayish oraliqlarini toping:

1) $f(x)=2-9x$; 2) $f(x)=\frac{1}{2}x-8$; 3) $f(x)=x^3-27x$;

4) $f(x)=\frac{x-1}{x}$; 5) $f(x)=x^2-2x+4$; 6) $f(x)=x(x^2-6)$;

7) $f(x)=-x^2+2x-3$; 8) $f(x)=\frac{1}{x^2}$; 9) $f(x)=x^4-2x^2$;

10) $f(x)=3x^4-8x^3+16$; 11) $f(x)=\frac{1}{1+x^2}$; 12) $f(x)=\sin x$;

13) $f(x)=\cos x$; 14) $f(x)=\mathrm{tg}\,x$; 15*) $f(x)=\sin 2x+\cos 2x$.

70. Funksiyaning statsionar nuqtalarini toping:

1) $f(x)=2x^2-3x+1$; 2) $f(x)=9x-\frac{1}{3}x^3$; 3*) $f(x)=|x-1|$;

4) $f(x)=x^2$; 5) $f(x)=8x^3+5x$; 6) $f(x)=3x-4$;

7*) $f(x)=|x|+1$; 8) $f(x)=2x^3+3x^2-6$; 9) $f(x)=3+8x^2-x^4$.

71. Funksiyaning lokal maksimum va lokal minimumlarini toping:

1) $f(x)=x^2-\frac{1}{2}x^4$; 2) $f(x)=(x-4)^8$; 3) $f(x)=4-3x^2-2x^3$;

4) $f(x)=\frac{5}{x}+\frac{x}{5}$; 5) $f(x)=x^4-2x^3+x^2-3$; 6) $f(x)=3\,\mathrm{tg}\,x$;

7) $f(x)=2\sin x+3$; 8) $f(x)=-5\cos x-7$; 9) $f(x)=x^4-x^3+4$.

72. Funksiyaning o'sish va kamayish oraliqlarini toping:

1) $f(x)=x^3-27x$; 2*) $f(x)=\frac{3x}{x^2+1}$; 3*) $f(x)=x+\frac{4}{x^2}$;

4) $f(x)=5\sin x+13$; 5) $f(x)=15\cos x-7$; 6) $f(x)=-3\mathrm{tg}\,x$.

73. Funksiyaning eng katta va eng kichik qiymatlarini toping:

1) $f(x)=x^4-8x^2+3,\ x\in[-4;1]$; 2) $f(x)=3x^5-5x^3+1,\ x\in[-2;2]$;

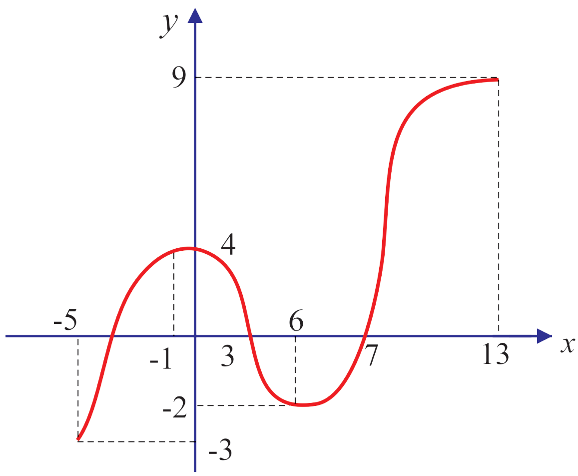
3) $f(x)=\frac{x}{x+1},\ x\in[1;2]$; 4) $f(x)=3x^3-6x^2-5x+8,\ x\in[-1;4]$.

74. Funksiyani tekshiring va grafigini yasang:

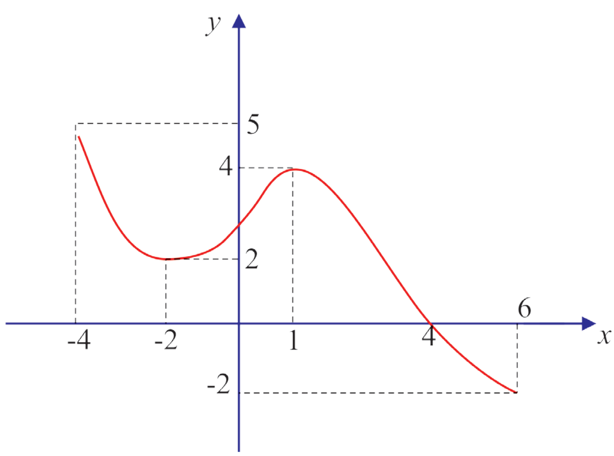
1) $y = x^3 - 6x^2 + 9x - 2;$ 2) $y = \frac{1}{5}x^5 - \frac{2}{3}x^3 + 1;$ 3) $y = x^4 - 4x^3 + 15.$

75*. Funksiya hosilasining grafigiga qarab (25, 26-rasmlar), quyidagilarni toping:

1) statsionar nuqtalarni; 2) o‘sish oraliqlarini;
3) kamayish oraliqlarini; 4) lokal maksimumlarini;
5) lokal minimumlarni.



25-rasm.



26-rasm.

Nazorat ishi namunasi

I variant

1. Hosilani toping: $f(x) = 20x^3 + 6x^2 - 7x + 3$.

2. $f(x) = x^2 - 5x + 4$ va $g(x) = \frac{x+1}{x-2}$ bo'lsa, $f(g(3))$ ni hisoblang.

3. $f(x) = x^3 - 5x^2 + 7x + 1$ funksiya uchun quyidagilarni toping:

1) statsionar nuqtalarni;
2) o'sish oraliqlarini;
3) kamayish oraliqlarini;
4) lokal maksimumlarini;
5) lokal minimumlarini.

4. Hosilani toping: $(3x + 5)^3 + \sin^2 x$.

5. $f(x) = \sqrt{1-3x}$ bo'lsa $f'\left(\frac{1}{4}\right)$ ni hisoblang.

II variant

1. Hosilani toping: $f(x) = 10x^3 + 16x^2 + 7x - 3$.

2. $f(x) = x^2 + 6x - 3$ va $g(x) = \frac{x-1}{x+2}$ bo'lsa, $f(g(3))$ ni hisoblang.

3. $f(x) = x^3 + 2x^2 - x + 3$ funksiya uchun quyidagilarni toping:

1) statsionar nuqtalarni;
2) o'sish oraliqlarini;
3) kamayish oraliqlarini;
4) lokal maksimumlarini;
5) lokal minimumlarini.

4. Hosilani toping: $(2x - 6)^3 + \cos^2 x$.

5. $f(x) = \sqrt{1-2x}$ bo'lsa, $f'\left(\frac{3}{8}\right)$ ni hisoblang.

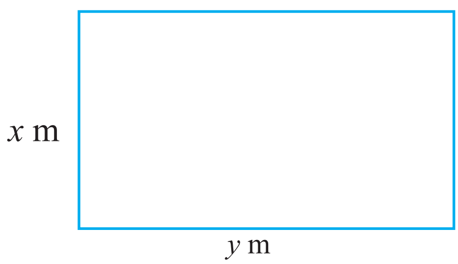
22–25 GEOMETRIK, FIZIK, IQTISODIY MAZMUNLI EKSTREMAL MASALALARNI YECHISHDA DIFFERENSIAL HISOB USULLARI

Geometrik mazmunli masalalar

1-masala. To‘g‘ri to‘rtburchak shaklidagi yer maydoni atrofini 100 m panjara bilan o‘rashmoqchi. Bu panjara eng ko‘pi bilan necha kvadrat mert yer maydonini o‘rashga yetadi?

△ Yer maydonining eni x m va bo‘yi y m bo‘lsin (27-rasm).

Masala shartiga ko‘ra yer maydonining perimetri $2x+2y=100$. Bundan $y=50-x$. Yer maydonining yuzi $S(x)=xy=x(50-x)=50x-x^2$. Masala $S(x)$ funksiyaning eng katta qiymatini topishga keltirildi. Avval $S(x)$ funksiyaning statsionar nuqtasini topamiz: $S'(x)=50-2x=0$, bundan $x=25$. $(-\infty;\ 25)$ oraliqda $S'(x)>0$ va



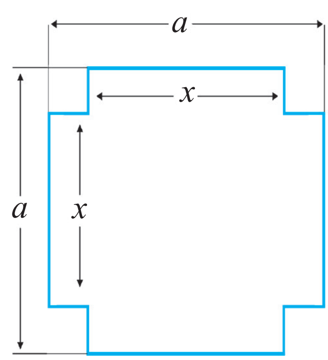
27-rasm.

$(25;+\infty)$ oraliqda $S'(x)<0$ bo‘lgani uchun $S(x)$ funksiya $x=25$ da eng katta qiymatga ega bo‘ladi va $S(25)=625$. Demak, 100 m panjara yordamida eng ko‘pi bilan 625 m^2 yer maydonini o‘rash mumkin. *Javob:* 625 m^2. ▲

Umuman, perimetri berilgan barcha to‘g‘ri to‘rtburchaklar ichida yuzasi eng kattasi kvadratdir.

2-masala. Tomoni a cm bo‘lgan kvadrat shaklidagi kartondan usti ochiq quti tayyorlashmoqchi. Bunda kartonning uchlaridan bir xil kvadratchalar kesib olinadi. Qutining hajmi eng katta bo‘lishi uchun uning asos tomoni uzunligi necha santimetr bo‘lishi kerak?

△ Kartonning uchlaridan bir xil kvadratchalar qirqib olinib, asosi x cm bo‘lgan ochiq quti yasalgan, desak (28-rasm), kesib olingan kvadratchaning tomoni $\frac{a-x}{2}$ cm bo‘ladi. Shuning uchun ochiq qutining hajmi $V(x)=\frac{a-x}{2}\cdot x\cdot x=$



28-rasm.

$= -\frac{x^3}{2} + \frac{ax^2}{2}$ cm^3. Demak, berilgan masala $V(x) = -\frac{x^3}{2} + \frac{ax^2}{2}$ funksiyaning [0; *a*] kesmadagi eng katta qiymatini topishga keldi. $V(x)$ funksiyaning statsionar nuqtalarini topamiz: $V'(x) = -\frac{3}{2}x^2 + ax = 0$.

Bu yerdan $x_1 = 0$, $x_2 = \frac{2}{3}a$ statsionar nuqtalar topiladi. Ravshanki, $V\left(\frac{2}{3}a\right) = \frac{2}{27}a^3$ va $V\left(\frac{2}{3}a\right) > V(0) = V(a) = 0$. Demak, $V(x)$ ning $[0; a]$ kesmadagi eng katta qiymati $\frac{2}{27}a^3$ bo'ladi.

Javob: ochiq qutining asos tomoni uzunligi $x = \frac{2}{3}a$ cm. ▲

Fizik mazmunli masalalar

3-masala. Moddiy nuqta $s(t) = -\frac{t^4}{12} + t^3$ qonuniyat bilan harakatlanmoqda ($s(t)$ metrda, t vaqt esa sekundda o'lchanadi). Quyidagilarni toping:

1) Eng katta tezlanishga erishiladigan vaqtni (t_0);
2) t_0 vaqtdagi oniy tezlikni;
3) t_0 vaqt ichida bosib o'tilgan yo'lni toping.

△ Moddiy nuqtaning tezligini topamiz:

$$v(t) = s'(t) = \left(-\frac{t^4}{12} + t^3\right)' = -\frac{t^3}{3} + 3t^2.$$

Fizikadan ma'lumki, tezlikdan olingan hosila tezlanishni beradi, ya'ni:

$$a(t) = v'(t) = -t^2 + 6t.$$

1) Eng katta tezlanishga ega bo'ladigan t_0 vaqtni aniqlash uchun $a(t) = v'(t) = -t^2 + 6t$ funksiyani maksimumga tekshiramiz. Avval

$a'(t) = -2t + 6 = 0$ tenglamani yechamiz, bundan $t_0 = 3$. (0; 3) oraliqda $a'(t) > 0$ va $(3; +\infty)$ oraliqda $a'(t) < 0$ bo'lgani uchun $t = 3$ da $a(t)$ eng katta qiymatga erishadi.

2) t_0 vaqtdagi oniy tezlikni hisoblaymiz: $v(3) = -\frac{3^3}{3} + 3 \cdot 3^2 = 18\frac{\text{m}}{\text{s}}$.

3) t_0 vaqt ichida bosib o'tilgan yo'l $s(t)=-\frac{t^4}{12}+t^3$ formulaga $t_0=3$ ni qo'yib hisoblanadi: $s(3)=-\frac{3^4}{12}+3^3=-\frac{27}{4}+27=\frac{81}{4}=20,25$ m.

Javob: 1) 3 s; 2) 18$\frac{\text{m}}{\text{s}}$; 3) 20,25 m. ▲

4-masala. Moddiy nuqta $s(t)=\frac{t^3}{3}-t^2+4t+50$ qonuniyat bilan harakatlanmoqda (s(t) masofa metrda, vaqt t sekundda o'lchanadi). Quyidagilarni toping:

1) eng kichik tezlikka erishiladigan vaqtni (t_0);
2) t_0 vaqtdagi tezlanishni;
3) t_0 vaqt ichida bosib o'tilgan yo'lni.

△ Moddiy nuqtaning tezligi va tezlanishini topamiz:

$$v(t)=s'(t)=\left(\frac{t^3}{3}-t^2+4t+50\right)'=t^2-2t+4,$$

$$a(t)=v'(t)=\left(t^2-2t+4\right)'=2t-2.$$

1) Eng kichik tezlikga erishiladigan t_0 vaqtni aniqlaymiz:

$$v'(t)=(t^2-2t+4)'=2t-2=0, \text{ bundan } t_0=1.$$

(0; 1) oraliqda $v'(t)<0$ va (1; + ∞) oraliqda $v'(t)>0$ bo'lgani uchun $t_0=1$ da $v(t)$ eng kichik qiymatga erishadi.

2) t_0 vaqtdagi tezlanishni hisoblaymiz: $a(1)=2\cdot1-2=0$ m/s².

3) t_0 vaqt ichida bosib o'tilgan yo'lni $s(t)=\frac{t^3}{3}-t^2+4t+50$ formulaga $t_0=1$ ni qo'yib hisoblanadi, ya'ni $s(1)=\frac{1^3}{3}-1^2+4\cdot1+50=53\frac{1}{3}$ m.

Javob: 1) 1 s; 2) 0 m/s²; 3) $53\frac{1}{3}$ m. ▲

5-masala. Havo shariga $t\in[0; 8]$ minut oralig'ida $V(t)=2t^3-3t^2+10t+2$ (m³) hajmda havo purkalmoqda. Quyidagilarni toping:

1) boshlang'ich vaqtdagi havo hajmini;
2) $t=8$ minutdagi havo hajmini;

3) $t=4$ minutdagi havo purkash tezligini;

△ 1) boshlang‘ich vaqtdagi havo hajmini topish uchun $V(t)=2t^3-3t^2+10t+2$ m^3 formulaga $t = 0$ qo‘yiladi, ya‘ni $V(0) = 2$ m^3.

2) $t=8$ minut vaqtdagi havo hajmini topish uchun $V(t)=2t^3-3t^2+10t+2$ m^3 formulaga $t = 8$ qo‘yiladi:

$$V(8) = 2\cdot 8^3-3\cdot 8^2+10\cdot 8+2 = 1024-192+80+2 = 914 \text{ m}^3;$$

3) havo purkash tezligini topamiz:

$$v'(t)=\left(2t^3-3t^2+10t+2\right)' = 6t^2-6t+10\left(\frac{\text{m}^3}{\text{min.}}\right).$$

Demak, $v'(4)=6\cdot 4^2-6\cdot 4+10=96-24+10=82\left(\frac{\text{m}^3}{\text{min}}\right)$.

Demak, $a(3)=12\cdot 3-6=30\left(\frac{\text{m}^3}{\text{min}^2}\right)$.

Javob: 1) 2 m^3; 2) 914 m^3; 3) $82\frac{\text{m}^3}{\text{min}}$. ▲

Iqtisodiy mazmunli masalalar

6-masala. Karima ko‘ylak tikish uchun buyurtma oldi. Bir oyda x ta ko‘ylak tiksa, $p(x)=-x^2+100x$ ming so‘m daromad qiladi. Quyidagilarni toping:

1) eng katta daromad olish uchun qancha ko‘ylak tikish kerak?

2) eng katta daromad qancha bo‘ladi?

△ 1) $p(x)=-x^2+100x$ funksiyani maksimumga tekshiramiz:

$p'(x)=(-x^2+100x)'=-2x+100=0$, bundan $x_0=50$. $(0;\ 50)$ kesmada $p'(x)>0$ va $(50;\ +\infty)$ oraliqda $p'(x)<0$ bo‘lgani uchun $x_0=50$ bo‘lganda funksiya eng katta qiymatga ega bo‘ladi. Demak, eng katta daromad olish uchun 50 ta ko‘ylak tikish kerak ekan.

2) Eng katta daromad qanchaligini topish uchun $p(x)=-x^2+100x$ ifodaga $x_0=50$ ni qo‘yamiz:

$p(50)=-50^2+100\cdot 50=-2500+5000=2500$(ming so‘m)$=2500000$ so‘m.

Javob: 1) 50 ta ko‘ylak; 2) 2 500 000 so‘m. ▲

Savol va topshiriqlar

Hosilani tatbiq qilib yechiladigan:
1) geometrik; 2) fizik; 3) iqtisodiy mazmunli masalaga misol keltiring.

Mashqlar

76. To‘g‘ri to‘rtburchak shaklidagi yer maydonining atrofini o‘rashmoqchi. 300 m panjara yordamida eng ko‘pi bilan necha kvadrat metr yer maydonini o‘rash mumkin?

77. To‘g‘ri to‘rtburchak shaklidagi yer maydonining atrofini o‘rashmoqchi. 480 m panjara yordamida eng ko‘pi bilan necha kvadrat metr yer maydonini o‘rash mumkin?

78*. Tomoni 120 cm bo‘lgan kvadrat shaklidagi kartondan usti ochiq quti tayyorlandi. Bunda kartonning uchlaridan bir xil kvadratchalar kesib olindi. Qutining hajmi eng katta bo‘lishi uchun kesib olingan kvadratchaning tomoni necha santimetr bo‘lishi kerak?

79*. Konserva banka silindr shaklida bo‘lib, uning to‘la sirti 216 π cm^2 ga teng. Bankaga eng ko‘p suv sig‘ishi (ketishi) uchun banka asosining radiusi va balandligi qanday bo‘lishi kerak?

80. To‘g‘ri to‘rtburchak shaklidagi maydonning yuzi 6400 m^2. Maydonning tomonlari qanday bo‘lganda uni o‘rash uchun eng kam panjara zarur bo‘ladi?

81*. Radiusi 5m bo‘lgan sharga eng kichik hajmli konus tashqi chizilgan. Konusning balandligini toping.

82*. Metalldan sig‘imi 13,5 l, asosi kvadratdan iborat bo‘lgan to‘g‘ri burchakli parallelepiped yasalmoqda. Idishning o‘lchamlari qanday bo‘lganda uni yasash uchun eng kam metall ketadi?

83. Moddiy nuqta $s(t) = -\frac{t^4}{4} + 5t^3$ qonuniyat bilan harakatlanmoqda ($s(t)$ metrda, vaqt t sekundda o‘lchanadi). Quyidagilarni toping:
1) eng katta tezlanishga erishiladigan t_0 vaqtni;
2) t_0 vaqtdagi oniy tezlikni;
3) t_0 vaqt ichida bosib o‘tilgan yo‘lni.

84. Moddiy nuqta $s(t) = -\frac{t^4}{2} + 12t^3$ qonuniyat bilan harakatlanmoqda ($s(t)$ m

da, vaqt t sekundda o‘lchanadi).
1) eng katta tezlanishga erishiladigan t_0 vaqtni;
2) t_0 vaqtdagi oniy tezlikni;
3) t_0 vaqt ichida bosib o‘tilgan yo‘lni toping.

85. Moddiy nuqta $s(t)=\frac{t^3}{9}-2t^2+40t+50$ qonuniyat bilan harakatlanmoqda ($s(t)$ metrda, vaqt t sekundda o‘lchanadi).
1) eng kichik tezlikka erishiladigan t_0 vaqtni;
2) t_0 vaqtdagi tezlanishni;
3) t_0 vaqt ichida bosib o‘tilgan yo‘lni toping.

86. Moddiy nuqta $s(t)=\frac{t^3}{2}-3t^2+8t+5$ qonuniyat bilan harakatlanmoqda ($s(t)$ metrda, vaqt t sekundda o‘lchanadi). Quyidagilarni toping:
1) eng kichik tezlikka erishiladigan t_0 vaqtni;
2) t_0 vaqtdagi tezlanishni;
3) t_0 vaqt ichida bosib o‘tilgan yo‘lni.

87. Havo shariga $t\in[0;\ 10]$ minut oralig‘ida $V(t)=5t^3+3t^2+2t+4$ (m^3) havo purkalmoqda.
1) boshlang‘ich vaqtdagi havo hajmini;
2) $t=10$ minutdagi havo hajmini;
3) $t=5$ minutdagi havo purkash tezligini toping;

88. Havo shariga $t\in[0;\ 15]$ minut oralig‘da $V(t)=t^3+13t^2+t+20$ (m^3) havo purkalmoqda.
1) boshlang‘ich vaqtdagi havo hajmini;
2) $t=15$ minutdagi havo hajmini;
3) $t=10$ minutdagi havo purkash tezligini toping;

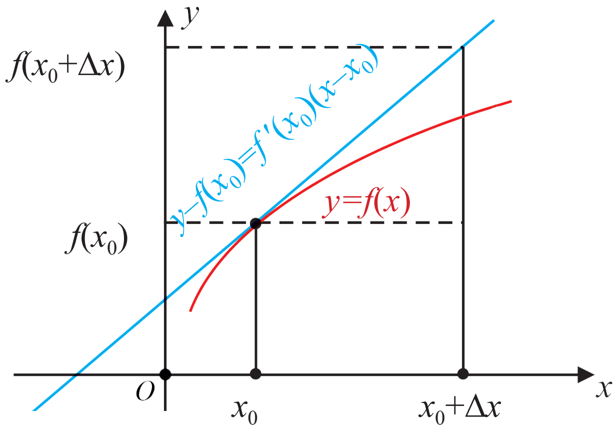
89. Muslima shim tikish uchun buyurtma oldi. U bir oyda x ta shim tiksa, $p(x)=-2x^2+120x$ ming so‘m daromad qiladi. Quyidagilarni toping:
1) daromadni eng katta qilish uchun qancha shim tikishi kerak?
2) eng katta daromad qancha bo‘ladi?

90. Muxlisa yubka tikish uchun buyurtma oldi. Bir oyda x ta yubka tiksa, $p(x)=-3x^2+96x$ (ming so‘m) daromad qiladi. Quyidagilarni toping:
1) daromadni eng katta qilish uchun qancha yubka tikish kerak?
2) eng katta daromad qancha bo‘ladi?

$y=f(x)$ funksiya x_0 nuqtada chekli $f'(x_0)$ hosilaga ega bo‘lsin.

x_0 abssissali nuqtada $y=f(x)$ funksiya grafigiga o‘tkazilgan urinma tenglamasi $y-f(x_0)=f'(x_0)(x-x_0)$ kabi yozilishini bilamiz.

x_0 nuqta yaqinida $y=f(x)$ funksiya grafigini urinmaning mos kesmasi bilan almashtirsa bo‘ladi (29-rasmga qarang):



29-rasm.

$x-x_0$ orttirmani Δx deb belgilasak (ya’ni $x=x_0+\Delta x$ deb olsak) quyidagi taqribiy munosabatga ega bo‘lamiz:

$$f(x)\approx f(x_0)+f'(x_0)(x-x_0), \text{ yoki}$$

$$f(x_0+\Delta x)\approx f(x_0)+f'(x_0)\cdot\Delta x \qquad (1)$$

(1) taqribiy formula *kichik orttirmalar formulasi* deb nomlanadi.

Izoh. x_0 nuqta sifatida $f(x_0), f'(x_0)$ qiymatlar oson hisoblanadigan nuqtani tanlab olish tavsiya etiladi. Shu bilan birga x nuqta x_0 ga qancha yaqin bo‘lsa, bunday almashtirish aniqroq bo‘lishini qayd etamiz.

Endi biz kichik orttirmalar formulasiga tayangan holda taqribiy hisoblashlarni bajaramiz.

1-misol. $f(x) = x^7-2x^6+3x^2-x+3$ funksiyaning $x = 2{,}02$ nuqtadagi qiymatini taqribiy hisoblang.

△ $x=2{,}02$ nuqtaga yaqin bo‘lgan $x_0=2$ nuqtani olsak, bu nuqtada $f(x)$ funksiya qiymati osonlikcha topiladi: $f(x_0)=f(2)=13$.

Bu funksiyaning hosilasini topamiz: $f'(x)=7x^6-12x^5+6x-1$.

U holda $f'(x_0)=f'(2)=75$, $\Delta x=x-x_0=2{,}02-2=0{,}02$ bo‘ladi.

Demak, (1) formulaga ko‘ra $f(2{,}02)=f(2+0{,}02)\approx 13+75\cdot 0{,}02=14{,}5$.

Kalkulator yoki boshqa hisoblash vositasi yordamida $f(2{,}02)\approx 14{,}57995$ qiymatni hosil qilishimiz mumkin. ▲

2-misol. $\sqrt{1{,}02}$ ildizning qiymatini taqribiy hisoblang.

△ $f(x)=\sqrt{x}$ funksiyani qaraymiz. Uning hosilasini topamiz:

$$f'(x)=\left(\sqrt{x}\right)'=\frac{1}{2\sqrt{x}}.$$

$x_0=1$ deb olsak, $f(x_0)=f(1)=\sqrt{1}=1$,

$$f'(x_0)=f'(1)=\frac{1}{2}\frac{1}{\sqrt{1}}=\frac{1}{2},\quad \Delta x=x-x_0=1{,}02-1=0{,}02 \text{ bo‘ladi.}$$

Demak, (1) formulaga ko‘ra

$$\sqrt{1{,}02}=\sqrt{1+0{,}02}\approx 1+\frac{1}{2}\cdot 0{,}02=1{,}01.$$

Kalkulator yoki boshqa hisoblash vositasi yordamida $\sqrt{1{,}02}\approx 1{,}0099504938...$ qiymatni hosil qilishimiz mumkin. ▲

3-misol. $\sqrt[3]{7.997}$ ning qiymatini taqribiy hisoblang.

△ $f(x)=\sqrt[3]{x}$ funksiyani qaraymiz. Uning hosilasini topamiz:

$$f'(x)=\frac{1}{3}x^{-\frac{2}{3}}.$$

$x_0=8$ deb olsak, $f(x_0)=f(8)=\sqrt[3]{8}=2$,

$$f'(x_0)=f'(8)=\frac{1}{3}8^{-\frac{2}{3}}=\frac{1}{3\sqrt[3]{8^2}}=\frac{1}{12},$$

$\Delta x=7{,}997-8=-0{,}003$ bo‘ladi.

Demak, (1) formulaga ko‘ra

$$\sqrt[3]{7,997}=\sqrt[3]{8+(-0,003)}\approx 2-\frac{0,003}{12}=1,9997.$$

Kalkulator yoki boshqa hisoblash vositasi yordamida

$\sqrt[3]{7.997}\approx 1,9997499687...$ qiymatni hosil qilishimiz mumkin. ▲

4-misol. sin 29° ning qiymatini taqribiy hisoblang.

△ $f(x)=\sin x$ funksiyani qaraymiz. Uning hosilasini topamiz: $f'(x)=\cos x$.

$x_0=\frac{\pi}{6}$ deb olsak, $f(x_0)=f\left(\frac{\pi}{6}\right)=\sin\frac{\pi}{6}=\frac{1}{2}$,

$f'(x_0)=f'\left(\frac{\pi}{6}\right)=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$, $\Delta x=\frac{29\pi}{180}-\frac{\pi}{6}=-\frac{\pi}{180}$ bo‘ladi.

Demak, (1) formulaga ko‘ra

$$\sin 29^\circ=\sin\left(\frac{\pi}{6}+\left(-\frac{\pi}{180}\right)\right)\approx\sin\frac{\pi}{6}-\frac{\sqrt{3}}{2}\frac{\pi}{180}=\frac{1}{2}-\frac{\sqrt{3}}{2}\frac{\pi}{180}\approx 0,484...\ .$$

Kalkulator yoki boshqa hisoblash vositasi yordamida $\sin 29^\circ\approx 0,4848096202...$ qiymatni hosil qilishimiz mumkin. ▲

5-misol. Logarifmlarni hisoblash uchun kichik orttirmalar formulasini keltiramiz.

△ $f(x)=\ln x$; $f'(x)=\frac{1}{x}$. (1) ga ko‘ra, $\ln(x_0+\Delta x)\approx\ln x_0+\frac{1}{x_0}\cdot\Delta x$ — kichik orttirmalar formulasini hosil qilamiz.

Agar $x_0=1$ va $\Delta x=t$ bo‘lsa, $\ln(1+t)\approx t$ bo‘ladi.
Bundan, masalan, $\ln 1,3907=\ln(1+0,3907)\approx 0,3907$ qiymatni olamiz.

Agar $x_0=0$, ya’ni $\Delta x=x-x_0=x$ bo‘lsa, (1) kichik orttirmalar formulasi

$$f(x)\approx f(0)+f'(0)x \qquad (2)$$

ko‘rinishni oladi. ▲

Sinfda bajariladigan topshiriq. (2) formulaga asoslanib, x yetarlicha kichik bo‘lganda

$\sin x\approx x$, $\operatorname{tg} x\approx x$, $e^x\approx 1+x$, $(1+x)^m\approx 1+mx$, jumladan, $\sqrt{1+x}\approx 1+\frac{1}{2}x$, $\sqrt[3]{1+x}\approx 1+\frac{1}{3}x$ taqribiy formulalarni hosil qiling.

6-misol. $\dfrac{1}{0{,}997^{30}}$ ifodani taqribiy hisoblang.

△ $(1+x)^m \approx 1+mx$ formuladan foydalanamiz:

$$\frac{1}{0{,}997^{30}} = (1-0{,}003)^{-30} \approx 1+(-30)(-0{,}003)=1+0{,}09=1{,}09.$$

Kalkulator yoki boshqa hisoblash vositasi yordamida $\dfrac{1}{0{,}997^{30}} \approx 1{,}0943223033...$ qiymatni hosil qilishimiz mumkin. ▲

$(1+x)^m \approx 1+mx$ taqribiy formuladan foydalanib ildizlarni tezkor hisoblash usulini taklif qilish mumkin.

Chindan ham, n – natural son bo‘lib, $|B|$ soni $|A^n|$ ga nisbatan yetarlicha kichik bo‘lsin.

U holda

$$\sqrt[n]{A^n+B} = A\left(1+\frac{B}{A^n}\right)^{\frac{1}{n}} \approx A\left(1+\frac{B}{nA^n}\right) \text{ yoki}$$

$$\sqrt[n]{A^n+B} \approx A+\frac{B}{nA^{n-1}}.$$

Masalan, $\sqrt[3]{131} = \sqrt[3]{125+6} = 5+\dfrac{6}{3\cdot 5^2} = 5{,}08.$

Kalkulator yoki boshqa hisoblash vositasi yordamida $\sqrt[3]{125} = 5{,}0788...$ qiymatni hosil qilishimiz mumkin.

(2) formulaga asoslanib, x yetarlicha kichik bo‘lganda $\cos x$ ning qiymatini taqribiy hisoblaylik. $(\cos x)' = -\sin x$ bo‘lgani uchun $f(x) \approx f(0)+f'(0)x$ formula $\cos x \approx \cos 0-(\sin 0)x=1$, ya’ni $\cos x \approx 1$ ko‘rinishni oladi. Ravshanki, bunday “taqribiy” formula bizni qanoatlantirmaydi. Shuning uchun, boshqacha yo‘l tutamiz. Asosiy trigonometrik ayniyatdan $\cos x = \pm\sqrt{1-\sin^2 x}$ tenglikni hosil qilamiz.

Yuqorida qayd etganimizdek, x yetarlicha kichik bo‘lganda $\sin x \approx x$ bo‘ladi. Demak, $\cos x = \sqrt{1-\sin^2 x} \approx \sqrt{1-x^2}$.

Ravshanki, x yetarlicha kichik bo‘lganda x^2 ham kichik bo‘ladi.

Demak, $\sqrt{1+x}\approx 1+\frac{1}{2}x$ formuladan $\sqrt{1-x^2}\approx 1-\frac{x^2}{2}$ formula bevosita kelib chiqadi, ya‘ni $\cos x\approx 1-\frac{x^2}{2}$ formula o‘rinli bo‘ladi.

7-misol. $\cos 44°$ ni taqribiy hisoblang.

△ $\cos(x-y)=\cos x\cos y+\sin x\sin y$ bo‘lgani uchun

$$\cos 44° = \cos\left(\frac{\pi}{4}-\frac{\pi}{180}\right)=\cos\frac{\pi}{4}\cos\frac{\pi}{180}+\sin\frac{\pi}{4}\sin\frac{\pi}{180}=$$

$$=\frac{\sqrt{2}}{2}\left(\cos\frac{\pi}{180}+\sin\frac{\pi}{180}\right).\quad \cos\frac{\pi}{180}\approx 1-\frac{1}{2}\left(\frac{\pi}{180}\right)^2=0{,}9998476...,$$

$$\sin\frac{\pi}{180}\approx\frac{\pi}{180}=0{,}0174532....$$

Demak, $\cos 44°\approx 0{,}7193403...$.

Kalkulator yoki boshqa hisoblash vositasi yordamida $\cos 44°\approx 0{,}7193339...$ qiymatni hosil qilamiz.

? Savol va topshiriqlar

1. Kichik orttirmalar formulasini yozing.
2. Kichik orttirmalar formulasining tatbiqiga oid misollar keltiring.

Mashqlar

91. $f(x)$ funksiyaning x_1 va x_2 nuqtalardagi qiymatini taqribiy hisoblang:

a) $f(x)=x^4+2x$, $x_1=2{,}016$, $x_2=0{,}97$;

b) $f(x)=x^5-x^2$, $x_1=1{,}995$, $x_2=0{,}96$;

d) $f(x)=x^3-x$, $x_1=3{,}02$, $x_2=0{,}92$;

e) $f(x)=x^2+3x$, $x_1=5{,}04$, $x_2=1{,}98$.

$(1+x)^m\approx 1+mx$ taqribiy formuladan foydalanib, sonli ifoda qiymatini hisoblang (**92–93**):

92. a) $1{,}002^{100}$; b) $0{,}995^6$; d) $1{,}03^{200}$; e) $0{,}998^{20}$.

93. a) $\sqrt{1{,}004}$; b) $\sqrt{25{,}012}$; d) $\sqrt{0{,}997}$; e) $\sqrt{4{,}0016}$.

Taqribiy formulalardan foydalanib, hisoblang (**94 – 97**):

94. a) tg 44°; b) cos 61°; d) sin 31°; e) ctg 47°.

95. a) $\cos\left(\frac{\pi}{6}+0,04\right)$; b) $\sin\left(\frac{\pi}{6}-0,02\right)$;

c) $\sin\left(\frac{\pi}{6}+0,03\right)$; d) $\operatorname{tg}\left(\frac{\pi}{6}-0,05\right)$.

96. a) $\frac{1}{1,003^{20}}$; b) $\frac{1}{0,996^{40}}$; d) $\frac{1}{2,0016^{3}}$; e) $\frac{1}{0,994^{5}}$.

97. a) ln0,9; b) $e^{0,015}$; d) $\frac{1}{0,994^{5}}$.

$y = f(x)$ ning ko‘rsatilgan nuqtadagi taqribiy qiymatini hisoblang

(**98 – 106**):

98. $y=\sqrt[3]{x^3+7x}$, $x=1,012$.

99. $y=\sqrt{x^2+x+3}$, $x=1,97$.

100. $y=x^3$, $x=1,021$.

101. $y=x^4$, $x=0,998$.

102. $y=\sqrt[3]{x^2}$, $x=1,03$.

103. $y=x^6$, $x=2,01$.

104*. $y=\sqrt{1+x+\sin x}$, $x=0,01$.

105*. $y=\sqrt[3]{3x+\cos x}$, $x=0,01$.

106*. $y=\sqrt[4]{2x-\sin(\pi x/2)}$, $x=1,02$.

29–32 HOSILA YORDAMIDA MODELLASHTIRISH

10-sinfda (**79 – 81** mavzu) bakteriyalar sonining ko'payish jarayonini o'rgandik. Endi bu hodisaga boshqacha yondashaylik.

1-masala. Har bir bakteriya ma'lum vaqtdan (bir necha soat yoki minutlardan) so'ng ikkiga bo'linadi va bakteriyalar soni ikki karra ortadi. Navbatdagi vaqtdan so'ng mazkur ikkita bakteriya ham ikkiga bo'linadi va populatsiya miqdori (bakteriyalar umumiy soni) yana ikki karra ortadi... Bu ko'payish jarayoni qulay (populatsiya uchun zarur resurslar, joy, oziqa, suv, energiya va hokazolar yetarli bo'lgan) sharoitlarda davom etaveradi, deylik.

Bakteriyalarning ***ko'payish tezligi*** bakteriyalar umumiy soniga proporsional deb faraz qilaylik.

Bakteriyalar populatsiyasining soni ixtiyoriy t vaqtga nisbatan qanday o'zgaradi?

△ $b(t)$ deb t vaqt oralig'idagi bakteriyalar populatsiyasining umumiy sonini belgilaylik.

Hosilaning ma'nosiga ko'ra, bakteriyalar ko'payish tezligi $b'(t)$ ga teng.

Farazimizga ko'ra, ixtiyoriy t vaqtda $b'(t)$ miqdor $b(t)$ miqdorga proporsional, ya'ni

$$b'(t)=kb(t) \qquad (1)$$

munosabat o'rinli. Bu yerda k – proporsionallik koeffitsiyenti.

$b_0 = b(0)$ – boshlang'ich $t = 0$ vaqtdagi populatsiya soni bo'lsin.

Ravshanki, $b(t)=b_0e^{kt}$ funksiya (1) ni qanoatlantiradi.

Chindan ham, $b'(t)=(b_0e^{kt})'=kb_0e^{kt}=kb(t)$.

Dastlab 10 million bakteriya bo'lsa (b_0=10 mln), bunday bakteriyalar soni bir soatdan so'ng $b(1)=10e^k=20$ (mln) ga teng bo'ladi, ya'ni $e^k=2$. Bundan $k=\ln 2$ ga ega bo'lamiz.

t vaqt oralig'idagi bakteriyalar populatsiyasining sonini topaylik:

$$b(t)=10e^{(\ln 2)t}=10\cdot 2^t \text{ (mln)}.$$

Bu natija 10-sinfda olingan natija bilan ustma-ust tushmoqda. ▲

Tarixiy ma'lumot. 18-asrda ingliz olimi Tomas Maltus yuqoridagi fikrlarga o'xshash fikr yuritib, yer yuzidagi aholi sonining o'sishi uchun

$$N'(t)=kN(t) \tag{2}$$

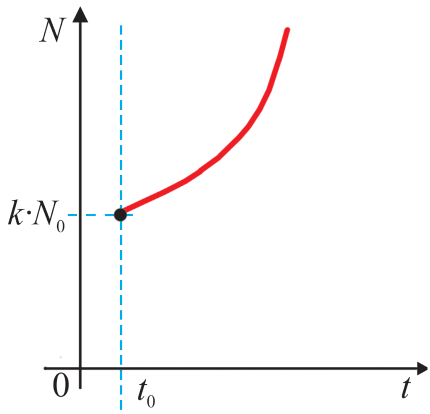
munosabatni hosil qildi, bu yerda $N(t)$ – vaqtning t momentidagi aholi soni.

$N_0=N(t_0)$ – boshlang'ich t_0 vaqtdagi aholi soni bo'lsin.

Bu holda $N(t)=N_0e^{k(t-t0)}$ funksiya (2) tenglamani qanoatlantiradi.

Chindan ham, $N'(t)= N_0(e^{k(t-t0)})'=kN_0e^{k(t-t0)}=kN(t)$.

$N(t)=N_0e^{k(t-t0)}$ qonuniyat aholining ***eksponensial o'sishini,*** ya'ni shiddatli, to'xtovsiz o'sish jarayonini ifodalashini inobatga olib, Tomas Maltus vaqt o'tishi bilan insoniyatga oziqa resurslari yetmasligini «bashorat" qilganligini qayd etamiz (30-rasmga qarang).



30-rasm.

2-masala. Ekologiya tirik organizmlarning tashqi muhit bilan o'zaro munosabatini o'rganadi. Ko'payish yoki turli sabablarga ko'ra nobud bo'lish bilan bog'liq bo'lgan populatsiyalar sonining o'zgarish tezligi vaqtga qanday bog'lanishda ekanini o'rganing.

△ $N(t)$ – vaqtning t momentidagi populatsiya soni bo'lsin, u holda agar vaqtning bir birligida populatsiyada tug'iladigan jonzotlar sonini A, nobud bo'ladiganlari sonini B desak, yetarli asos bilan aytish mumkinki, N ning vaqtga nisbatan o'zgarish tezligi

$$N'(t) =A-B \tag{3}$$

munosabatni qanoatlantiradi.

Tadqiqotchilar A va B ning N ga bog'liqligini quyidagicha tavsiflaydilar.

a) Eng sodda hol: $A = aN(t)$, $B = bN(t)$. Bu yerda a va b – vaqtning bir birligida tug'ilish va nobud bo'lish koeffitsiyentlari.

Bu holda (3) munosabatni

$$N'(t) = (a-b)N(t) \tag{4}$$

ko'rinishda yozish mumkin.

$N_0 = N(t_0)$ – boshlang'ich t_0 vaqtdagi populatsiya soni bo'lsin.

Bu holda $N(t) = N_0 e^{(a-b)(t-t_0)}$ funksiya (4) ni qanoatlantiradi (tekshiring).

b) $A = aN(t)$, $B = bN^2(t)$ hol ham uchraydi.

Bunda

$$N'(t) = aN(t) - bN^2(t) \tag{5}$$

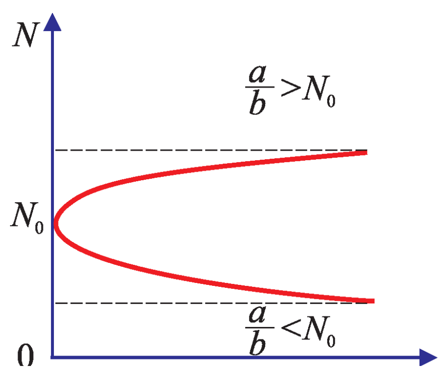
munosabat hosil bo'ladi.

Tekshirish mumkinki, $N(t) = \dfrac{N_0 a/b}{N_0 + [a/b - N_0]e^{-a(t-t_0)}}$ funksiya (5) tenglamani qanoatlantiradi. ▲

(4) munosabatni 1845-yilda belgiyalik demograf-olim Ferxyulst populatsiyadagi ichki kurashni hisobga olgan holda kashf qildi. Bu natija Maltusning (2) munosabatiga nisbatan populatsiyaning rivojlanishini aniqroq tavsiflaydi.

Populatsiyaning o'sish-kamayishi a va b sonlarga qanday bog'liq bo'ladi, degan savol tug'ilishi tabiiy.

31-rasmda $\dfrac{a}{b} > N_0$ va $\dfrac{a}{b} < N_0$ hollar uchun $N(t) = \dfrac{N_0 a/b}{N_0 + [a/b - N_0]e^{-a(t-t_0)}}$ ko'rinishdagi funksiya grafiklari tasvirlangan:



31-rasm.

Ko‘rinib turibdiki, vaqt kechishi bilan populatsiya soni $\frac{a}{b}$ soniga yaqinlashar ekan. Mazkur holat *to‘yinish* deb nomlangan hodisani bildiradi.

Chizmada tasvirlangan egri chiziq Maltus tomonidan *logistik egri chiziq* deb nomlanib, u inson turmushining turli sohalarida uchrab turadi.

Funksiyaning hosilasini shu funksiya bilan bog‘lovchi $y'(x)=F(x;\ y)$ ko‘rinishdagi munosabat *differensial tenglama* deyiladi.

Yuqorida keltirilgan (1) – (5) munosabatlar differensial tenglamalarga misollardir.

Differensial tenglamani qanoatlantiradigan har qanday funksiya uning yechimi deyiladi. Oliy matematikada muayyan shartlarda $y'(x)=F(x,\ y)$ ko‘rinishdagi differensial tenglamaning $y(x_0)=y_0$ boshlang‘ich shartni qanoatlantiradigan yagona $y(x)$ yechimi mavjudligi isbot qilingan.

3-masala. Vaqtning t momentida sotilayotgan mahsulot haqida xabardor bo‘lgan xaridorlar soni $x(t)$ ning vaqtga bog‘liqligini o‘rganing. (Bu masala reklama samaradorligini aniqlashda muhim.)

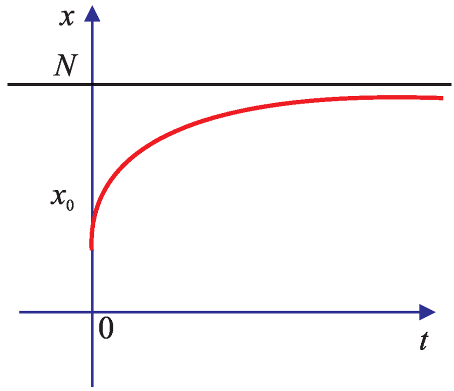
△ Barcha xaridorlar sonini N deb belgilasak, sotilayotgan mahsulotdan bexabarlar soni $N-x(t)$ bo‘ladi.

Mahsulot haqida xabardor bo‘lgan xaridorlar sonining o‘sish tezligi $x(t)$ ga va $N–x(t)$ ga proporsional deb hisoblasak, quyidagi differensial tenglamaga ega bo‘lamiz:

$x'(t)=kx(t)(N–x(t))$, bu yerda $k > 0$ – proporsionallik koeffitsiyenti.

Bu tenglamaning yechimi $x(t)=\frac{N}{1+Pe^{-Nkt}}$ dan iborat, bunda $P=\frac{1}{e^{NC}}$, C – o‘zgarmas son.

Ravshanki, har qanday holatda t vaqt kechishi bilan Pe^{-Nkt} had kichiklashib boraveradi va bundan $x(t)=\frac{N}{1+Pe^{-Nkt}}$ ifodaning qiymati N ga yaqinlashadi (32-rasmga qarang). ▲



32-rasm.

4-masala. Massasi m, issiqlik sig‘imi c o‘zgarmas bo‘lgan jism boshlang‘ich momentda T_0 temperaturaga ega bo‘lsin. Havo temperaturasi o‘zgarmas va τ ($T > \tau$) ga teng. Jismning cheksiz kichik vaqt ichida bergan issiqligi jism va havo temperaturalari orasidagi farqqa, shuningdek, vaqtga proporsional ekanligini e’tiborga olgan holda, jismning sovish qonunini toping.

△ Sovish davomida jism temperaturasi T_0 dan τ gacha pasayadi. Vaqtning t momentida jism temperaturasi $T(t)$ ga teng bo‘lsin. Cheksiz kichik vaqt oralig‘ida jism bergan issiqlik miqdori, yuqorida aytilganiga ko‘ra,

$$Q'(t) = -k(T - \tau)$$

ga teng, bu yerda k – proporsionallik koeffitsiyenti.

Ikkinchi tomondan, fizikadan ma’lumki, jism T temperaturadan τ temperaturagacha soviganda beradigan issiqlik miqdori $Q = mc(T(t) - \tau)$ ga teng. Hosilani hisoblaymiz:

$$Q'(t) = mcT'(t). \tag{6}$$

$Q'(t)$ uchun topilgan har ikkala ifodani taqqoslab, $mcT'(t) = -k(T - \tau)$ differensial tenglamani hosil qilamiz.

$$T(t) = \tau + Ce^{-\frac{k}{mc}t}$$

funksiya (6) differensial tenglamani qanoatlantiradi (o‘zingiz tekshiring!), bu yerda C – ixtiyoriy o‘zgarmas son.

Boshlang‘ich shart ($t = 0$ da $T = T_0$) C ni topishga imkon beradi:

$$C = T_0 - \tau.$$

Shuning uchun jismning sovish qonuni quyidagi ko‘rinishda yoziladi:

$$T(t) = \tau + (T_0 - \tau)\, e^{-\frac{k}{mc}}.$$

Javob: $T(t) = \tau + (T_0 - \tau)\, e^{-\frac{k}{mc}}$ ▲.

5-masala. Tandirdan olingan (uzilgan) nonning temperaturasi 20 minut ichida 100° dan 60° gacha pasayadi. Tashqi muhit temperaturasi 25°. Nonning temperaturasi qancha vaqtda 30° gacha pasayadi?

△ Yuqoridagi masalaning yechimidan foydalanib, nonning sovish qonunini quyidagi ko‘rinishda yoza olamiz:

$$T(t) = \tau + (T_0 - \tau)\, e^{-\frac{k}{mc}} = 25 + (100 - 25)e^{at} = 25 + 75e^{at},$$

bu yerda a – noma’lum koeffitsiyent.

a ni topish uchun $t = 20$ da $T(20) = 60$ tenglikdan foydalanamiz:

$$T(20) = 25 + 75e^{20a} = 60,$$

$$75e^{20a} = 35, \quad (e^a)^{20} = \frac{35}{75} = \frac{7}{15}, \quad e^a = \left(\frac{7}{15}\right)^{\frac{1}{20}}.$$

Demak, nonning sovushi $T = 25 + 75\left(\frac{7}{15}\right)^{\frac{t}{20}}$ qonuniyatga bo‘ysunar ekan.

Nonning temperaturasi 30° gacha pasayish vaqtini topamiz:

$$30 = 25 + 75\left(\frac{7}{15}\right)^{\frac{t}{20}}, \quad \left(\frac{7}{15}\right)^{\frac{t}{20}} = \frac{5}{75} = \frac{1}{15},$$

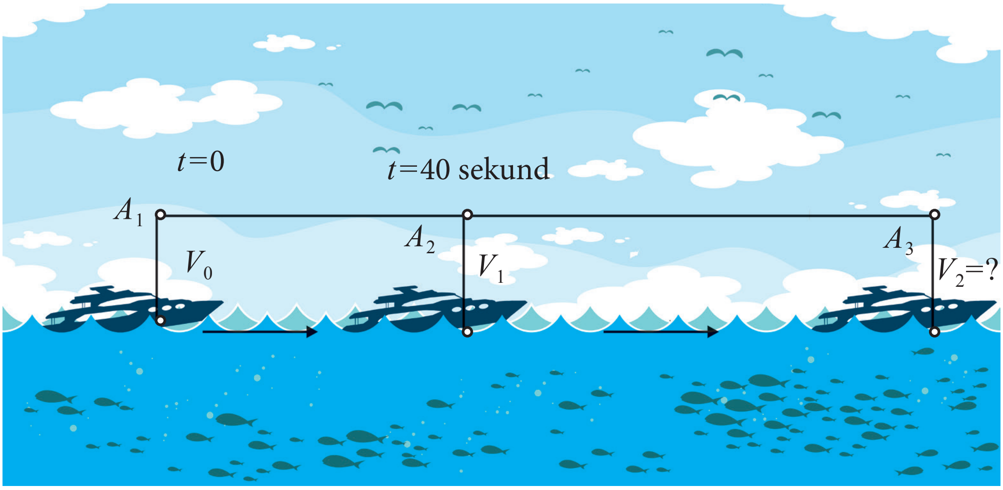
$$\ln\left(\frac{7}{15}\right)^{\frac{t}{20}} = \frac{t}{20}(\ln(7) - \ln(15))$$

bo‘lgani uchun $t^* = \dfrac{-20\ln 15}{\ln 7 - \ln 15} \approx \dfrac{-20 \cdot 2,7081}{-0,762} \approx 71.$

Javob: 1 soat-u 11 minutda nonning temperaturasi 30° gacha pasayadi. ▲

6-masala. Motorli qayiq turg‘un suvda 20 km/soat tezlik bilan harakatlanmoqda. Ma’lum vaqtdan keyin motor ishdan chiqdi. Motor to‘xtagandan 40 sekund vaqt o‘tgach qayiqning tezligi 8 km/soat bo‘ldi.

Suvning qarshiligi tezlikka proporsional bo‘lsa, motor to‘xtagandan 2 minut vaqt o‘tgach qayiq tezligini toping.



33-rasm.

△ Qayiqqa $F=-kv$ kuch ta’sir qilmoqda. Nyuton qonuniga ko‘ra $F=mv'(t)$ Bundan $mv'(t)=-kv$.

Bu tenglamani $v(t)=Ce^{-\frac{k}{m}t}$ ko‘rinishdagi funksiya qanoatlantiradi. $t=0$ da $v=20$ shartidan $C=20$ kelib chiqadi.

Bundan $v(t)=20e^{-\frac{r}{m}t}$. $t=40\ \text{s}=\frac{1}{90}$ soat bo‘lganda qayiqning tezligi 8 km/soatga teng, bundan $8=20e^{-\frac{r}{m}\cdot\frac{1}{90}}$ yoki $e^{\frac{r}{m}}=\left(\frac{5}{2}\right)^{90}$ hamda

$$t=2\min=\frac{1}{30}\ \text{soat bo‘lganidan}\ v=20\left[\left(\frac{5}{2}\right)^{90}\right]^{-\frac{1}{30}}=20\left(\frac{5}{2}\right)^{-3}=\frac{32}{25}\approx 1{,}28$$

(km/s) ekanini topamiz.

Javob: Motor to‘xtaganidan 2 minut vaqt o‘tgach, qayiqning tezligi taxminan 1,28 km/soat ga teng bo‘ladi. ▲

7-masala. Radioaktiv yemirilish natijasida radioaktiv modda massasi $m(t)$ ning vaqtga nisbatan o‘zgarish qonuniyatini toping. Bu yerda $m(t)$ gramm, t – yillarda o‘lchanadi.

△ Yemirilish tezligi massaga proporsional deb faraz qilsak,

$$m'(t) = -\alpha\, m(t) \qquad (7)$$

differensial tenglamaga ega bo‘lamiz. $m(t)=Ce^{-\alpha t}$ funksiya bu tenglamaning yechimi ekanligini tekshirish mumkin.

$m(t_0) = m_0$ boshlang‘ich shartdan $m(t) = m_0 e^{-\alpha\,(t-t_0)}$ qonuniyatga ega bo‘lamiz. *Javob:* $m(t) = m_0 e^{-\alpha\,(t-t_0)}$. ▲

Iqtisodiy modellar. Talab va taklif iqtisodiyotning fundamental (asosiy) tushunchalaridir.

Talab (tovarlar va xizmatlarga talab) – xaridor, iste’molchining bozordagi muayyan tovarlarni, ne’matlarni sotib olish istagi; bozorga chiqqan va pul imkoniyatlari bilan ta’minlangan ehtiyojlari.

Talab miqdorining o‘zgarishiga bir qancha omillar ta’sir qiladi. Ularning orasida eng muhimi narx omilidir. Tovar narxining pasayishi sotib olinadigan tovar miqdorining o‘sishi va aksincha, narxning o‘sishi xarid miqdorining kamayishiga olib keladi.

Taklif — muayyan vaqtda va muayyan narxlar bilan bozorga chiqarilgan va chiqarilishi mumkin bo‘lgan tovarlar va xizmatlar miqdori bilan ifoda etiladi; taklif – ishlab chiqaruvchi (sotuvchi)larning o‘z tovarlarini bozorda sotishga bo‘lgan istagi. Bozorda tovar narxi bilan uning taklif miqdori o‘rtasida bevosita bog‘liqlik mavjud: narx qanchalik yuqori bo‘lsa, boshqa sharoitlar o‘zgarmagan hollarda, sotish uchun shuncha ko‘proq tovar taklif etiladi, yoki aksincha, narx pasayishi bilan taklif hajmi qisqaradi.

Talab va taklifninng tub mazmuni ularning narx orqali o‘zaro aloqadorlikda mavjud bo‘lishidir. Bu aloqadorlik — talab va taklif qonuni bozor iqtisodiyotining obyektiv qonuni hisoblanadi. Talab va taklif qonuniga ko‘ra, bozordagi taklif va talab faqat miqdoran emas, balki o‘zining tarkibi jihatidan ham bir-biriga mos kelishi kerak, shundagina bozor muvozanatiga erishiladi. Bu qonun ayirboshlash qonuni bo‘lib, bozorni boshqaruvchi va tartiblovchi kuch darajasiga ko‘tariladi. Unga ko‘ra bozordagi talab o‘zgarishlari darhol ishlab chiqarishga yetkazilishi kerak. Bozordagi talab va taklif nisbatiga qarab ishlab chiqarish sur’atlari va tuzilmasi tashkil topadi.

Quyidagi *masalani* qaraylik.

Fermer uzoq muddat davomida mevalarni bozorda sotishga chiqarib keladi. Har hafta yakunida u narxning o‘zgarish tezligini kuzatib, kelgusi haftaga chiqariladigan mevalarning yangi narxini chamalaydi.

Xuddi shunday iste'molchilar ham narxning o'zgarish tezligini kuzatib, kelgusi haftaga sotib olinadigan mevalarning miqdorini belgilaydilar.

Kelgusi haftadagi mevalarning narxini p orqali, narxning o'zgarish tezligini esa p' orqali belgilaylik.

Taklif ham, talab ham tovar narxi bilan uning o'zgarish tezligiga bog'liq ekanligini ishonch bilan aytishimiz mumkin. Bu bog'lanish qanday bo'ladi?

△ Bunday bog'lanishlarning eng sodda ko'rinishi quyidagicha bo'lar ekan: $y=ap'+bp+c$, bu yerda a, b, c – haqiqiy sonlar.

Masalan, q orqali talabni, s orqali esa taklifni belgilasak, ular uchun yuqoridagi bog'lanishlar $q=4p'-2p+39$, $s=44p'+2p-1$ tenglamalar yordamida ifodalanishi mumkin.

Bu holda talab va taklifning o'zaro tengligi $4p'-2p+39=44p'+2p-1$ munosabat yordamida ifodalanadi.

Bu tenglikdan $p'=-\dfrac{p-10}{10}$ ko'rinishdagi differensial tenglamani hosil qilamiz.

Agar boshlang'ich narxni $p(0)=p_0$ deb belgilasak, narx $p=(p_0-10)e^{-\frac{t}{10}}+10$ qonuniyat bilan o'zgarishini hosil qilamiz. ▲

Investitsiya. Faraz qilaylik, qandaydir mahsulot p narx bilan sotiladi, $Q(t)$ funksiya t vaqt mobaynida ishlab chiqarilgan mahsulot miqdori o'zgarishini bildiradi desak, u holda t vaqt davomida $pQ(t)$ ga teng daromad olinadi. Aytaylik, olingan daromadning bir qismi mahsulot ishlab chiqarish investitsiyasiga sarf bo'lsin, ya'ni

$$I(t)=mpQ(t), \qquad (8)$$

m – investitsiya normasi, o'zgarmas son va $0<m<1$.

Agar bozor yetarlicha ta'minlangan va ishlab chiqarilgan mahsulot to'la sotilgan degan tasavvurdan kelib chiqilsa, bu holat ishlab chiqarish tezligining yana oshishiga olib keladi.

Ishlab chiqarish tezligi esa investitsiyaning o'sishiga proporsional, ya'ni

$$Q'=l\cdot I(t), \qquad (9)$$

bu yerda l – proporsionallik koeffitsiyenti.

(8) formulani (9) ga qo'yib

$$Q' = kQ, \quad k = lmp \tag{10}$$

differensial tenglamani hosil qilamiz.

C – ixtiyoriy oʻzgarmas son boʻlganda $Q = Ce^{kt}$ koʻrinishdagi funksiya (10) differensial tenglamani qanoatlantiradi.

Faraz qilaylik, boshlangʻich moment $t = t_0$ da mahsulot ishlab chiqarish hajmi Q_0 berilgan. U holda bu shartdan oʻzgarmas C ni topish mumkin:

$$Q_0 = Ce^{kt_0}, \text{ bundan } C = Q_0 e^{-kt_0}.$$

Natijada ishlab chiqarish hajmi $Q = Q_0 e^{k(t-t_0)}$ qonuniyat bilan oʻzgarishini bilib olamiz.

? Savol va topshiriqlar

1. Bakteriyalarning ma'lum vaqtdan soʻng ikkiga boʻlina borishi jarayonini hosila yordamida modellashtiring.
2. Tomas Maltusning yer yuzidagi aholi soni oʻsishiga oid masalasini tushuntiring.
3. Tomas Maltusning logistik egri chizigʻini tushuntiring.
4. Reklama samaradorligiga oid masalani hosila yordamida modellashtiring.

Mashqlar

Matndagi 4-masala yechimidan foydalanib, mashqlarni bajaring **(107–108)**:

107. Temperaturasi 25°C boʻlgan metall parchasi pechga qoʻyildi. Pechning temperaturasi 25°C dan boshlab minutiga 20°C tezlik bilan tekis ravishda koʻtarila boshladi. Pech va metall temperaturasining farqi T°C boʻlganda, metall minutiga $10 \cdot T$°C tezlik bilan isitila boshlaydi. Metall parchasini 30 minutdan keyingi temperaturasini toping.

108. Jismning boshlangʻich temperaturasi 5°C. Jism N minut davomida 10°C gacha isidi. Atrof-muhit temperaturasi 25°C boʻlib turibdi. Jism qachon 20°C gacha isiydi?

Matndagi 7-masala yechimidan foydalanib, mashqlarni bajaring:

109. Tajribalarga koʻra 1 yil davomida radiyning har bir grammidan 0,44 mg modda yemiriladi

a) necha yildan soʻng mavjud radiyning 20 foizi yemiriladi?

b) mavjud radiyning 400 yildan soʻng necha foizi qoladi?

Matndagi 6-masalani yechishdagi mulohazalardan foydalanib, mashqlarni bajaring **(110 – 111):**

110. Qayiq suvning qarshiligi ta'siri ostida o'z harakatini sekinlashtiradi. Suvning qarshiligi qayiq tezligiga proporsional. Qayiqning boshlang'ich tezligi 1,5 m/s. 4 sekunddan so'ng uning tezligi 1 m/s ni tashkil qildi.Necha sekunddan so'ng qayiqning tezligi 2 marta kamayadi?

111. 10 l hajmdagi idish havo bilan to'ldirilgan (80% azot, 20% kislorod). Shu idishga 1 sekundda 1 litr tezlikda azot purkalmoqda. U uzluksiz ravishda aralashib, shu tezlikda idishdan chiqmoqda. Qancha vaqtdan so'ng idishda 95% azotli aralashma hosil bo'ladi?

Ko'rsatma: $y(t)$ bilan t vaqtdagi azot ulushini belgilasak, $y(t)$ funksiya $y' \cdot V = a(1-y)$ munosabatni qanoatlantiradi deylik. Bu yerda V – isitish hajmi, a – purkash tezligi.

Nazorat ishi namunasi

1. Asosi kvadrat bo'lgan to'g'ri burchakli parallelepiped shaklidagi usti ochiq metall idish yasashmoqchi. Idish hajmi 270 l bo'lishi kerak. Idishning o'lchamlari qanday bo'lganda uni yasashda eng kam metall ketadi?

2. Moddiy nuqta $s(t) = -\frac{t^4}{4} + 72t^3$ qonuniyat bilan harakatlanmoqda ($s(t)$ metrda, t vaqt sekundda o'lchanadi).

1) eng katta tezlanishga erishadigan vaqtni (t_0);

2) t_0 vaqtdagi oniy tezlikni;

3) t_0 vaqt mobaynida bosib o'tilgan yo'lni toping.

3. Taqribiy hisoblash formulasidan foydalanib $\ln 0{,}92$ ni toping.

4. Taqribiy hisoblash formulasidan foydalanib sin(–1; 2)ni toping.

5. Mahsulot ishlab chiqaruvchi tadbirkorning kunlik daromadi quyidagi formula bilan hisoblanadi:

$P(x) = -3x^2 + 42x - 6$ (ming so'm), bu yerda x – mahsulotlar soni.

Quyidagilarni aniqlang:

1) eng katta daromad olish uchun tadbirkor nechta mahsulot ishlab chiqarishi kerak?

2) tadbirkorning eng katta daromadi necha so'mni tashkil qiladi?

MASALALAR YECHISH

112. Moddiy nuqta harakatining qonuni $s=s(t)$ ga ko‘ra uning eng katta yoki eng kichik tezligini toping:

1) $s=13t$; | 2) $s=17t-5$; | 3) $s=t^2+5t+18$; | 4) $s=t^3+2t^2+5t+8$;
5) $s=2t^3+5t^2+6t+3$; | 6) $s=13t^3+2t^2$; | 7) $s=t^3+t^2+3$.

113. Berilgan funksiya grafigiga: 1) $x_0=-1$; 2) $x_0=2,2$; 3) $x_0=0$ abssissali nuqtada o‘tkazilgan urinmani toping:

1) $f(x)=12x^2+5x+1$; | 2) $f(x)=13x+4$; | 3) $f(x)=60$; | 4) $f(x)=x^3+4x$.

114. Berilgan funksiya uchun $y=-7x+2$ to‘g‘ri chiziqqa parallel bo‘lgan urinma tenglamasini yozing:

1) $f(x)=5x^3-2x^2+16$; | 2) $f(x)=-4x^2+5x+3$; | 3) $f(x)=-8x+5$.

115. Berilgan $f(x)$ va $g(x)$ funksiyalar grafiklarining urinmalari parallel bo‘ladigan nuqtalarini toping:

1) $f(x)=2x^2-3x+4$, $g(x)=12x-8$;
2) $f(x)=18x+19$, $g(x)=-15x+18$;
3) $f(x)=2x+13$, $g(x)=4x-19$;
4) $f(x)=2x^3$, $g(x)=4x^2$;
5) $f(x)=2x^3+3x^2$, $g(x)=15x-17$;
6) $f(x)=2x^4$, $g(x)=4x^3$.

116. 1) $y=\frac{1}{x}$ funksiya grafigining $x=-\frac{1}{2}$ nuqtadan o‘tuvchi urinmasi tenglamasini tuzing.

2) $y=x^2$ parabolaning $x=1$ va $x=3$ abssissalarga mos nuqtalari tutashtirilgan. Parabolaning ushbu 2 nuqtani tutashtiruvchi kesmaga parallel bo‘lgan urinmasi qaysi nuqtadan o‘tadi?

3) Moddiy nuqta $s(t)=\frac{2}{9}\cdot\sin\frac{\pi t}{2}+3$ qonuniyat bilan harakatlanmoqda (s – santimetrda, t – sekundda). Moddiy nuqtaning 1-sekunddagi tezlanishini toping.

117. Funksiyaning ko‘rsatilgan nuqtadagi hosilasini hisoblang:

1) $f(x)=x^2-15$, $x_0=-\frac{1}{2}$; 2) $f(x)=3\cos x$, $x_0=-\pi$;

3) $f(x)=\frac{3}{x}$, $x_0=-2$;
4) $f(x)=-\sin x$, $x_0=-\frac{\pi}{3}$.
5) $f(x)=x^3-4$, $x_0=5$;
6) $f(x)=\sin x$, $x_0=\frac{\pi}{6}$;
7) $f(x)=\frac{1}{x^3}$, $x_0=-2$;
8) $f(x)=\cos 5x$, $x_0=\frac{\pi}{4}$;
9) $f(x)=-\cos 2x$, $x_0=-\frac{\pi}{8}$.

118. Ko‘rsatilgan vaqtdagi tezlik va tezlanishni toping:

1) $s(t)=5t^2-t+50$ $t_0=2$;
2) $s(t)=t^3+12t^2+1$, $t_0=1$;
3) $s(t)=2t+t^3$, $t_0=5$;
4) $s(t)=8\sin t$, $t_0=\frac{\pi}{2}$.

119. Funksiyaning abssissasi ko‘rsatilgan nuqtadagi hosilasini hisoblang:

1) $f(x)=x^2-15$, $x_0=\frac{1}{2}$;
2) $f(x)=3\cos x$, $x_0=\pi$;
3) $f(x)=\frac{3}{x}$, $x_0=2$;
4) $f(x)=-\sin x$, $x_0=\frac{\pi}{3}$.
5) $f(x)=x^3-4$, $x_0=-5$;
6) $f(x)=\sin x$, $x_0=-\frac{\pi}{6}$;
7) $f(x)=\frac{1}{x^3}$, $x_0=2$;
8) $f(x)=\cos 5x$, $x_0=-\frac{\pi}{4}$;
9) $f(x)=-\cos 2x$, $x_0=\frac{\pi}{8}$;
10) $f(x)=\sin 2x$, $x_0=\frac{\pi}{4}$.

120. Ko‘rsatilgan vaqtdagi tezlik va tezlanishni toping:

1) $s(t)=3t^2-2t+10$, $t_0=2$;
2) $s(t)=t^3-6t^2+1$, $t_0=1$;
3) $s(t)=5t+2t^3$, $t_0=5$;
4) $s(t)=8\cos t$, $t_0=\frac{\pi}{2}$.

Berilgan funksiyaning hosilasini toping (**121–122**):

121. 1) $f(x)=-x^2+x+30$; 2) $f(x)=\sin x-\cos x$; 3) $f(x)=\sqrt{x}-\frac{1}{x}$;
4) $f(x)=4^x-\sin x$; 5) $f(x)=8\cos x$; 6) $f(x)=\ln x-10x^2+x-1$.

122. 1) $y=x^4$; | 2) $y=\frac{x-1}{x+2}$; | 3) $y=x-\frac{20}{x}$; | 4) $y=x^2\ln x$;

5) $y=x^3\sin x$; | 6) $y=e^x\sin x$; | 7) $y=\frac{x+1}{4x^2}$; | 8) $y=2(10x-1)\sin x$.

123. Berilgan funksiyalar uchun $f'(-\frac{\pi}{2})$, $f'(\frac{\pi}{4})$ sonlarni hisoblang:

1) $f(x)=e^x\cos x$; 2) $f(x)=3x+1$; 3) $f(x)=2x^2+x+3$;
4) $f(x)=\sin x+x^2$; 5) $f(x)=\sin x+\cos x$; 6) $f(x)=\sin x$;
7) $f(x)=\cos x+x^4$; 8) $f(x)=\sin 3x+\cos 3x$.

124. Moddiy nuqta $x(t)=-\frac{t^3}{6}+6t^2+15$ qonuniyat bilan harakatlanmoqda.
1) tezlanish nol bo'lgan t_0 vaqtni; 2) shu t_0 vaqtdagi tezlikni toping.

125*. $f(x)=x^2-13x+2$ funksiya Ox o'qi bilan qanday burchak ostida kesishadi?

126. $f'(0)$ sonni toping: 1) $f(x)=x^6-4x^3+4$; | 2) $f(x)=(x+10)^6$.

127. $y'(x)$ ni toping: 1) $y(x)=\sin^2 x$; | 2) $y(x)=\cos^2 x$; | 3) $y(x)=\operatorname{tg}^2 x$.

128. Funksiyaning o'sish va kamayish oraliqlarini toping:

1) $f(x)=3+7x$; 2) $f(x)=x^3+17x$; 3) $f(x)=\frac{1}{4}x+18$;

4) $f(x)=\frac{x+21}{x}$; 5) $f(x)=x^2+5x-14$; 6) $f(x)=x(x^2+8)$;

7) $f(x)=-x^2-4x+6$; 8) $f(x)=-\frac{1}{x^2}$;

9) $f(x)=x^3-12x^2-17x-23$; 10) $f(x)=3x^4+18x^3-6$;

11) $f(x)=x^3-5x^2+19x+22$; 12) $f(x)=x^4+7x^2$.

129. Funksiyaning statsionar nuqtalarini toping:

1) $f(x)=3x^2-7x+9$; | 2) $f(x)=19x-\frac{1}{7}x^3$; 3) $f(x)=5x^3$;

4) $f(x)=8x^2$; 5) $f(x)=7x-14$; 6) $f(x)=27-x^3$;

7) $f(x)=12x^3+13x^2-16$; 8) $f(x)=x^3-6x^2+9$.

130. Funksiyaning lokal maksimum va lokal mimimumlarini toping:

1) $f(x)=x^2-\frac{1}{4}x^4$; 2) $f(x)=14+13x^2-12x^3$;

3) $f(x)=x^4-3x^3+x^2+9$; 4) $f(x)=2x^4-x^3+7$.

131. Funksiyaning o‘sish, kamayish oraliqlari hamda lokal maksimum va minimumlarini toping:

1) $f(x)=x^3-64x$; 2) $f(x)=2x^3-24$; 3) $f(x)=4x^3-108x$.

132. Funksiyaning eng katta va eng kichik qiymatlarini toping:

1) $f(x)=x^4-3x^2+2$, $x\in[-4; 1]$; 2) $f(x)=x^5+6x^3+1$, $x\in[-1; 2]$;

3) $f(x)=\frac{x}{x+4}$, $x\in[1; 5]$; 4) $f(x)=x^3+6x^2+5x+8$, $x\in[-3; 4]$.

133. Funksiyaning grafigini yasang:

1) $y=x^3-2x^2+3x-2$; 2) $y=\frac{1}{5}x^5+\frac{2}{3}x^3$; 3) $y=x^4+4x^3$.

134. To‘g‘ri to‘rtburchak shaklidagi ekin maydonining atrofini o‘rash uchun 1000 metr panjara sotib olindi. Bu panjara yordamida eng ko‘pi bilan necha kvadrat metr maydonni o‘rab olish mumkin?

135. Tomoni 16 dm bo‘lgan kvadrat shaklidagi kartondan usti ochiq quti tayyorlandi. Bunda kartonning uchlaridan bir xil kvadratchalar kesib olindi. Qutining hajmi eng katta bo‘lishi uchun uning asosi necha santimetr bo‘lishi kerak?

136*. Konserva banka silindr shaklida bo‘lib, uning to‘la sirti 512π cm^2 ga teng. Bankaga eng ko‘p suv sig‘ishi uchun banka asosining radiusi va balandligi qanday bo‘lishi kerak?

137. To‘g‘ri to‘rtburchak shaklidagi maydoning yuzi 3600 m^2. Maydonning tomonlari qanday bo‘lganda uni o‘rash uchun eng kam panjara zarur bo‘ladi?

138*. Radiusi 8 dm bo‘lgan sharga eng kichik hajmli konus tashqi chizilgan. Shu konus balandligini toping.

139*. Asosi kvadrat bo‘lgan to‘g‘ri burchakli parallelepiped shaklidagi ochiq metall idishga 32 *l* suyuqlik ketadi. Idishning o‘lchamlari qanday bo‘lganda uni yasashga eng kam metall sarflanadi?

140. Moddiy nuqta $s(t)=-\frac{t^4}{4}+10t^3$ qonuniyat bilan harakatlanmoqda ($s(t)$ metrda, t sekundda o‘lchanadi).

1) eng katta tezlanishga erishadigan (t_0) vaqtni;
2) t_0 vaqtdagi oniy tezlikni;
3) t_0 vaqtda bosib o‘tilgan yo‘lni toping.

141. Havo shariga $t\in[0;10]$ minut oralig‘da $V(t) = t^3+3t^2+2t+4$ m³ havo purkalmoqda.

1) boshlang‘ich vaqtdagi havo hajmini;
2) $t = 10$ minutdagi havo hajmini;
3) $t = 5$ minutdagi havo purkash tezligini toping.

142. Akrom shim tikish uchun buyurtma oldi. Bir oyda x ta shim tiksa, $p(x)=-2x^2+240x$ (ming so‘m) daromad qiladi.

1) daromadni eng katta qilish uchun qancha shim tikish kerak?
2) eng katta daromad necha so‘m bo‘ladi?

143. Funksiyaning hosilasini toping:

1) $y=e^{3x}$;	2) $y=e^{\sin x}$;	3) $y=\sin(3x+2)$;	4) $y=(2x+1)^4$;
5) $y=\frac{x-2}{x^2+1}$;	6) $y=\frac{\ln x}{x}$;	7) $y=\operatorname{arctg}2x$;	8) $y=x^2\cdot\cos x$.

144. $f(x)=e^{2x}$ va $g(x)=4x+2$ funksiyalar uchun $F(x)$ murakkab funksiyani tuzing:

1) $F(x)=f(g(x))$; 2) $F(x)=f(x)^{g(x)}$;

3) $F(x)=g(f(x))$; 4) $F(x)=\sqrt{g(g(x))}$.

145. Murakkab funksiyaning hosilasini toping:

1) $y=(x^2+1)^5$; 2) $y=\ln\cos x$;

3) $y=\sqrt{5x-7}$; 4) $y=\sqrt{\operatorname{tg}(2x-3)}$;

5) $y=\operatorname{arctg}(3x-4)$; 6*) $y=\sin(\operatorname{arctg}2x)$;

7) $y=\sin^3 x+\cos^3 x$; 8*) $y=e^{\sin(\cos x)}$.

146. Funksiyaning o‘sish va kamayish oraliqlarini toping:

1) $y=2+x-x^2$; 2) $y=\dfrac{\sqrt{x}}{x+100}$ $(x\geq 0)$;

3) $y=3x-x^3$; 4) $y=2x-\sin x$;

5) $y=\dfrac{2x}{1+x^2}$; 6) $y=\dfrac{x^2}{2^x}$;

7) $y=(x-1)^3$; 8) $y=(x-1)^4$.

147. Funksiyaning statsionar nuqtalari, lokal maksimum va lokal minimumlarini toping:

1) $y=x^3-6x^2+9x-4$; 2) $y=\dfrac{2x}{1+x^2}$;

3) $y=x+\dfrac{1}{x}$; 4) $y=\sqrt{2x-x^2}$.

148. Funksiyaning ko‘rsatilgan oraliqdagi eng katta va eng kichik qiymatlarini toping:

1) $f(x)=2^x$, $[-1;\ 5]$; 2) $f(x)=x^2-4x+6$, $[-3;\ 10]$;

3) $f(x)=x+\dfrac{1}{x}$, $[0{,}01;\ 100]$; 4) $f(x)=\sqrt{5-4x}$, $[-1;\ 1]$;

5) $f(x)=\cos x$, $\left[-\dfrac{\pi}{2};\ \pi\right]$; 6) $f(x)=\left|x^2-3x+2\right|$, $[-10;\ 10]$;

7) $f(x)=\sin x$, $\left[\dfrac{\pi}{2};\ \pi\right]$; 8) $f(x)=\left|x^2+3x+2\right|$, $[-15;\ 10]$.

149. Funksiyani tekshiring va grafigini yasang:

1) $y=3x-x^3$; 2) $y=1+x^2-\dfrac{x^4}{2}$; 3) $y=(x+1)(x-2)^2$;

4) $y=x+\dfrac{1}{x}$; 5) $y=\sqrt{16-x^2}$; 6) $y=\sqrt{x^2-9}$;

7) $y=x^2-5|x|+6$; 8) $y=\dfrac{1}{4}x^4-\dfrac{1}{2}x^2$.

II BOB. INTEGRAL VA UNING TATBIQLARI

37–39 BOSHLANG‘ICH FUNKSIYA VA ANIQMAS INTEGRAL TUSHUNCHALARI

Agar nuqta harakat boshlanganidan boshlab t vaqt mobaynida $s(t)$ masofani o‘tgan bo‘lsa, uning oniy tezligi $s(t)$ funksiyaning hosilasiga teng ekanini bilasiz: $v(t)=s'(t)$. Amaliyotda *teskari masala:* nuqtaning berilgan harakat tezligi $v(t)$ bo‘yicha uning bosib o‘tgan yo‘li $s(t)$ ni topish masalasi ham uchraydi. Shunday $s(t)$ funksiyani topish kerakki, uning hosilasi $v(t)$ bo‘lsin. Agar $s'(t)=v(t)$ bo‘lsa, $s(t)$ funksiya $v(t)$ funksiyaning *boshlang‘ich funksiyasi* deyiladi. Umuman, shunday ta’rif kiritish mumkin:

> Agar $(a;\ b)$ ga tegishli ixtiyoriy x uchun $F'(x)=f(x)$ bo‘lsa, $F(x)$ funksiya $(a;\ b)$ oraliqda $f(x)$ ning *boshlang‘ich funksiyasi* deyiladi.

1-misol. a – berilgan biror son va $v(t)=at$ bo‘lsa, $s(t)=\frac{1}{2}at^2$ funksiya $v(t)$ funksiyaning boshlang‘ichidir, chunki $s'(t)=(\frac{at^2}{2})'=at=v(t)$.

2-misol. $f(x)=x^2,\ x\in(-\infty;\ \infty)$, bo‘lsa, $F(x)=\frac{1}{3}x^3$ funksiya $f(x)$ ning $(-\infty;\ \infty)$ dagi boshlang‘ich funksiyasi bo‘ladi, chunki

$$F'(x)=(\frac{1}{3}x^3)'=\frac{1}{3}\cdot 3x^2=x^2=f(x).$$

3-misol. $f(x)=\frac{1}{\cos^2 x}$, bunda $x\neq\frac{\pi}{2}+k\pi,\ k\in Z$, funksiya uchun $F(x)=\mathrm{tg}x$ boshlang‘ich funksiya bo‘ladi, chunki $(\mathrm{tg}x)'=\frac{1}{\cos^2 x}$.

4-misol. $f(x)=\frac{1}{x}, x>0$, bo‘lsa, $F(x)=\ln x$ funksiya $\frac{1}{x}$ ning boshlang‘ich funksiyasi bo'ladi, chunki $F'(x)=(\ln x)' = \frac{1}{x}$.

1-masala. $F_1(x)=\frac{x^4}{4}$, $F_2(x)=\frac{x^4}{4}+17$, $F_3(x)=\frac{x^4}{4}-25$ funksiyalar ayni bir $f(x)=x^3$ funksiyaning boshlang'ich funksiyalari ekanini isbotlang.

△ Hosilalar jadvaliga muvofiq yoza olamiz:

1) $F_1'(x)=(\frac{x^4}{4})'=4\cdot\frac{x^3}{4}=x^3=f(x)$;

2) $F_2'(x)=(\frac{x^4}{4}+17)'=(\frac{x^4}{4})'+(17)'=4\cdot\frac{x^3}{4}+0+x^3=x^3=f(x)$;

3) $F_3'(x)=(\frac{x^4}{4}-25)'=(\frac{x^4}{4})'-(25)'=4\cdot\frac{x^3}{4}-0=x^3=f(x)$.

Bu masaladan shunday xulosaga kelish mumkin: ixtiyoriy $F(x)=\frac{x^4}{4}+C$ funksiya (C – biror o'zgarmas son) ham $f(x)=x^3$ uchun boshlang'ich funksiya bo'la oladi. Chindan ham, $F'(x)=(\frac{x^4}{4}+C)'=(\frac{x^4}{4})'+C'=4\cdot\frac{x^3}{4}+0=x^3=f(x)$. ▲

Bu masaladan yana shunday xulosaga kelish mumkin: berilgan $f(x)$ funksiya uchun uning boshlang'ich funksiyasi bir qiymatli aniqlanmaydi.

Agar $F(x)$ funksiya $f(x)$ ning biror oraliqlari boshlang'ich funksiyasi bo'lsa, $f(x)$ funksiyaning barcha boshlang'ichlari $F(x)+C$ (C – ixtiyoriy o'zgarmas son) ko'rinishida yoziladi.

$F(x)+C$ ko'rinishidagi barcha funksiyalar to'plami $f(x)$ ning *aniqmas integrali* deyiladi va $\int f(x)dx$ kabi belgilanadi.

Demak, $\int f(x)dx=F(x)+C$.

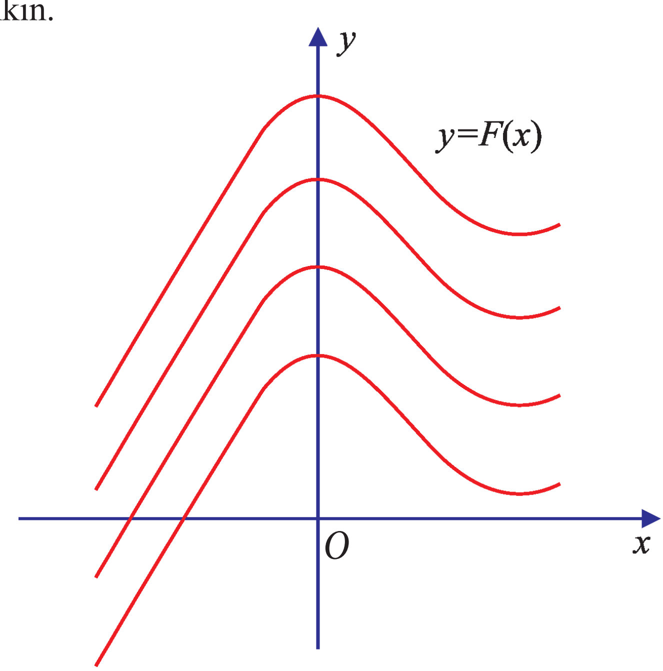
$\int$ – integral belgisi, $f(x)$ – integral ostidagi funksiya, $f(x)dx$ esa integral ostidagi ifoda deyiladi.

5-misol. $\int a^x dx=\frac{a^x}{\ln a}+C$, chunki hosilalar jadvaliga ko'ra,

$$(\frac{a^x}{\ln a}+C)'=(a^x)'\cdot\frac{1}{\ln a}+C'=a^x\cdot\ln a\cdot\frac{1}{\ln a}+0=a^x.$$

6-misol. $\int x^k dx = \frac{x^{k+1}}{k+1} + C,\ k \neq -1,$ chunki $(\frac{x^{k+1}}{k+1} + C)' = \frac{1}{k+1} \cdot (x^{k+1})' + C' =$
$= \frac{k+1}{k+1} \cdot x^k + 0 = x^k$. $k = -1$ bo‘lsa, $x > 0$ da 4-misolga ko‘ra, $\int \frac{dx}{x} = \ln x + C.$

$y = F(x) + C$ funksiyaning grafigi $y = F(x)$ funksiya grafigini Oy o‘q bo‘ylab siljitishdan hosil qilinadi (1-rasm). O‘zgarmas son C ni tanlash hisobiga boshlang‘ich funksiya grafigining berilgan nuqta orqali o‘tishiga erishish mumkin.



1-rasm.

2-masala. $f(x)=x^2$ funksiyaning grafigi $A(3;\ 10)$ nuqtadan o‘tadigan boshlang‘ich funksiyasini toping.

△ $f(x) = x^2$ funksiyaning barcha boshlang‘ich funksiyalari $F(x) = \frac{x^3}{3} + C$ ko‘rinishda bo‘ladi, chunki $F'(x) = (\frac{x^3}{3} + C)' = \frac{1}{3} \cdot 3x^2 + C' = x^2 + 0 = x^2$.

O‘zgarmas son C ni $F(x) = \frac{x^3}{3} + C$ funksiyaning grafigi (3; 10) nuqtadan o‘tadigan qilib tanlaymiz: $x=3$ da $F(3)=10$ bo‘lishi kerak. Bundan

$10=\frac{3^3}{3}+C$, $C=1$. Demak, izlanayotgan boshlang‘ich funksiya $F(x)=\frac{x^3}{3}+1$ bo‘ladi. *Javob:* $\frac{x^3}{3}+1$. ▲

3-masala. $f(x)=\sqrt[3]{x}$ funksiyaning grafigi $A(8;15)$ nuqtadan o‘tadigan boshlang‘ich funksiyasini toping.

△ $f(x)=\sqrt[3]{x}$ ning barcha boshlang‘ich funksiyalari $F(x)=\frac{3}{4}\cdot x^{\frac{4}{3}}+C$ ko‘rinishida bo‘ladi, chunki

$$F'(x)=\left(\frac{3}{4}\cdot x^{\frac{4}{3}}+C\right)'=\frac{3}{4}(x^{\frac{4}{3}})'+C'=\frac{3}{4}\cdot\frac{4}{3}\cdot x^{\frac{1}{3}}+C'=x^{\frac{1}{3}}+0=\sqrt[3]{x}.$$

O‘zgarmas son C ni shunday tanlaymizki, $F(x)=\frac{3}{4}x^{\frac{4}{3}}+C$ funksiyaning grafigi $A(8, 15)$ nuqtadan o‘tsin, ya’ni $F(8)=15$ tenglik bajarilsin. $x^{\frac{4}{3}}=x\sqrt[3]{x}$ ekanidan $15=\frac{3}{4}\cdot 8\cdot\sqrt[3]{8}+C$, bundan $C=3$. Demak, izlanayotgan boshlang‘ich funksiya $F(x)=\frac{3}{4}x\sqrt[3]{x}+3$ bo‘ladi. *Javob:* $\frac{3}{4}x\sqrt[3]{x}+3$. ▲

4*-masala. $\int\frac{dx}{x}=\ln|x|+C$ ekanini ko‘rsating.

△ $x>0$ da $\int\frac{dx}{x}=\ln x+C$, chunki $(\ln x+C)'=\frac{1}{x}+0=\frac{1}{x}$;

$x<0$ da $\int\frac{dx}{x}=\ln(-x)+C$, chunki $(\ln(-x)+C)'=\frac{(-1)}{(-x)}+0=\frac{1}{x}$. ▲

? Savol va topshiriqlar

1. Boshlang‘ich funksiya nima? Misollar keltiring.
2. Berilgan $f(x)$ funksiya uchun boshlang‘ich funksiya bir qiymatli topiladimi? Nima uchun?
3. Boshlang‘ich funksiya $F(x)$ ning grafigini berilgan nuqtadan o‘tishiga qanday qilib erishish mumkin? Misolda tushuntiring.

Mashqlar

1. Haqiqiy sonlar to‘plami $R=(-\infty;\ \infty)$ da $f(x)$ funksiya uchun $F(x)$ funksiyaning boshlang‘ich funksiya ekanini isbotlang:

1) $F(x)=x^2-\sin 2x+2018$, $\quad f(x)=2x-2\cos 2x$;

2) $F(x)=-\cos\frac{x}{2}-x^3+28$, $\quad f(x)=\frac{1}{2}\sin\frac{x}{2}-3x^2$;

3) $F(x)=2x^4+\cos^2 x+3x$, $\quad f(x)=8x^3-\sin 2x+3$;

4) $F(x)=3x^5+\sin^2 x-7x$, $\quad f(x)=15x^4+\sin 2x-7$.

Quyidagi funksiyalarning barcha boshlang‘ich funksiyalarini, hosilalar jadvalidan foydalanib toping (**2 – 6**):

2. 1) $f(x)=x^2\cdot\sqrt{x}$; 2) $f(x)=6x^5$; 3) $f(x)=x^{10}$; 4) $f(x)=\frac{2}{3}\cdot\sqrt{x}$;

5) $f(x)=\sin x$; | 6) $f(x)=\cos x$; | 7) $f(x)=\sin 2x$; | 8) $f(x)=\cos 2x$;

3. 1) $f(x)=4^x$; 2) $f(x)=\pi^x$; 3) $f(x)=e^x$; 4) $f(x)=a^x$;

5) $f(x)=a^{2x}$; 6) $f(x)=e^{\pi x}$; 7) $f(x)=10^{3x}$; 8) $f(x)=e^{2x+3}$.

4. 1) $f(x)=\frac{1}{2x+3}$; 2) $f(x)=\frac{1}{4x-5}$; 3) $f(x)=\frac{1}{2x+7}$;

4) $f(x)=\frac{1}{ax}$; 5) $f(x)=\frac{1}{ax+b}$; 6) $f(x)=\frac{a}{ax-b}$.

5. 1) $f(x)=\sin 3x$; 2) $f(x)=\sin(2x+5)$; 3) $f(x)=\sin(4x+\pi)$;

4) $f(x)=\cos 5x$; 5) $f(x)=\cos(3x-2)$; 6) $f(x)=\cos(2x+\frac{\pi}{2})$.

6. 1) $f(x)=\frac{1}{x^2}$; | 2) $f(x)=\frac{1}{x^5}$; | 3) $f(x)=(3x+2)^2$; | 4) $f(x)=(2x-1)^3$.

7. Berilgan $f(x)$ funksiya uchun uning ko‘rsatilgan A nuqtadan o‘tuvchi boshlang‘ich funksiyasini toping:

1) $f(x)=2x+3$, $\quad A(1;\ 5)$; $\quad$ 2) $f(x)=-x^2+2x+5$, $\quad A(0;\ 2)$;

3) $f(x)=\sin x$, $\quad A(0;\ 3)$; $\quad$ 4) $f(x)=\cos x$, $\quad A(\frac{\pi}{2};\ 5)$.

Berilgan $f(x)$ funksiya uchun uning shunday boshlang‘ich funksiyasini topingki, bu boshlang‘ich funksiyaning grafigi y to‘g‘ri chiziq bilan faqat bitta umumiy nuqtaga ega bo‘lsin (**8 – 9**):

8. 1) $f(x)=4x+8$, $y=3$; 2) $f(x)=3-x$, $y=7$,

3) $f(x)=4{,}5x+9$, $y=6{,}8$; 4) $f(x)=2x-6$, $y=1$.

9*. $f(x)=ax+b$, $y=k$.

Ko‘rsatma: $F(x)=\dfrac{ax^2}{2}+bx+C$, masala shartidan va $\dfrac{ax^2}{2}+bx+C=k$ kvadrat tenglamadan C ni toping. $C=\dfrac{2ak+b^2}{2a}=k+\dfrac{b^2}{2a}$ bo‘ladi.

10*. $f(x)$ uchun uning shunday boshlang‘ich funksiyasini topingki, bu boshlang‘ich funksiyaning grafigi ko‘rsatilgan nuqtalardan o‘tsin:

1) $f'(x)=\dfrac{16}{x^3}$, $A(1;\ 10)$ va $B(4;\ -2)$;

2) $f'(x)=\dfrac{54}{x^4}$, $A(-1;\ 4)$ va $B(3;\ 4)$;

3) $f'(x)=6x$, $A(1;6)$ va $B(3;30)$;

4) $f'(x)=20x^3$; $A(1;9)$ va $B(-1;7)$.

Ko‘rsatma: Berilgan $f'(x)$ bo‘yicha $f(x)+C_1$ topiladi. So‘ngra $f(x)+C_1$ uchun boshlang‘ich funksiyasi $F(x)=\int f(x)dx+C_1x+C_2$ topiladi. Berilgan nuqtalar koordinatalarini oxirgi tenglikka qo‘yib, C_1 va C_2 sonlarni topish uchun chiziqli tenglamalar sistemasiga kelinadi.

11*. Berilgan $f(x)$ funksiya uchun uning shunday boshlang‘ich funksiyasini topingki, bu boshlang‘ich funksiyaning grafigi bilan $f(x)$ hosilasining grafigi abssissasi ko‘rsatilgan nuqtada kesishsin:

1) $f(x)=(3x-2)^{\frac{1}{3}}$, $x_0=1$; 2) $f(x)=(4x+5)^{\frac{1}{4}}$, $x_0=-1$;

3) $f(x)=(7x-5)^{\frac{1}{7}}$, $x_0=1$; 4) $f(x)=(kx+b)^{\frac{1}{k}}$, $x_0=\dfrac{1-b}{k}$.

12. Berilgan $f(x)$ funksiya uchun ko‘rsatilgan nuqtadan o‘tuvchi boshlang‘ich funksiyani toping:

1) $f(x)=\frac{5}{x-2}$, $A(3;\ 7)$; 2) $f(x)=\frac{3}{x+1}$, $A(0;\ 1)$;

3) $f(x)=\cos x$, $A(\frac{\pi}{2};\ 8)$; 4) $f(x)=\sin x$, $A(\pi;\ 10)$.

13. $F(x)$ funksiya son o‘qida $f(x)$ funksiyaning boshlang‘ich funksiyasi ekanini ko‘rsating:

1) $F(x)=k\cdot e^{\frac{x}{k}}$, $f(x)=e^{\frac{x}{k}}$, $k\neq 0$;

2) $F(x)=C+\sin kx$, $f(x)=k\cdot\cos kx$, C – o‘zgarmas son;

3) $F(x)=C+\cos kx$, $f(x)=-k\cdot\sin kx$, C – o‘zgarmas son;

4) $F(x)=\frac{1}{5}\sin(5x+12)$, $f(x)=\cos(5x+12)$.

14. $f(x)$ funksiyaning ko‘rsatilgan nuqtadan o‘tuvchi boshlang‘ich funksiyasini toping:

1) $f(x)=\sin 3x$, $A(\frac{\pi}{3};\ \frac{1}{3})$; 2) $f(x)=\cos 5x$, $A(\frac{\pi}{2};\ \frac{4}{5})$;

3) $f(x)=\cos\frac{x}{2}$, $A(\frac{\pi}{3};\ 1)$; 4) $f(x)=\sin\frac{x}{3}$, $A(\pi;\ \frac{9}{2})$.

15. $f(x)$ funksiya uchun uning berilgan tenglamalar sistemasining yechimi $(x_0;\ y_0)$ nuqtadan o‘tuvchi boshlang‘ich funksiyasini toping:

1) $f(x)=3x^2$; $\begin{cases}\log_2 x+\log_2 y=3,\\ 4\log_2 x-\log_2 y=2;\end{cases}$

2) $f(x)=4x^3$; $\begin{cases}5^x+5^y=30,\\ 3\cdot 5^x-2\cdot 5^y=15;\end{cases}$

3) $f(x)=\cos x$; $\begin{cases}x+y=\frac{3\pi}{2},\\ 4x-3y=-\pi;\end{cases}$

4) $f(x)=\frac{1}{5x+e}$; $\begin{cases}2^x+3^y=4,\\ 3\cdot 2^x-3^y=0.\end{cases}$

40–43 INTEGRALLAR JADVALI. INTEGRALLASHNING ENG SODDA QOIDALARI

Integrallar jadvalini hosilalar jadvali yordamida tuzish mumkin.

№	**Funksiya** $f(x)$	**Boshlang'ich funksiya** $F(x)+C$
1	$x^p, \quad p \neq -1$	$\frac{x^{p+1}}{p+1}+C$
2	$1/x$	$\ln\lvert x\rvert+C$
3	e^x	e^x+C
4	$\sin x$	$-\cos x+C$
5	$\cos x$	$\sin x+C$
6	$(kx+b)^p, \quad p \neq -1, \quad k \neq 0$	$\frac{(kx+b)^{p+1}}{k(p+1)}+C$
7	$\frac{1}{kx+b}, \quad k \neq 0$	$\frac{1}{k}\ln\lvert kx+b\rvert+C$
8	$e^{kx+b}, \; k \neq 0$	$\frac{1}{k}e^{kx+b}+C$
9	$\sin(kx+b), \; k \neq 0$	$-\frac{1}{k}\cos(kx+b)+C$
10	$\cos(kx+b), \quad k \neq 0$	$\frac{1}{k}\sin(kx+b)+C$
11	$1/\cos^2 x$	$\mathrm{tg}x+C$
12	$1/\sin^2 x$	$-\mathrm{ctg}x+C$
13	a^x	$\frac{a^x}{\ln a}+C$
14	$\frac{1}{1+x^2}$	$\mathrm{arctg}x+C$
15	$f(kx+b)$	$\frac{1}{k}F(kx+b)+C$
16	$f(g(x))g'(x)$	$F(g(x))+C$

Biror X oraliqda aniqlangan $F(x)$ funksiya $f(x)$ funksiyaning boshlang‘ich funksiyasi bo‘lishi uchun ikkala $F(x)$ va $f(x)$ funksiya ham ayni shu X oraliqda aniqlangan bo‘lishi kerak.

Masalan, $\frac{1}{5x-8}$ funksiyaning $5x-8>0$, ya’ni $x>1{,}6$ oraliqdagi integrali, jadvalga muvofiq, $\frac{1}{5}\ln(5x-8)+C$ ga teng.

Differensiyalash qoidalaridan foydalanib, *integrallash qoidalarini* bayon qilish mumkin.

$F(x)$ va $G(x)$ funksiyalar biror oraliqda, mos ravishda, $f(x)$ va $g(x)$ funksiyalarning boshlang‘ich funksiyalari bo‘lsin. Ushbu qoidalar o‘rinlidir:

1-qoida: $a\cdot F(x)$ funksiya $a\cdot f(x)$ funksiyaning boshlang‘ich funksiyasi bo‘ladi, ya’ni

$$\int a\cdot f(x)dx = a\cdot F(x)+C.$$

2-qoida: $F(x)\pm G(x)$ funksiya $f(x)\pm g(x)$ funksiyaning boshlang‘ich funksiyasi bo‘ladi, ya’ni:

$$\int(f(x)\pm g(x))dx=\int f(x)dx\pm\int g(x)dx=F(x)\pm G(x)+C.$$

1-misol. $f(x)=5\sin(3x+2)$ funksiyaning integralini toping.

△ Bu funksiyaning integralini 1-qoida va integrallar jadvalining 9-bandiga muvofiq topamiz:

$$\int f(x)dx=\int 5\sin(3x+2)dx=5\int\sin(3x+2)dx=$$

$$=5\cdot\left(-\frac{1}{3}\cos(3x+2)\right)+C=-\frac{5}{3}\cos(3x+2)+C,$$

chunki integrallar jadvaliga ko‘ra

$$\int\sin(3x+2)dx=-\frac{1}{3}\cos(3x+2)+C.$$

Javob: $-\frac{5}{3}\cos(3x+2)+C.$ ▲

2-misol. $f(x)=8x^7+2\cos 2x$ funksiyaning integralini toping.

△ Bu funksiyaning integralini 1- va 2-qoidalar hamda integrallar jadvalining 1- va 10-bandiga muvofiq topamiz:

$$\int f(x)dx=\int(8x^7+2\cos 2x)dx=8\int x^7dx+2\int\cos 2xdx$$

$$=8\cdot\frac{1}{8}x^8+2\cdot\frac{1}{2}\sin 2x+C=x^8+\sin 2x+C.$$

Javob: $x^8+\sin 2x+C.$ ▲

3-misol. $\int\frac{xdx}{x^2+8}$ integralni hisoblang.

△ Bu kabi misollarni yechishda *o'zgaruvchini almashtirish* qulay.

Agar $x^2+8=u$ deyilsa, $du=2xdx$, $xdx=\frac{1}{2}du$ bo'ladi. U holda

$$\int\frac{xdx}{x^2+8}=\frac{1}{2}\int\frac{du}{u}=\frac{1}{2}\ln|u|+C=\frac{1}{2}\ln(x^2+8)+C.$$

Tekshirish: Topilgan boshlang'ich funksiyadan hosila olinsa, integral ostidagi funksiya $\frac{x}{x^2+8}$ hosil bo'lishi kerak. Chindan ham,

$$\left(\frac{1}{2}\ln(x^2+8)+C\right)'=\frac{1}{2}(\ln(x^2+8))'+C'=\frac{1}{2}\cdot\frac{1}{x^2+8}\cdot(x^2+8)'=\frac{1}{2}\cdot\frac{2x}{x^2+8}=\frac{x}{x^2+8}.$$

Javob: $\frac{1}{2}\cdot\ln(x^2+8)+C.$ ▲

4-misol. $\int e^{\sin x}\cos xdx$ integralni hisoblang.

△ $\sin x=t$ almashtirish bajaramiz. U holda $dt=\cos xdx$ va berilgan integral $\int e^tdt$ ko'rinishni oladi. Integrallar jadvallarining 3-bandiga ko'ra $\int e^tdt=e^t+C$ bo'ladi. Demak, $\int e^{\sin x}\cos xdx=e^{\sin x}+C.$

Tekshirish. $(e^{\sin x}+C)'=(e^{\sin x})'+C'=e^{\sin x}(\sin x)'+0=e^{\sin x}\cos x$ – berilgan integral ostidagi funksiyani hosil qildik.

Javob: $e^{\sin x}+C.$ ▲

5-misol. $\int \sin 5x \cdot \cos 3x dx$ integralni hisoblang.

△ Bunda $2\sin 5x \cdot \cos 3x = \sin 8x + \sin 2x$ ayniyat yordam beradi. U holda

$$\int \sin 5x \cos 3x dx = \frac{1}{2}\int \sin 8x dx + \frac{1}{2}\int \sin 2x dx =$$

$$= \frac{1}{16}(-\cos 8x) + \frac{1}{4}(-\cos 2x) + C = -\frac{\cos 8x}{16} - \frac{\cos 2x}{4} + C.$$

Javob: $-\dfrac{\cos 8x}{16} - \dfrac{\cos 2x}{4} + C.$ ▲

6*-misol. $\int \cos mx \cos nx dx$ integralni hisoblang.

△ $\cos mx \cos nx = \frac{1}{2}(\cos(m+n)x + \cos(m-n)x)$ ayniyatgavaintegrallash jadvalining 10-bandiga muvofiq:

$$\int \cos mx \cos nx dx = \frac{1}{2}\int \cos(m+n)x dx + \frac{1}{2}\int \cos(m-n)x dx =$$

$$= \frac{1}{2}\cdot\frac{\sin(m+n)x}{m+n} + \frac{1}{2}\cdot\frac{\sin(m-n)x}{m-n} + C.$$

Javob: $\dfrac{1}{2}\cdot\dfrac{\sin(m+n)x}{m+n} + \dfrac{1}{2}\cdot\dfrac{\sin(m-n)x}{m-n} + C.$ ▲

7-misol. $\int \dfrac{dx}{x^2 - 5x + 6}$ integralni hisoblang.

△ Integral ostidagi funksiya uchun quyidagi tengliklar o‘rinlidir:

$$\frac{1}{x^2 - 5x + 6} = \frac{1}{(x-2)(x-3)} = \frac{(x-2)-(x-3)}{(x-2)(x-3)} = \frac{1}{x-3} - \frac{1}{x-2}.$$

Bundan

$$\int \frac{dx}{x^2 - 5x + 6} = \int \left(\frac{1}{x-3} - \frac{1}{x-2}\right)dx = \int \frac{dx}{x-3} - \int \frac{dx}{x-2} =$$

$$= \ln|x-3| - \ln|x-2| + C = \ln\left|\frac{x-3}{x-2}\right| + C,$$

Javob: $\ln\left|\dfrac{x-3}{x-2}\right| + C.$ ▲

8-misol. $\int \frac{dx}{1+\cos x}$ integralni hisoblang.

△ Bu integralni hisoblash uchun $1+\cos x=2\cos^2\frac{x}{2}$ va $\int \frac{dx}{\cos^2 x}=\mathrm{tg}x+C$ ekanidan foydalanamiz. U holda

$$\int \frac{dx}{1+\cos x}=\int \frac{dx}{2\cos^2\frac{x}{2}}=\frac{1}{2}\cdot 2\cdot \mathrm{tg}\frac{x}{2}+C=\mathrm{tg}\frac{x}{2}+C.$$

Tekshirish: $(\mathrm{tg}\frac{x}{2}+C)'=(\mathrm{tg}\frac{x}{2})'+C'=\frac{1}{\cos^2\frac{x}{2}}\cdot(\frac{x}{2})'+0=\frac{1}{2}\cdot\frac{1}{\cos^2\frac{x}{2}}=\frac{1}{1+\cos x}$ integral ostidagi funksiya hosil bo‘ldi.

Javob: $\mathrm{tg}\frac{x}{2}+C.$ ▲

9-misol. $\int \sin^2 2x dx$ integralni hisoblang.

△ Integralni hisoblash uchun $2\sin^2 2x=1-\cos 4x$ ayniyatdan foydalanamiz.

$$\int \sin^2 2x dx=\int \frac{1}{2}(1-\cos 4x)dx=\frac{1}{2}\int dx-\frac{1}{2}\int \cos 4x dx=\frac{x}{2}-\frac{1}{2}\cdot\frac{1}{4}\sin 4x+C=\frac{x}{2}-\frac{1}{8}\sin 4x+C.$$

Javob: $\frac{x}{2}-\frac{1}{8}\sin 4x+C.$ ▲

? Savol va topshiriqlar

1. Integrallar jadvalidagi o‘zingiz xohlagan 4 ta misolni tanlang va uni isbotlang.
2. Integrallashning sodda qoidalarini bayon qiling. Misollarda tushuntiring.
3. O‘zgaruvchi almashtirish usuli nima? $\int e^{\cos 2x}\sin 2x dx$ integralni hisoblashda shu usulni qo‘llang va misolni yechish jarayonini tushuntiring.

Mashqlar

Berilgan funksiyaning boshlang‘ich funksiyalaridan birini toping (**16 – 18**):

16. 1) $3x^5-4x^3$; 2) $8x^7-5x^4$; 3) $\frac{4}{x}-\frac{4}{x^2}$; 4) $\frac{5}{x^4}+\frac{3}{x^5}$;

5) $\sqrt[3]{x}+3\sqrt[3]{x}$; 6) $7\sqrt[3]{x}-5\sqrt{x}$; 7) $5x^4+4x^3-2x^2$.

17. 1) $5\cos x-3\sin x$; 2) $7\sin x+4\cos x$; 3) $2\cos x-a^x$;

4) $5e^x+2\cos x+1$; 5) $4+2\cdot e^{-x}-7\sin x$; 6) $\frac{6}{\sqrt[3]{x}}+\frac{4}{x}-e^{-x}$.

18. 1) $(x-2)^3$; 2) $(x+5)^4$; 3) $\frac{1}{\sqrt{x-5}}$ 4) $\frac{6}{\sqrt[3]{x+7}}$;

5) $4\cos(x+5)+\frac{8}{x-7}$; 6) $2\sin(x-3)-\frac{4}{x-2}$; 7) $(3x+7)^4+\frac{1}{x^5}$.

Berilgan funksiyaning barcha boshlang‘ich funksiyalarini toping (**19 – 20**):

19. 1) $\cos(5x+3)$; 2) $\sin(7x-6)$; 3) $\cos(\frac{2x}{3}+1)$;

4) $\sin(\frac{5x}{7}-2)$; 5) $e^{\frac{2x+3}{4}}$; 6) e^{3-2x};

7) $\frac{4}{\cos^2 x}$; 8) $\frac{3}{\cos^2 4x}$; 9) $\frac{5}{\sin^2 5x}$.

20. 1) $\frac{4}{x^5}-(1-2x)^3$; 2) $(3x+2)^4-\frac{1}{x^6}$; 3) $x+\frac{2}{\cos^6 x}-1$;

4) $2x-\frac{3}{\sin^2 x}+6$; 5) $(1+3x)(x-1)$; 6) $\frac{1}{2}\cdot\sqrt[3]{x^2}+2\sin(3x-1)$.

21. Berilgan $f(x)$ funksiya uchun grafigi $A(x;y)$ nuqtadan o‘tadigan boshlang‘ich funksiyani toping:

1) $f(x)=\sin 4x,\ A(\frac{\pi}{4};\ 7)$; 2) $f(x)=\cos 5x,\ A(\frac{\pi}{4};\ 4)$;

3) $f(x)=3x^2+\frac{2}{\sqrt{x+2}},\ A(-1;0)$; 4) $f(x)=4x^3-\frac{1}{2\sqrt{x-1}}\ A(2;0)$;

5) $f(x)=\cos^2 3x+\sin^2 3x+\frac{1}{4}\sin 4x,\ A(\frac{\pi}{8};\ \frac{\pi}{8});$

6) $f(x)=\mathrm{tg}x\cdot\mathrm{ctg}x-2\cos\frac{x}{2},\ A(2\pi;\ 2\pi);$

7) $f(x)=\frac{2}{\sqrt{5-2x}}+4x,\ A(2;6);$ 8) $f(x)=6x^2-\frac{1}{2\sqrt{2-x}},\ A(-2;\ 4).$

Integrallarni toping (**22 – 28**):

22. 1) $\int(x^3-\sin 2x-3)dx;$ 2) $\int(x^4+\cos 3x+4)dx;$

3) $\int(x^2-\sin\frac{x}{2}+\cos\frac{x}{2})dx;$ 4) $\int(4x^3+\cos\frac{x}{3}+\sin\frac{x}{3})dx.$

23*. 1) $\int(\frac{8}{\sin^2 x}+6\cos^2 x+2)dx;$ 2) $\int(\frac{6}{\cos^2 x}-8\sin^2 x+3)dx;$

3) $\int\sin 2x\cos 2xdx;$ 4) $\int(\sin 3x\cos x+\cos 3x\sin x)dx;$

5) $\int(\sin 2x\cdot\sin 4x+\cos 2x\cos x)dx;$ 6) $\int\cos^2 5xdx.$

24*. 1) $\int\sin 5x\cos 3xdx;$ 2) $\int\cos 2x\cos 3xdx;$ 3) $\int\sin 7x\sin 3xdx.$

25*. 1) $\int\frac{x}{x+1}dx;$ 2) $\int\frac{dx}{x^2-7x+12};$ 3) $\int\frac{(x-3)dx}{x^2-4x+3};$ 4) $\frac{(x+4)dx}{x^2-16}.$

26. 1) $\int\frac{x^5+x^3-2}{x^2+1}dx;$ 2) $\frac{x^2-1}{x^2+1}dx;$ 3) $\int\frac{dx}{1+\cos 2x};$

4) $\frac{dx}{1-\cos 2x};$ 5) $\int\frac{dx}{4(x^2-4)};$ 6) $\int(1-2\sin^2 5x)dx.$

27. 1) $\int(x^3-1)^4x^2dx;$ 2) $\int\frac{xdx}{(1+x^2)^3};$ 3) $\int\frac{\mathrm{tg}x}{\cos^3 x}dx;$

4) $\int\frac{\mathrm{ctg}x}{\sin^2 x}dx;$ 5) $\int\sin^3 xdx;$ 6) $\int\cos^3 xdx.$

28*. 1) $\int\frac{xdx}{\sqrt{x-1}};$ 2) $\int x\cdot\sqrt{x-4}dx;$ 3) $\int\frac{(x-1)dx}{\sqrt{x+1}};$

4) $\int(\mathrm{tg}^2x+\mathrm{tg}^4x)dx;$ 5) $\int(\mathrm{ctg}^2x+\mathrm{ctg}^4x)dx.$

Berilgan $f(x)$ funksiya uchun grafigi $A(x;\ y)$ nuqtadan o'tadigan boshlang'ich funksiyani toping **(29 – 30)**:

29. 1) $f(x)=\frac{3}{2}\cdot\cos\frac{x}{3}$, $A(\pi;\ 4)$;

2) $f(x)=\frac{3}{5}\cdot\sin 5x$, $A(\frac{\pi}{2};\ 3)$;

3) $f(x)=2\sin 5x+2\cos\frac{x}{2}$, $A(\frac{\pi}{3};\ 0)$;

30. 1) $f(x)=3x^2-2x+8$, $A(1;\ 9)$;

2) $f(x)=4x^3-3x^2+2x+1$, $A(-1;\ 4)$;

3) $f(x)=5x^4+3x^2+2$, $A(-2;\ 1)$.

31. Integralni toping:

1) $\int(x^2-1)(x+2)dx$; 2) $\int(x+2)(x^2-9)dx$; 3) $\int(x^2+1)(x^3-1)dx$;

4) $\int\frac{1-4x^2+\sqrt{1-2x}}{1-2x}dx$; 5) $\int\frac{9x^2-4-\sqrt{3x+2}}{3x+2}dx$;

6) $\int(e^{5-2x}-2^x)dx$; 7) $\int(e^{3x+2}+10^x)dx$.

32. Integralni hisoblang:

1) $\int\frac{dx}{x^2+6x+10}$; 2) $\int\frac{dx}{x^2-4x+5}$; 3) $\int\frac{dx}{x^2+10x+26}$.

Namuna: $I=\int\frac{dx}{x^2+4x+5}$ integralni hisoblang.

△ $I=\int\frac{dx}{x^2+4x+5}=\int\frac{dx}{1+(x+2)^2}$; $x+2=u$ deyilsa, $1+(x+2)^2=1+u^2$ $x'=u'$ va integrallar jadvalining 14–15 bandlariga ko'ra

$$I=\int\frac{du}{1+u^2}=\text{arctg}u+C=\text{arctg}(x+2)+C.$$

Tekshirish:

$$(\text{arctg}(x+2)+C)'=(\text{arctg}(x+2))'+C'=\frac{1}{1+(x+2)^2}+0=$$
$$=\frac{1}{1+(x+2)^2}=\frac{1}{x^2+4x+5}.$$

Javob: $\text{arctg}(x+2)+C$.▲

Integrallash qoidalaridan yana biri *bo'laklab integrallashdir*.

3-qoida*. Agar biror X oraliqda $f(x)$ va $g(x)$ funksiyalar uzluksiz $f'(x)$ va $g'(x)$ hosilaga ega bo'lsa, u holda

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx \qquad (1)$$

formula o'rinlidir. Bu formula *bo'laklab integrallash formulasi* deyiladi.

Bu formulaning isboti $f(x)$ va $g(x)$ funksiyalar ko'paytmasini differensiallash qoidasi $(f(x)g(x))'=f'(x)g(x)+f(x)g'(x)$ va $\int f'(x)dx=f(x)+C$ ekanidan kelib chiqadi.

Formuladan *foydalanish yo'rig'i*: 1) integral ostidagi ifoda $f(x)$ va $g'(x)$ lar ko'paytmasi ko'rinishida yozib olinadi; 2) $g'(x)$ va $g(x)f'(x)$ ifodalarning integrallarini oson (qulay) hisoblanadigan qilib olish nazarda tutiladi.

1-misol. $\int x\cdot e^x dx$ integralni hisoblang.

△ Bu yerda $f(x)=x$, $g'(x)=e^x$ deb olish qulay, chunki

$g(x)=\int g'(x)dx=\int e^x dx=e^x$, $\quad f'(x)=1$. U holda (1) ga asosan,

$\int xe^x dx = x\cdot e^x - \int e^x dx = x\cdot e^x - e^x + C.$

Demak, $\int xe^x dx = e^x\cdot(x-1)+C.$

Javob: $e^x(x-1)+C$. ▲

2-misol. $\int \ln x dx$ integralni hisoblang.

△ Integral ostidagi $\ln x$ funksiyani $f(x)=\ln x$ va $g'(x)=1$ larning ko'paytmasi deb hisoblaymiz: $\ln x=f(x)\cdot g'(x)$.

U holda $f'(x)=\dfrac{1}{x}$, $g(x)=\int 1\cdot dx = x+C.$

(1) formulaga ko'ra,

$$\int \ln x dx = x\ln x - \int x\cdot\frac{1}{x}dx = x\ln x - \int dx = x\ln x - x + C =$$

$$= x(\ln x - 1) + C = x\cdot(\ln x - \ln e) + C = x\cdot\ln\frac{x}{e} + C.$$

Demak, $\int \ln x dx = x \cdot \ln \frac{x}{e} + C.$

Tekshirish:

$$(x\ln\frac{x}{e}+C)' = (x\ln\frac{x}{e})' + C' = x'\cdot\ln\frac{x}{e} + x(\ln\frac{x}{e})' + 0 =$$

$$= \ln\frac{x}{e} + x\cdot\frac{e}{x}\cdot\frac{1}{e} = \ln x - \ln e + 1 = \ln x - 1 + 1 = \ln x.$$

Javob: $x \cdot \ln \frac{x}{e} + C.$ ▲

3-misol. $\int x \cos x dx$ integralni hisoblang.

△ Integralni hisoblash uchun $f(x)=x,\ g'(x)=\cos x$ deyish qulay. U holda $f'(x)=1,\ \ g(x)=\int \cos x dx = \sin x$ (bu yerda boshlang'ich funksiyalardan bittasini oldik, shuning uchun o'zgarmas son C ni yozmadik). Bo'laklab integrallash formulasiga muvofiq,

$$\int x\cos x dx = x\cdot\sin x - \int \sin x dx = x\sin x + \cos x + C.$$

Javob: $x \sin x + \cos x + C.$ ▲

Integrallarni hisoblang (**33 – 35**):

33*. 1) $\int x\sin x dx$; 2) $\int x^2 \cos x dx$; 3) $\int x\ln x dx$; 4) $\int 2x\ln x dx$.

34*. 1) $\int x\cos 2x dx$; 2) $\int x\sin 3x dx$; 3) $\int x\sin\frac{x}{3} dx$; 4) $\int x\cos\frac{x}{4} dx$.

35*. 1) $\int 2^x \cdot x dx$; 2) $\int 3^x \cdot x dx$; 3) $\int 5^x \cdot x dx$; 4) $\int \mathrm{tg}^2 nx dx$;

5) $\int \frac{2^{x+1}-5^{x-1}}{10^x} dx$; 6) $\int \frac{e^{3x}+1}{e^x+1} dx$; 7) $\int (3^x+4^x)^2 dx$;

8) $\int (e^{-x}+e^{-2x}) dx$; 9) $\int \frac{e^{4x}-1}{e^{2x}-1} dx$; 10) $\int \frac{e^x dx}{\pi+e^x}$;

11) $\int x\cdot e^{-x^2} dx$; 12) $\int \frac{dx}{e^x+e^{-x}}$; 13) $\int \frac{\ln^2 x}{x} dx$.

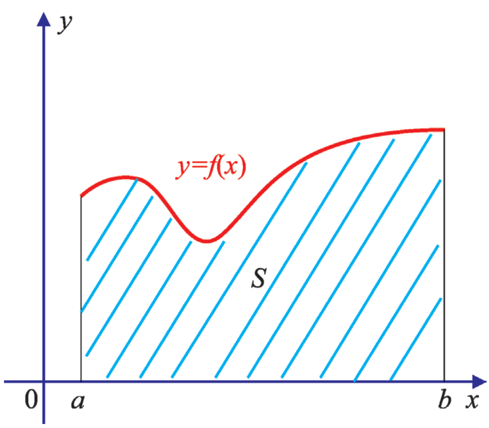
44–46

ANIQ INTEGRAL. NYUTON–LEYBNIS FORMULASI

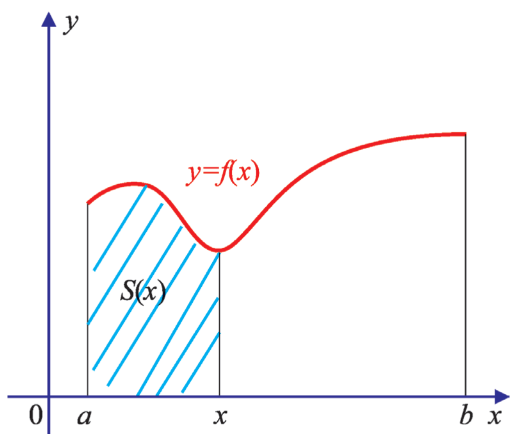
2-rasmda tasvirlangan shakl *egri chiziqli trapetsiya* deyiladi. Bu shakl yuqoridan $y = f(x)$ funksiyaning grafigi bilan, quyidan $[a, b]$ kesma bilan, yon tomonlardan esa $x=a$, $x=b$ to‘g‘ri chiziqlarning kesmalari bilan chegaralangan. $[a; b]$ kesma egri chiziqli trepetsiyaning *asosi* deyiladi.

Egri chiziqli trapetsiyaning yuzini qaysi formulaga ko‘ra hisoblaymiz, degan savol tug‘iladi.

Bu yuzni S deb belgilaylik. S yuzni $f(x)$ funksiyaning boshlang‘ich funksiyasi yordamida hisoblash mumkin ekan. Shunga oid mulohazalarni keltiramiz.



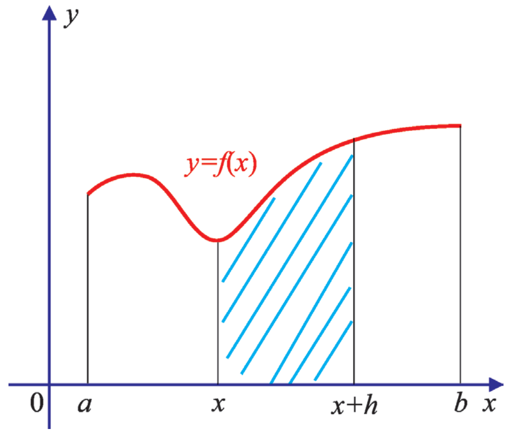
2-rasm.



3-rasm.

$[a; x]$ asosli egri chiziqli trapetsiyaning yuzini $S(x)$ deb belgilaymiz (3-rasm), bunda x shu $[a; b]$ kesmadagi istalgan nuqta: $x=a$ bo‘lganda $[a; x]$ kesma nuqtaga aylanadi, shuning uchun $S(a)=0$; $x=b$ da $S(b) = S$.

$S(x)$ ni $f(x)$ funksiyaning boshlang‘ich funksiyasi bo‘lishini, ya’ni $S'(x) = f(x)$ ekanini ko‘rsatamiz.



4-rasm.

△ $S(x+h) - S(x)$ ayirmani ko‘raylik, bunda $h>0$ ($h<0$ hol ham xuddi shunday ko‘riladi). Bu ayirma asosi $[x;\ x+h]$ bo‘lgan egri chiziqli trapetsiyaning yuziga teng (4-rasm). Agar h son kichik bo‘lsa, u holda bu yuz taqriban $f(x)\cdot h$ ga teng, ya’ni $S(x+h)-S(x)\approx f(x)\cdot h$. Demak,

$$\frac{S(x+h)-S(x)}{h}\approx f(x).$$

Bu taqribiy tenglikning chap qismi $h\to 0$ da hosilaning ta‘rifiga ko‘ra $S'(x)$ ga intiladi. Shuning uchun $h\to 0$ da $S'(x)=f(x)$ tenglik hosil bo‘ladi. Demak, $S(x)$ yuz $f(x)$ funksiya uchun boshlang‘ich funksiyasi ekan.▲

Boshlang‘ich funksiya $S(x)$ dan ixtiyoriy boshqa boshlang‘ich $F(x)$ funksiya o‘zgarmas songa farq qiladi, ya’ni

$$F(x)=S(x)+C.$$

Bu tenglikdan $x=a$ da $F(a) = S(a)+C$ va $S(a) = 0$ bo‘lgani uchun $C=F(a)$. U holda (1) tenglikni quyidagicha yozish mumkin:

$S(x)=F(x)-F(a)$. Bundan $x=b$ da $S(b)=F(b)-F(a)$ ekanini topamiz.

Demak, *egri chiziqli trapetsiyaning yuzini* (2-rasm) quyidagi formula yordamida hisoblash mumkin:

$$S=F(b)-F(a), \qquad (2)$$

bunda $F(x)$ – berilgan $f(x)$ funksiyaning istalgan boshlang‘ich funksiyasi.

Shunday qilib, egri chiziqli trapetsiyaning yuzini hisoblash $f(x)$ funksiyaning $F(x)$ boshlang‘ich funksiyasini topishga, ya’ni $f(x)$ funksiyani integrallashga keltiriladi.

$F(b) - F(a)$ ayirma $f(x)$ funksiyaning $[a; b]$ kesmadagi *aniq integrali* deyiladi va bunday belgilanadi: $\int_a^b f(x)dx$

(o‘qilishi: “*a* dan *b* gacha integral ef iks de iks“), ya’ni

$$\int_a^b f(x)dx = F(b) - F(a). \qquad (3)$$

(3) formula *Nyuton–Leybnis formulasi* deb ataladi.

(2) va (3) formulaga muvofiq:

$$S = \int_a^b f(x)dx. \qquad (4)$$

Integralni hisoblashda, odatda, quyidagicha belgilash kiritiladi:

$F(b)-F(a)=F(x)\Big|_a^b$. U holda (3) formulani shunday yozish mumkin:

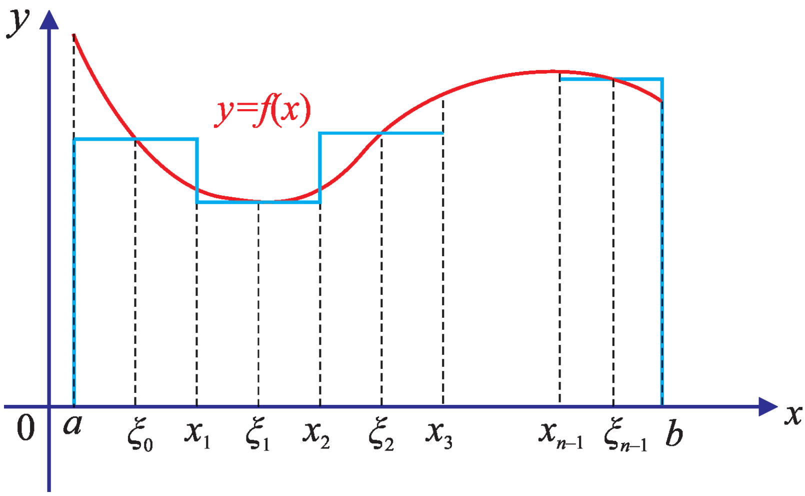
$$S = \int_a^b f(x)dx = F(x)\Big|_a^b. \qquad (5)$$

Shu o‘rinda qisqacha *tarixiy ma’lumotni* aytish joiz.

Egri chiziqlar bilan chegaralangan shakl yuzini hisoblash masalasi aniq integral tushunchasiga olib kelgan. Uzluksiz $f(x)$ funksiya aniqlangan $[a; b]$ kesma $a=x_0, x_1.., x_{n-1}, ... x_n= b$ nuqtalar yordamida o‘zaro teng $[x_k; x_{k+1}]$ (k=0, 1, ... n–1) kesmalarga bo‘lingan va har bir $[x_k; x_{k+1}]$ kesmadan ixtiyoriy ξ_k nuqta olingan. $[x_k; x_{k+1}]$ kesma uzunligi $\Delta x_k=x_{k+1}-x_k$ ni berilgan $f(x)$ funksiyaning ξ_k nuqtadagi qiymati $f(\xi_k)$ ga ko‘paytirilgan va ushbu

$$S_n=f(\xi_0)\Delta x_0+f(\xi_1)\Delta x_1+\ldots+f(\xi_{k-1})\Delta_{n-1} \qquad (6)$$

yig‘indisi tuzilgan, bunda har bir qo‘shiluvchi asosi Δx_k va balandligi $f(\xi_k)$ bo‘lgan to‘g‘ri to‘rtburchakning yuzidir. S_n yig‘indi egri chiziqli trapetsiyaning yuzi S ga taqriban teng: $S_n \approx S$ (5-rasm).



5-rasm.

(6) yig‘indi $f(x)$ funksiyaning $[a; b]$ kesmadagi *integral yig‘indisi* deyiladi. Agar n cheksizlikka intilganda ($n \to \infty$), Δx_k nolga intilsa ($\Delta x_k \to 0$), u holda S_n integral yig‘indi biror songa intiladi. Ayni shu son $f(x)$ funksiyaning $[a; b]$ kesmadagi *integrali* deb ataladi.

1-misol. 6-rasmda tasvirlangan egri chiziqli trapetsiyaning yuzini toping.

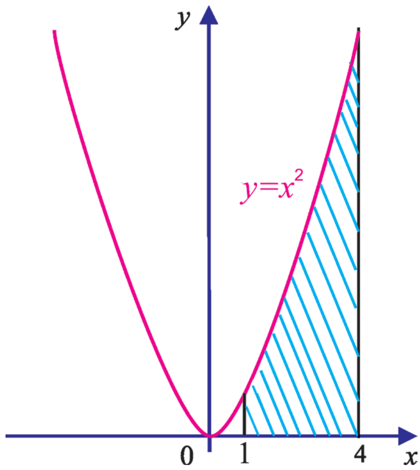
△ (4) formulaga muvofiq $S = \int\limits_1^4 x^2 dx$. Bu integralni Nyuton–Leybnis formulasi (3) yordamida hisoblaymiz. $f(x) = x^2$ funksiyaning boshlang‘ich funksiyalaridan biri $F(x) = \frac{x^3}{3}$ ekani ravshan. Demak,

$$S = \int\limits_1^4 x^2 dx = \frac{x^3}{3}\Bigg|_1^4 = \frac{1}{3}(4^3 - 1^3) = \frac{1}{3} \cdot 63 = 21 \text{ (kv. birlik).}$$

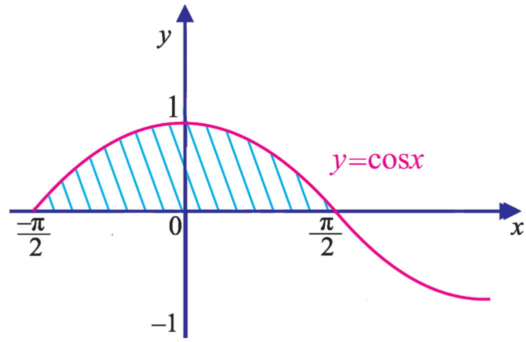
Javob: $S = 21$ kv. birlik. ▲

2-misol. 7-rasmdagi shtrixlangan soha yuzini toping.

△ Shtrixlangan soha egri chiziqli trapetsiya bo‘lib, u yuqoridan $y = \cos x$ funksiyaning grafigi, pastdan esa $\left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$ kesma bilan chegaralangan. $y = \cos x$ – juft funksiya, soha Oy o‘qqa nisbatan simmetrik. Shu ma'lumotlarga ko‘ra, soha yuzi $\int_0^{\frac{\pi}{2}} \cos x dx$ yuzining ikki barobariga teng deyish mumkin.



6-rasm.



7-rasm.

Nyuton–Leybnis formulasi va (5) formulaga koʻra:

$$S=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos x dx=\sin x\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}=\sin\frac{\pi}{2}-\sin\left(\frac{-\pi}{2}\right)=1-(-1)=1+1=2 \text{ (kv. birlik).}$$

Javob. 2 kv.birlik. ▲

3-misol. $\int_0^{\pi}\cos x dx$ aniq integralni hisoblang.

△ Nyuton–Leybnis formulasi va (5) formulasiga koʻra:

$$\int_0^{\pi}\cos x dx=\sin x\Big|_0^{\pi}=\sin\pi-\sin 0=0.$$

Javob: 0. ▲

4-misol. $\int_{-1}^{2}(2x^2-3x+4)dx$ aniq integralni hisoblang.

△ Nyuton–Leybnis formulasi va (5) formulaga koʻra:

$$\int_{-1}^{2}(2x^2-3x+4)dx=\left(\frac{2}{3}x^3-\frac{3}{2}x^2+4x\right)\Big|_{-1}^{2}=\frac{22}{3}-\left(-\frac{37}{6}\right)=\frac{81}{6}=13,5 \text{(kv. birlik).}$$

Javob: 13,5 kv. birlik. ▲

5-misol. $S=\int_0^{\frac{\pi}{3}}\sin^2\left(3x+\frac{\pi}{6}\right) dx$ aniq integralni hisoblang.

△ Avval aniqmas integralni topamiz:

$$\int \sin^2(3x+\frac{\pi}{6})dx=\frac{1}{2}\int(1-\cos(6x+\frac{\pi}{3}))dx=\frac{1}{2}\cdot(x-\frac{1}{6}\sin(6x+\frac{\pi}{3})).$$

U holda $S=\frac{1}{2}(x-\frac{1}{6}\sin(6x+\frac{\pi}{3}))\Big|_0^{\frac{\pi}{3}}=\frac{1}{2}(\frac{\pi}{3}-\frac{1}{6}\sin(2\pi+\frac{\pi}{3}))-\frac{1}{2}(0-\frac{1}{6}\sin\frac{\pi}{3})=$

$=\frac{\pi}{6}-\frac{1}{12}\sin\frac{\pi}{3}+\frac{1}{12}\sin\frac{\pi}{3}=\frac{\pi}{6}.$

Javob: $S=\frac{\pi}{6}$. ▲

6-misol. $\int_2^6\sqrt{2x-3}\,dx$ aniq integralni hisoblang.

△ Avval aniqmas integralni topamiz:

Integrallar jadvaliga ko‘ra $\int\sqrt{2x-3}dx=\frac{1}{3}\cdot(2x-3)^{\frac{3}{2}}+C.$ U holda

$$\int_2^6\sqrt{2x-3}dx=\frac{1}{3}\cdot(2x-3)^{\frac{3}{2}}\Big|_1^6=\frac{1}{3}\cdot\left((2\cdot6-3)^{\frac{3}{2}}-(2\cdot2-3)^{\frac{3}{2}}\right)=\frac{1}{3}\cdot(27-1)=\frac{26}{3}=8\frac{2}{3}.$$

Javob: $8\frac{2}{3}$. ▲

Aniq integral quyidagi xossalarga ega:

1. $\int_a^a f(x)dx=0.$ Chindan ham, $\int_a^a f(x)dx=F(a)-F(a)=0.$

2. $\int_a^b f(x)dx=-\int_b^a f(x)dx.$

▲ $\int_a^b f(x)dx=F(b)-F(a);\ \int_b^a f(x)dx=F(a)-F(b)=-\big(F(b)-F(a)\big).$

Demak, $-\int_b^a f(x)dx=F(b)-F(a)=\int_a^b f(x)dx.$ ▲

3. a, b, c – haqiqiy sonlar bo‘lsa, $\int_b^a f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ (aniq integralning additivlik xossasi).

4. $f(x)$, $x \in R$, juft funksiya bo‘lsa, u holda $\int_{-a}^{a} f(x)dx = 2\cdot\int_0^a f(x)dx$.

5. Agar $f(x) \geq 0$, $x \in [a,b]$ bo‘lsa, $\int_a^b f(x)dx \geq 0$ bo‘ladi.

6. $x \in [a,b]$ da $f(x) < g(x)$ bo‘lsa, u holda $\int_a^b f(x)dx \leq \int_a^b g(x)dx$ bo‘ladi.

? Savol va topshiriqlar

1. Aniq integral nima?
2. Egri chiziqli trapetsiya yuzini hisoblash masalasini ayting. Misollarda tushuntiring.
3. Nyuton–Leybnis formulasi nima? Uning mazmun-mohiyatini ayting.
4. Aniq integralning xossalarini ayting. Misollarda tushuntiring.

Mashqlar

Aniq integrallarni hisoblang (**36 – 41**):

36. 1) $\int_0^2 3x^2dx$; 2) $\int_0^2 2xdx$; 3) $\int_{-1}^4 5xdx$; 4) $\int_1^2 8\cdot x^3dx$;

5) $\int_1^e \frac{1}{x}dx$; 6) $\int_3^4 \frac{1}{x^2}dx$; 7) $\int_1^2 \frac{1}{x^4}dx$; 8) $\int_0^1 \sqrt{2x}dx$;

9) $\int_1^4 \frac{2}{\sqrt{x}}dx$; 10) $\int_8^{27} \frac{dx}{\sqrt[3]{x}}$; 11) $\int_{-1}^3 \frac{dx}{\sqrt{2x+3}}$; 12) $\int_0^3 x\sqrt{x+1}dx$.

37. 1) $\int_{\frac{\pi}{2}}^{\pi} \cos(2x+\frac{\pi}{4})dx$; 2) $\int_{-\pi}^{\pi} \sin^2 2xdx$;

3) $\int_0^{\frac{\pi}{6}} \sin 3x\cos 3xdx$; 4) $\int_0^{\frac{\pi}{8}} (\cos^2 2x - \sin^2 2x)dx$.

38. 1) $\int_0^{\ln 2} e^{2x}dx;$ 2) $\int_0^2 e^{4x}dx;$ 3) $\int_1^3 (e^{2x} - e^x)dx.$

39. 1) $\int_{-1}^{1} (x^2 + 3x))(x-1)dx;$ 2) $\int_{-1}^{0} (x+2)(x^2-3)dx;$

3) $\int_1^3 (x+\frac{1}{x})^2 dx;$ 4) $\int_{-2}^{-1} \frac{1}{x^2}(1-\frac{1}{x})dx.$

40*. 1) $\int_1^6 \frac{dx}{\sqrt{3x-2}};$ 2) $\int_0^3 \frac{dx}{\sqrt{x+1}};$ 3) $\int_0^{\frac{\pi}{8}} (\sin^4 2x + \cos^4 2x)dx.$

41*. 1) $\int_1^5 x^2 \cdot \sqrt{x-1}dx;$ 2) $\int_1^5 \frac{x^2-6x+10}{x-3}dx;$ 3) $\int_0^1 \frac{x^2+2x+4}{x+1}dx.$

42*. 1) Shunday a va b sonlarni topingki, $f(x)=a\cdot 2^x+b$ funksiya $f'(1)=2,$

$\int_0^3 f(x)dx = 7$ shartlarni qanoatlantirsin.

2) $\int_1^b (b-4x)dx \geq 6-5b$ tengsizlik bajariladigan barcha $b>1$ sonlarni toping.

43*. 1) $\int_1^2 (b^2 + (4-4b)x + 4x^3)dx \leq 12$ tengsizlik bajariladigan barcha b sonlarni toping.

2) Qanday $a>0$ sonlar uchun $\int_{-a}^{a} e^x dx > \frac{3}{2}$ tengsizlik bajariladi?

44. $f(x)$ funksiyani a ning ixtiyoriy qiymatida tengliklar bajariladigan qilib tanlang:

1) $\int_0^a f(x)dx = 2a^2 - 3a;$ 2) $\int_0^a f(x)dx = 4a - a^2;$

3) $\int_0^a f(x)dx = \frac{1}{3}a^3 - \frac{3}{2}a^2;$ 4) $\int_0^a f(x)dx = a^2 + a + \sin a.$

Integrallarni hisoblang (**45 – 46**):

45. 1) $\int_0^1 (e^{-x}+1)^2 dx;$ 2) $\int_{-2}^{-1} 10^x \cdot 2^{-x} dx;$ 3) $\int_0^1 (e^{-x}-1)^2 dx;$

4) $\int_{-3}^{-1} 3^{-x} 6^x dx;$ 5) $\int_{\ln 2}^{\ln 3} e^{-3x} dx;$ 6) $\int_{\ln 3}^{\ln 5} e^{2x} dx.$

46*. 1) $\int_0^1 \frac{2^x+3^x}{6^{x+1}} dx;$ 2) $\int_0^1 \frac{2^{x-1}+5^{x-1}}{10^x} dx;$ 3) $\int_0^{\sqrt{e-1}} \frac{2xdx}{x^2+1};$

4) $\int_{\sqrt{3}}^{\sqrt{e+2}} \frac{2xdx}{x^2-2};$ 5) $\int_0^1 \frac{3^x+4^x}{12^x} dx;$ 6) $\int_0^2 4^{-x} \cdot 8^x dx.$

47. $x=a,\ x=b$ to‘g‘ri chiziqlar, Ox o‘qi va $y=f(x)$ funksiya grafigi bilan chegaralangan egri chiziqli trapetsiyaning yuzini toping. Mos rasm chizing:

1) $a=1,\quad b=2,\ f(x)=x^3;$ 2) $a=2,\ b=4,\ f(x)=x^2;$

3) $a=-2,\ b=1,\ f(x)=x^2+2;$ 4) $a=1,\ b=2,\ f(x)=x^3+2;$

5) $a=\frac{\pi}{3},\ b=\frac{2\pi}{3},\ f(x)=\sin x;$ 6) $a=\frac{\pi}{4},\ b=\frac{\pi}{2},\ f(x)=\cos x.$

48. Ox o‘qi va berilgan parabola bilan chegaralangan shaklning yuzini toping:

1) $y=9-x^2;$ 2) $y=16-x^2;$ 3) $y=-x^2+5x-6;$

4) $y=-x^2+7x-10;$ 5) $y=-x^2+4x;$ 6) $y=-x^2-3x.$

Quyidagi chiziqlar bilan chegaralangan shaklning yuzini toping. Mos rasm chizing (**49 – 50**):

49. 1) $y=-x^2+2x,\ y=0;$ 2) $y=-x^2+3x+18,\ y=0;$

3) $y=2x^2+1,\ y=0,\ x=-1,\ x=1;$ 4) $y=-x^2+2x,\ y=x.$

50. 1) $y=-2x^2+7x,\ y=3{,}5-x;$ 2) $y=x^2,\ y=0,\ x=3;$

3) $y=x^2,\ y=0,\ y=-x+2;$ 4) $y=2\sqrt{x},\ y=0,\ x=1,\ x=4.$

5) $y=\frac{1}{a}\cdot x^2, y=a\cdot\sqrt{x};$ 6) $y=2^x, y=2, x=0;$

7) $y=|\lg x|, y=0, y=2, x=0.$

Nazorat ishi namunasi
I variant

1. $f(x)=\frac{x^3}{2}-\cos 3x$ funksiyaning barcha boshlang‘ich funksiyalarini toping.

2. Agar $F(\frac{3}{2})=1$ bo‘lsa, $f(x)=\frac{6}{(4-3x)^2}$ funksiyaning boshlang‘ich funksiyasi $F(x)$ ni toping.

3. Hisoblang: $\int_{-1}^{2}(x^2-6x+9)dx.$

4. Hisoblang: $\int_{0}^{\pi}\sin\frac{x}{3}dx.$

5. Ox o‘qi, $x=-1$ va $x=2$ to‘g‘ri chiziqlar va $y=9-x^2$ parabola bilan chegaralangan egri chiziqli trapetsiyaning yuzini hisoblang.

II variant

1. $f(x)=\frac{x^4}{3}+\sin 4x$ funksiyaning barcha boshlang‘ich funksiyalarini toping.

2. Agar $F\left(\frac{1}{2}\right)=2$ bo‘lsa, $f(x)=\frac{3}{(2-5x)^3}$ funksiyaning boshlang‘ich funksiyasi $F(x)$ ni toping.

3. Hisoblang: $\int_{-3}^{1}(x^2+7x-8)dx$.

4. Hisoblang: $\int_{-\pi}^{\pi}\cos\frac{x}{2}dx$

5. Ox o‘qi, $x=-2$ va $x=3$ to‘g‘ri chiziqlar va $y=x^2-1$ parabola bilan chegaralangan egri chiziqli trapetsiyaning yuzini hisoblang.

JAVOBLAR
I BOB

1. *a*) Puls chastotasi – bu yurakning bir minutda qancha urishini ko'rsatuvchi belgi. Demak, bir minutda Madinaning yuragi 67 marta uradi. *b*) 4020. **2.** *a*) $\approx 0{,}00150\,\frac{\text{xato}}{\text{so'z}}$. Sifat ortdi; *b*) ≈ 0,15. **3.** Ma'ruf unumliroq ishlagan. **4.** ***a)*** $\approx 0{,}000177\,\frac{\text{mm}}{\text{km}}$. **5.** $89\frac{\text{km}}{\text{soat}}$ yoki $89\frac{\text{m}}{\text{s}}$. **6.** a) $0{,}1\ \frac{m}{s}$; b) $0{,}9\ \frac{m}{s}$; c) $0{,}5\ \frac{m}{s}$. **7.** 1) a) $3{,}1\ \frac{dona}{g}$; $4{,}22\ \frac{dona}{g}$; b) doza 2 grammdan 8 grammgacha oshirilganda hasharotlar soni tez kamayadi, keyin esa kamayishi sust bo'ladi. **8.** a) 7; b) 7; c) 11; d) 16; e) 0; f) 5. **9.** a) 5; b) 7; c) c. **10.** a) –2; b) 7; c) –1; d) 1. **11.** a) –3; b) –5; c) –1 d) 6; e) –4; f) –8; g) 1; h) 2; i) 5. **13.** a) $3x^2$; b) $-\frac{1}{x^2}$; c) $\frac{1}{2\sqrt{x}}$; d) 0. **15.** a) 2; b) $6x + 5$; c) $6x^2+8x+6$. **16*.** a) $f'(x)=a$; b) $f'(x)=2ax + b$; c) $f'(x)=3ax^2 + 2bx + c$. **20.** 1) $4x^3$; 2) $-2x^{-3}$; 3) $-3x^{-4}$. **21.** 2) $-x^{-2}+1$; 4) $4x^3+3x^2+2x-1+x^{-2}+2x^{-3}$. **22.** 2) 1; 4) $-\frac{1}{(2\sqrt{x}(\sqrt{x}-1)^2)}$.

23. 2) 53,25. **24.** 2) –3; 4) 2. **25.** 2) $-\frac{4}{x^2}+\frac{1}{4}$; 4) $2x-\frac{2}{x^3}$. **26.** 2) $3(x+2)^2$; 4) $2x$.

27. 3) $-\frac{2x^9+4x^3}{(x^6-1)^2}$; 4) $-\frac{1}{(x-1)^2}$; 6) $4x^3 - 4$; 8) $7x^6+3x^2-3x^{-4}-7x^{-8}$. **28.** 2) 0; 4) $\frac{1}{\cos^2 x}$; 6) $\frac{1}{x\ln 2}$; 8) $1+\ln x$; 10) $2e^x-\frac{1}{x}-\frac{1}{x^2}$. **29.** 2) $2e^x\cos x$; 4) $\frac{1-\ln x}{x^2}$; 6) $5+\frac{1}{2\sqrt{x}}+\frac{1}{3\sqrt[3]{x^2}}$; 8) $3(2+x)^2$. **30.** 2) 11. **31.** 2) 0. **32.** 2) $-\frac{1}{\cos^2 x}$; 4) $-\frac{1}{\sin^2 x\cos^2 x}$; 6) $2x\sin x+x^2\cdot\cos x$; 8) $x\cos x$. **33.** 2) 1. **34.** 2) $n\pi$, $n\in z$; 4) 1. **35.** 1) $\frac{1}{x^2}-1$; 2) $4x^2-1$. **36.** 2) $\frac{1+x^2}{1-x^2}$; 4) $\frac{x+2}{x}$. **37.** 2) x^4; 4) x^2-1. **38.** 2) x^3+3x^2+3x+1; 4) $x^6 + 1$.

39. x^2-2x. **43.** 2) $e^{\sin x}\cos x$; 4) $\sin 2x$; 6) $\frac{4}{4x-1}$; 8) $20(2x-1)^9$. **44.** 3) $-\text{tg}x$; 8) $-30x^2\cos^{29}x\cdot\sin x+2x\cos^{30}x$; 9) $\frac{5\text{ctg}x}{x}-\frac{5\ln x}{\sin^2 x}$. **45.** 2) $y=3x-4$; $y=3x-4$; $y=3x-4$. 4) $y=-x-2$; $y=8x+16$; $y=-4x$. **46.** 2) $y=7x-6$. **47.** 4) 0 va $\frac{2}{3}$; 6) 0 va $\frac{3}{4}$. **48.** 1) $y=x-2$; $y=-17x-11$; $y=-5x+1$. **49.** 2) 0,1 ; 0,331 . **50.** 2) a) 0,2718; b) 9,06; 4) a) 0,938127;

b) 31,2709. **51.** 2) a) 0; b) 0; 4) a) 0,119401; b) 11,9401 . **52.** 1) 4; 2) –7; 3) 6; 4) 19/28; 5) 0. **53.** 2) 29; 4) $32x–3$; 6)$18–2x$; 8) $48x^2+10x–2$. **54.** 1) a) 15; b) 15; c) 15; d) 15; 4) a) –29; b) 12; c) 5; d) –1. **55.** 2) $3(x+2)^2$; 4) $1–x^{-2}$. **56.** 1) 12; 2) 3. **57.** 15 m/s. **58.** 3) $\frac{1}{5\sqrt[5]{x^4}}+\mathrm{tg}x+\frac{x}{\cos^2 x}-\frac{1}{x\ln 3}$; 10) $7^x x^7\ln7+7^x\cdot 7x^6$; 12) $\frac{\sqrt{2}}{2}-\cos x$; 14) $8–2^x$. **59.** 2) 4; 4) 2. **60.** 2) Ø. **61.** 1 va 2 . **62.** 2) $–2x^{-3}–1$. **63.** 2) 2,75. **64.** 2) $\frac{x^2+16x-24}{(x+8)^2}$; 4) $6x^2+8x+5$; 6) $14x+12$. **65.** 2) $\frac{-2x^7-4x^5-5x^4+21x^2+7}{(x^5+7)^2}$.

66. 2) $e^{5x}(4\cos x–6\sin x)$; 4) $\frac{1-2\ln x}{x^3}$. **67.** 2) –4; 4) $-\frac{1}{\sin^2 1}-\frac{1}{20}$.

68. 1) $2x\sin x+x^2\cos x$; 2) $-\frac{\mathrm{tg}x}{\ln 15}$; 4) $\frac{35\mathrm{tg}^{34}x}{\cos^2 x}$; 8) $(2x–10)\ln\cos x–(x^2–10x+7)\mathrm{tg}x$.

69. 3) o‘sish: $(–\infty; –3) \cup (3; –\infty)$ kamayish: $(–3;3)$.
4) o‘sish: $(–\infty; 0) \cup (0; +\infty)$; kamayish: Ø.
6) o‘sish: $(-\infty;\sqrt{2})\cup(\sqrt{2};+\infty)$; kamayish: $-\sqrt{2};\sqrt{2}$.
8) o‘sish: $(–\infty; 0)$; kamayish: $(0; +\infty)$.
9) o‘sish: $(–1; 0) \cup (1; +\infty)$; kamayish: $(–\infty; –1) \cup (1; +\infty)$.
10) o‘sish: $(2; +\infty)$; kamayish: $(–\infty; 2)$.
14) o‘sish: $(-\frac{\pi}{2}+n\pi;\frac{\pi}{2}+n\pi),\ n\in\mathbb{Z}$; kamayish: Ø.

70. 2) –3; 3 . 4) 0. 6) Ø. 8) 0; –1.

71. 2) lokal minimum $x=4$; lokal maksimum mavjud emas.
4) lokal minimum $x=5$; lokal maksimum $x=-5$.
6) lokal minimum $x=0,75$; lokal maksimum mavjud emas.
8) lokal minimum $x=2n\pi,\ n\in\mathbb{Z}$; lokal maksimum $x=\pi+2n\pi,\ n\in\mathbb{Z}$.

72. 2) o‘sadi $(–1; 1)$; kamayadi: $(–\infty; –1) \cup (1; +\infty)$.
4) o‘sadi: $(-\frac{\pi}{2}+n\pi;\frac{\pi}{2}+n\pi),\ n\in\mathbb{Z}$; kamayadi: $(-\frac{\pi}{2}+2n\pi;\frac{3\pi}{2}+2n\pi),\ n\in\mathbb{Z}$;
6) o‘sadi: Ø ; kamayadi: $(-\frac{\pi}{2}+n\pi;\frac{\pi}{2}+n\pi),\ n\in\mathbb{Z}$.

73. 2) eng katta qiymat: 57; eng kichik qiymat: –55.
4) eng katta qiymat: 84; eng kichik qiymat: $-\frac{28}{9}$.

76. 5625m^2. **80.** 80 m. **83.** 1) 5 s; 2) 250 m/s; 3) $\frac{1875}{4}$m.

87. 1) 4m^3; 2) $5324\ \text{m}^3$; 3) $407\ \frac{\text{m}^3}{\text{min}}$;

89. 1) 30 ta; 2) 1800000 so‘m .

91. d) 24,52, –0,1; e) 40,52, 9,86. **93.** g) 2,0004. **94.** e) 0,9302.

95. d) 0,526. **96.** d) 0,1247. **112.** 1) eng katta 13; eng kichik 13; 3) eng katta mavjud emas; eng kichik 5; 5) eng katta mavjud emas; eng kichik $\frac{11}{6}$.

113. 2) $y=13x+4; y=13x+4; y=13x+4$. **114.** 1) mavjud emas. **115.** 3) mavjud emas.

117. 1) –1; 2) 0; 3) $-\frac{3}{4}$; 4) $-\frac{1}{2}$; 5) 75; 6) $\frac{\sqrt{3}}{2}$; 7) $-\frac{3}{16}$; 8) $\frac{5\sqrt{2}}{2}$; 9) $-\sqrt{2}$.

118. 1) 19; 10; 2) 27;30; 3) 77; 30; 4) 0; –8.

119. 1) 1; 2) 0; 3) $-\frac{3}{4}$; 4) $-\frac{1}{2}$; 5) 75; 6) $\frac{\sqrt{3}}{2}$; 7) $-\frac{3}{16}$; 8) $\frac{5\sqrt{2}}{2}$; 9) $\sqrt{2}$; 10) 0.

120. 1) 10; 6. 2) 15; 18. 3) 225; 80.

121. 1) $-2x+1$; 2) $\cos x+\sin x$; 4) $4^x\ln 4-\cos x$; 6) $\frac{1}{x}-20x+1$. **122.** 1) $4x^3$; 3) $1+\frac{20}{x^2}$; 6) $e^x(\sin x+\cos x)$; 8) $20\sin x+2(10x-1)\cos x$.

123. 1) $\frac{1}{\sqrt{e^{\pi}}}$; 0; 2) 3; 3; 3) $-2\pi+1$; $\pi+1$. 4) $-\pi$; $\frac{\pi}{2}+\frac{\sqrt{2}}{2}$; 5) 1; 0; 6) 0; $\frac{\sqrt{2}}{2}$; 7) $1-\frac{\pi^3}{2}$; $-\frac{\sqrt{2}}{2}+\frac{\pi^3}{16}$. 8) 3; $-3\sqrt{2}$.

124. 1) 12; 2)72. **126.** 1) 0; 2) 600 000. **127.** 2) $-\sin 2x$.

128. 2) o‘sish: $(-\infty; +\infty)$; kamayish: Ø.
4) o‘sish: Ø; kamayish: $(-\infty; 0) \cup (0; +\infty)$.
6) o‘sish: $(-\infty; +\infty)$; kamayish: Ø.
8) o‘sish: $(0; +\infty)$; kamayish: $(-\infty; 0)$.

129. 2) $\sqrt{\frac{133}{3}}$; $-\sqrt{\frac{133}{3}}$. 4) 0; 6) 3; –3; 8) 0; $-\frac{13}{18}$.

130. 2) lokal minimum: $x=9$. lokal maxsimum: mavjud emas.

131. 2) eng katta: 81; eng kichik: –6. **134.** 62 500 m^2.

143. 1) $3e^{3x}$; 2) $e^{\sin x}\cos x$; 3) $3\cos(3x+2)$; 4) $8(2x+1)^3$;

144. 1) e^{8x+4}; 2) e^{8x^2+4x}; 3) $4e^{2x}+2$; 4) $\sqrt{16x+10}$.

145. 1) $10x(x^2+1)^4$; 3) $\frac{5}{2\sqrt{5x-7}}$; 8) $-e^{\sin(\cos x)}\cdot\cos(\cos x)\cdot\sin x$.

146. 1) o‘sadi: $(-\infty; 0,5)$; kamayadi: $(0,5; -\infty)$;
3) o‘sadi: $(-1; 1)$; kamayadi: $(-\infty; -1) \cup (1; +\infty)$.
4) o‘sadi: $(-\infty; +\infty)$; kamayadi: Ø.
7) o‘sadi: $(-\infty; +\infty)$; kamayadi: Ø.
8) o‘sadi: $(1; +\infty)$; kamayadi: $(-\infty; 1)$.

147. 1) statsionar nuqtalari: 1 va 3; lokal maksimum: 0; lokal minimum: –4.

II BOB

2. 2) x^6+C; 4) $x^{\frac{3}{2}}+C$; 6) $\sin x+C$; 8) $\frac{1}{2}\sin 2x+C$. **3.** 2) $\frac{\pi^x}{\ln\pi}+C$; 4) $\frac{a^x}{\ln a}+C$; 6) $\frac{e^{\pi x}}{\pi}+C$. **4.** 4) $\frac{1}{a}\ln x+C$. **5.** 4) $\frac{1}{5}\sin 5x+C$; 6) $\frac{1}{2}\cos 2x+C$.

6. 4) $\frac{1}{8}(2x-1)^4+C$. **7.** 2) $-\frac{1}{3}x^3+x^2+5x+2$; 4) $\sin x+4$. **8.** 1) $2x^2+8x+11$; 2) $-\frac{x^2}{2}+3x+2{,}5$; 3) $\frac{9}{4}x^2+9x+15{,}8$; 4) $x^2-6x+10$. **10.** 1) $\frac{8}{x}-2x+4$; 2) $\frac{9}{x^2}+2x-3$; 3) x^3-x+6; 4) x^5+7x+1. **11.** 1) $\frac{1}{4}\cdot(3x-2)^{\frac{4}{3}}+\frac{3}{4}$; 2) $\frac{1}{5}\cdot(4x+5)^{\frac{5}{4}}+\frac{4}{5}$; 3) $\frac{1}{8}\cdot(7x-5)^{\frac{8}{7}}+\frac{7}{8}$; 4) $\frac{1}{k+1}\cdot(kx+b)^{\frac{k+1}{k}}+\frac{k}{k+1}$.

12. 1) $5\ln|x-2|+7$; 2) $3\ln|x+1|+1$; 3) $\sin x+7$; 4) $-\cos x+9$. **14.** 2) $\frac{1}{5}\cdot\sin 5x+\frac{3}{5}$; 4) $-3\cos\frac{x}{3}+6$. **15.** 1) x^3-4; 2) x^4-15. **16.** 2) x^8+x^5; 4) $-\frac{5}{3}\cdot\frac{1}{x^3}-\frac{3}{4}\cdot\frac{1}{x^4}$.

17. 2) $-7\cos x+4\sin x$; 4) $5e^x+2\sin x$. **18.** 2) $\frac{1}{5}(x+5)^5$; 4) $9\cdot(x+1)^{\frac{2}{3}}$; 6) $-2\cos(x-3)-4\ln|x-2|$. **19.** 2) $-\frac{1}{7}\cdot\cos(7x-6)+C$; 4) $-\frac{7}{5}\cos(\frac{5x}{7}-2)+C$; 6) $-\frac{1}{2}\cdot e^{3-2x}+C$. **20.** 2) $\frac{1}{15}\cdot(3x+2)^5+\frac{1}{5}x^{-5}+C$; 4) $x^2+3\operatorname{ctg}x+6x+C$. **21.** 2) $\frac{1}{5}\sin 5x+3\frac{4}{5}$;

4) $x^4-\sqrt{x-1}-15$. **22.** 2) $\frac{1}{5}x^5+\frac{1}{3}\sin 3x+4x+C$; 4) $x^4+3\sin\frac{x}{3}-3\cdot\cos\frac{x}{3}+C$.

23. 2) $\frac{-1}{4}\cos 4x+C$. **24.** 1) $\frac{-1}{16}\cos 8x-\frac{1}{4}\cos 4x$. **25.** 2) $\ln\left|\frac{x-4}{x-3}\right|+C$; 4) $\ln|x-4|+C$.

26. 2) $x-\operatorname{arctg}x+C$; 4) $-\frac{1}{2}\operatorname{ctg}x+C$. **27.** 2) $-\frac{1}{4(1+x^2)^2}+C$; 4) $-\frac{1}{2}\operatorname{ctg}^2x+C$.

28. 2) $\frac{8}{3}(x-4)^{\frac{3}{2}}+\frac{2}{5}(x-4)^{\frac{5}{2}}+C$; 4) $\frac{1}{3}\operatorname{tg}^3x+C$. **29.** 2) $-\frac{3}{25}\cos 5x+3$. **31.** 4) $x+x^2-\sqrt{1-2x}+C$. **33.** 1) $\sin x-x\cos x+C$; 2) $x^2\cdot\sin x-2\sin x+2x\cos x+C$;

3) $\frac{1}{2}\cdot x^2\ln x-\frac{1}{4}x^2+C$;   4) $x\cdot \operatorname{arctg}x-\frac{1}{2}\ln(1+x^2)+C$.

34. 1) $\frac{1}{2}\cdot(x\sin 2x+\frac{1}{2}\cos 2x)+C$;   3) $9\sin\frac{x}{3}-3x\cdot\cos\frac{x}{3}+C$.

36. 4) 30. **37.** 4) $\frac{1}{4}$. **38.** 2) $\frac{1}{4}\cdot(e^8-1)$. **39.** $\frac{1}{8}$. **40.** 2) 2. **41.** 1,5+ln2. **42.** 1) $a=\frac{1}{\ln 2}$, $b=\frac{7(\ln^2 2-1)}{3\ln^2 2}$;   2) $b=2$. **43.** 1) $b=3$; 2) $a>\ln 2$. **44.** 1) $f(x)=4x-3$;   2) $f(x)=4-2x$;   3) $f(x)=x^2-3x$; 4) $f(x)=1+2x+\cos x$. **45.** 2) $\frac{4}{5\ln 5}$; 6) 8. **46.** 2) $\frac{0,4}{\ln 5}+\frac{0,1}{\ln 2}$; 4) 1. **47.** 2) $\frac{56}{3}$; 4) $1-\frac{\sqrt{2}}{2}$. **48.** 2) $85\frac{1}{3}$. **49.** 1) $\frac{4}{3}$;   2) 121,5;   3) $\frac{10}{3}$;   4) $\frac{1}{6}$.

50. 1) 9;   2) 9;   3) 4,5;

Foydalanilgan va tavsiya etiladigan adabiyotlar

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